SA-STUDENT

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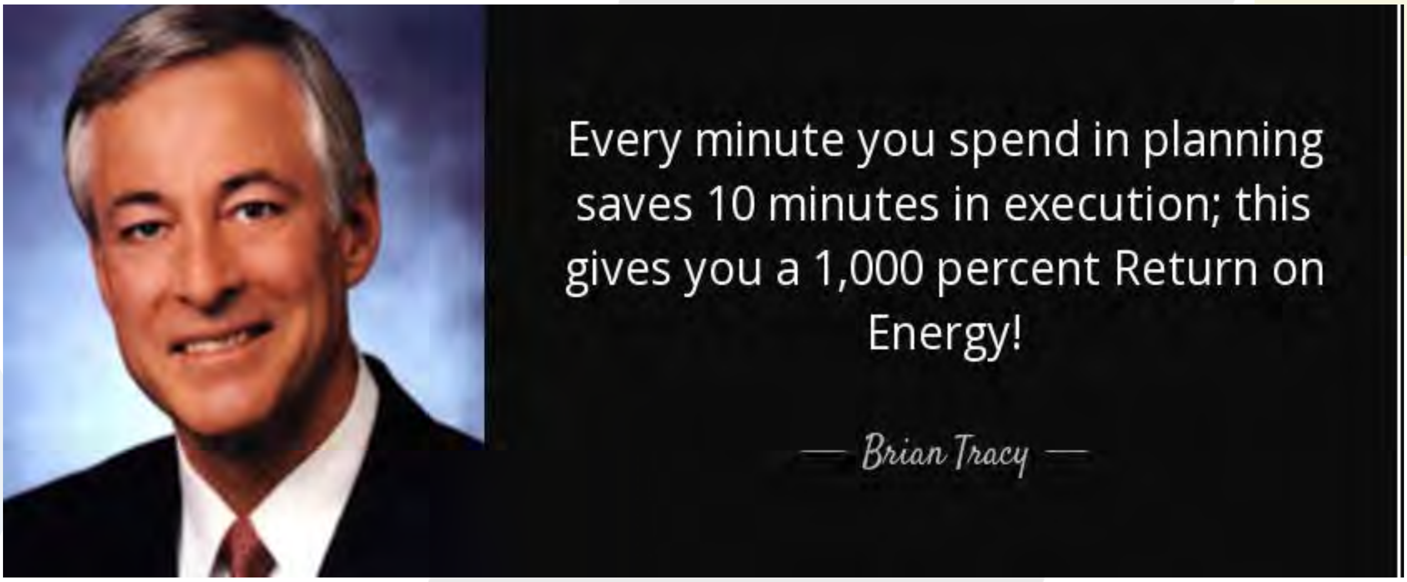


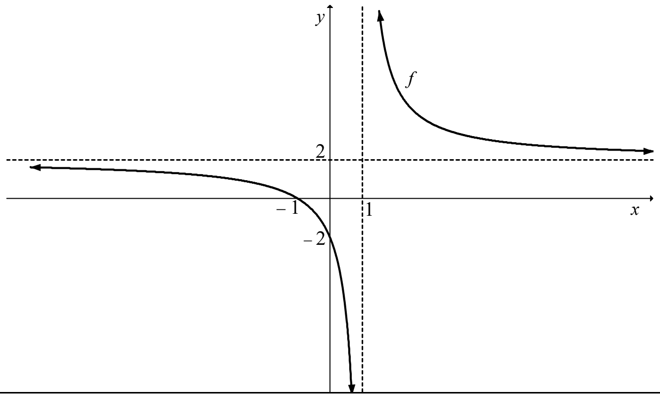
QUESTION 3/*VRAAG* 3

3.1	$x \;;\; x \;;\; T_3 \;;\; \ldots$ first differences: $0 \;;\; T_3 - x$ second difference: 10	
	$2a = 10 \qquad 3a + b = 0$ $a = 5 \qquad b = -15$	✓ $2a = 10$ ✓ $3a + b = 0$
	$T_3 - x - 0 = 10$ $\therefore T_3 = x + 10$	✓ $T_3 = x + 10$
	$2x + T_3 = 28$ $2x + x + 10 = 28$ $3x = 18$ $x = 6$	✓ $2x + T_3 = 28$ ✓ $x = 6$
	$a + b + c = 6$ $5 - 15 + c = 6$ $c = 16$	✓ $5 - 15 + c = 6$ (6)
	$\therefore T_n = 5n^2 - 15n + 16$	
	OR/*OF*	**OR/*OF***
	$2a = 10$ $\therefore a = 5$	✓ $2a = 10$
	$T_1 = a + b + c \qquad T_2 = 4a + 2b + c \qquad T_3 = 9a + 3b + c$ $= 5 + b + c \qquad = 20 + 2b + c \qquad = 45 + 3b + c$	
	$5 + b + c = 20 + 2b + c$ $b = -15$	✓ $5 + b + c = 20 + 2b + c$
	$T_1 = -10 + c \qquad T_2 = -10 + c \qquad T_3 = c$	✓ $T_1 = -10 + c$ ✓ $T_2 = -10 + c$
	$T_1 + T_2 + T_3 = -10 + c - 10 + c + c$ $28 = 3c - 20$ $c = 16$	✓ $28 = 3c - 20$ ✓ $c = 16$ (6)

3.2	$T_n = 5n^2 - 15n + 16$ $216 = 5n^2 - 15n + 16$ $5n^2 - 15n - 200 = 0$ $n^2 - 3n - 40 = 0$ $(n-8)(n+5) = 0$ $n = 8$ or $n \neq -5$ $\therefore T_8 = 216$	✓ equating ✓ standard form ✓ $n = 8$ (3)
		[9]

QUESTION 4/*VRAAG* 4

4.1.1	decreasing	✓ decreasing (1)
4.1.2	$y = \left(\frac{1}{3}\right)^x$ $x = \left(\frac{1}{3}\right)^y$ $\therefore y = \log_{\frac{1}{3}} x$ **OR/OF** $y = 3^{-x}$ $x = 3^{-y}$ $\therefore y = -\log_3 x$	✓ swop x and y ✓ answer (2) **OR/OF** ✓ swop x and y ✓ answer (2)
4.1.3	$x > 0; x \in R$	✓ answer (1)
4.1.4	$y = -5$	✓ answer (1)
4.2.1	$x = 1$ $y = 2$	✓ $x = 1$ ✓ $y = 2$ (2)
4.2.2	$\frac{4}{x-1} + 2 = 0$ $4 = -2x + 2$ $2x = -2$ $x = -1$	✓ let $y = 0$ ✓ $x = -1$ (2)

4.2.3		✓ asymptotes ✓ x-intercept ✓ y-intercept ✓ shape (4)
4.2.4	$\frac{4}{x-1} \geq -2$ $\frac{4}{x-1} + 2 \geq 0$ $x \leq -1$ or $x > 1$	✓ $x \leq -1$ ✓ $x > 1$ (2)
4.2.5	$y = -x + c$ $2 = -3 + c$ $c = 5$ $y = -x + 5$ **OR/*OF*** $y = -x + c$ $2 = -1 + c$ $c = 3$ $y = -x + 3$ $y = -(x-2) + 3$ $y = -x + 5$ **OR/OF** $y = -(x+p) + q$ $y = -((x-2) + (-1)) + 2$ $y = -x + 5$	✓ intersection of axes at (3 ; 2) ✓ subst (3 ; 2) and $m = -1$ ✓ $y = -x + 5$ (3) **OR/*OF*** ✓✓ $-(x-2)+3$ ✓ $y = -x + 5$ (3) **OR/*OF*** ✓✓ $y = -((x-2) + (-1)) + 2$ ✓ $y = -x + 5$ (3)
		[18]

QUESTION 3/*VRAAG* 3

3.1	$3a+b=7$ $3+b=7$ $b=4$ **OR/OF** $T_2-T_1=7$ $4+2b+9-(1+b+9)=7$ $b=4$	✓ $3a+b=7$ ✓ $3+b=7$ (2) **OR/OF** ✓ $T_2-T_1=7$ ✓ substitution (2)
3.2	$T_n=n^2+4n+9$ $T_{60}=(60)^2+4(60)+9$ $=3849$ Answer only: full marks	✓ substitution ✓ answer (2)
3.3	14 ; 21 ; 30 ; 41; First difference: 7 ; 9 ; 11 ; … Common 2nd difference: 2 $T_p=2p+5$ Answer only: full marks **OR/*OF*** First difference: 7 ; 9 ; 11 ; … $T_n=a+(n-1)d$ $T_p=7+(p-1)(2)$ $T_p=2p+5$	✓first difference ✓ 2 ✓ $2p+5$ (3) **OR/*OF*** ✓first difference ✓ 2 ✓ $2p+5$ (3)
3.4	$157=2p+5$ $p=76$ $\therefore$ Between T_{76} and T_{77} **OR/OF** $T_{n+1}-T_n=157$ $(n+1)^2+4(n+1)+9-(n^2+4n+9)=157$ $n^2+2n+1+4n+4+9-n^2-4n-9=157$ $2n=152$ $n=76$ $\therefore$ Between T_{76} and T_{77}	✓$157=2p+5$ ✓ $p=76$ ✓ T_{76} and T_{77} (3) **OR/OF** ✓ $T_{n+1}-T_n=157$ ✓ $n=76$ ✓ T_{76} and T_{77} (3)
		[10]

QUESTION 4/*VRAAG 4*

4.1.1	$p=-1$ and $q=2$	✓ $p=-1$ ✓ $q=2$ (2)
4.1.2	$\frac{1}{x-1}+2=0$ $-2x+2=1$ $x=\frac{1}{2}$ $\left(\frac{1}{2}\ ;\ 0\right)$	✓ $=0$ ✓ answer (2)
4.1.3	$x=\frac{1}{2}-3$ $=\frac{-5}{2}$ Answer only: full marks	✓ -3 ✓ $x=\frac{-5}{2}$ (2)
4.1.4	$y=x+t$ $2=1+t$ $t=1$	✓subst $(1\ ;\ 2)$ ✓ $t=1$ (2)
4.1.5	$-2\le\frac{1}{x-1}$ Answer only: full marks $\frac{1}{x-1}+2\ge 0$ $\therefore x\le\frac{1}{2}$ or $x>1$ **OR/*OF*** $x\in\left(-\infty\ ;\frac{1}{2}\right]$ or $(1\ ;\infty)$	✓ $\frac{1}{x-1}+2\ge 0$ ✓ $x\le\frac{1}{2}$ ✓ $x>1$ (3)
4.2.1	$y=-5$	✓ answer (1)
4.2.2	$x=\frac{-b}{2a}=\frac{-(-4)}{2(1)}=2$ $f(2)=2^2-4(2)-5=-9$ $\therefore D(2\ ;-9)$ **OR/*OF*** $f^{/}(x)=2x-4$ $2x-4=0$ $x=2$ $f(2)=2^2-4(2)-5=-9$ $\therefore D(2\ ;-9)$	✓ $x=2$ ✓ $y=-9$ (2) **OR/*OF*** ✓ $x=2$ ✓ $y=-9$ (2)

4.2.3	$q=-5$ $-9=a(2)^2-5$ $-4=4a$ $a=-1$ $\therefore g(x)=-2^x-5$	✓ $q=-5$ ✓ substitution of (2 ; – 9) ✓ $a=-1$ (3)
4.2.4	$y\in(-\infty\,;-5)$ OR $y<-5; y\in R$	✓answer (1)
4.2.5	$k<-9$	✓ – 9 ✓ $k<-9$ (2)
		[20]

QUESTION/*VRAAG* 3

3.1.1	$T_n = ar^{n-1}$ $T_{10} = 1\,024\left(\frac{1}{4}\right)^{10-1}$ $\therefore T_{10} = \frac{1}{256}$ ANSWER ONLY: FULL MARKS	✓substitution into the correct formula ✓ answer (2)
3.1.2	$\sum_{p=0}^{8} 256\left(4^{1-p}\right) = 1\,024 + 256 + 64 + \ldots$ $S_n = \frac{a\left[1-r^n\right]}{1-r}$ $S_9 = \frac{1024\left[1-\left(\frac{1}{4}\right)^9\right]}{1-\frac{1}{4}}$ $S_9 = \frac{87\,381}{64}$ $= 1\,365{,}33$	✓ 1024 ✓ $n = 9$ ✓substitution into the correct formula ✓answer (4)
	OR/OF $\sum_{p=0}^{8} 256\left(4^{1-p}\right)$ $= 1\,024 + 256 + 64 + 16 + 4 + 1 + \frac{1}{4} + \frac{1}{16} + \frac{1}{64}$ $S_9 = \frac{87\,381}{64}$ $= 1\,365{,}33$	**OR/OF** ✓ 1024 ✓ rest of expansion ✓ $n = 9$ terms ✓ answer (4)
3.2	$-t^2 - 6t - 9\,;\ \frac{t^3 + 9t^2 + 27t + 27}{2}$ $-\left(t^2 + 6t + 9\right);\ \frac{1}{2}(t+3)\left(t^2 + 6t + 9\right)$ $-(t+3)^2\ ;\ \frac{1}{2}(t+3)^3$ $r = \frac{-(t+3)}{2}$ $-1 < \frac{-t-3}{2} < 1$ $-2 < -t - 3 < 2$ $1 < -t < 5$ $-5 < t < -1$	✓ $r = \frac{\frac{t^3 + 9t^2 + 27t + 27}{2}}{-t^2 - 6t - 9}$ ✓ $-(t^2 + 6t + 9)$ ✓ $\frac{1}{2}(t+3)(t^2 + 6t + 9)$ ✓ $-1 < \frac{-t-3}{2} < 1$ ✓ answer (5)
		[11]

QUESTION 4

4.1	$10 = a\left(\frac{1}{3}\right)^{-2} + 7$ $3 = 9a$ $\therefore a = \frac{1}{3}$	✓ subs (–2 ; 10) ✓ simplification ✓ answer (3)
4.2	$y = g(0)$ $y = \frac{1}{3} \times \left(\frac{1}{3}\right)^{0} + 7$ $y = \frac{22}{3} = 7{,}33$ $\therefore \left(0\ ;\ \frac{22}{3}\right)$ ANSWER ONLY: FULL MARKS	✓ substitution of $x = 0$ ✓ answer (2)
4.3.1	Translation by 1 unit to the right and 7 units downwards	✓ 1 unit right ✓ 7 units downwards (2)
4.3.2	$h(x) = \left(\frac{1}{3}\right)^{x}$ ANSWER ONLY: FULL MARKS $h^{-1}: \quad x = \left(\frac{1}{3}\right)^{y}$ $y = \log_{\frac{1}{3}}(x)$ **OR/*OF*** $y = -\log_{3}(x)$	✓ swap x and y ✓ answer (2)
		[9]

QUESTION 3

3.1	–82	✓ answer (1)
3.2	–145 ; –122 ; –101 ; . . . 23 21 –2 –2 $2a=-2 \quad \therefore\ a=-1$ $3a+b=23 \quad \therefore\ 3(-1)+b=23 \quad \therefore\ b=26$ $a+b+c=-145 \ \therefore\ -1+26+c=-145 \ \therefore\ c=-170$ $\therefore T_n=-n^2+26n-170$ **OR/OF** $2a=-2 \quad \therefore\ a=-1$ $c=-145+(-2)-23=-170$ $\therefore T_n=-n^2+bn-170$ $-145=-1+b-170$ $b=26$ $\therefore T_n=-n^2+26n-170$	✓ $2a=-2$ ✓ $3(-1)+b=23$ ✓ $-1+26+c=-145$ (3) **OR/OF** ✓ $2a=-2$ ✓ $c=-145+(-2)-23$ ✓ $-145=-1+b-170$ (3)
3.3	$T_n=bn+c$ $T_n=-2n+25$ or $T_n=a+(n-1)d$ $=23+(n-1)(-2)$ $=25-2n$ $-2n+25=-121$ $-2n=-146$ $n=73$ Between T_{73} and T_{74} **OR/*OF*** $T_{n+1}-T_n=-(n+1)^2+26(n+1)-170-(-n^2+26n-170)$ $-121=-2n+25$ $n=73$ Between T_{73} and T_{74}	✓ $T_n=-2n+25$ ✓ $T_n=-121$ ✓ $n=73$ ✓answer (4) **OR/*OF*** ✓ $T_n=-2n+25$ ✓ $T_n=-121$ ✓ $n=73$ ✓ answer (4)
3.4	$n=\dfrac{-b}{2a}=\dfrac{-26}{2(1)}=13$ $T_{13}=-1$ $\therefore$ add 2 **OR/*OF*** $T'_n=-2n+26=0$ $n=13$ $T_{13}=-(13)^2+26(13)-170=-1$ $\therefore$ add 2	✓13 ✓ $T_{13}=-1$ ✓ add 2 (3) **OR/*OF*** ✓13 ✓ $T_{13}=-1$ ✓ add 2 (3)
		[11]

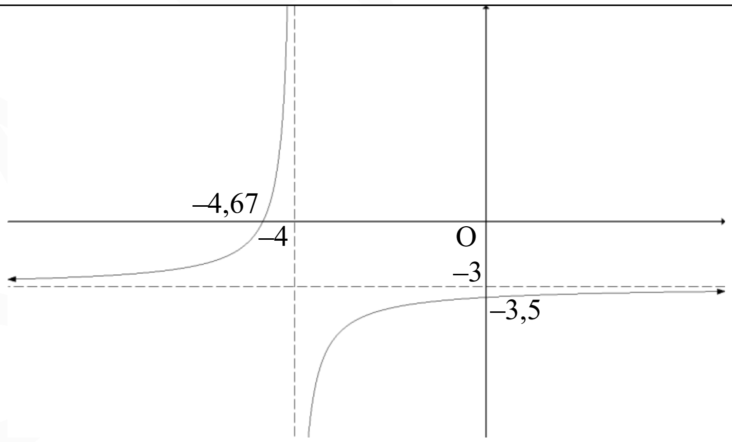
QUESTION/*VRAAG* 4

4.1	$a = 5$ and/*en* $d = 2$ $T_{51} = 5 + (51-1)(2)$ $= 105$	✓ a and d ✓ substitution into correct formula ✓ answer (3)
4.2	$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$ $S_{51} = \frac{51}{2}\left[2(5) + (51-1)2\right]$ or/*of* $S_{51} = \frac{51}{2}\left[5 + 105\right]$ $= 2\ 805$ $= 2\ 805$	✓ substitution into correct formula ✓ answer (2)
4.3	$\sum_{n=1}^{5\,000}(2n+3) = 5 + 7 + 9 + ... + 10\ 003$	✓ expansion (1)
4.4	$T_1 = -3$ $T_{4\,999} = -2(4\,999) - 1 = -9\,999$ $\therefore \sum_{n=1}^{5000}(2n+3) + \sum_{n=1}^{4999}(-2n-1)$ = (5 + 7 + 9 + … + 9 999 + 10 001 + 10 003) + (–3 – 5 – 7 – 9 – … – 9 999) = 10 001 + 10 003 –3 = 20 001 **OR/OF** $S_{4\,999} = \frac{4\,999}{2}\left[2(-3) + (4\,999-1)(-2)\right] = -24\ 999\ 999$ $S_{5\,000} = \frac{5000}{2}\left((2)(5) + (5\,000-1)(2)\right) = 25\ 020\ 000$ $\sum_{n=1}^{5\,000}(2n+3) + \sum_{n=1}^{4\,999}(-2n-1) = 25\ 020\ 000 - 24\ 999\ 999$ $= 20\ 001$	✓ $T_1 = -3$ ✓ $T_{4999} = -9\,999$ ✓both expansions ✓ answer (A) (4) **OR/OF** ✓ $T_1 = -3$ ✓ $S_{4\,999} = -24\ 999\ 999$ ✓ $S_{5\,000} = 25\ 020\ 000$ ✓ answer (A) (4)
		[10]

QUESTION/*VRAAG* 3

3.1.1	$T_n = ar^{n-1} = 2000\left(\frac{1}{5}\right)^{n-1}$	✓ $2000\left(\frac{1}{5}\right)^{n-1}$ (1)
3.1.2	$T_7 = 2000\left(\frac{1}{5}\right)^{7-1} = \frac{16}{125}$	✓ $\frac{16}{125}$ (1)
3.1.3	$\frac{16}{15625} = 2000\left(\frac{1}{5}\right)^{n-1}$ $\frac{1}{1953125} = \left(\frac{1}{5}\right)^{n-1}$ $\left(\frac{1}{5}\right)^{9} = \left(\frac{1}{5}\right)^{n-1}$ **OR** $n-1 = \log_{\frac{1}{5}} \frac{1}{1953125}$ $n-1=9$ $n=10$	✓ equating ✓ same base / use of log ✓ answer (3)
3.2	$S_\infty = 27 = \frac{a}{1-r}$ $S_3 = \frac{a(1-r^3)}{1-r} = 26$ $27(1-r^3) = 26$ $1-r^3 = \frac{26}{27}$ $r^3 = \frac{1}{27}$ $\therefore r = \frac{1}{3}$ **OR/OF** $S_\infty = 27 = \frac{a}{1-r}$ $a = 27(1-r)$ But $a + ar + ar^2 = 26$ $a(1+r+r^2) = 26$ $27(1-r)(1+r+r^2) = 26$ $(1-r)(1+r+r^2) = \frac{26}{27}$ $r^2 + r + 1 - r^3 - r^2 - r = \frac{26}{27}$ $-r^3 + 1 = \frac{26}{27}$ $r^3 = \frac{1}{27}$ $\therefore r = \frac{1}{3}$	✓ $S_\infty = 27 = \frac{a}{1-r}$ ✓ $S_3 = \frac{a(1-r^3)}{1-r} = 26$ ✓substitution ✓ $r = \frac{1}{3}$ (4) **OR/OF** ✓ $a = 27(1-r)$ ✓ $a + ar + ar^2 = 26$ ✓substitution ✓ $r = \frac{1}{3}$ (4)
		[9]

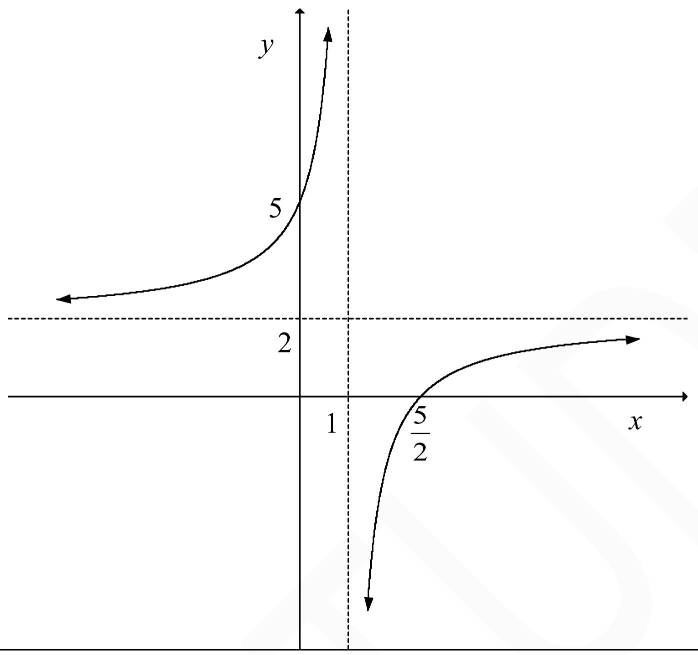
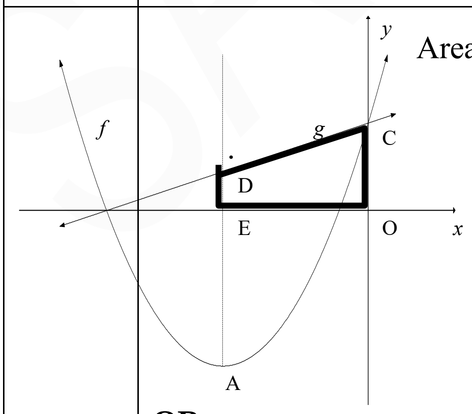
QUESTION/*VRAAG* 4

4.1	$x+1=-x-7$ $2x=-8$ $x=-4$ $\therefore y=-3$ $\therefore f(x)=\dfrac{-2}{x+4}-3$ $\therefore p=4$ and $q=-3$ **OR/OF** $p+q=1$ ……(1) $-p+q=-7$ $q=p-7$ …….(2) subs. (2) into (1) $p+p-7=1$ $2p=8$ $p=4$ $q=-3$	✓ $x+1=-x-7$ ✓ $2x=-8$ ✓ $x=-4$ ✓ $y=-3$ (4) **OR/OF** ✓ $p+q=1$ ✓ $q=p-7$ ✓ substitution ✓ simplification (4)
4.2	$y=\dfrac{-2}{x+4}-3$ $0=\dfrac{-2}{x+4}-3$ $-2-3(x+4)=0$ $-3x-14=0$ $\therefore x=-\dfrac{14}{3}$	✓ $y=0$ ✓ $x=-\dfrac{14}{3}$ (2)
4.3		✓ horizontal asymptote ✓ vertical asymptote ✓ y intercept ✓ shape (4)
		[10]

QUESTION/*VRAAG* 3

3.1	$\sum_{k=1}^{\infty} 4.3^{2-k} = 12 + 4 + \frac{4}{3} + \ldots$ $r = \frac{4}{12} = \frac{1}{3}$ $-1 < \frac{1}{3} < 1$ $\therefore$ series is convergent ($-1 < r < 1$)	✓ $12 + 4 + \frac{4}{3} + ..$ or $36\left(\frac{1}{3}\right)^k$ ✓ value of r ✓ $-1 < r < 1$ (3)
3.2	$\sum_{k=p}^{\infty} 4.3^{2-k} = 4.3^{2-p} + 4.3^{1-p} + 4.3^{-p} + \ldots$ $a = 4.3^{2-p}$ $r = \frac{1}{3}$ $S_\infty = \frac{a}{1-r}$ $\frac{2}{9} = \frac{4.3^{2-p}}{1-\frac{1}{3}}$ $4.3^{2-p} = \frac{4}{27}$ $3^{2-p} = 3^{-3}$ $2 - p = -3$ $p = 5$	✓ expression for a ✓ substitution of a, r and S_∞ ✓ simplification $\left(4.3^{2-p} = \frac{4}{27}\right)$ ✓ $3^{2-p} = 3^{-3}$ ✓ answer (5)
		[8]

QUESTION/*VRAAG* 4

4.1.1	$x=1$ $y=2$	✓ $x=1$ ✓ $y=2$ (2)
4.1.2	$y=mx+c$ $2=-1+c$ $c=3$ $y=-x+3$ **or** $y-y_1=m(x-x_1)$ $y-2=-1(x-1)$ $y-2=-x+1$ $y=-x+3$ **or** $y=-(x-p)+q$ $=-(x-1)+2$ $y=-x+3$	✓ substitution of $m=-1$ and $(1\,;2)$ ✓ answer (2)
4.1.3	 	✓ vertical asymptote: $x=1$ and horizontal asymptote: $y=2$ ✓ x-intercept: $\frac{5}{2}$ ✓ y-intercept: 5 ✓ shape (A) (4)
4.2.1	$(-5\,;-8)$	✓ $x=-5$ ✓ $y=-8$ (2)
4.2.2	$y\geq -8$ **or** $[-8;\infty)$	✓ answer (1)
4.2.3	$m=-5$ $n=g(-5)$ $=\frac{1}{2}(-5)+\frac{9}{2}$ $=2$	✓ $m=-5$ ✓substitution ✓ $n=2$ (3)
4.2.4		
 	Area trapezium $=\frac{1}{2}(\text{DE}+\text{OC})\times\text{OE}$ $=\frac{1}{2}(2+4,5)\times 5$ $=\frac{65}{4}$ or 16,25 **OR**	✓ method ✓correct substitution ✓answer (3) **OR**

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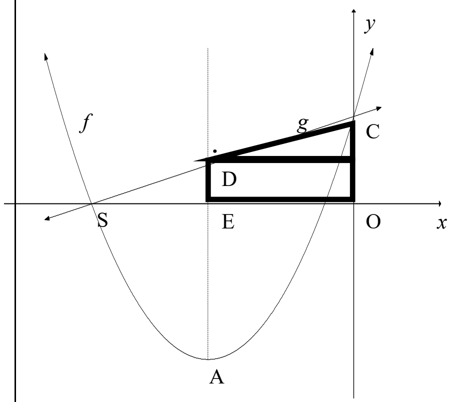


Area $\Delta = \frac{1}{2}b.h$

$= \frac{1}{2}(5)\left(\frac{5}{2}\right)$

$= \frac{25}{4}$

Area rect $= b.h$

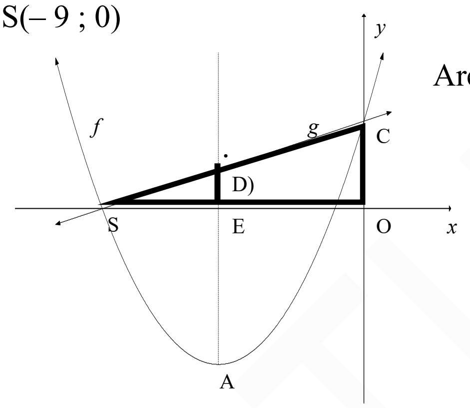
$= (5)(2)$

$= 10$

Area trapezium = $\frac{25}{4} + 10 = \frac{65}{4}$ or 16,25

OR

S(– 9 ; 0)



Area Δ SOC = $\frac{1}{2}b.h$

$= \frac{1}{2}(9)\left(\frac{9}{2}\right)$

$= \frac{81}{4}$

Area Δ SED = $\frac{1}{2}b.h = \frac{1}{2}(4)(2) = 4$

Area trapezium = area Δ SOC – Area Δ SED

$= \frac{81}{4} - 4$

$= \frac{65}{4}$ or 16,25

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✓ method

✓correct substitution

✓answer (3)

OR

✓ method

✓correct substitution

✓answer (3)

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<td>4.2.5</td>
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$g^{-1}: \ x = \frac{1}{2}y + \frac{9}{2}$

$g^{-1}: \ y = 2x - 9$

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✓ changing x and y

✓answer

(2)

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4.2.6	$f(x)=\frac{1}{2}(x+5)^2-8$ $f(x)=\frac{1}{2}(x^2+10x+25)-8$ $f(x)=\frac{1}{2}x^2+5x+4{,}5$ $f^{/}(x)=x+5$ $h(x)=2x-9+k$ $x+5=2$ $x=-3 \quad y=-6$ $(-3\,;-6)$ **OR** $f(x)=h(x)$ $\frac{1}{2}(x+5)^2-8=2x-9+k$ $\frac{1}{2}x^2+3x+\frac{27}{2}-k=0$ $x=\frac{-3}{2\left(\frac{1}{2}\right)}=-3 \qquad b^2-4ac=0$ $y=-6$ $(-3\,;-6)$	✓ $f^{/}(x)$ ✓ $x+5=2$ ✓ $x=-3$ ✓ $y=-6$ (4) **OR** ✓ equating ✓ turning point / $\triangle=0$ ✓ $x=-3$ ✓ $y=-6$ (4)
		[23]

QUESTION/*VRAAG* 3

3.1	$\sum_{y=3}^{10}\frac{1}{y-2}-\sum_{y=3}^{10}\frac{1}{y-1}$ $=\left(\frac{1}{1}+\frac{1}{2}+\frac{1}{3}+....+\frac{1}{8}\right)-\left(\frac{1}{2}+\frac{1}{3}+....+\frac{1}{8}+\frac{1}{9}\right)$ $=1-\frac{1}{9}$ $=\frac{8}{9}$	✓ $\left(\frac{1}{1}+\frac{1}{2}+\frac{1}{3}+....+\frac{1}{8}\right)$ ✓ $\left(\frac{1}{2}+\frac{1}{3}+....+\frac{1}{8}+\frac{1}{9}\right)$ ✓answer (3)
3.2	$\left(\frac{1}{3}\times\frac{2}{3}\right)+\left(\frac{2}{3}\times\frac{2}{3}\right)+\left(1\times\frac{2}{3}\right)+....+\left(4\times\frac{2}{3}\right)$ $=\frac{2}{9}+\frac{4}{9}+\frac{2}{3}+....+\frac{8}{3}$ $a=\frac{2}{9}$ and $d=\frac{2}{3}-\frac{4}{9}=\frac{2}{9}$ $S_n=\frac{n}{2}[2a+(n-1)d]$ OR $S_n=\frac{n}{2}(a+l)$ $S_{12}=\frac{12}{2}\left[2\left(\frac{2}{9}\right)+(12-1)\frac{2}{9}\right]$ $S_{12}=\frac{12}{2}\left(\frac{2}{9}+\frac{8}{3}\right)$ $=\frac{52}{3}\text{ m}^2$ $=\frac{52}{3}\text{ m}^2$ $\therefore$ for both sides $=2\times\frac{52}{3}=\frac{104}{3}=34{,}67\text{m}^2$ **OR/OF** $\frac{2}{9}\times(1+2+3+4+5+6+7+8+9+10+11+12)\times2$ $=34{,}67\text{ m}^2$ **OR/OF** $T_1=\frac{2}{9}\times12=\frac{8}{3}$ $l=\frac{2}{9}\times1=\frac{2}{9}$ $2S_{12}=2\left(\frac{12}{2}\right)\left(\frac{8}{3}+\frac{2}{9}\right)$ $=34{,}67\text{ m}^2$	✓✓a ✓d ✓ substitution into the correct formula ✓ answer ✓ answer for both sides (6) **OR/OF** ✓✓ a ✓✓ (1 + …. + 12) ✓ ×2 ✓ answer (6) **OR/OF** ✓✓ a ✓ $T_1=\frac{8}{3}$ ✓ $l=\frac{2}{9}$ ✓ substitution into correct formula ✓ answer (6)
		[9]

QUESTION/*VRAAG* 4

4.1	$p=-1$	✓ $p=-1$ (1)
4.2	$y=\dfrac{a}{x-1}$ $-3=\dfrac{a}{0-1}$ $a=3$ $y=x^2+bx-3$ $0=(1)^2+(1)b-3$ $b=2$	✓coordinates D(0 ; –3) ✓substitute (0 ; –3) ✓substitute (1 ; 0) (3)
4.3	$y=x^2+2x-3$ axis of sym: $x=\dfrac{-b}{2a}$ $x=\dfrac{-2}{2(1)}$ $x=-1$ $y=(-1)^2+2(-1)-3=-4$ $C(-1;-4)$ **OR/OF** $\dfrac{dy}{dx}=0$ $2x+2=0$ $x=-1$ $y=(-1)^2+2(-1)-3=-4$ $C(-1;-4)$	✓substitution ✓ $x=-1$ ✓substitution ✓$y=-4$ (4) **OR/OF** ✓derivative ✓ $x=-1$ ✓substitution ✓$y=-4$ (4)
4.4	$y\in[-4;\infty)$ or $y\geq-4$	✓ –4 ✓ answer (2)
4.5	$m=\tan 45°=1$ $y=mx+c$ $-4=(1)(-1)+c$ $c=-3$ $y=x-3$	✓gradient ✓subs m and $(-1;-4)$ ✓equation (3)
4.6	No, the line passes through C and D **OR/OF** No, a tangent through turning point C will have a gradient of 0	✓ No ✓ reason (2) **OR/OF** ✓ No ✓ reason (2)

4.7	$f(m-x)=f[-(x-m)]$ f is reflected in the y-axis and translated 1 unit to the left and 4 units upwards. Therefore: $m=-1$ $q=4$ **OR/OF** Substitute $x=0$ and $q=4$ for one x- intercept $h(x)=(m-x)^2+2(m-x)-3+q$ $h(0)=(m-0)^2+2(m-0)-3+4$ $0=m^2+2m+1$ $0=(m+1)^2$ $m=-1$ $q=4$	✓✓value of m ✓✓ value of q (4) **OR/OF** ✓✓value of m ✓✓ value of q (4)
		[19]

QUESTION/*VRAAG* 3

3.1.1	$p + 6 - (2p + 3) = p - 2 - (p + 6)$ $-p + 3 = -8$ $p = 11$	✓ equating i.t.o p ✓ simplifying (2)
3.1.2	$T_n = 25 + (n-1)(-8) = 33 - 8n$ $33 - 8n < -55$ $-8n < -88$ $n > 11$ $\therefore$ Term 12 will be the first term smaller than -55 $\therefore$ *Term 12 sal die eerste term kleiner as – 55 wees.*	✓ subst into T_n formula ✓ $n > 11$ ✓ $n = 12$ (3)
3.2	$S_6 = \frac{n}{2}[a+l] = \frac{6}{2}[(x-3)+(x-18)]$ $= 6x - 63$ $S_9 = \frac{n}{2}[a+l] = \frac{9}{2}[(x-3)+(x-27)]$ $= 9x - 135$ $6x - 63 = 9x - 135$ $3x = 72$ $x = 24$ $\therefore S_{15} = \frac{n}{2}[a+l] = \frac{15}{2}[(x-3)+(x-45)]$ $= \frac{15}{2}[2x - 48]$ $= \frac{15}{2}[2(24) - 48] = 0 = \text{RHS}$ **OR/*OF*** $\sum_{k=7}^{9}(x-3k) = 0$ $(x-21)+(x-24)+(x-27) = 0$ $\therefore 3x - 72 = 0$ $3x = 72$ $x = 24$ $\sum_{k=1}^{15}(24-3k)$ $= 21 + 18 + 15 + ... + -21.$ $S_n = \frac{n}{2}[a+l)]$ $= \frac{15}{2}[21 - 21]$ $= 0 = \text{RHS}$ **OR/*OF***	✓ $6x - 63$ ✓ $9x - 135$ ✓ 24 ✓ $\frac{15}{2}[(x-3)+(x-45)]$ ✓ substitution of x (5) **OR/*OF*** ✓ expansion ✓ $3x - 72 = 0$ ✓ 24 ✓ substitution of x ✓ sum of 15 terms (5) **OR/*OF***

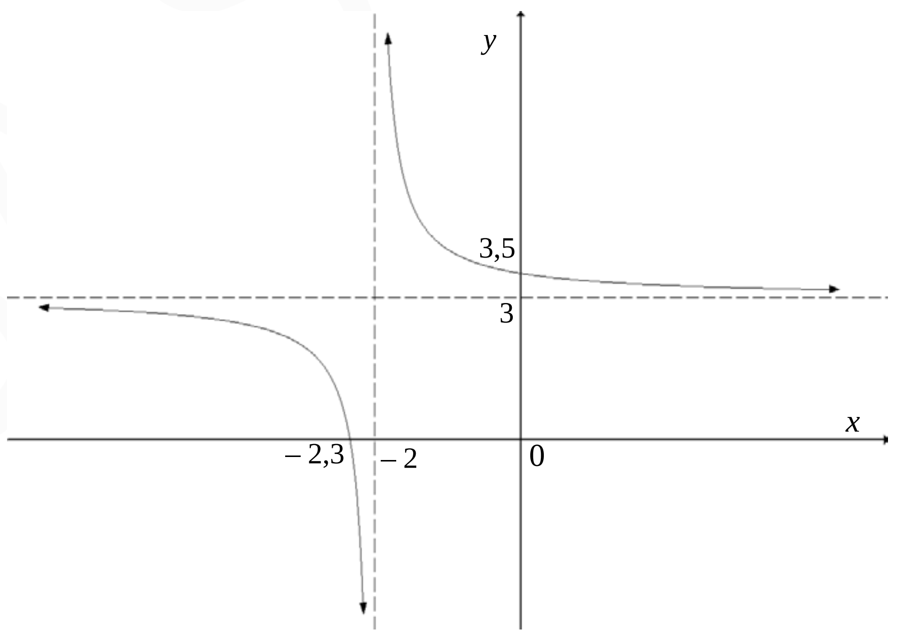
	$(x-3)+(x-6)+(x-9)+(x-12)+(x-15)+(x-18)$ $=(x-3)+(x-6)+(x-9)+(x-12)+(x-15)+(x-18)$ $+(x-21)+(x-24)+(x-27)$ $\therefore 3x-72=0$ $3x=72$ $x=24$ $\sum_{k=1}^{15}(24-3k)$ $=21+18+15+.....+-21.$ $S_n=\frac{n}{2}[a+l]$ $=\frac{15}{2}[21-21]$ $=0=\text{RHS}$	✓ expansion ✓ $3x-72=0$ ✓ 24 ✓ substitution of x ✓ sum of 15 terms (5) **[10]**

QUESTION/*VRAAG* 4

4.1	$y>0$ **OR/*OF*** $y\in(0\ ;\ \infty)$	✓answer (1) **OR/*OF*** ✓answer (1)
4.2	$g: y=\left(\frac{1}{2}\right)^x$ $g^{-1}: x=\left(\frac{1}{2}\right)^y$ $y=\log_{\frac{1}{2}} x$ or $y=-\log_2 x$ or $y=\log_2 \frac{1}{x}$	✓ $x=\left(\frac{1}{2}\right)^y$ ✓ equation (2)
4.3	Yes. The vertical line test cuts g^{-1} once *Ja. Die vertikale lyn toets sny g^{-1} slegs eenkeer.* **OR/*OF*** Yes. For every x-value there is a unique y-value *Ja. Vir elke x-waarde is daar 'n unieke y-waarde* **OR/*OF*** Yes. g is a one-to-one function / *Ja. g is 'n een-tot-een funksie* **OR/*OF*** Yes. The horizontal line cuts g only once *Ja. Die horisontale lyn sny g slegs een keer*	✓ yes ✓ valid reason (2) **OR/*OF*** ✓ yes ✓ valid reason (2) **OR/*OF*** ✓ yes ✓ valid reason (2) **OR/*OF*** ✓ yes ✓ valid reason (2)

4.4.1	$y = -\log_2 x$ $2 = -\log_2 a$ $a = 2^{-2} = \frac{1}{4}$ or $a = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$	✓ correct subst into correct formula (a ; 2) ✓answer (2)
4.4.2	$M^{/}\left(2;\frac{1}{4}\right)$ or $M^{/}(2;a)$	✓ answer (1)
4.5	$M^{//}\left(-1;\frac{9}{4}\right)$	✓ -1 ✓✓ $\frac{9}{4}$ (3)
		[11]

QUESTION/*VRAAG* 5

5.1.1	$x = -2$ $y = 3$	✓ answer ✓ answer (2)
5.1.2	$\left(0\ ;\frac{7}{2}\right)$	✓ answer (1)
5.1.3	$\frac{1}{x+2} + 3 = 0$ $1 + 3(x+2) = 0$ $3x = -7$ $x = -\frac{7}{3}$ x-intercept $\left(-\frac{7}{3}\ ;0\right)$	✓ $y = 0$ ✓ answer (2)
5.1.4		✓asymptotes at $y = 3$ and $x = -2$ ✓intercepts at $y = 3{,}5$ and $x = -2{,}3$ ✓shape (reasonable representation in correct quadrants) (3)

QUESTION/*VRAAG* 3

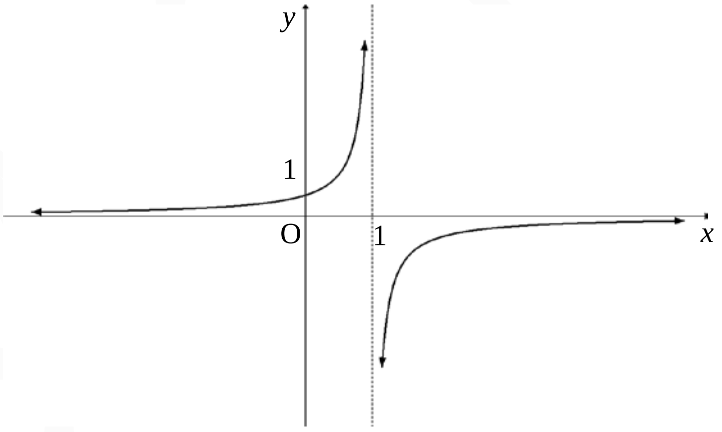
3.1	$r=\frac{1}{2}$ and $S_\infty=6$ $S_\infty=\frac{a}{1-r}$ $6=\frac{a}{1-\frac{1}{2}}$ $a=3$	✓substitution ✓answer (2)
3.2	$T_n=ar^{n-1}$ $T_8=3\left(\frac{1}{2}\right)^7$ $T_8=\frac{3}{128}$	✓✓ $T_8=3\left(\frac{1}{2}\right)^7$ (2)
3.3	$\sum_{k=1}^{n}3(2)^{1-k}=5{,}8125$ $3+\frac{3}{2}+\frac{3}{4}+...=5{,}8125$ $S_n=\frac{a(1-r^n)}{1-r}=5{,}8125$ $\frac{3\left[1-\left(\frac{1}{2}\right)^n\right]}{1-\frac{1}{2}}=5{,}8125$ $6\left[1-\left(\frac{1}{2}\right)^n\right]=5{,}8125$ $\left(\frac{1}{2}\right)^n=\frac{1}{32}=0{,}03125$ $2^{-n}=2^{-5}$ or $n\log\frac{1}{2}=\log\frac{1}{32}$ $n=5$ $\quad$ $n=5$	✓ $r=\frac{1}{2}$ ✓substitution ✓simplification ✓answer (4)

3.4	$\sum_{k=1}^{20} 3(2)^{1-k} = p$ $3+\frac{3}{2}+\frac{3}{4}+........+3.2^{-19} = p$ $\sum_{k=1}^{20} 24(2)^{-k}$ $=12+6+3+.....+24.2^{-20}$ $=4\left(3+\frac{3}{2}+\frac{3}{4}+.....+3.2^{-19}\right)$ $= 4p$	✓expansion ✓ expansion ✓ answer (3)
	OR/OF	**OR/OF**
	$\sum_{k=1}^{20} 3(2)^{1-k} = p$ $\sum_{k=1}^{20} 6(2)^{-k} = p$ $\therefore \sum_{k=1}^{20} 24(2)^{-k} = 4p$	✓ $\sum_{k=1}^{20} 6(2)^{-k} = p$ ✓ $\sum_{k=1}^{20} 4\times 6(2)^{-k}$ ✓$4p$ (3)
	OR/OF $\sum_{k=1}^{20} 24(2)^{-k} = \sum_{k=1}^{20} 4\times 3\times 2(2)^{-k}$ $= 4\sum_{k=1}^{20} 3\times 2(2)^{-k}$ $= 4\sum_{k=1}^{20} 3\times (2)^{1-k} = 4p$	**OR/OF** ✓ $\sum_{k=1}^{20} 4\times 3\times 2(2)^{-k}$ ✓ $4\sum_{k=1}^{20} 3\times 2(2)^{-k}$ ✓$4p$ (3)
	OR/OF $S_{20} = \frac{3\left(\left(\frac{1}{2}\right)^{20} -1\right)}{\frac{1}{2}-1} = 6 = p$ $S_{20} = \frac{12\left(\left(\frac{1}{2}\right)^{20} -1\right)}{\frac{1}{2}-1} = 24$ $24 = 4\times 6 = 4p$	**OR/OF** ✓ substitution and answer ✓substitution and answer ✓$4p$ (3)
		[11]

QUESTION/*VRAAG* 4

4.1	Yes For every x-value there is only one corresponding y value **OR/OF** One to one mapping (vertical line test)	✓answer ✓reason (2)
4.2	R(–12 ; –6)	✓answer (1)
4.3	$f(x) = ax^2$ substitute $(-6\,;-12)$ $-12 = a(-6)^2$ $a = \frac{-1}{3}$	✓substitution ✓answer (2)
4.4	$f : y = -\left(\frac{1}{3}\right)x^2$ $f^{-1} : x = -\left(\frac{1}{3}\right)y^2$ $y^2 = -3x$ $y = \pm\sqrt{-3x}$ Only $y = -\sqrt{-3x}$ and $x \le 0$	✓swapping x and y ✓ $y^2 = -3x$ ✓ $y = -\sqrt{-3x}$ (3)
		[8]

QUESTION/*VRAAG* 5

5.1	Domain: $x \in R\,;\ x \neq 1$ **OR/OF** $x \in (-\infty;1) \cup (1;\infty)$	✓answer (1)
5.2	$x = 1$ $y = 0$	✓$x = 1$ ✓$y = 0$ (2)
5.3		✓ y intercept ✓vertical asymptote ✓shape (3)
5.4	$x \geq 0$; $x \neq 1$ **OR/OF** $0 \leq x < 1$ or $x > 1$ **OR/OF** $x \in [0;1) \cup (1;\infty)$	✓$x \geq 0$ ✓$x \neq 1$ (2) **OR/OF** ✓ $0 \leq x < 1$ ✓ $x > 1$
		[8]

QUESTION/*VRAAG* 3

3.1	$-1\ ;\ 2\ ;\ 5$ $T_n = -1+(n-1)(3)$ $= 3n-4$	✓ $3n$ ✓ -4 (2)
3.2	$T_{43} = 3(43)-4$ $= 125$ **OR/ OF** $T_{43} = -1+(43-1)(3)$ $= 125$ NOTE: Answer only 2 / 2	✓ subs of 43 ✓ answer (2)
3.3	$T_n = 3n-4$ $S_n = \sum_{k=1}^{n} T_k = -1+2+5+\dots\dots\dots+3n-4$ $S_n = \frac{n}{2}[-1+3n-4]$ or $S_n = \frac{n}{2}[-2+(n-1)3]$ $= \frac{n}{2}[3n-5]$ $= \frac{3n^2-5n}{2}$ **OR/*OF*** $T_n = 3n-4$ $\sum_{k=1}^{n} T_k = 3(1)-4+3(2)-4+3(3)-4+\dots\dots\dots+3n-4$ $= 3(1+2+3+\dots\dots+n)-4n$ $= \frac{3n(n+1)}{2}-4n$ $= \frac{3n^2-5n}{2}$	✓ $S_n = \sum_{k=1}^{n} T_k$ ✓ substitution into correct formula ✓ $\frac{n}{2}[3n-5]$ or $\frac{3n^2-5n}{2}$ **OR/*OF*** ✓ $(1)-4+3(2)-4+3(3)-4+\dots\dots\dots+3n-4$ ✓ $3(1+2+3+\dots\dots+n)-4n$ ✓ $\frac{3n^2-5n}{2}$ (3)

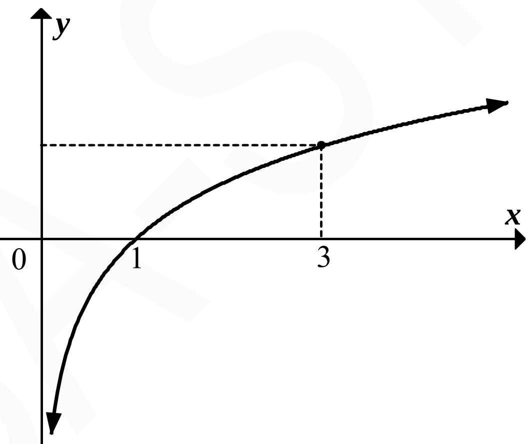
3.4	$T_{11} = (T_{11} - T_{10}) + (T_{10} - T_9) + (T_9 - T_8) + \ldots + (T_3 - T_2) + (T_2 - T_1) + T_1$ $125 = 29 + 26 + 23 + \ldots 2 + T_1$ $= \frac{10}{2}(29 + 2) + T_1$ $= 155 + T_1$ $T_1 = -30$ NOTE: Answer only 1 / 6 If they only use $3n - 4$ breakdown 0 / 6 **OR/*OF*** $T_n = an^2 + bn + c$ $\therefore T_{11} = 121a + 11b + c = 125$ $T_n - T_{n-1} = an^2 + bn + c - \left[a(n-1)^2 + b(n-1) + c\right]$ $= an^2 + bn + c - an^2 + 2an - a - bn + b - c$ $= 2an + b - a$ $T_n - T_{n-1} = 3n - 4$ $2a = 3$ and $b - a = -4$ $a = \frac{3}{2}$ and $b = -\frac{5}{2}$ $121a + 11b + c = 125$ $121\left(\frac{3}{2}\right) + 11\left(-\frac{5}{2}\right) + c = 125$ $c = -29$ $T_n = \frac{3}{2}n^2 - \frac{5}{2}n - 29$ $T_1 = \frac{3}{2}(1)^2 - \frac{5}{2}(1) - 29$ $= -30$	✓✓ generating sum ✓ $29 + 26 + 23 + \ldots 2$ ✓ $\frac{10}{2}(29 + 2)$ ✓ 155 ✓ -30 **OR/*OF*** ✓ $121a + 11b + c = 125$ ✓ calculating $T_n - T_{n-1}$ in terms of a, b and c ✓ $a = \frac{3}{2}$ ✓ $b = -\frac{5}{2}$ ✓ $c = -29$ ✓ -30 (6) **[13]**

QUESTION/*VRAAG* 4

4.1	E(4 ; –9)	✓ $x = 4$ ✓ $y = -9$ (2)
4.2	$f(x) = (x-4)^2 - 9$ $(x-4)^2 - 9 = 0$ $(x-4)^2 = 9$ $x - 4 = \pm 3$ $x = 7$ or $x = 1$ A(1 ; 0) **OR/*OF*** $f(x) = (x-4)^2 - 9$ $0 = x^2 - 8x + 16 - 9$ $0 = x^2 - 8x + 7$ $(x-7)(x-1) = 0$ $x = 7$ or $x = 1$ A(1 ; 0)	✓ $y = 0$ ✓ $x - 4 = \pm 3$ ✓ A(1 ; 0) **OR/*OF*** ✓ $y = 0$ ✓ $(x-7)(x-1)$ ✓ A(1 ; 0) (3)
4.3	C$(0\,;7)$ M$(8\,;7)$ NOTE: Answer only 3 / 3	✓C$(0\,;7)$ ✓ $x = 8$ ✓ $y = 7$ (3)
4.4	C$(0\,;7)$ D$(4\,;0)$ $m = \frac{7-0}{0-4}$ or $m = \frac{0-7}{4-0}$ or $0 = 4m + 7$ $m = -\frac{7}{4}$ $m = -\frac{7}{4}$ $m = -\frac{7}{4}$ $y - 0 = -\frac{7}{4}(x-4)$ $y = -\frac{7}{4}x + 7$	✓ D$(4\,;0)$ ✓ $m = -\frac{7}{4}$ ✓ $y = -\frac{7}{4}x + 7$ (3)
4.5	$g:\ y = -\frac{7}{4}x + 7$ $g^{-1}: x = -\frac{7}{4}y + 7$ $4x = -7y + 28$ $7y = -4x + 28$ $y = -\frac{4}{7}x + 4$ **OR/*OF***	✓ interchange x and y ✓ simplification ✓ $y = -\frac{4}{7}x + 4$ **OR/*OF***

	g^{-1} is the straight line through (0 ; 4) and (7 ; 0) $y = mx + 4$ $0 = 7m + 4$ $y = -\frac{4}{7}x + 4$	✓ straight line through (0 ; 4) and (7 ; 0) ✓ substitution ✓ $y = -\frac{4}{7}x + 4$ (3)
4.6	$x \, . \, f(x) \le 0$ $\therefore x \le 0$ or $1 \le x \le 7$	✓✓ $x \le 0$ ✓✓ $1 \le x \le 7$ (4) **[18]**

QUESTION/*VRAAG* 5

5.1	$a^0 = 1$ T(0 ; 1)	✓ $x = 0$ ✓ $y = 1$ (2)
5.2	$g(x) = a^x$ $9 = a^2$ $a = 3$ $a > 0$	✓ substitution ✓ $a = 3$ (2)
5.3	$y = \left(\frac{1}{3}\right)^x$ or $y = 3^{-x}$	✓✓ $y = \left(\frac{1}{3}\right)^x$ (2)
5.4	$3^0 < 3^{\log_3 x} < 3^1$ $1 < x < 3$ **OR**  $1 < x < 3$	✓ $1 < x$ ✓ $x < 3$ (2) ✓ $1 < x$ ✓ $x < 3$ (2) **[8]**

QUESTION/*VRAAG* 3

3.1	$r = 0{,}94;\quad a = 100$ $T_3 = ar^2$ $= 100(0{,}94)^2$ $= 88{,}36$ km	✓ $r = 0{,}94$ ✓ answer (2)
3.2	$S_n = \dfrac{a(r^n - 1)}{r - 1}$ $750 = \dfrac{100(0{,}94^n - 1)}{0{,}94 - 1}$ $\dfrac{750(-0{,}06)}{100} = 0{,}94^n - 1$ $0{,}94^n = 1 - \dfrac{9}{20}$ or $\left(\dfrac{47}{50}\right)^n = \dfrac{11}{20}$ $0{,}94^n = 0{,}55$ $n = \dfrac{\log 0{,}55}{\log 0{,}94}$ $= 9{,}66$ He will pass the halfway point on the 10th day *Hy sal die halfpadmerk verbysteek op die 10de dag*	✓ substitution into correct formula ✓ $0{,}94^n = 0{,}55$ ✓ use of logarithms ✓ answer (4)
3.3	$S_\infty = \dfrac{a}{1 - r}$ $1500 < \dfrac{100}{1 - r}$ $1 - r < \dfrac{100}{1500}$ $r > \dfrac{14}{15}$ or 93,33%	✓ use of S_∞ formula ✓ substitution ✓ answer (3) **[9]**

QUESTION/*VRAAG* 4

4.1	$0 < x \leq 1$ or $(0\,;1]$	✓✓ answer (2)
4.2	$p = \log_{\frac{4}{3}} \frac{16}{9}$ $\left(\frac{4}{3}\right)^p = \frac{16}{9}$ $\left(\frac{4}{3}\right)^p = \left(\frac{4}{3}\right)^2$ $p = 2$	✓ substitution ✓ $\left(\frac{4}{3}\right)^2$ ✓ answer (3)
4.3	$f : y = \log_{\frac{4}{3}} x$ $f^{-1} : x = \log_{\frac{4}{3}} y$ $y = \left(\frac{4}{3}\right)^x$	✓ $x = \log_{\frac{4}{3}} y$ ✓ $y = \left(\frac{4}{3}\right)^x$ (2)
4.4	$y > 0$ or $y \in (0; \infty)$	✓✓ answer (2)
4.5	$\left(-2\,; \frac{16}{9}\right)$	✓ –2 ✓ $\frac{16}{9}$ (2) **[11]**

QUESTION/*VRAAG* 3

3.1	$a + ar = 2$ $a(1+r) = 2$ $a = \frac{2}{1+r}$	✓ $a + ar = 2$ ✓ $a = \frac{2}{1+r}$ (2)
	OR/*OF* $\frac{a}{1-r} - 2 = \frac{1}{4}$ $4a - 8(1-r) = 1 - r$ $4a - 8 + 8r = 1 - r$ $4a = 9 - 9r$ $a = \frac{9-9r}{4}$	✓ $\frac{a}{1-r} - 2 = \frac{1}{4}$ ✓ $a = \frac{9-9r}{4}$ (2)
	OR/*OF* $S_n = \frac{a(r^n - 1)}{r-1}$ $2 = \frac{a(r^2-1)}{r-1}$ $2 = \frac{a(r-1)(r+1)}{r-1}$ $2 = a(r+1)$ $a = \frac{2}{r+1}$	**OR/*OF*** ✓ $2 = \frac{a(r^2-1)}{r-1}$ ✓ $a = \frac{2}{1+r}$ (2)
	OR/*OF* $\frac{ar^2}{1-r} = \frac{1}{4}$ $a = \frac{1-r}{4r^2}$	**OR/*OF*** ✓ $\frac{ar^2}{1-r} = \frac{1}{4}$ ✓ $a = \frac{1-r}{4r^2}$ (2)

3.2	$S_\infty = T_1 + T_2 + \sum_{n=3}^{\infty} T_n$ $S_\infty = 2 + \frac{1}{4}$ $\frac{a}{1-r} = 2 + \frac{1}{4}$ $\frac{a}{1-r} = \frac{9}{4}$ $\left(\frac{2}{1+r}\right) \times \left(\frac{1}{1-r}\right) = \frac{9}{4}$ $\frac{2}{1-r^2} = \frac{9}{4}$ $8 = 9 - 9r^2$ $9r^2 = 1$ $r = \frac{1}{3}$ $a = \frac{3}{2}$	✓ $S_\infty = 2 + \frac{1}{4}$ ✓ $\frac{a}{1-r} = \frac{9}{4}$ ✓ substitution of a into the correct formula ✓ $9r^2 = 1$ ✓ $r = \frac{1}{3}$ ✓ $a = \frac{3}{2}$ (6)
	OR/*OF* $S_\infty = T_1 + T_2 + \sum_{n=3}^{\infty} T_n$ $S_\infty = 2 + \frac{1}{4}$ $\frac{a}{1-r} = 2 + \frac{1}{4}$ $\frac{a}{1-r} = \frac{9}{4}$ $4a = 9 - 9r$ $r = \frac{9-4a}{9}$ $a + a\left(\frac{9-4a}{9}\right) = 2$ $9a + 9a - 4a^2 = 18$ $2a^2 - 9a + 9 = 0$ $(a-3)(2a-3) = 0$ $a = \frac{3}{2}$ or $a = 3$ $r = \frac{1}{3}$ or $r = -\frac{1}{3}$ N/A	**OR/*OF*** ✓ $S_\infty = 2 + \frac{1}{4}$ ✓ $\frac{a}{1-r} = \frac{9}{4}$ ✓ $r = \frac{9-4a}{9}$ ✓ substitution of a into the correct formula ✓ $a = \frac{3}{2}$

		✓ $r=\frac{1}{3}$ (6)
	OR/*OF* $r=\frac{2-a}{a}$ $\frac{ar^2}{1-r}=\frac{1}{4}$ $4ar^2=1-r$ $4a\left(\frac{2-a}{a}\right)^2=1-\frac{2-a}{a}$ $16-16a+4a^2=2a+2$ $2a^2-9a+9=0$ $(2a-3)(a-3)=0$ $a=\frac{3}{2}$ $\quad a\neq 3$ $r=\frac{1}{3}$ $\quad r\neq-\frac{1}{3}$	**OR/*OF*** ✓ $r=\frac{2-a}{a}$ ✓ $\frac{ar^2}{1-r}=\frac{1}{4}$ ✓ substitution of a ✓ $(2a-3)(a-3)=0$ ✓ $a=\frac{3}{2}$ ✓ $r=\frac{1}{3}$ (6)
	OR/*OF* $S_\infty=T_1+T_2+\sum_{n=3}^{\infty}T_n$ $S_\infty=2+\frac{1}{4}$ $\frac{a}{1-r}=2+\frac{1}{4}$ $\frac{a}{1-r}=\frac{9}{4}$ $\left(\frac{1-r}{4r^2}\right)\times\left(\frac{1}{1-r}\right)=\frac{9}{4}$ $\frac{1}{4r^2}=\frac{9}{4}$ $4=36r^2$ $9r^2=1$ $r=\frac{1}{3}$ $a=\frac{3}{2}$	**OR/*OF*** ✓ $S_\infty=2+\frac{1}{4}$ ✓ $\frac{a}{1-r}=\frac{9}{4}$ ✓ substitution of a ✓ $9r^2=1$ ✓ $r=\frac{1}{3}$ ✓ $a=\frac{3}{2}$ (6)
		[8]

QUESTION/*VRAAG* 4

4.1	$f(x)=-ax^2+bx+6$ $f^{/}(x)=-2ax+b$ $-2ax+b=3$ at $x=-1$ $2a+b=3 \quad [1]$ $f(-1)=\frac{7}{2}$ $-a-b+6=\frac{7}{2}$ $-2a-2b+12=7$ $2a+2b=5 \quad [2]$ $[2]-[1]$ $b=2$ $2a+2=3$ $a=\frac{1}{2}$	✓$-2ax+b$ ✓✓ $2a+b=3$ ✓ $-a-b+6=\frac{7}{2}$ ✓solve simultaneously (5)
	OR/*OF* $f^{/}(x)=-2ax+b$ $3=2a+b$ $b=3-2a$ $\frac{7}{2}=-a(-1)^2+(3-2a)(-1)+6$ $a+3=\frac{7}{2}$ $a=\frac{1}{2}$ $b=2$	✓$-2ax+b$ ✓✓ $2a+b=3$ ✓ $\frac{7}{2}=-a(-1)^2+(3-2a)(-1)+6$ ✓solve simultaneously (5)
4.2	$f(x)=-\frac{1}{2}x^2+2x+6$ $x-$intercepts: $-\frac{1}{2}x^2+2x+6=0$ $-x^2+4x+12=0$ $x^2-4x-12=0$ $(x-6)(x+2)=0$ $(-2\,;0) \quad (6\,;0)$	✓ $-\frac{1}{2}x^2+2x+6=0$ ✓$(-2\,;0)$ ✓ $(6\,;0)$ (3)

<table>
<tr><td>4.3</td><td>

$f(x)=-\frac{1}{2}x^2+2x+6$

$f'(x)=0$ or $x=-\frac{b}{2a}$ or $x=\frac{-2+6}{2}$

$-x+2=0$ $x=-\frac{2}{2.\left(-\frac{1}{2}\right)}$ $x=2$

$x=2$ $x=2$

$y=-\frac{1}{2}(2)^2+2(2)+6$

$=-2+4+6$

$=8$

TP $(2;8)$

OR/OF

$y=-\frac{1}{2}(x^2-4x-12)$

$=-\frac{1}{2}\left[(x-2)^2-4-12\right]$

$=-\frac{1}{2}(x-2)^2+8$

TP $(2;8)$

</td><td>

✓ $-x+2$ / $-\frac{2}{2.\left(-\frac{1}{2}\right)}$ / $\frac{-2+6}{2}$

✓ $x=2$

✓ $y=8$

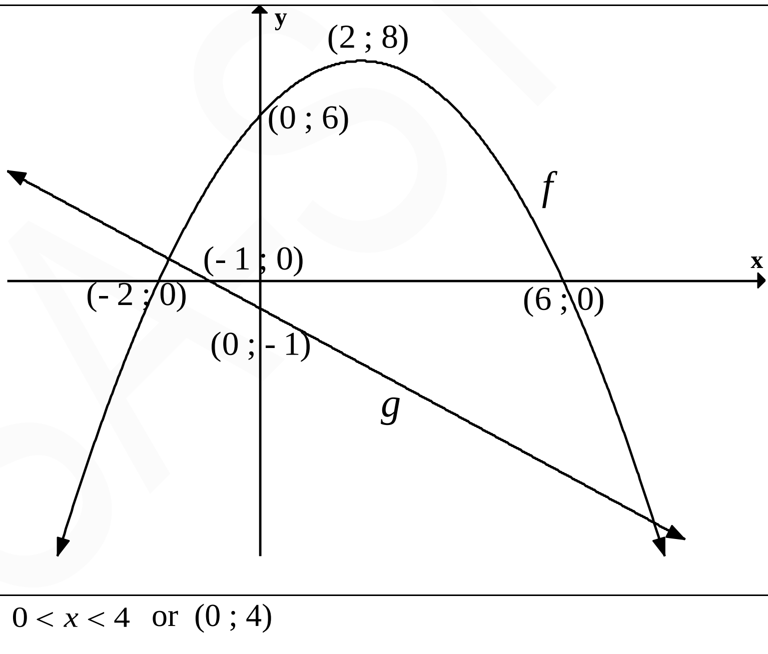
OR/*OF*

✓ $-\frac{1}{2}(x-2)^2+8$

✓ $x=2$
✓ $y=8$

(3)

</td></tr>
<tr><td>4.4
4.6</td><td>



</td><td>

4.4:
f:
✓shape
✓*x*- intercepts
✓*y*- intercept
✓(2 ; 8)

(4)

4.6:
g:

✓ *x*- intercept
✓ *y*- intercept

(2)

</td></tr>
<tr><td>4.5</td><td>$0 < x < 4$ or $(0 ; 4)$</td><td>✓ 4
✓✓ $0 < x < 4$ (3)</td></tr>
<tr><td>4.7</td><td>

$x \le -2$ or $-1 \le x \le 6$

OR/OF

$(-\infty ; -2]$ or $[-1; 6]$

</td><td>✓ $x \le -2$
✓✓ $-1 \le x \le 6$
(3)
[23]</td></tr>
</table>

2.2.2	$S_n = \frac{n}{2}[2a+(n-1)d]$ $30\ 500 = \frac{n}{2}[2(100)+(n-1)(150)]$ $61\ 000 = n(150n+50)$ $61\ 000 = 150n^2+50n$ $3n^2+n-1\ 220=0$ $(3n+61)(n-20)=0$ $n=-\frac{61}{3}$ or $n=20$ N/A $x=100+(20-1)(150)$ $=\text{R } 2\,950$	✓substitute 30 500, a and d into sum formula for AS ✓ simplification ✓ factors or quad formula ✓ $n=20$ ✓ substitution T_n of AS ✓ 2 950 (6) **[15]**

QUESTION/*VRAAG* 3

3.1	First differences: 17; 15 Second difference: –2 $T_n = an^2+bn+c$ $a=\frac{\text{second difference}}{2}=\frac{-2}{2}=-1$ $3a+b=17$ $3(-1)+b=17$ $b=20$ $a+b+c=0$ $-1+20+c=0$ $c=-19$ $T_n=-n^2+20n-19$ **OR / OF** First differences: 17; 15 $T_n = T_1+(n-1)d_1+\frac{(n-1)(n-2)}{2}d_2$ $=(0)+(n-1)(17)+\frac{(n-1)(n-2)}{2}(-2)$ $=17n-17-n^2+3n-2$ $=-n^2+20n-19$	✓17; 15 ✓value of a ✓ value of b ✓value of c ✓17; 15 ✓value of a ✓ value of b ✓value of c (4)

3.2	$56 = -n^2 + 20n - 19$ $n^2 - 20n + 75 = 0$ $(n-15)(n-5) = 0$ $n = 5$ or $n = 15$	✓ $T_n = 56$ ✓factors ✓both answers (3)
3.3	$\sum_{n=5}^{10} T_n - \sum_{n=11}^{15} T_n$ $= T_5 + T_6 + T_7 + T_8 + T_9 + T_{10} - T_{11} - T_{12} - T_{13} - T_{14} - T_{15}$ $= (T_5 - T_{15}) + (T_6 - T_{14}) + ... + (T_9 - T_{13}) + T_{10}$ $= T_{10}$ because by symmetry $T_5 = T_{15}$; $T_6 = T_{14}$ .... $T_{10} = -(10)^2 + 20(10) - 19$ $= 81$ **OR/*OF*** T_5 T_{10} T_{15} 0; 17; 32; 45; 56; 65; 72; 77; 80; 81; 80; 77; 72; 65; 56 Hence, $\sum_{n=5}^{10} T_n - \sum_{n=11}^{15} T_n$ $= (56 + 65 + 72 + 77 + 80 + 81) - (80 + 77 + 72 + 65 + 56)$ $= 81$	✓✓ symmetry of terms ✓ T_{10} ✓ 81 (4) ✓ writing out the symmetry of terms ✓ $56 + 65 + 72 + 77 + 80 + 81$ ✓ $80 + 77 + 72 + 65 + 56$ ✓ 81 (4) **[11]**

QUESTION/*VRAAG* 4

4.1	A (4; 3)	✓(4; 3) (1)
4.2	$y=\frac{6}{-4}+3$ $=\frac{3}{2}$ $B\left(0\,;\frac{3}{2}\right)$	✓ $x=0$ ✓ $y=\frac{3}{2}$ (2)
4.3	$0=\frac{6}{x-4}+3$ $-3=\frac{6}{x-4}$ $-3(x-4)=6$ $-3x+12=6$ $x=2$ C(2 ; 0)	✓$y=0$ ✓ $x=2$ (2)
4.4	Average gradient $=\frac{0-\frac{3}{2}}{2-0}$ $=-\frac{3}{4}$	✓ $\frac{0-\frac{3}{2}}{2-0}$ ✓ $-\frac{3}{4}$ (2)
4.5	$y=-x+7$ **OR/*OF*** $m=-1$ $\therefore y-3=-(x-4)$ $y=-x+7$	✓ $m=-1$ ✓ $y=-x+7$ **OR/*OF*** ✓ $m=-1$ ✓ $y=-x+7$ (2) **[9]**

QUESTION/*VRAAG* 3

3.1.1	24	✓ 24 (1)
3.1.2	6 6 9 15 0 3 6 3 3 $2a = 3$ $3a + b = 0$ $a + b + c = 6$ $a = \frac{3}{2}$ $b = -\frac{9}{2}$ $c = 9$ $T_n = \frac{3}{2}n^2 - \frac{9}{2}n + 9$ **OR/OF** $T_n = T_1 + (n-1)d_1 + \frac{(n-1)(n-2)d_2}{2}$ $= 6 + (n-1)(0) + \frac{(n-1)(n-2)(3)}{2}$ $= 6 + \frac{n^2 - 3n + 2}{1}\left(\frac{3}{2}\right)$ $= 6 + \frac{3}{2}n^2 - \frac{9}{2}n + 3$ $= \frac{3}{2}n^2 - \frac{9}{2}n + 9$	✓ $a = \frac{3}{2}$ ✓ $b = -\frac{9}{2}$ ✓ $c = 9$ ✓ $T_n = \frac{3}{2}n^2 - \frac{9}{2}n + 9$ (4) ✓formula ✓substitution ✓simplifying ✓ $T_n = \frac{3}{2}n^2 - \frac{9}{2}n + 9$ (4)
3.1.3	$\frac{3}{2}n^2 - \frac{9}{2}n + 9 = 3249$ $3n^2 - 9n + 18 = 6498$ $3n^2 - 9n - 6480 = 0$ $n^2 - 3n - 2160 = 0$ $(n+45)(n-48) = 0$ $n \neq -45$ *or* $n = 48$	✓ equating general term to 3249 ✓ standard form ✓ factors ✓ $n \neq -45$ or $n = 48$ (4)
3.2	$-1\ ;\ 2\sin 3x\ ;\ 5;\ ....$ $2\sin 3x + 1 = 5 - 2\sin 3x$ $4\sin 3x = 4$ $\sin 3x = 1$ $3x = 90^\circ$ $x = 30^\circ$	✓ $2\sin 3x + 1 = 5 - 2\sin 3x$ ✓ $\sin 3x = 1$ ✓ $3x = 90^\circ$ ✓ $x = 30^\circ$ (4) **[13]**

QUESTION/*VRAAG* 4

4.1	U(1 ; 0)	✓ (1 ; 0) (1)
4.2	$x=1$ $y=1$	✓ $x=1$ ✓ $y=1$ (2)
4.3	$\frac{2}{x-1}+1=0$ $2=-x+1$ $x=-1$ $T(-1;0)$	✓ $y=0$ ✓ $x=-1$ (2)
4.4	$f(x)=\log_5 x$ $h:\ x=\log_5 y$ $y=5^x$	✓ change x and y ✓ $y=5^x$ (2)
4.5	$y=0$	✓ answer (1)
4.6	$V(\sqrt{2}+1;\sqrt{2}+1)$ $V(2{,}41;2{,}41)$ **OR / OF** $x=\frac{2}{x-1}+1$ $x^2-x=2+x-1$ $x^2-2x-1=0$ $x=\frac{2\pm\sqrt{4-4(1)(-1)}}{2}$ $=\frac{2\pm\sqrt{8}}{2}$ $=\frac{2\pm2\sqrt{2}}{2}$ $=1\pm\sqrt{2}$ $V(1+\sqrt{2}\ ;1+\sqrt{2})$ **OR / OF** $x-1=\frac{2}{x-1}$ $(x-1)^2=2$ $x=1\pm\sqrt{2}$ $V(1+\sqrt{2}\ ;1+\sqrt{2})$	✓✓ $\sqrt{2}+1$ ✓✓ $\sqrt{2}+1$ (4) ✓ $x=\frac{2}{x-1}+1$ ✓subs into correct formula ✓ $x=\sqrt{2}+1$ ✓ $y=\sqrt{2}+1$ (4) ✓ $x-1=\frac{2}{x-1}$ ✓ $(x-1)^2=2$ ✓ $x=\sqrt{2}+1$ ✓ $y=\sqrt{2}+1$ (4)
4.7	$T^{/}(3;2)$	✓ $x=3$ ✓ $y=2$ (2) **[14]**

QUESTION/*VRAAG* 3

3.1.1	$-1\ ;\ x\ ;\ 3\ ;\ x+8\ ;\ ...$ $-1 \qquad x \qquad 3 \qquad x+8$ $x+1 \qquad 3-x \qquad x+5$ $-2x+2 \qquad 2x+2$ $-2x+2=2x+2$ $4x=0$ $x=0$	✓ $x+1;3-x$ and $x+5$ ✓ calculating second differences ✓ $-2x+2=2x+2$ ✓ $x=0$ (4)
3.1.2	First differences/*Eerste verskille*: 1 ; 3 ; 5 ; …… $S_n=\frac{n}{2}[2(1)+(n-1)(2)]$ $=n^2$ $250<n^2$ $n>\sqrt{250}$ $\therefore n>15{,}8$ The sum of the 16 first differences will be greater than 250. Therefore the 17th term of the quadratic number pattern is the first satisfying this condition./*Die som van* 16 *eerste verskille sal groter as* 250 *wees. Gevolglik sal die 17de term van die kwadratiese getalpatroon die eerste wees wat aan die voorwaarde voldoen.*	✓ $S_n=n^2$ ✓ $S_n>250$ ✓ $n>15{,}8$ ✓ $n=17$ (4)
3.2.1	$21+21(0{,}85)+21(0{,}85)^2+.........$ $T_n=ar^{n-1}$ $T_{10}=(21)(0{,}85)^9$ $=4{,}86$ cm	✓ $n=10$; $r=0{,}85$ or $\frac{17}{20}$ ✓ substitution into correct formula ✓ answer (3)

3.2.2	$S_n=\frac{a(1-r^n)}{1-r}$ $S_{15}=\frac{21(1-(0{,}85)^{15})}{1-0{,}85}$ $=127{,}77$ Area of the page = 30 x 21 = 630 Percentage of paper covered in grey ink: $=\frac{127{,}77}{630}\times 100\%$ $=20{,}28\%$	✓ $n=15$ ✓ 127,77 ✓ 630 ✓ 20,28 (4) **[15]**

QUESTION/*VRAAG* 4

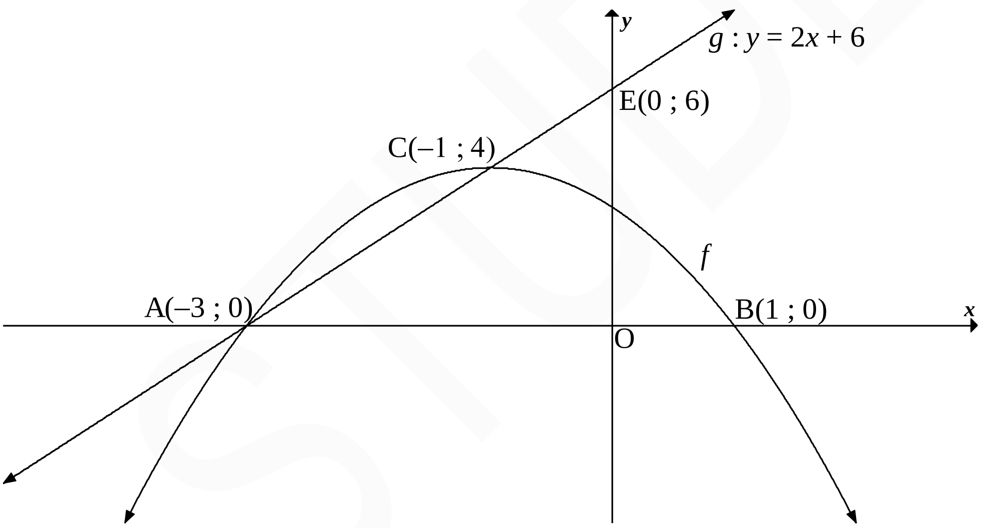
4.1	$y = 0$	✓ $y = 0$ (1)
4.2	R(0 ; 1)	✓ answer (1)
4.3	$y = a^x$ $9 = a^2$ $\therefore a = 3$	✓ substitution ✓ $a = 3$ (2)
4.4	DP $= 2 - b$ $y = 3^x$ $\frac{1}{81} = 3^b$ $3^{-4} = 3^b$ $b = -4$ DP $= 2 - (-4)$ $= 6$ units	✓ $\frac{1}{81} = 3^b$ ✓ 3^{-4} or use of logs ✓ $b = -4$ ✓ DP $= 6$ units (4)
4.5	$h(x+2) + k = 0$ $h(x+2) = -k$ $0 < -k < \frac{1}{81}$ $-\frac{1}{81} < k < 0$	✓✓ $-k < \frac{1}{81}$ or $k > -\frac{1}{81}$ ✓ $-\frac{1}{81} < k < 0$ (3)
		[11]

QUESTION/*VRAAG* 3

3.1	$-1 \quad 2 \quad 9 \quad 20$ $3 \quad 7 \quad 11$ $4 \quad 4$ $2a = 4$ $a = 2$ $3a + b = 3$ $b = -3$ $a + b + c = -1$ $c = 0$ $T_n = 2n^2 - 3n$ **OR/OF** $T_n = T_1 + (n-1)d_1 + \frac{(n-1)(n-2)}{2}d_2$ $= (-1) + (n-1)(3) + \frac{(n-1)(n-2)}{2}(4)$ $= -1 + 3n - 3 + 2n^2 - 6n + 4$ $= 2n^2 - 3n$	✓2[nd] difference = 4 ✓ $a = 2$ ✓ $b = -3$ ✓ $T_n = 2n^2 - 3n$ (4) ✓ formula ✓ 2[nd] difference = 4 ✓simplifying ✓ $T_n = 2n^2 - 3n$ (4)
3.2	$T_n = 2n^2 - 3n$ $T_{48} = 2(48)^2 - 3(48)$ $= 4464$	✓ substitution ✓ answer (2)
3.3	$3 + 7 + 11.......$ $S_n = \frac{n}{2}[2a + (n-1)d]$ $= \frac{n}{2}[2(3) + (n-1)4]$ $= \frac{n}{2}[6 + 4n - 4]$ $= 2n^2 + n$	✓ $a = 3$ ✓ $d = 4$ ✓ substitution into correct formula (3)

3.4	$S_{69} = 9591$ and $T_1 = -1$ (of the original sequence/*van die oorspronklike ry*) $9591 + (-1) = 9590$ $S_{69} + T_1 = 9590$ The 70^{th} term of the original sequence will have a value of 9590 **OR/OF** $2n^2 - 3n = 9590$ $2n^2 - 3n - 9590 = 0$ $(n-70)(2n+137) = 0$ $n = 70$ $T_{70} = 9590$	✓ $(9591)+(-1)$ ✓70 (2) ✓ $2n^2 - 3n - 9590 = 0$ ✓70 (2) **[11]**

QUESTION/*VRAAG* 4



4.1	(0 ; 3)	✓ (0 ; 3) (1)
4.2	$x = -\frac{b}{2a}$ or $-2x - 2 = 0$ $= -\frac{(-2)}{2(-1)}$ $\therefore x = -1$ $= -1$ $y = -(-1)^2 - 2(-1) + 3$ or $y = \frac{4ac - b^2}{4a}$ $= 4$ $= \frac{4(-1)(3)-(-2)^2}{4(-1)}$ $C(-1;4)$	✓ $x = -\frac{(-2)}{2(-1)}$ or $-2x - 2 = 0$ ✓ simplification ✓ in the context of a turning point $-(-1)^2 - 2(-1) + 3$ $\frac{4(-1)(3)-(-2)^2}{4(-1)}$ (3)

4.3	B$(1\,;\,0)$ By symmetry/*Deur simmetrie* A$(-3\,;\,0)$ **OR/OF** $x^2+2x-3=0$ $(x+3)(x-1)=0$ $x=-3$ or $x=1$ A$(-3\,;0)$	✓A$(-3\,;\,0)$ (1) ✓A$(-3\,;\,0)$ (1)
4.4	Equation of g: $m=\dfrac{4-0}{-1+3}$ $=2$ $y=2x+q$ **OR/OF** $y-0=2(x+3)$ $0=2(-3)+q$ or $4=2(-1)+q$ $y=2x+6$ $q=6$ or $y-4=2(x+1)$ $y=2x+6$ E(0 ;6) C(–1 ;4) $\text{CE}=\sqrt{(0+1)^2+(6-4)^2}$ $=\sqrt{5}$ units/2,24 units	✓ $m=2$ ✓subs of A(–3;0) or C (–1;4) ✓ $y=2x+6$ ✓E(0 ;6) ✓substitution into distance formula ✓answer (6)
4.5	$f'(x)=-2x-2.$ But $m_{\tan}=2$ $-2x-2=2$ $x=-2$ $f(-2)=3$ $y=2x+k$ $3=2(-2)+k$ $k=7$ **OR/OF** $-x^2-2x+3=2x+k$ $-x^2-4x+3-k=0$ $x^2+4x-3+k=0$ For equal roots : $\Delta=b^2-4ac=0$ $(-4)^2-4(-1)(3-k)=0$ $(4)^2-4(1)(k-3)=0$ $16+12-4k=0$ or $16-4k+12=0$ $k=7$ $k=7$	✓$-2x-2$ ✓$-2x-2=2$ ✓$x=-2$ ✓ $y=3$ ✓answer (5) ✓$-x^2-2x+3=2x+k$ ✓standard form ✓$b^2-4ac=0$ ✓substitution ✓answer (5)

4.6	g: $y = 2x + 6$ g^{-1}: $x = 2y + 6$ $2y = x - 6$ $y = \frac{x-6}{2}$ or $y = \frac{x}{2} - 3$	✓ $x = 2y + 6$ ✓ $y = \frac{x-6}{2}$ or $y = \frac{x}{2} - 3$ (2)
4.7	$g(x) \geq g^{-1}(x)$ $2x + 6 \geq \frac{x-6}{2}$ $4x + 12 \geq x - 6$ $3x \geq -18$ $x \geq -6$	✓ $2x + 6 \geq \frac{x-6}{2}$ ✓ $4x + 12 \geq x - 6$ ✓ $x \geq -6$ (3) **[21]**

QUESTION/*VRAAG* 5

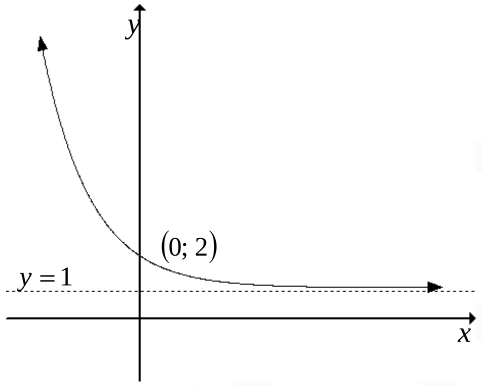
5.1	$r = 2$	✓$r = 2$ (1)
5.2	$g(x) = 2^x + 2$ $g(0) = 2^0 + 2 = 3$ B(0 ; 3) $3 = \frac{3}{0 - p} + 2$ $p = -3$	✓ $g(0) = 2^0 + 2$ ✓ $y = 3$ ✓substitute B(0 ; 3) and $q = 2$ ✓ $p = -3$ (4)
5.3	at A: $x = -3$ $y = 2^{-3} + 2 = 2\frac{1}{8}$ $A\left(-3 ; 2\frac{1}{8}\right)$ or $A\left(-3 ; \frac{17}{8}\right)$ or $A(-3 ; 2{,}125)$	✓ at A : $x = -3$ (p-value) ✓ substitute $x = -3$ into exponential eqution ✓ y-value (3)
5.4	$-3 < x \leq 0$ **OR/ OF** $(-3 ; 0]$	✓ $-3 < x$ ✓ $x \leq 0$ (2)
5.5	$f(x) = \frac{3}{x+3} + 2$ $f(x-2) = \frac{3}{x-2+3} + 2$ $h(x) = \frac{3}{x+1} + 2$	✓substitution of $x - 2$ ✓ $h(x) = \frac{3}{x+1} + 2$ (2) **[12]**

QUESTION/*VRAAG* 3

3.1	$r=\frac{70}{100}$ $=\frac{7}{10}$ $T_n=ar^{n-1}$ $11{,}76=100\left(\frac{7}{10}\right)^{n-1}$ $\left(\frac{7}{10}\right)^{n-1}=\frac{11{,}76}{100}$ $n-1=\log_{\frac{7}{10}}\left(\frac{11{,}76}{100}\right)$ $n-1=6$ $n=7$ During the 7th year/*In die 7de jaar* **OR/*OF*** $r=\frac{70}{100}$ $=\frac{7}{10}$ $T_n=ar^{n-1}$ $11{,}76=100(0{,}7)^{n-1}$ $0{,}7^{n-1}=\frac{11{,}76}{100}$ $=0{,}1176$ $(n-1)\log 0{,}7=\log 0{,}1176$ $n-1=\frac{\log 0{,}1176}{\log 0{,}7}$ $n-1=6$ $n=7$ During the 7th year/*In die 7de jaar*	✓value of r ✓substitution in formula for T_n ✓use of logarithms ✓answer (4) ✓value of r ✓substitution in formula for T_n ✓use of logarithms ✓answer (4)
3.2	$h(n)=130+(100+70+49+\ldots\text{to } n \text{ terms})$ $=130+\frac{100\left(1-(0{,}7)^n\right)}{1-0{,}7}$ $=130+\frac{100\left(1-(0{,}7)^n\right)}{0{,}3}$	✓130 ✓ $100+70+49+\ldots\text{to } n \text{ terms}$ ✓answer (3)

3.3	Eventual height of the tree/*Uiteindelike hoogte van die boom* $= 130 + \frac{100}{1-0,7}$ $= 463,33$ mm OR $\frac{1390}{3}$ mm	✓✓$130 + \frac{100}{1-0,7}$ ✓answer (3) **[10]**

QUESTION/*VRAAG* 4

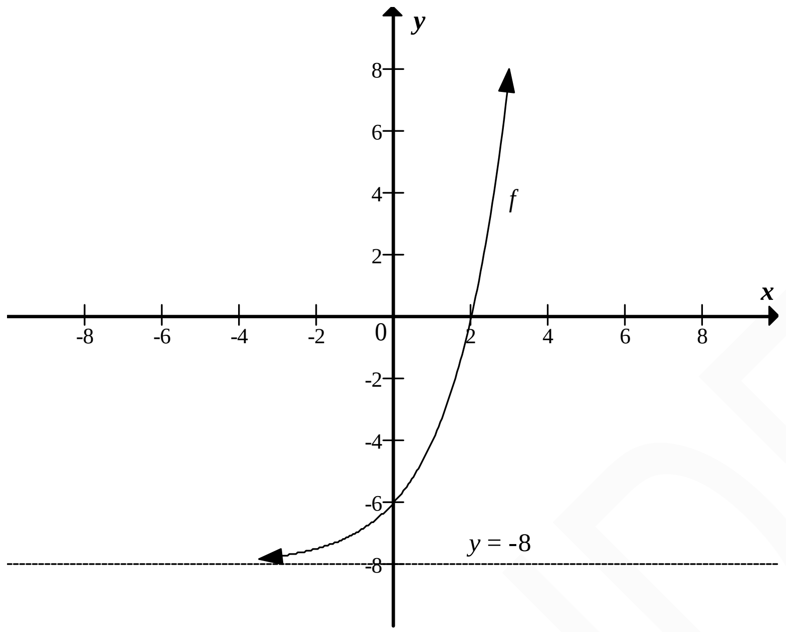
4.1	$(0\ ;2)$	✓answer (1)
4.2	 	✓shape ✓$(0;2)$ ✓asymptote (3)
4.3	$f(-2) = 5$ $f(1) = 2^{-1} + 1 = \frac{3}{2}$ Average gradient $= \frac{f(1) - f(-2)}{1-(-2)}$ $= \frac{\frac{3}{2} - 5}{3}$ $= -\frac{7}{6}$	✓ $f(-2) = 5$ ✓ $f(1) = \frac{3}{2}$ ✓ answer (3)
4.4	Since the asymptote of f is $y = 1$, the asymptote of $h(x) = 3f(x)$ will be $y = 3$. *Omdat die asimptoot van f* $y = 1$ *is,* *sal die asimptoot van* $h(x) = 3f(x)$ $y = 3$ *wees.*	✓answer (1) **[8]**

QUESTION/*VRAAG* 3

3.1	$d = 8$ $T_k = a + (k-1)d$ $= -3 + (k-1)(8)$ $= -3 + 8k - 8$ $= 8k - 11$	✓ d value ✓answer (2)
3.2	$\sum_{k=1}^{n}(8k-11)$ **OR/*OF*** $\sum_{k=0}^{n-1}(8(k+1)-11) = \sum_{k=0}^{n-1}(8k-3)$	✓for general term ✓lower and upper values in sigma notation (2)
3.3	$S_n = \frac{n}{2}[2a + (n-1)d]$ $= \frac{n}{2}[2(-3) + (n-1)(8)]$ $= \frac{n}{2}[-6 + 8n - 8]$ $= \frac{n}{2}[8n - 14]$ $= n(4n-7)$ $= 4n^2 - 7n$ **OR/*OF*** $S_n = \frac{n}{2}[2a + (n-1)d]$ $= \frac{n}{2}[2(-3) + (n-1)(8)]$ $= \frac{n}{2}[-6 + 8n - 8]$ $= \frac{n}{2}[8n - 14]$ $= 4n^2 - 7n$ **OR/*OF*** $S_n = \frac{n}{2}[a + l]$ $= \frac{n}{2}[-3 + 8n - 11]$ $= \frac{n}{2}[8n - 14]$ $= 4n^2 - 7n$	✓formula ✓substitution ✓ $\frac{n}{2}[8n-14]$ (3) ✓formula ✓substitution ✓ $\frac{n}{2}[8n-14]$ (3) ✓formula ✓substitution ✓ $\frac{n}{2}[8n-14]$ (3)

	OR/*OF* S_1 S_2 S_3 S_4 -3 2 15 36 5 13 21 8 8 $S_n = an^2 + bn + c$ $a = \frac{8}{2}$ $a = 4$ $S_1 = 4 + b + c = -3$ $b + c = -7$ ....(1) $S_2 = 16 + 2b + c = 2$ $2b + c = -14$.............(2) $b = -7$................(2) – (1) $c = 0$ Hence $S_n = 4n^2 - 7n$	$S_2 = -3 + 5 = 2$ $S_3 = 2 + 13 = 15$ $S_4 = 15 + 21$ ✓calculates S_1, S_2, S_3 and S_4, ✓ $a = 4$ ✓solves simultaneously for *b* and *c*. (3)
3.4.1	$Q_6 = -6 - 3 + 5 + 13 + 21 + 29$	✓✓ answer (2)
3.4.2	$Q_{129} = -6 + S_{128}$ $= -6 + 4(128)^2 - 7(128)$ $= 64634$ **OR/*OF*** Q_1 Q_2 Q_3 Q_4 -6 -9 -4 9 -3 5 13 8 8 8 $Q_n = an^2 + bn + c$ $a = 4$ $Q_1 = 4 + b + c = -6$ $b + c = -10$ ....(1) $Q_2 = 16 + 2b + c = -9$ $2b + c = -25$...............(2) $b = -15$................(2) – (1) $c = 5$ Hence $Q_n = 4n^2 - 15n + 5$ $Q_{129} = 4(129)^2 - 15(129) + 5$ $= 64\ 634$	✓✓ $-6 + 4(128)^2 - 7(128)$ ✓answer (3) ✓$a = 4$ ✓ $Q_n = 4n^2 - 15n + 5$ ✓answer (3) **[12]**

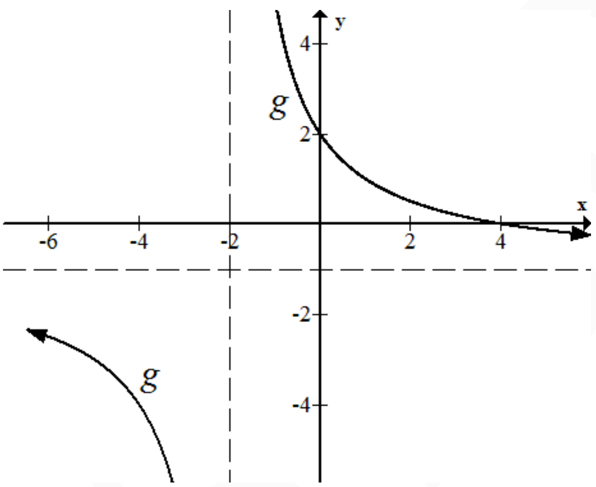
QUESTION/*VRAAG* 4

Given: $f(x)=2^{x+1}-8$		
4.1	$y=-8$	✓ $y=-8$ (1)
4.2		✓x-intercept ✓y-intercept ✓shape ✓asymptote (only if the graph does not cut the asymptote) (4)
4.3	$g(x)=2^{-x+1}-8$ **OR/*OF*** $g(x)=\left(\frac{1}{2}\right)^{x-1}-8$	✓answer (1) ✓answer (1) **[6]**

QUESTION 3

3.1	$r=\dfrac{40,5}{45}=0,9$ $T_{12}=45(0,9)^{12-1}$ $=14,12147682...$ $=14,12$	✓$r = 0,9$ ✓ substitution into correct formula/*substitusie in korrekte formule* ✓ answer/*antwoord* (3)
3.2	$r=0,9$ $-1<0,9<1$	✓answer/*antwoord* (1)
3.3	$S_\infty=\dfrac{45}{1-0,9}$ $S_\infty=450$	✓ substitution/*substitusie* ✓ 450 (2)
3.4	$S_\infty-S_n<1$ $S_\infty-S_n=450-\dfrac{45\left(1-(0,9)^n\right)}{1-0,9}$ $S_\infty-S_n=450-450\left(1-(0,9)^n\right)$ $450(0,9)^n<1$ $(0,9)^n<\dfrac{1}{450}$ $\log(0,9)^n<\log\dfrac{1}{450}$ $n.\log(0,9)<\log\dfrac{1}{450}$ $n>\dfrac{\log\frac{1}{450}}{\log(0,9)}$ $n>57,98...$ Smallest value/*Kleinste waarde:* $n = 58$	✓ $450-\dfrac{45\left(1-(0,9)^n\right)}{1-0,9}$ ✓ $(0,9)^n=\dfrac{1}{450}$ ✓ introducing/*gebruik* logs ✓making n the subject/*maak n die onderwerp* ✓$n = 58$ (5) **[11]**

QUESTION/*VRAAG* 4

4.1	$x = -2$ $y = -1$	✓ $x = -2$ ✓ $y = -1$ (2)
4.2.1	$g(0) = \frac{6}{0+2} - 1$ $= 2$ y-intercept/*afsnit* (0 ; 2)	✓ answer/*antwoord* (1)
4.2.2	$0 = \frac{6}{x+2} - 1$ $1 = \frac{6}{x+2}$ $x + 2 = 6$ $x = 4$ x-intercept/*afsnit* (4 ; 0)	✓ equating to/*stel gelyk aan* 0 ✓ answer/*antwoord* (2)
4.3		✓ asymptotes/*asimptote* ✓ intercepts/*afsnitte* ✓ shape/*vorm* (3)
4.4	$y + 1 = -(x + 2)$ $y = -x - 3$ **OR/*OF*** Using general formula/*Gebruik algemene formule*: $y = -(x + p) + q$ $y = -(x + 2) - 1$ $y = -x - 3$	✓ $m = -1$ ✓ substitution of (–2 ; –1) ✓ answer (3) ✓ formula/*formule* ✓ substitution of p and q values/*substitusie van p- en q-waardes* ✓ answer/*antwoord* (3)
4.5	$x > -2$	✓✓ answer (2) **[13]**

QUESTION/*VRAAG* 3

3.1.1	$-1 \; ; \; -7 \; ; \; -11 \; ; \; p \; ; \ldots$ First differences: $-6 \; ; \; -4 \; ; \; p+11$ Second differences: $2 \; ; \; 2$ $p+11-(-4)=2$ $p+15=2$ $p=-13$	✓$p + 15 = 2$ ✓$p = -13$ (2)
	OR/*OF* $-1 \; ; \; -7 \; ; \; -11 \; ; \; p \; ; \ldots$ First differences: $-6 \; ; \; -4 \; ; \; p+11$ Second differences: $2 \; ; \; 2$ $p+11=-2$ $p=-13$	✓first differences/ *eerste verskille* ✓$p = -13$ (2)
3.1.2	$2a=2$ $a=1$ $3a+b=-6$ $3(1)+b=-6$ $b=-9$ $a+b+c=-1$ $1-9+c=-1$ $c=7$ $T_n=n^2-9n+7$	✓$a=1$ ✓$b=-9$ ✓$c=7$ ✓answer/*antwoord* (4)
	OR/*OF* $T_n=T_1+(n-1)d_1+\frac{(n-1)(n-2)d_2}{2}$ $=-1+(n-1)(-6)+\frac{(n-1)(n-2)(2)}{2}$ $=-1-6n+6+\frac{2n^2-6n+4}{2}$ $=n^2-9n+7$	✓formula/*formule* ✓substitution of first and second differences/*substitusie van eerste en tweede verskille* ✓simplification/*vereenvoudiging* ✓answer/*antwoord* (4)

	OR/*OF* $7; -1 \;;\; -7 \;;\; -11 \;;\; p \;;\; \ldots$ $-8 \qquad -6 \qquad -4 \qquad p+11$ $2 \qquad 2 \qquad 2$ $T_0 = 7 = c$ $2a = 2 \quad \therefore a = 1$ $3a + b = -6 \quad \therefore b = -9$ $T_n = n^2 - 9n + 7$ **OR/OF** $a = \frac{1}{2}(2) = 1$ $\therefore T_n = n^2 + bn + c$ $T_1 = -1 \;\; \therefore 1 + b + c = -1 \quad ....(1)$ $T_2 = -7 \;\; \therefore 4 + 2b + c = -7 \; ....(2)$ $(2)-(1): \quad 3 + b = -6$ $\therefore b = -9$ sub in $(1): c = 7$ $\therefore T_n = n^2 - 9n + 7$	✓*c*-value/*c-waarde* ✓*a*-value/*a-waarde* ✓*b*-value/*b-waarde* ✓answer/*antwoord* (4) ✓*a*-value/*a-waarde* ✓*b*-value/*b-waarde* ✓*c*-value/*c-waarde* ✓answer/*antwoord* (4)

3.1.3	The sequence of first differences is/*Die reeks van eerste verskille is*: $-6\ ;-4\ ;-2\ ;0\ ;\ ...$ $-6+(n-1)(2)=96$ $n=52$ $\therefore$ two terms are/*twee terme is*: $T_{52}=52^2-9(52)+7=2243$ $T_{53}=53^2-9(53)+7=2339$	✓$-6+(n-1)(2)=96$ ✓52 ✓2 243 ✓2 339 (4)
	OR/OF The sequence of first differences is/*Die reeks van eerste verskille is*: $-6\ ;-4\ ;-2\ ;0\ ;\ ...$ The formula for the sequence of first differences/*Die formule vir die reeks van eerste verskille* is $T_n=2n-8$ 1st difference/*1ste verskil*: $2n-8=96$ $2n=104$ $n=52$ $\therefore$ two terms are/*twee terme is*: $T_{52}=52^2-9(52)+7=2243$ $T_{53}=53^2-9(53)+7=2339$	✓$2n-8=96$ ✓52 ✓2 243 ✓2 339 (4)
	OR/*OF* $T_n-T_{n-1}=96$ $\left(n^2-9n+7\right)-\left[(n-1)^2-9(n-1)+7\right]=96$ $n^2-9n+7-n^2+2n-1+9n-9-7=96$ $2n=106$ $n=53$ $T_{52}=52^2-9(52)+7=2243$ $T_{53}=53^2-9(53)+7=2339$	✓$T_n-T_{n-1}=96$ ✓53 ✓2 243 ✓2 339 (4)
	OR/OF $T_{n+1}-T_n=96$ $\left[(n+1)^2-9(n+1)+7\right]-\left[n^2-9n+7\right]=96$ $n^2+2n+1-9n-9+7-n^2+9n-7=96$ $2n=104$ $n=52$ $T_{52}=52^2-9(52)+7=2243$ $T_{53}=53^2-9(53)+7=2339$	✓$T_{n+1}-T_n=96$ ✓52 ✓2 243 ✓2 339 (4)

3.2.1	$T_{12} = 16\left(\frac{1}{4}\right)^{12-1}$ $= \frac{1}{4^9}$ or 4^{-9} or $\frac{1}{2^{18}}$ or 2^{-18}	✓ $a = 16$ and $r = \frac{1}{4}$ ✓subst. into correct formula/ *subt in korrekte formule* ✓ answer/*antwoord* (3)
3.2.2	$S_{10} = \frac{16\left(1-\left(\frac{1}{4}\right)^{10}\right)}{1-\frac{1}{4}}$ $= 21{,}33$ **OR/OF** $S_{10} = \frac{16\left(\left(\frac{1}{4}\right)^{10}-1\right)}{\frac{1}{4}-1}$ $= 21{,}33$	✓ substitution into correct formula /*substitusie in korrekte formule* ✓ answer/*antwoord* (2) ✓ substitution into correct formula /*substitusie in korrekte formule* ✓ answer/*antwoord* (2)
3.3	$\left(1+\frac{1}{2}\right)\left(1+\frac{1}{3}\right)\left(1+\frac{1}{4}\right)...\left(1+\frac{1}{99}\right)$ $= \left(\frac{\cancel{3}}{2}\right)\left(\frac{\cancel{4}}{\cancel{3}}\right)\left(\frac{\cancel{5}}{\cancel{4}}\right)\left(\frac{\cancel{6}}{\cancel{5}}\right)...\left(\frac{100}{\cancel{99}}\right)$ $= \left(\frac{100}{2}\right)$ $= 50$ **OR/OF** $\left(1+\frac{1}{2}\right)\left(1+\frac{1}{3}\right)\left(1+\frac{1}{4}\right)...\left(1+\frac{1}{99}\right)$ $T_1 = \left(1+\frac{1}{2}\right) = \frac{3}{2}$ $T_2 = \frac{3}{2}\left(1+\frac{1}{3}\right) = \frac{3}{2}\times\frac{4}{3} = 2$ $T_3 = 2\left(1+\frac{1}{4}\right) = 2\times\frac{5}{4} = \frac{5}{2}$ $\frac{3}{2}, 2, \frac{5}{2}$ ... is an arithmetic sequence with $a = \frac{3}{2}$ and $d = \frac{1}{2}$ $\therefore T_{98} = \frac{3}{2} + (98-1)\frac{1}{2}$ $= \frac{100}{2} = 50$	✓ improper fractions/ *onegte breuke* ✓ $\left(1+\frac{1}{99}\right)$ or $\left(\frac{100}{99}\right)$ ✓✓ answer/*antwoord* (4) ✓$\left(1+\frac{1}{99}\right)$ ✓ giving the first three terms / *gee die eerste drie terme* ✓✓answer /*antwoord* (4) **[19]**

QUESTION/*VRAAG* 4

4.1	$p=1$ $q=1$	✓p value /*waarde* ✓q value /*waarde* (2)
4.2	$0=\frac{2}{x+1}+1$ $-x-1=2$ $x=-3$ **OR/*OF*** Reflect (0 ; 3) across $y = -x$ to get T(–3 ; 0) $x=-3$ *Reflekteer* (0 ; 3) *om* $y = -1$ *om* T(–3 ; 0) *te kry* $x=-3$	✓ $0=\frac{2}{x+1}+1$ ✓ $x=-3$ (2) ✓ reflect across/*reflekteer om* $y = -x$ ✓ $x=-3$ (2)
4.3	Shifting g five units to the left shifts (– 1 ; 0) five units to the left. $x = -6$	✓ answer/*antwoord* (1)
4.4	$\frac{2}{x+1}+1=x$ $2+x+1=x^2+x$ $x^2=3$ $\therefore x=\sqrt{3}$ since at S, $x>0$ $y=\sqrt{3}=1{,}73...$ $\text{OS}^2=x^2+y^2=3+3=6$ $\therefore \text{OS}=\sqrt{6}=2{,}45$ units/*eenhede* **OR/*OF***	✓ equating both graphs/*stel grafieke gelyk* ✓ $x^2=3$ ✓ $x=\sqrt{3}$ and $y=\sqrt{3}$ ✓ $\text{OS}^2=6$ ✓ answer/*antwoord* (5)

	Translate *g* one unit down and one unit to the right/*Transleer g een eenheid af en een eenheid na regs* The new equation/*Die nuwe vergelyking* : $p(x)=\frac{2}{x}$ Therefore the image of S is $S^{/}\left(\sqrt{2};\sqrt{2}\right)$/ *Daarom is die beeld van S nou* $S^{/}\left(\sqrt{2};\sqrt{2}\right)$ Now translate *p* back to *g/Transleer p terug na g*: $S\left(\sqrt{2}-1;\sqrt{2}+1\right)$ $OS^2=\left(\sqrt{2}-1\right)^2+\left(\sqrt{2}+1\right)^2=2-2\sqrt{2}+1+2+2\sqrt{2}+1$ $\therefore OS=\sqrt{6}=2{,}45$ units/*eenhede*	✓ $p(x)=\frac{2}{x}$ ✓✓coord. of/*koörd. van* $S^{/}$ ✓coord. of/*koörd. van* S ✓answer/*antwoord* (5)
4.5	*k* < 3 will give roots with opposite signs/ *k* < 3 *sal wortels met teenoorgestelde tekens gee*	✓*k* < 3 (1) **[11]**

QUESTION/*VRAAG* 3

3.1.1	$w-3;\ 2w-4;\ 23-w$ $(2w-4)-(w-3)=(23-w)-(2w-4)$ $w-1=27-3w$ $4w=28$ $w=7$	✓ $(2w-4)-(w-3)$ $=(23-w)-(2w-4)$ ✓ $w=7$ (2)
3.1.2	Sequence is: 4 ; 10 ; 16 First difference / *Eerste verskil* = 6 **OR** $d=w-1$ $=6$	✓ answer (1) ✓ answer (1)
3.2	$T_{50}=3+(4+10+16+\ldots \text{ to } 49 \text{ terms})$ $T_{50}=3+\frac{49}{2}[2(4)+(49-1)(6)]$ $=3+7252$ $=7255$ **OR** $2a=6$ $a=3$ $3a+b=4$ $3(3)+b=4$ $b=-5$ $a+b+c=3$ $3-5+c=3$ $c=5$ $T_n=3n^2-5n+5$ $T_{50}=3(50)^2-5(50)+5$ $=7255$	✓ $T_{50}=3+$ sum of 49 linear terms ✓ $a=4$ ✓ $n=49$ ✓ 7252(sum of 49 terms) ✓ answer (5) ✓ $a=3$ ✓ $b=-5$ ✓ $c=5$ ✓ substitution 50 ✓ answer (5) **[8]**

QUESTION/*VRAAG* 4

4.1	$S_n = p\left(1-\left(\frac{1}{2}\right)^n\right)$ $a = p\left[1-\left(\frac{1}{2}\right)^1\right]$ $= \frac{p}{2}$ $r = \frac{1}{2}$ $\therefore 10 = \frac{\frac{p}{2}}{1-\frac{1}{2}}$ $5 = \frac{p}{2}$ $p = 10$ **OR** $\left(\frac{1}{2}\right)^n \to 0$ as $n \to \infty$ $\therefore S_\infty = p$ $p = 10$	✓ $a = \frac{p}{2}$ ✓ $r = \frac{1}{2}$ ✓ substitute in correct formula ✓ answer (4)
4.2	$r = \frac{1}{2}$ $\frac{a}{1-\frac{1}{2}} = 10$ $a = 5$ $T_2 = ar = \frac{5}{2}$ **OR**	✓ $r = \frac{1}{2}$ ✓ substitution ✓ $a = 5$ ✓ answer

	$S_n = 10 - 10.2^{-n}$ $a = T_1$ $= S_1$ $= 10 - 10.2^{-1}$ $= 10 - \frac{10}{2}$ $= 5$ $T_2 = S_2 - T_1$ $= 10 - 10.2^{-2} - 5$ $= 10 - \frac{10}{4} - 5$ $= \frac{5}{2}$	✓ $S_1 = 5$ ✓ $a = 5$ ✓ $T_2 = S_2 - T_1$ ✓ answer (4)
	OR $T_2 = S_2 - S_1$ $= p\left(1 - \left(\frac{1}{2}\right)^2\right) - p\left(1 - \frac{1}{2}\right)$ $= \frac{p}{4}$ $= \frac{10}{4}$ $= \frac{5}{2}$	✓ $T_2 = S_2 - S_1$ ✓ substitution ✓ $\frac{p}{4}$ ✓ answer (4) **[8]**

QUESTION/*VRAAG* 3

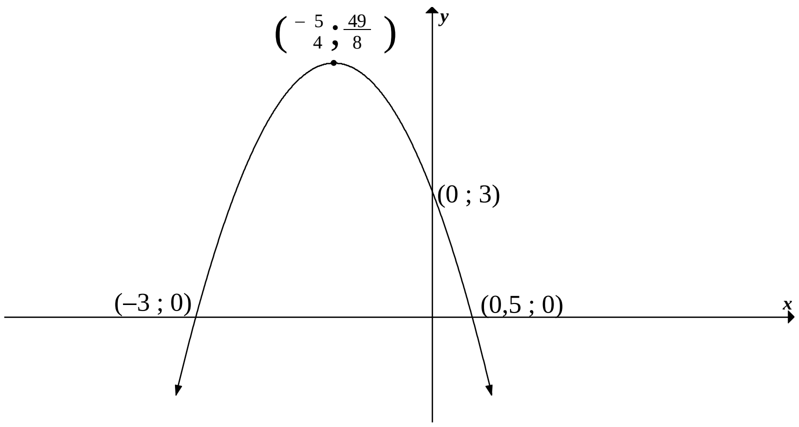
3.1.1	$T_n = 4n - 7$ **OR** $T_n = -3 + (n-1)(4)$	✓4n ✓ –7 (2) ✓ -3 ✓ $(n-1)(4)$ (2)
3.1.2	$T_4 = 9$ $T_5 = 13$ $T_6 = 17$ $T_7 = 21$	✓ any TWO consecutive answers correct ✓ last TWO answers correct (2)
3.1.3	0 ; 1 ; 2 ; 0 ; 1 ; 2 ; 0	2 marks for all 7 correct **OR** 1 mark for only **first** / **last** 3 correct **OR** 0 marks if less than 3 correct (2)
3.1.4	Multiples of 3 in the pattern are : $-3 ; 9 ; 21$ $T_n = -3 + 12(n-1)$ $T_n = 12n - 15$ $393 = 12n - 15$ $12n = 408$ $n = 34$ or $T_n = a + (n-1)d$ $393 = -3 + (n-1)(12)$ $393 = 12n - 15$ $12n = 408$ $n = 34$ $S_n = \frac{n}{2}[a + L]$ $S_{34} = \frac{34}{2}[-3 + 393]$ $S_{34} = 6630$ or $S_n = \frac{n}{2}[2a + (n-1)d]$ $S_{34} = \frac{34}{2}[2(-3) + 33(12)]$ $S_{34} = 6630$ **NOTE:** • If the candidate does not show the working to get to *n* = 34: no penalty • If a candidate sums the whole sequence: 0/5 marks • Answer only: max 1/5 marks	✓ $12n - 15$ ✓ $393 = 12n - 15$ ✓ $n = 34$ ✓ subs *a* = – 3 and *d* = 12 into correct formula ✓ $S_{34} = 6630$ (5)
3.2.1	T_1 T_2 T_3 T_4 T_5 1 5 12 22 35 4 7 10 13 3 3 3 $T_5 = 35$	✓✓ answer (2)

	OR The sequence is 1, 5, 12, 22, 35. Therefore $T_5 = 35$ **OR** $T_5 = 22 + 13 = 35$	✓✓ answer (2) ✓✓ answer (2)
3.2.2	$T_{50} = T_1 + \frac{49}{2}[2(4) + 48(3)]$ $= 1 + 3724$ $= 3725$ **NOTE:** • Answer only: max 1 mark • If the candidate calculates the general formula in 3.2.1, they can be awarded 5/5 marks in 3.2.2 **OR** $2a = 3$ $a = \frac{3}{2}$ $3\left(\frac{3}{2}\right) + b = 4$ $b = -\frac{1}{2}$ $\left(\frac{3}{2}\right) + \left(-\frac{1}{2}\right) + c = 1$ $c = 0$ $T_n = \frac{3}{2}n^2 - \frac{1}{2}n$ $T_{50} = \frac{3}{2}(50)^2 - \frac{1}{2}(50)$ $= 3725$ **OR** $T_1 = 1$ $T_2 - T_1 = 4$ $T_3 - T_2 = 7$ $T_4 - T_3 = 10$ ... $T_{50} - T_{49} = ?$ Add both sides $T_{50} = 1 + 4 + 7 + 10 + ...$ to 50 terms $= \frac{50}{2}(2 + 49(3))$ $= 3725$	✓ $a = 4$ ✓ $d = 3$ ✓ $n = 49$ ✓ substitution into correct formula ✓ answer (5) ✓ $a = \frac{3}{2}$ ✓ $b = -\frac{1}{2}$ ✓ $c = 0$ ✓ subs $n = 50$ ✓ answer (5) ✓✓ expansion ✓ $T_{50} = 1 + 4 + 7 + 10 + ...$ to 50 terms ✓ subs into correct formula ✓ answer (5) **[18]**

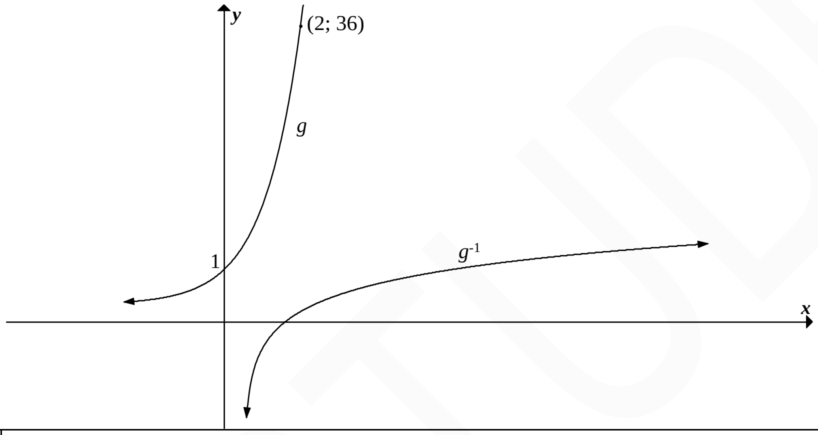
QUESTION/*VRAAG* 4

Given: $f(x) = -2x^2 - 5x + 3$

4.1	$(0\ ;\ 3)$ **OR** $x = 0$ and $y = 3$ • If there is evidence that the candidate has indicated that $x = 0$ and $y = 3$: 1 mark • If candidate just states $y = 3$: 1 mark	✓ answer in coordinate form (1) ✓ both values correct (1)
4.2	$0 = -2x^2 - 5x + 3$ $0 = 2x^2 + 5x - 3$ $0 = (2x - 1)(x + 3)$ $x = \frac{1}{2}$ or -3	✓$y = 0$ ✓factors ✓x-values (3)
4.3	$f(x) = -2x^2 - 5x + 3$ $x = -\frac{b}{2a}$ $= -\frac{(-5)}{2(-2)}$ $= -\frac{5}{4}$ or $-1\frac{1}{4}$ or $f'(x) = 0$ $-4x - 5 = 0$ $x = -\frac{5}{4}$ or $-1\frac{1}{4}$ or $\frac{\frac{1}{2} - 3}{2}$ $= \frac{-2{,}5}{2}$ $= -\frac{5}{4}$ $y = f\left(-\frac{5}{4}\right)$ $= -2\left(-\frac{5}{4}\right)^2 - 5\left(-\frac{5}{4}\right) + 3$ $= \frac{49}{8}$ or $6\frac{1}{8}$ or Hence the turning point of f is $\left(-\frac{5}{4}\ ;\ \frac{49}{8}\right)$ or $(-1{,}25\ ;\ 6{,}13)$ **OR** $f(x) = -2x^2 - 5x + 3$ $= -2\left(x^2 + \frac{5x}{2} - \frac{3}{2}\right)$ $= -2\left(x + \frac{5}{4}\right)^2 + \frac{25}{8} + \frac{24}{8}$ $= -2\left(x + \frac{5}{4}\right)^2 + \frac{49}{8}$ Hence the turning point of f is $\left(-\frac{5}{4}\ ;\ \frac{49}{8}\right)$	✓ $x = -\frac{(-5)}{2(-2)}$ or $f'(x) = 0$ or $\frac{\frac{1}{2} - 3}{2}$ ✓x-coordinate ✓ y-coordinate (3) ✓ $-2\left(x + \frac{5}{4}\right)^2 + \frac{49}{8}$ ✓x-coordinate ✓ y-coordinate (3)

4.4		✓shape ✓intercepts with the axes ✓ turning point (3) **[10]**

QUESTION/*VRAAG* 5

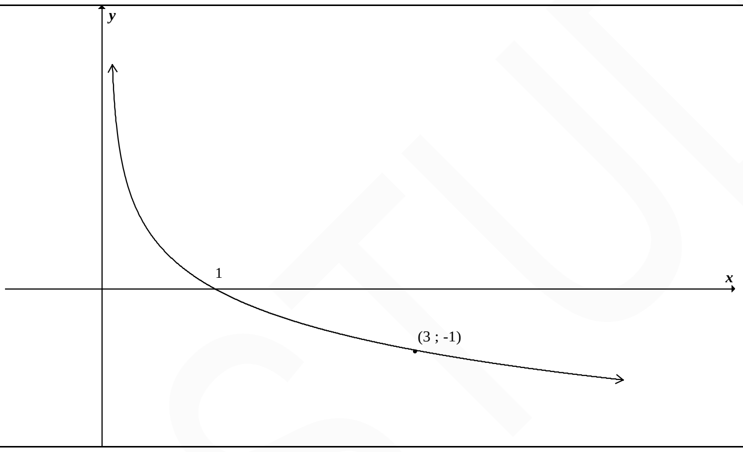
Given: $g(x) = k^x$ and $y = g^{-1}(x)$ 		
5.1.1	$k^2 = 36$ $k = 6$	✓ $k^2 = 36$ ✓answer (2)
5.1.2	g: $y = 6^x$ g^{-1}: $x = 6^y$ $y = \log_6 x$ or $y = \frac{\log x}{\log 6}$ **NOTE:** Answer only : 2/2 marks	✓ $x = 6^y$ ✓ $y = \log_6 x$ or $y = \frac{\log x}{\log 6}$ (2)
5.1.3	$0 < x \leq 1$ **OR** $(0;1]$	✓ $0 < x$ ✓ $x \leq 1$ (2)
5.1.4	$x > 3$ **OR** $(3;\infty)$	✓ answer (1)

QUESTION 3

3.1	Jacob calculated that the sequence is geometric or exponential. Vusi calculated that the sequence is quadratic. **OR** Jacob has multiplied each term by 3 to get the next term. Vusi sees it as a sequence with a constant second difference. **OR** Jacob calculated that the sequence is geometric or exponential. Vusi calculated that the sequence can be seen as a combination of exponential and cubic sequences.	✓Jacob (geometric/exponential) ✓ Vusi (quadratic) (2) ✓ Jacob (multiplied each term by 3) ✓ Vusi (constant second difference) (2) ✓Jacob (geometric/exponential) ✓ Vusi (exponential and cubic combined) (2)
3.2.1	$T_n = 3^n$ **OR** $T_n = 3.3^{n-1}$	✓answer (1) ✓answer (1)
3.2.2	3 9 27 57 6 18 30 12 12 $2a = 12$ $3a + b = 6$ $a + b + c = 3$ $a = 6$ $18 + b = 6$ $6 - 12 + c = 3$ $b = -12$ $c = 9$ $T_n = 6n^2 - 12n + 9$ **OR** $2a = 12$ $a = 6$ $T_0 = c = 9$ $T_n = an^2 + bn + 9$ $3 = 6(1)^2 + b(1) + 9$ $b = -12$ $T_n = 6n^2 - 12n + 9$ **OR**	✓ $a = 6$ ✓ method ✓ $b = -12$ ✓ $c = 9$ (4) ✓ $a = 6$ ✓ $c = 9$ ✓ method ✓ $b = -12$ (4)

	$2a = 12$ $a = 6$ $T_n = 6n^2 + bn + c$ $T_1 = 3 = 6(1)^2 + b(1) + c$ i.e. $3 = 6 + b + c$ $T_2 = 9 = 6(2)^2 + b(2) + c$ i.e. $9 = 24 + 2b + c$ $6 = 18 + b$ $b = -12$ $c = 9$ $T_n = 6n^2 - 12n + 9$	✓ $a = 6$ ✓ method ✓ $b = -12$ ✓ $c = 9$ (4)
	OR $T_n = 3^n + k(n-1)(n-2)(n-3)$ $57 = 3^4 + k(3)(2)(1)$ $6k = -24$ $k = -4$ $T_n = 3^n - 4(n-1)(n-2)(n-3)$	✓✓ $T_n = 3^n + k(n-1)(n-2)(n-3)$ ✓ substitution ✓ answer (4) **[7]**

QUESTION 4

4.1	**R** **OR** $(-\infty;\infty)$	✓answer (1)
4.2	$y=0$	✓ $y=0$ (1)
4.3	$x=\left(\frac{1}{3}\right)^{y}$ $y=\log_{\frac{1}{3}} x$ **OR** $x=\left(\frac{1}{3}\right)^{y}$ $x=3^{-y}$ $-y=\log_3 x$ $y=-\log_3 x$	✓ $x=\left(\frac{1}{3}\right)^{y}$ ✓ $y=\log_{\frac{1}{3}} x$ (2) ✓ $x=\left(\frac{1}{3}\right)^{y}$ ✓ $y=-\log_3 x$ (2)
4.4		✓ shape ✓intercept at (1 ; 0) ✓any other correct point (3)
4.5	$x=-2$	✓✓ $x=-2$ (2)
4.6	$\text{LHS}=[f(x)]^2-[f(-x)]^2$ $=\left[\left(\frac{1}{3}\right)^{x}\right]^2-\left[\left(\frac{1}{3}\right)^{-x}\right]^2$ $=3^{-2x}-3^{2x}$ $\text{RHS}=f(2x)-f(-2x)$ $=\left(\frac{1}{3}\right)^{2x}-\left(\frac{1}{3}\right)^{-2x}$ $=3^{-2x}-3^{2x}$ $\therefore$ LHS = RHS $[f(x)]^2-[f(-x)]^2=f(2x)-f(-2x)$	✓ $\left[\left(\frac{1}{3}\right)^{x}\right]^2-\left[\left(\frac{1}{3}\right)^{-x}\right]$ ✓ $3^{-2x}-3^{2x}$ ✓ $\left(\frac{1}{3}\right)^{2x}-\left(\frac{1}{3}\right)^{-2x}$ (3) **[12]**

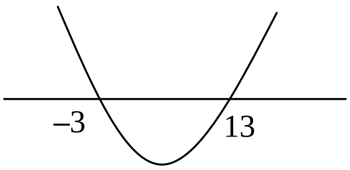
	OR $S_{11} = -110$ n: 12, 13, 14, 15, 16, 17, 18, 19, 20 T_n: –34, –38, –42, –46, –50, –54, –58, –62, –66 S_n: –144, –182, –224, –270, –320, –374, –432, –494, –560 $\therefore n = 20$	✓ $S_{11} = -110$ ✓ sequence expanded ✓✓ series calculated ✓✓ answer (6) **[10]**

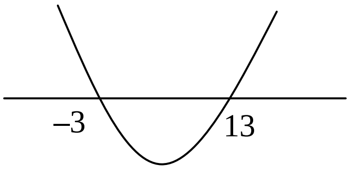
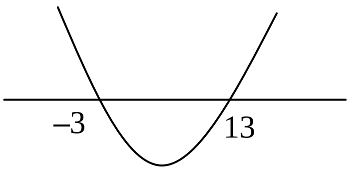
n	12	13	14	15	16	17	18	19	20
T_n	–34	–38	–42	–46	–50	–54	–58	–62	–66
S_n	–144	–182	–224	–270	–320	–374	–432	–494	–560

QUESTION 3

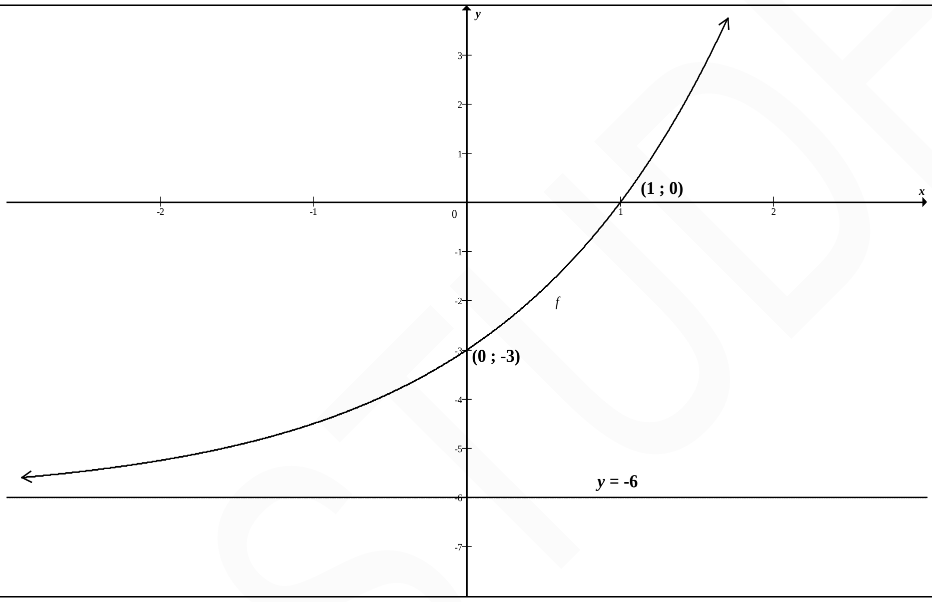
3.1.1	$T_n = ar^{n-1}$ $= 27\left(\frac{1}{3}\right)^{n-1}$ **Note:** The final answer can also be written as 3^{4-n} **or** $\left(\frac{1}{3}\right)^{n-4}$	✓ $a = 27$ and $r = \frac{1}{3}$ ✓ substitute into correct formula (2)
3.1.2	$-1 < r < 1$ or $\lvert r \rvert < 1$ **Note:** If candidate concludes series is not convergent, award 0 marks. **OR** The common ratio (r) is $\frac{1}{3}$ which is between –1 and 1. **OR** $-1 < \frac{1}{3} < 1$	✓ answer (1) ✓ answer (1) ✓ answer (1)
3.1.3	$S_\infty = \frac{a}{1-r}$ $= \frac{27}{1-\frac{1}{3}}$ $= \frac{81}{2}$ or 40,5 or 41 **Note:** If $r > 1$ or $r < -1$ is substituted then 0/2 marks.	✓ substitution ✓ answer (2)

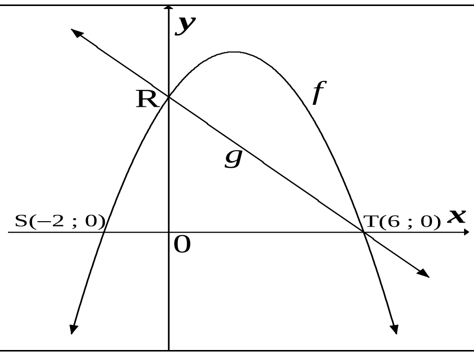
3.2	Let V be the volume of the first tank. $\frac{V}{2};\frac{V}{4};\frac{V}{8}.......$ $S_{19}=\frac{\frac{V}{2}\left[1-\left(\frac{1}{2}\right)^{19}\right]}{1-\frac{1}{2}}$ $=\frac{524287}{524288}V$ $=0,9999980927\ V$ $<V$ Yes, the water will fill the first tank without spilling over. **Note:** If candidate lets the volume of the first tank be a specific value (instead of a variable) and his/her argument follows correctly, award 4/4 marks **Note:** If candidate answers 'Yes' only with no justification: 1/4 marks	✓ $\frac{V}{2}$ ✓ substitute into correct formula ✓ answer ✓ conclusion (4)
	OR Let V be the volume of the first tank. $\frac{V}{2};\frac{V}{4};\frac{V}{8}.......$ $S_{19}=\frac{\frac{V}{2}\left[1-\left(\frac{1}{2}\right)^{19}\right]}{1-\frac{1}{2}}$ $=V\left[1-\left(\frac{1}{2}\right)^{19}\right]$ $<V\cdot 1$ $=V$ Yes, the water will fill the first tank without spilling over.	✓ $\frac{V}{2}$ ✓ substitute into correct formula ✓ observes that $\left[1-\left(\frac{1}{2}\right)^{19}\right]<1$ ✓ conclusion (4)
	OR Let V be the volume of the first tank. $\frac{V}{2};\frac{V}{4};\frac{V}{8}.......$ $S_{\infty}=\frac{\frac{V}{2}}{1-\frac{1}{2}}$ $=V$ Since the first tank will hold the water from infinitely many tanks without spilling over, certainly: Yes, the first tank will hold the water from the other 19 tanks without spilling over.	✓ $\frac{V}{2}$ ✓ substitute into correct formula ✓✓ correct argument (4)

	OR If the tanks are emptied one by one, starting from the second, each tank **will fill only half the remaining space**, so the first tank can hold all the water from the other 19 tanks.	✓ Yes (explicit or understood from the argument.) ✓✓✓ argument (4)
3.3.1	$T_n = -2(n-5)^2 + 18$ Term 1 = – 14 Term 2 = 0 Term 3 = 10	✓ – 14 ✓ 0 ✓ 10 (3)
3.3.2	Term 5 **OR** $n = 5$ **OR** T_5	✓ answer (1)
3.3.3	Second difference = $2a$ Second difference = $2(-2)$ Second difference = -4 **OR** $\begin{array}{ccccc} -14 & & 0 & & 10 \\ & 14 & & 10 & \\ & & -4 & & \end{array}$ Second difference = -4	✓ subs – 2 into $2a$ ✓ answer (2) ✓ first differences ✓second difference (2)
3.3.4	**Note:** Answer only award 2/6 marks $-2(n-5)^2 + 18 < -110$ $-2(n-5)^2 + 128 < 0$ $-2n^2 + 20n - 50 + 128 < 0$ $-2n^2 + 20n + 78 < 0$ $n^2 - 10n - 39 > 0$ $(n-13)(n+3) > 0$ $\begin{array}{ccccc} + & 0 & - & 0 & + \\ \hline & -3 & & 13 & \end{array}$  $n < -3$ or $n > 13$ $n \geq 14$; $n \in \mathrm{N}$ **OR** $n > 13$; $n \in \mathrm{N}$ **OR**	✓ $T_n < -110$ ✓standard form ✓factors ✓ critical values ✓ inequalities ✓ $n > 13$ (accept: $n \geq 14$) (6)

	$-2(n-5)^2+18<-110$ $-2(n-5)^2+128<0$ $(n-5)^2-64>0$ $[(n-5)-8][(n-5)+8]>0$ $(n-13)(n+3)>0$ + 0 − 0 + -3 13  $n<-3$ or $n>13$ $n\geq 14$; $n\in N$ **OR** $n>13$; $n\in N$	✓ $T_n<-110$ ✓ $(n-5)^2-64>0$ ✓factors ✓ critical values ✓ inequalities ✓ $n>13$ (accept: $n\geq 14$) (6)
	OR $-2(n-5)^2+18<-110$ $-2(n-5)^2<-128$ $(n-5)^2>64$ $n-5<-8$ or $n-5>8$ $n<-3$ or $n>13$ $n\geq 14$; $n\in N$ **OR** $n>13$; $n\in N$	✓ $T_n<-110$ ✓ $2(n-5)^2>128$ ✓ 8 and – 8 ✓ $n-5>8$ ✓ $n-5<-8$ ✓ $n>13$ (accept: $n\geq 14$) (6)
	OR $T_n=-2(n-5)^2+18$ $T_n=-2n^2+20n-32$ $-2n^2+20n-32<-110$ $-2n^2+20n-78<0$ $n^2-10n-39>0$ $(n-13)(n+3)>0$ + 0 − 0 + -3 13  $n<-3$ or $n>13$ $n\geq 14$; $n\in N$ **OR** $n>13$; $n\in N$	✓ $T_n<-110$ ✓standard form ✓factors ✓ critical values ✓ inequalities ✓ $n>13$ (accept: $n\geq 14$) (6)
	OR –14 ; 0 ; 10 ; 16 ; 18 ; 16 ; 10 ; 0 ; –14 ; –32 ; –54 ; –80 ; –110 $n\geq 14$; $n\in N$	✓✓✓✓ expansion ✓✓ conclusion of $n\geq 14$ (accept $n>13$) (6) **[21]**

QUESTION 4

4.1.1	$y = 3.2^0 - 6$ $y = 3 - 6$ $y = -3 \qquad (0\,;-3)$	✓ answer (1)
4.1.2	$0 = 3.2^x - 6$ $3.2^x = 6$ $2^x = 2^1$ $x = 1 \qquad (1\,;0)$ **Note:** If a candidate interchanges question 4.1.1 and 4.1.2: 0/3 marks **Note:** If a candidate says that $3.2^x = 6^x$ (i.e. wrong mathematics) s/he will arrive at correct answer BUT award max 1/2	✓ $y = 0$ ✓ x-value (2)
4.1.3		✓intercepts ✓ asymptote ✓ shape (3)
4.1.4	$y > -6 \qquad$ OR $\qquad (-6\,;\infty)$	✓ answer (1)

4.2		
4.2.1	$y = -2x + d$ $0 = (-2)(6) + d$ $d = 12$ **OR** $y - y_1 = m(x - x_1)$ $y - 0 = -2(x - 6)$ $y = -2x + 12$ $\therefore d = 12$ **OR** Since $m = -2$ and $m = \dfrac{-d}{6}$ $-2 = \dfrac{-d}{6}$ $d = 12$	✓substitution ✓ answer (2) ✓substitution ✓ answer (2) ✓substitution ✓ answer (2)
4.2.2	$y = a(x-6)(x+2)$ $12 = a(0-6)(0+2)$ $a = -1$ $y = -(x^2 - 4x - 12)$ $= -x^2 + 4x + 12$ **Note:** No marks for answer only. **OR** $y = ax^2 + bx + 12$ $0 = a(-2)^2 + b(-2) + 12$ i.e. $0 = 4a - 2b + 12$ $0 = a(6)^2 + b(6) + 12$ i.e. $0 = 36a + 6b + 12$ $0 = 24b - 96$ $b = 4$ $0 = 4a - 2(4) + 12$ $a = -1$ $y = -x^2 + 4x + 12$	✓ $y = a(x-6)(x+2)$ ✓ subs R(0 ; 12) ✓ a-value ✓ $y = -x^2 + 4x + 12$ (4) ✓ $y = ax^2 + bx + 12$ ✓subs $S(-2;0)$ and $T(6;0)$ ✓ b-value ✓ $y = -x^2 + 4x + 12$ (4)

	OR $y = a(x-2)^2 + q$ $0 = a(-2-2)^2 + q$ or $0 = a(6-2)^2 + q$ i.e. $0 = 16a + q$ $12 = a(0-2)^2 + q$ i.e. $12 = 4a + q$ $12 = -12a$ $a = -1$ $q = 16$ $y = -(x-2)^2 + 16$ $= -(x^2 - 4x + 4) + 16$ $= -x^2 + 4x + 12$	✓ $y = a(x-2)^2 + q$ ✓ subs R(0 ; 12) and S(– 2 ; 0) (or T(6 ; 0)) ✓ a-value ✓ $y = -x^2 + 4x + 12$ (4)
	OR $y = a(x-6)(x+2)$ $= a(x^2 - 4x - 12)$ $= -(x^2 - 4x - 12)$ $= -x^2 + 4x + 12$	✓ $y = a(x-6)(x+2)$ ✓ expand ✓ a-value ✓ $y = -x^2 + 4x + 12$ (4)
4.2.3	$\frac{dy}{dx} = 0$ $-2x + 4 = 0$ $x = 2$ $y = -(2)^2 + 4(2) + 12$ $= 16$ TP of f is (2 ; 16) **OR** $x = -\frac{b}{2a}$ $= -\frac{4}{2(-1)}$ $= 2$ $y = -(2)^2 + 4(2) + 12$ $= 16$ TP of f is (2 ; 16) **OR** $f(x) = -(x-2)^2 + 16$ TP of f is (2 ; 16)	✓x-value ✓ y-value (2) ✓ x-value ✓ y-value (2) ✓ x-value ✓ y-value (2)

	OR $x = \dfrac{-2+6}{2}$ $= 2$ $y = -(2)^2 + 4(2) + 12$ $= 16$ TP of f is (2 ; 16)	✓ x-value ✓ y-value (2)
4.2.4	$k < 16$ **OR** $(-\infty;16)$	✓✓ answer (2)
4.2.5	Maximum value of $h(x) = 3^{f(x)-12}$ occurs at max value of $f(x)$ Maximum value = 3^{16-12} $= 81$ **OR** Maximum value of $h(x) = 3^{f(x)-12}$ occurs at max value of $f(x)$ $h(2) = 3^{f(2)-12}$ $= 3^{16-12}$ $= 3^4$ or 81 **OR** $f(x) - 12 = -x^2 + 4x$ $= x(4-x)$ which has a maximum value of $f(2) = 4$ $\therefore$ Maximum value of $h(x)$ is 3^4 or 81	✓✓ subs 16 for $f(x)$ ✓ 3^4 or 81 (3) ✓✓ subs 16 for $f(x)$ ✓ 3^4 or 81 (3) ✓✓ subs $f(2) = 4$ ✓ 3^4 or 81 (3) **[20]**

QUESTION 3

3.1.1	$r=-\frac{1}{2}$ $T_4=1\left(-\frac{1}{2}\right)$ $=-\frac{1}{2}$	✓ $r=-\frac{1}{2}$ ✓ answer (2)
3.1.2	$T_n=4\left(-\frac{1}{2}\right)^{n-1}$ $\frac{1}{64}=4\left(-\frac{1}{2}\right)^{n-1}$ $\frac{1}{256}=\left(-\frac{1}{2}\right)^{n-1}$ $\left(-\frac{1}{2}\right)^{8}=\left(-\frac{1}{2}\right)^{n-1}$ $8=n-1$ $n=9$ **OR** $T_n=-8\left(-\frac{1}{2}\right)^{n}$ $\frac{1}{64}=-8\left(-\frac{1}{2}\right)^{n}$ $\frac{1}{256}=\left(-\frac{1}{2}\right)^{n}$ $\left(-\frac{1}{2}\right)^{8}=\left(-\frac{1}{2}\right)^{n-1}$ $8=n-1$ $n=9$	✓ $4\left(-\frac{1}{2}\right)^{n-1}$ ✓ substitution ✓ $\frac{1}{256}=\left(-\frac{1}{2}\right)^{n-1}$ ✓ answer (4)
	OR $T_4=-\frac{1}{2}$ $T_5=\frac{1}{4}$ $T_6=-\frac{1}{8}$ $T_7=\frac{1}{16}$ $T_8=-\frac{1}{32}$ $T_9=\frac{1}{64}$ $n=9$	✓ T_5 and T_6 ✓ T_7 ✓ T_8 ✓ answer (4)
3.1.3	$S_\infty=\frac{a}{1-r}$ $=\frac{4}{1-\left(-\frac{1}{2}\right)}$ $=\frac{8}{3}$	✓ substitution into correct formula ✓ answer (2)

3.2	For a geometric sequence: $\frac{x+1}{1}=\frac{x-3}{x+1}$ $x^2+2x+1=x-3$ $x^2+x+4=0$ $x=\frac{-1\pm\sqrt{1-4(1)(4)}}{2(1)}$ $x=\frac{-1\pm\sqrt{-15}}{2}$ **OR** $x^2+2x+1=x-3$ $x^2+x+4=0$ $\left(x+\frac{1}{2}\right)^2+\frac{15}{4}=0$ $\left(x+\frac{1}{2}\right)^2+\frac{15}{4}\geq\frac{15}{4}>0$ Solution is non-real. There is no x-value that makes the sequence geometric.	✓ $\frac{T_2}{T_1}=\frac{T_3}{T_2}$ ✓ standard form ✓ subs in quadratic formula ✓ non-real/no x-values (4)
	OR	
	For a geometric sequence: $\frac{x+1}{1}=\frac{x-3}{x+1}$ $x^2+2x+1=x-3$ $x^2+x+4=0$ $b^2-4ac=1-4(1)(4)$ $=-15$ Solution is non-real There is no x-value that makes the sequence geometric.	✓ $\frac{T_2}{T_1}=\frac{T_3}{T_2}$ ✓ standard form ✓ subs in discriminant ✓ non-real/no x-values (4) **[12]**

QUESTION 4

4.1	17 10 5 2 -7 -5 -3 2 2 $r = 1$ $s = 2$	✓ complete pattern ✓ $r = 1$ ✓ $s = 2$ (3)
	OR **NOTE:** Candidates may do 4.2 first. $d(n) = n^2 - 10n + 26$ $r = d(5)$ $= (5)^2 - 10(5) + 26$ $= 1$ $s = d(6)$ $= (6)^2 - 10(6) + 26$ $= 12$	✓ $d(n) = n^2 - 10n + 26$ ✓ $r = 1$ ✓ $s = 2$ (3)
	OR $d(n) = (n-5)^2 + 1$ $r = d(5)$ $= (5-5)^2 + 1$ $= 1$ $s = d(6)$ $= (6-5)^2 + 1$ $= 2$	✓ $d(n) = (n-5)^2 + 1$ ✓ $r = 1$ ✓ $s = 2$ (3)
4.2	$2a = 2$ $a = 1$ $3a + b = -7$ $\therefore 3(1) + b = -7$ $b = -10$ $\therefore a + b + c = 17$ $1 - 10 + c = 17$ $c = 26$ $\therefore d(n) = n^2 - 10n + 26$ **OR**	✓ $a = 1$ ✓ method ✓ $b = -10$ ✓ $c = 26$ (4)

	$a+b+c=17$ $4a+2b+c=10$ $3a+b=-7$ $9a+3b=-21$ $9a+3b+c=5$ $-21+c=5$ $c=26$ $a+b=-9$ $4a+2b=-16$ $2a+2b=-18$ $2a=2$ $a=1$ $b=-10$ $d(n)=n^2-10n+26$ **OR** $a+b+c=17$ $4a+2b+c=10$ $9a+3b+c=5$ $3a+b=-7$ $5a+b=-5$ $2a=2$ $a=1$ $3(1)+b=-7$ $b=-10$ $(1)-10+c=17$ $c=26$ $d(n)=n^2-10n+26$	✓ method ✓ $a = 1$ ✓ $c = 26$ ✓ $b = -10$ (4)
	OR $2a=2$ $a=1$ $c=26$ $d(n)=n^2+bn+26$ $17=(1)^2+b+26$ $b=-10$ $d(n)=n^2-10n+26$	✓ method ✓ $a = 1$ ✓ $c = 26$ ✓ $b = -10$ (4)
	OR $\mathrm{d(n)}=\frac{\mathrm{n}-1}{2}\left[2(\text{first difference})+(\mathrm{n}-2)(\text{second difference})\right]+\mathrm{d(1)}$ $d(n)=\frac{n-1}{2}\left[2(-7)+(n-2)(2)\right]+17$ $d(n)=\frac{n-1}{2}\left[-18-2n\right]+17$ $d(n)=(n-1)(-9-n)+17$ $d(n)=n^2-10n+26$ **OR**	✓ method ✓ $a = 1$ ✓ $c = 26$ ✓ $b = -10$ (4)

	$d(n)=(n-1)d(2)-(n-2)d(1)+\text{second difference}\times\frac{(n-1)(n-2)}{2}$ $d(n)=(n-1)(10)-(n-2)(17)+\frac{2(n-1)(n-2)}{2}$ $d(n)=10n-10-17n+34+(n-1)(n-2)$ $d(n)=n^2-10n+26$ **OR** $d(n)=(n-5)^2+1$ $\quad =n^2-10n+26$ $a=1$ $b=-10$ $c=26$	✓ method ✓ $a = 1$ ✓ $c = 26$ ✓ $b = -10$ (4) ✓ method ✓ $a = 1$ ✓ $c = 26$ ✓ $b = -10$ (4)
4.3	$d(8)=(8)^2-10(8)+26$ $\quad =10$ m **OR** By symmetry $d(8)$ $=d(5+3)$ $=d(5-3)$ $=d(2)$ $=10$ **OR** 17, 10, 5, 2, 1, 2, 5, 10 so $d(8) = 10$	✓ subs $t = 8$ ✓ answer (2) ✓ method ✓ answer (2) ✓ method ✓ answer (2)
4.4	Since the distance from P is decreasing for $n < 5$ the athlete is moving towards P. Since the distance from P is increasing for $n > 5$, the athlete is moving away from P. **OR** It is sufficient to show that d is decreasing when $n < 5$ and increasing when $n > 5$ $d(n)=n^2-10n+26$ $d'(n)=2n-10$ $d'(n)=2(n-5)$ For $n < 5$, $2(n-5)<0$ $d'(n)<0 \therefore$ decreasing For $n > 5$, $2(n-5)>0$ $d'(n)>0 \therefore$ increasing	✓✓ decreasing Moving towards ✓✓ increasing Moving away ✓ $d'(n)=2n-10$ ✓ $2(n-5)<0$ ✓ decreasing ✓ increasing (4) **[13]**

QUESTION 3

3.1	21; 24 **Note:** If candidate writes $T_8 = 21$ $T_7 = 24$: award 1/2 marks	✓ 21 ✓ 24 (2)
3.2	$T_{2k} = 3.2^{k-1}$ and so $T_{52} = 3.2^{26-1} = 100663296$ $T_{2k-1} = 3 + 6(k-1) = 6k-3$ and so $T_{51} = 6(26) - 3 = 153$ $T_{52} - T_{51} = 100663296 - 153$ $= 100663143$ **Note:** If candidate writes out all 52 terms **and** gets correct answer: award 5/5 marks **Note:** If candidate used $k = 52$: max 2/5	✓ 3.2^{k-1} ✓ T_{52} ✓ $6k-3$ ✓ T_{51} ✓ answer (5)
	OR Consider sequence P: 3 ; 6 ; 12 ... $P_n = 3.2^{n-1}$ $P_{26} = 3.2^{26-1} = 100663296$ Consider sequence Q: 3 ; 9 ; 15 ... $Q_n = 6n-3$ $Q_{26} = 6(26) - 3 = 153$ $T_{52} - T_{51} = P_{26} - Q_{26}$ $= 100663296 - 153$ $= 100663143$ **Note:** if candidate interchanges order i.e. does $T_{51} - T_{52}$: max 4/5 marks **Note:** writes out all 52 terms and subtracts $T_{51} - T_{52}$: max 4/5 marks	✓ $P_n = 3.2^{n-1}$ ✓ P_{26} ✓ $Q_n = 6n-3$ ✓ Q_{26} ✓ answer (5)

3.3	For all $n \in \mathbf{N}$, $n = 2k$ or $n = 2k-1$ for some $k \in \mathbf{N}$ If $n = 2k$: $T_n = T_{2k} = 3.2^{k-1}$ If $n = 2k-1$: $T_n = T_{2k-1}$ $= 6k - 3$ $= 3(2k-1)$ In either case, T_n has a factor of 3, so is divisible by 3. **Note:** If a candidate only illustrates divisibility by 3 with a specific finite part of the sequence, not the general term: 0/2 marks	✓ factors 3.2^{k-1} ✓ factors $3(2k-1)$ (2)
	OR $P_n = 3.2^{n-1}$ Which is a multiple of 3 $Q_n = 6n - 3$ $= 3(2n-1)$ Which is also a multiple of 3 Since $T_n = Q_{2k-1}$ or $T_n = P_{2k}$ for all $n \in \mathbf{N}$, T_n is always divisible by 3	✓ factors 3.2^{n-1} ✓ factors $3(2n-1)$ (2)
	OR The odd terms are odd multiples of 3 and the even terms are 3 times a power of 2. This means that all the terms are multiples of 3 and are therefore divisible by 3.	✓ odd multiples of 3 ✓ 3 times a power of 2 (2) **[9]**

QUESTION 4

4.1	The second, third, fourth and fifth terms are 1 ; – 6 ; T_4 and – 14 First differences are: – 7 ; T_4 + 6 ; – 14 – T_4 So $T_4 + 6 + 7 = -14 - 2T_4 - 6$ $T_4 = -11$ d = – 11 + 6 + 7 = 2 or – 14 + 22 – 6 = 2 **Note:** Answer only (i.e. d = 2) with no working: 3 marks **Note:** Candidate gives $T_4 = -11$ and $d = 2$ only: award 5/5 marks	✓ – 7 ✓ T_4 + 6 ✓ – 14 – T_4 ✓ setting up equation $T_5 - T_2 = (T_5 - T_4) + (T_4 - T_3) + (T_3 - T_2)$ ✓ answer (5)
	OR $\begin{array}{ccccccc} T_2 & & T_3 & & T_4 & & T_5 \\ 1 & & -6 & & & & -14 \\ & -7 & & -7+d & & -7+2d & \\ & & d & & d & & \end{array}$ $T_5 - T_2 = (T_5 - T_4) + (T_4 - T_3) + (T_3 - T_2)$ $-15 = (-7+2d) + (-7+d) + -7$ $-15 = -21 + 3d$ $6 = 3d$ $d = 2$ **Note:** Candidate uses trial and error **and** shows this: award 5/5 marks	✓ – 7 ✓ – 7 + d ✓ – 7 + 2d ✓ setting up equation $T_5 - T_2 = (T_5 - T_4) + (T_4 - T_3) + (T_3 - T_2)$ ✓ answer (5)
	OR $4a + 2b + c = 1$ $9a + 3b + c = -6$ $5a + b = -7$ $25a + 5b + c = -14$ $16a + 2b = -8$ $10a + 2b = -14$ $6a = 6$ $a = 1$ $d = 2a = 2$	✓ $4a + 2b + c = 1$ ✓ $9a + 3b + c = -6$ ✓ $25a + 5b + c = -14$ ✓ solved simultaneously ✓ answer (5)
	OR $\begin{array}{ccccccccc} T_1 & & 1 & & -6 & & T_4 & & -14 \\ & 1 - T_1 & & -7 & & T_4+6 & & -14 - T_4 & \\ & & T_1 - 8 & & T_4+13 & & -20-2T_4 & & \end{array}$ $T_4 + 13 = -20 - 2T_4$ $3T_4 = -33$ $T_4 = -11$ $d = -11 + 13$ $d = 2$	✓ – 7 ✓ $T_4 + 6$ ✓ $-14 - T_4$ ✓ setting up equation ✓ answer (5)

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OR

$$\begin{array}{ccccccccc} T_1 & & T_2 & & T_3 & & T_4 & & T_5\\ x & & 1 & & -6 & & y & & -14\\ & 1-x & & -7 & & y+6 & & -14-y & \\ & & -8+x & & y+13 & & -20-2y & & \end{array}$$

$$\begin{aligned} y+13 &= -20-2y\\ 3y &= -33\\ y &= -11 \end{aligned}$$

Second difference = $y+13=-11+13=2$

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✓ -7
✓ $y+6$
✓ $-14-y$

✓ setting up equation
✓ answer
(5)

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$$\begin{array}{ccccc} T_1 & & 1 & & -6\\ & -9 & & -7 & \\ & & 2 & & \end{array}$$

Note: Answer only: award 2/2 marks

Note:
If incorrect d in 4.1, 2/2 CA marks for $T_1=d+8$ (since $1-T_1=-7-d$)

$T_1=10$

OR

$$\begin{aligned} a &= 1\\ 5a+b &= -7\\ 5(1)+b &= -7\\ b &= -12\\ a+b+c &= 1\\ 4(1)+2(-12)+c &= 1\\ c &= 21\\ T_n &= n^2-12n+21\\ T_1 &= (1)^2-12(1)+21\\ &= 10 \end{aligned}$$

OR

$$\begin{aligned} T_4+13 &= -8+T_1\\ -11+13 &= -8+T_1\\ T_1 &= 10 \end{aligned} \qquad \textbf{OR} \qquad \begin{aligned} y+13 &= -8+x\\ -11+13 &= -8+x\\ x &= 10 \end{aligned}$$

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✓ method
✓ $T_1=10$
(2)

✓ method
✓ $T_1=10$
(2)

✓ method
✓ $T_1=10$
(2)
[7]

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QUESTION 3

3.1	$S_\infty = 8 + \frac{8}{\sqrt{2}} + \ldots$ $r = \frac{1}{\sqrt{2}}$ and $s_\infty = \frac{a}{1-r}$ $= \frac{8}{1-\frac{1}{\sqrt{2}}}$ $= \frac{8\sqrt{2}}{\sqrt{2}-1}$ $= \frac{8\sqrt{2}(\sqrt{2}+1)}{(\sqrt{2}-1)(\sqrt{2}+1)}$ $= 8\sqrt{2}\sqrt{2} + 8\sqrt{2}$ $= 16 + 8\sqrt{2}$	✓ $r = \frac{1}{\sqrt{2}}$ ✓ substitution ✓ rationalisation ✓ simplification (4)
	OR	
	$S_\infty = 8 + \frac{8}{\sqrt{2}} + \ldots$ $r = \frac{1}{\sqrt{2}}$ and $s_\infty = \frac{a}{1-r}$ $= \frac{8}{1-\frac{1}{\sqrt{2}}}$ $= \frac{8\left(1+\frac{1}{\sqrt{2}}\right)}{\left(1-\frac{1}{\sqrt{2}}\right)\left(1+\frac{1}{\sqrt{2}}\right)}$ $= \frac{8\left(1+\frac{1}{\sqrt{2}}\right)}{\frac{1}{2}}$ $= 16\left(1+\frac{1}{\sqrt{2}}\right)$ $= 16 + \frac{16\sqrt{2}}{2}$ $= 16 + 8\sqrt{2}$	✓ $r = \frac{1}{\sqrt{2}}$ ✓ substitution ✓ rationalisation ✓ simplification (4)

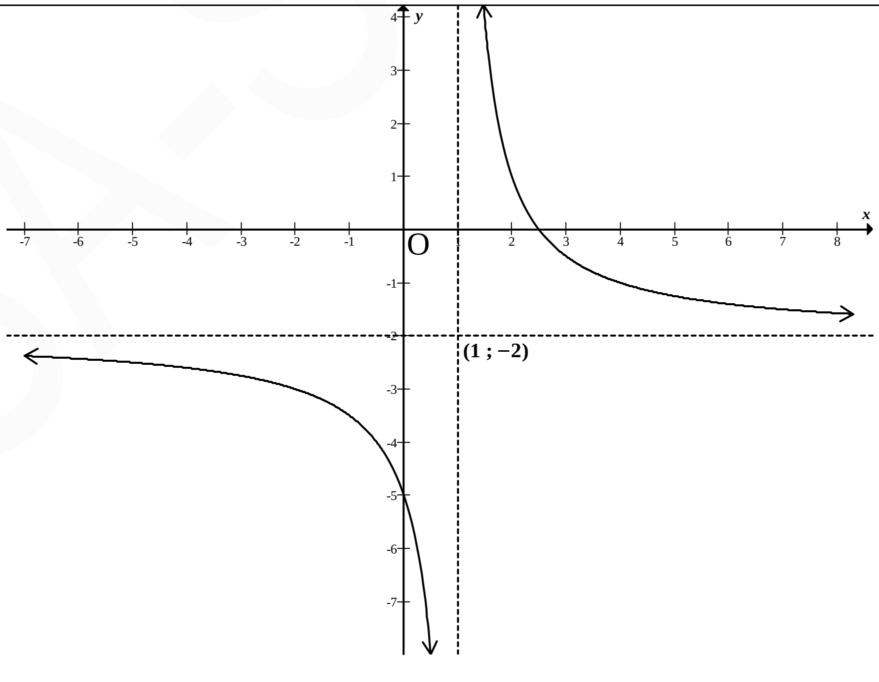
3.2.1	$5 + 15 + 45 + \ldots + T_{20}$ $= \sum_{n=1}^{20} 5(3)^{n-1}$ **OR** $5 + 15 + 45 + \ldots + T_{20}$ $= 5\sum_{n=0}^{19} (3)^{n}$ **OR** $5 + 15 + 45 + \ldots + T_{20}$ $= 5\sum_{i=l}^{l+19} (3)^{i-l}$ for any $l \in Z$	✓ ✓answer (2) ✓ ✓answer (2) ✓ ✓answer (2)
3.2.2	$5 + 15 + 45 + \ldots + T_{20}$ $= \frac{5(3^{20} - 1)}{3 - 1}$ $= 8\ 716\ 961\ 000$	✓ formula ✓ substitution ✓ answer (3) **[9]**

QUESTION 4

4.1.1	$S_{23} = \frac{23}{2}(5(23) + 9)$ $= 1426$	✓ substitution ✓ answer (2)
4.1.2	$T_{23} = S_{23} - S_{22}$ $= 1426 - \frac{22}{2}(5(22) + 9)$ $= 1426 - 1309$ $= 117$	✓ statement ✓ $S_{22} = 1309$ ✓ answer (3)
4.2	Arithmetic Sequence: $12 ; 12 + d ; 12 + 2d$ Geometric Sequence: $12 ; 12r ; 12r^2$ $12 + d = 12r$ $d = 12r - 12$ $12 + 12r + 12r^2 = 12 + 12 + d + 12 + 2d + 3$ $12r^2 = 12 + 2(12r - 12) + 3$ $12r^2 = 12 + 24r - 24 + 3$ $12r^2 - 24r + 9 = 0$ $4r^2 - 8r + 3 = 0$ $(2r - 3)(2r - 1) = 0$ $r = \frac{3}{2}$ or $r = \frac{1}{2}$	 ✓ equation ✓ equation ✓ standard form ✓ factors ✓ answers (6)

	OR The 3rd term of GP = 3 + 3rd term of AP $12r^2 = 3 + 12 + 2d$ $12r^2 = 15 + 24r - 24$ $12r^2 - 24r + 9 = 0$ $4r^2 - 8r + 3 = 0$ $(2r-3)(2r-1) = 0$ $r = \frac{3}{2}$ *or* $r = \frac{1}{2}$	✓ equation ✓ equation ✓ standard form ✓ factors ✓ answers **[11]**

QUESTION 5

5.1	$x = 1$ $y = -2$	✓✓ answers (2)
5.2	y-intercept: $y = \frac{3}{0-1} - 2 = -5$ x-intercept: $\left(\frac{5}{2}\,;\,0\right)$ $0 = \frac{3}{x-1} - 2$ $2 = \frac{3}{x-1}$ $2x - 2 = 3$ $2x = 5$ $x = \frac{5}{2}$	✓ y = -5 ✓ substitute y = 0 ✓ answer (3)
5.3		✓ asymptotes ✓ y–intercept ✓ shape (3)

- Consistent Accuracy will apply as a general rule.
- If a candidate does a question twice and does not delete either, mark the FIRST attempt.
- If a candidate does a question, crosses it out and does not re-do it, mark the deleted attempt.

QUESTION 3

3.1	$-1 + 2 + 5 + \ldots$ **OR** $-1 ; 2 ; 5$	✓ all three terms (1)
3.2	$S_n = -1 + 2 + 5 + 8 + \ldots\text{to } 100 \text{ terms}$ $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$ $S_{100} = \frac{100}{2}\left[2(-1) + (100-1)(3)\right]$ $= 50[-2 + 297]$ $= 14\,750$ Answer only: 4 / 4 **OR** $S_n = -1 + 2 + 5 + 8 + \ldots\text{to } 100 \text{ terms}$ $T_{100} = 3(100) - 4$ $= 296$ $S_n = \frac{n}{2}\left[T_1 + T_{100}\right]$ $S_{100} = \frac{100}{2}\left[-1 + 296\right]$ $= 50[\,295]$ $= 14\,750$ **NOTE**: If $S_n = -1 + 2 + 5 + 8 + \ldots\text{to } 99 \text{ terms}$ $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$ $S_{99} = \frac{99}{2}\left[2(-1) + (99-1)(3)\right]$ $= \frac{99}{2}[-2 + 294]$ $= 14454$ **Then 3 / 4** Apply consistent accuracy. This is the answer if series is $2 + 5 + 8 +$ $S_n = 2 + 5 + 8 + \ldots\text{to } 100 \text{ terms}$ $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$ $S_{100} = \frac{100}{2}\left[2(2) + (100-1)(3)\right]$ $= 50[4 + 297]$ $= 15050$ **Then 4 / 4**	✓ formula ✓ $n = 100$ ✓ substitution ✓ answer (4) **[5]**

- Consistent Accuracy will apply as a general rule.
- If a candidate does a question twice and does not delete either, mark the FIRST attempt.
- If a candidate does a question, crosses it out and does not re-do it, mark the deleted attempt.

QUESTION 4

4.1	The first differences are 1; – 1; – 3; – 5; …. These form a linear pattern $T_n = 1 + (n-1)(-2)$ $= 3 - 2n$ **OR** T_n **=** $-2n + 3$ **ANSWER ONLY:** Full marks	✓ pattern ✓ $d = -2$ ✓ answer (3)
4.2	Between the 35th and 36th terms of the quadratic sequence lies the 35th first difference 35th first difference = 3 – 2(35) = – 67 **OR** From the quadratic sequence: $P_{36} = -1158$ and $P_{35} = -1091$ 35th first difference = – 1158 – (– 1091) = – 67 If substitute and get $T_{35} = -2(35) + 3 = -67$ and $T_{36} = -2(36) + 3 = -69$, leading to the answer – 2 then **1 / 2**	✓ substitution of 35 into $T_n = -2n + 3$ ✓ answer (2) ✓ $P_{36} = -1158$ and $P_{35} = -1091$ ✓ answer (2)
4.3	Second difference of terms is – 2 . $P_n = an^2 + bn + c$ $a = -1.$ $3a + b = 1$ $-3 + b = 1$ $b = 4$ $a + b + c = -3$ $-1 + 4 + c = -3$ $c = -6$ $P_n = -n^2 + 4n - 6$ If the general term has been worked out correctly in 4.2 and not redone in 4.3 but answer just written down then **4 / 4** **OR**	✓ $a = -1$ ✓ substitution ✓ $b = 4$ ✓ $c = -6$ (4)

- Consistent Accuracy will apply as a general rule.
- If a candidate does a question twice and does not delete either, mark the FIRST attempt.
- If a candidate does a question, crosses it out and does not re-do it, mark the deleted attempt.

4.3 contd	Second difference of terms is -2 . $P_n = an^2 + bn + c$ $a = -1.$ $P_0 = -6 = c$ $P_n = -n^2 + bn - 6$ $-3 = -(1)^2 + (1)b - 6$ $b = 4$ $P_n = -n^2 + 4n - 6$ **OR** $P_n = \frac{n-1}{2}\left[2(\textit{first first difference}) + (n-2)(\textit{second difference})\right] + P_1$ $P_n = \frac{n-1}{2}[2(1) + (n-2)(-2)] - 3$ $P_n = n - 1 - (n-2)(n-1) - 3$ $P_n = n - 1 - n^2 + 3n - 2 - 3$ $P_n = -n^2 + 4n - 6$ **OR** $P_n = (n-1)P_2 - (n-2)P_1 + 2nd\ difference\frac{(n-1)(n-2)}{2}$ $P_n = (n-1)(-2) - (n-2)(-3) - 2\frac{(n-1)(n-2)}{2}$ $P_n = -2n + 2 + 3n - 6 - n^2 + 3n - 2$ $P_n = -n^2 + 4n - 6$ **OR** $P_n = \frac{(n-2)(n-3)T_1 - 2(n-1)(n-3)T_2 + (n-2)(n-1)T_3}{2}$ $P_n = \frac{(n^2 - 5n + 6)(-3) - 2(n^2 - 4n + 3)(-2) + (n^2 - 3n + 2)(-3)}{2}$ $P_n = \frac{-3n^2 + 15n - 18 + 4n^2 - 16n + 12 - 3n^2 + 9n - 6}{2}$ $P_n = -n^2 + 4n - 6$ **OR**	

- Consistent Accuracy will apply as a general rule.
- If a candidate does a question twice and does not delete either, mark the FIRST attempt.
- If a candidate does a question, crosses it out and does not re-do it, mark the deleted attempt.

4.3 contd	$P_2 - P_1 = T_1$ $P_3 - P_2 = T_2$ $P_4 - P_3 = T_3$ $P_n - P_{n-1} = T_{n-1}$ $P_n - P_1 = T_1 + T_2 + ... + T_{n-1}$ $P_n - P_1 = \frac{n-1}{2}\left[2(1) + (n-2)(-2)\right]$ $P_n - (-3) = (n-1)(3-n)$ $P_n = -n^2 + 4n - 6$	
4.4	Maximum value of T_n is $\frac{4(-1)(-6) - 4^2}{4(-1)} = -2$ The maximum value is negative and hence the sequence can not have any positive terms as the function is maximum valued	✓ max value – 2 ✓ explanation (2)
	OR $-n^2 + 4n - 6$ $= -(n-2)^2 + 4 - 6$ $= -(n-2)^2 - 2$ The function has a maximum-value of – 2 and therefore the pattern will never have positive values.	✓ max value – 2 ✓ explanation (2)
	OR $T_n = -n^2 + 4n - 6$ $\frac{d}{dn}(T_n) = -2n + 4$ $0 = -2n + 4$ $n = 2$ $T_2 = -(2)^2 + 4(2) - 6$ $= -2$ The function has a maximum-value of – 2 and therefore the pattern will never have positive values.	✓ max value – 2 ✓ explanation (2)
	OR As the sequence decreases from the second term onwards and the second term is negative, the sequence will never have a positive term.	✓✓ answer (2)
	OR $T_n = -n^2 + 4n - 6$ $\frac{d}{dn}(T_n) = -2n + 4$ $\frac{d}{dn}(T_n) < 0$ for $n > 2$ and $T_2 < 0$ so the sequence decreases and stays negative	✓✓ answer (2) **[11]**