SA-STUDENT

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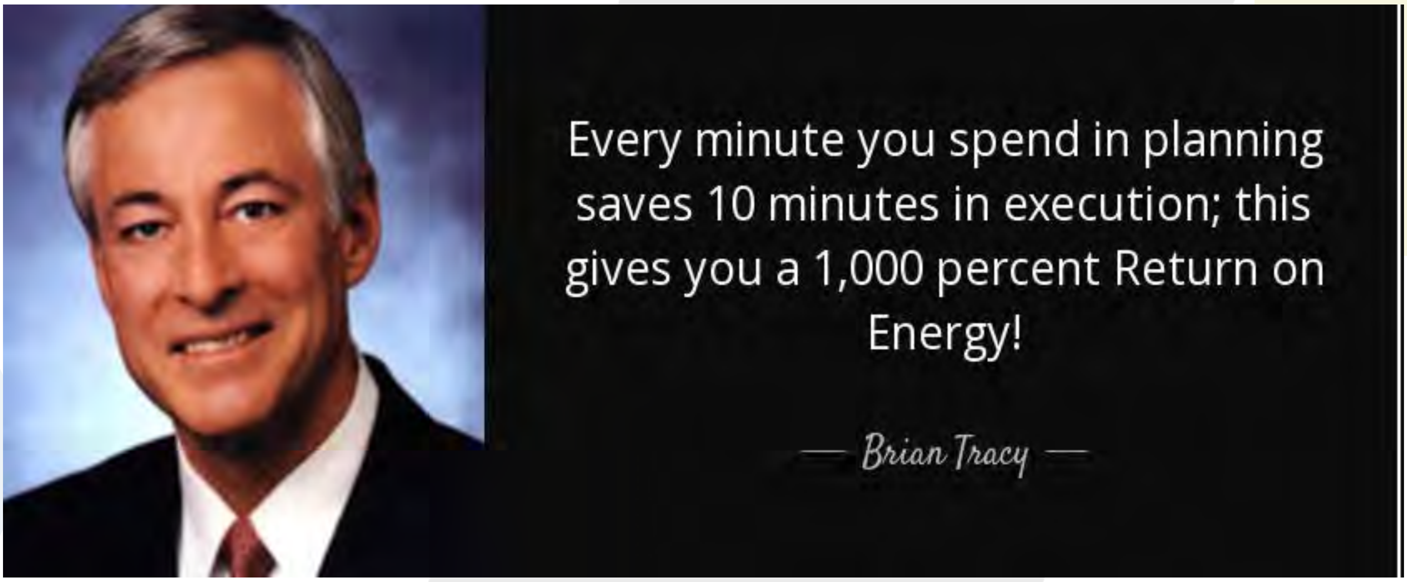

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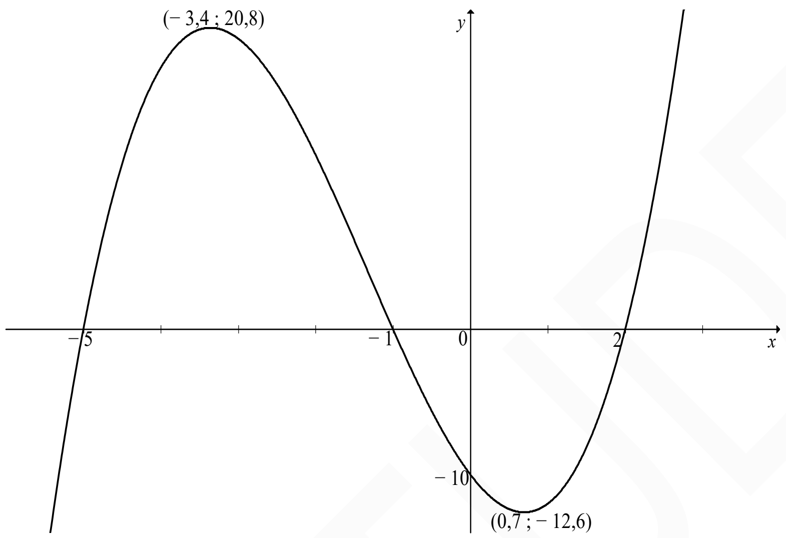
QUESTION 8/*VRAAG* 8

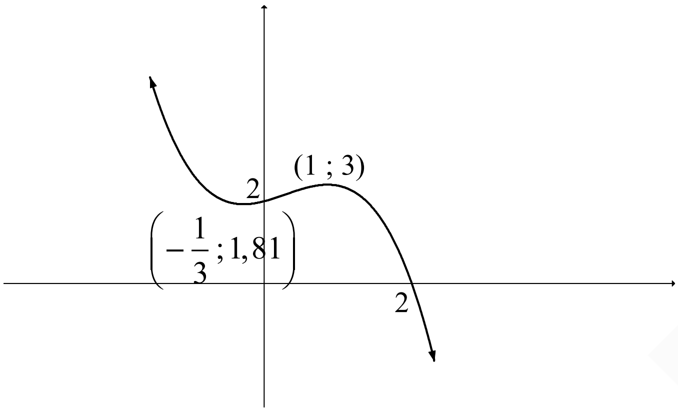
8.1	$y = -10$	✓ answer (1)
8.2	$f(x) = x^3 + 4x^2 - 7x - 10$ $f(2) = 2^3 + 4(2)^2 - 7(2) - 10 = 0$	✓substitution of $x = 2$ ✓ $f(2) = 0$ (2)
8.3	$f(x) = (x-2)(x^2+6x+5)$ $f(x) = (x-2)(x+5)(x+1)$	✓ $(x-2)$ ✓ $(x+5)$ ✓ $(x+1)$ (3)
8.4		✓ x- intercepts ✓ y-intercept ✓ sketching the graph with turning points in 2^{nd} and 4^{th} quadrant (3)
8.5.1	$x \in (-3{,}4\ ;\ 0{,}7)$ **OR/*OF*** $-3{,}4 < x < 0{,}7$	✓✓ $x \in (-3{,}4\ ;\ 0{,}7)$ (2)
8.5.2	$f(x) = x^3 + 4x^2 - 7x - 10$ $f^{/}(x) = 3x^2 + 8x - 7$ $f^{//}(x) = 6x + 8 = 0$ $\therefore x = -\frac{8}{6} = -\frac{4}{3} = -1{,}33$ **OR/*OF*** $\frac{-3{,}4 + 0{,}7}{2} = -1{,}35 = -1{,}35$	✓ $f^{//}(x) = 6x + 8$ ✓ answer (2) **OR/*OF*** ✓ substitution ✓ answer (2)
8.5.3	$x \le -3{,}4$ or $-1{,}33 \le x \le 0{,}7$ **OR/*OF*** $x \in (-\infty\ ;\ -3{,}4] \cup [-1{,}33\ ;\ 0{,}7]$	✓ $x \le -3{,}4$ (A) ✓✓ $-1{,}33 \le x \le 0{,}7$ (A 0,7) (3)
		[16]

QUESTION 9/VRAAG 9

9.1	Perimeter of the square = $12-6x$ Side length of square $=\dfrac{12-6x}{4}=\dfrac{6-3x}{2}=3-\dfrac{3}{2}x$	✓ $12-6x$ ✓ answer (2)
9.2	$V=\left(\dfrac{6-3x}{2}\right)^2(4x)$ $=\left(\dfrac{36-36x+9x^2}{4}\right)(4x)$ $=36x-36x^2+9x^3$ $V(x)=36x-36x^2+9x^3$ $V'(x)=36-72x+27x^2$ $36-72x+27x^2=0$ $9x^2-24x+12=0$ $3x^2-8x+4=0$ $(3x-2)(x-2)=0$ $x=\dfrac{2}{3}$ or $x=2$ $V\left(\dfrac{2}{3}\right)=36\left(\dfrac{2}{3}\right)-36\left(\dfrac{2}{3}\right)^2+9\left(\dfrac{2}{3}\right)^3$ $=\dfrac{32}{3}\text{ m}^3=10{,}67\text{ m}^3$	✓ $\left(\dfrac{6-3x}{2}\right)^2(4x)$ ✓ $\left(\dfrac{36-36x+9x^2}{4}\right)$ ✓ $36x-36x^2+9x^3$ ✓ V' ✓ $V'=0$ ✓ values ✓ answer (7)
		[9]

QUESTION 8/*VRAAG* 8

8.1	$f^{/}(x)=mx^2+nx+k$ $f^{/}(x)=m\left(x+\frac{1}{3}\right)(x-1)$ $1=m\left(0+\frac{1}{3}\right)(0-1)$ $1=-\frac{1}{3}m$ $\therefore m=-3$ $f^{/}(x)=-3\left(x+\frac{1}{3}\right)(x-1)$ $f^{/}(x)=-3\left(x^2-\frac{2}{3}x-\frac{1}{3}\right)$ $f^{/}(x)=-3x^2+2x+1$ $\therefore n=2$ $\therefore k=1$	✓ substitution of $\left(-\frac{1}{3}\ ;0\right)$ and (1 ; 0) ✓ substitution of (0 ; 1) ✓ $m=-3$ ✓ $f^{/}(x)=-3\left(x^2-\frac{2}{3}x-\frac{1}{3}\right)$ ✓ $n=2$ ✓ $k=1$ (6)
	OR/OF $k=1$ $0=m+n+1$ and $\frac{1}{9}m-\frac{1}{3}n+1=0$ $m+n=-1$ (1) $m-3n=-9$ (2) (1) – (2) $4n=8$ $\therefore n=2$ $m+2=-1$ $\therefore m=-3$	**OR/OF** ✓ $k=1$ ✓ $m+n=-1$ ✓ $m-3n=-9$ ✓ $4n=8$ ✓ $n=2$ ✓ $m=-3$ (6)
8.2.1	$f(x)=-x^3+x^2+x+2$ $f\left(-\frac{1}{3}\right)=\frac{49}{27}=1,81$ $\text{T.P}\left(-\frac{1}{3}\ ;\frac{49}{27}\right)$ $f(1)=3$ $\text{T.P}(1\ ;3)$	✓ x-coordinates of the TP ✓ $\text{T.P}\left(-\frac{1}{3}\ ;\frac{49}{27}\right)$ ✓ $\text{T.P}(1\ ;3)$ (3)

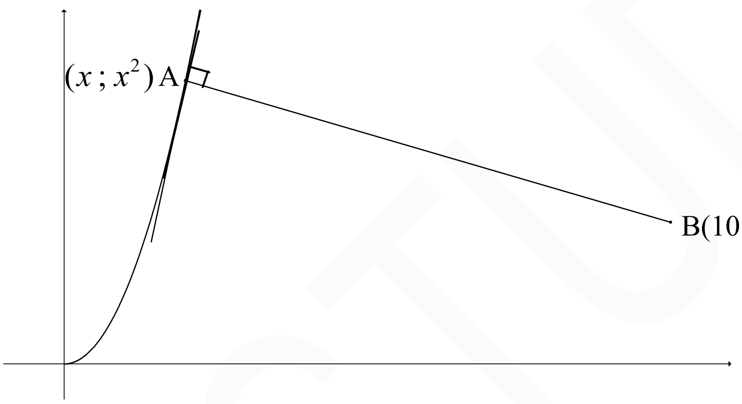
<table>
<tr><td>8.2.2</td><td>$f(x) = -x^3 + x^2 + x + 2$
$-x^3 + x^2 + x + 2 = 0$
$(x-2)(-x^2 - x - 1) = 0$
$x = 2$ or no solution

</td><td>✓ $x = 2$

✓one x-intercept
✓ two turning points
✓y-intercept
✓ shape: neg cubic
(5)</td></tr>
<tr><td>8.3.1</td><td>$a = \dfrac{-\frac{1}{3} + 1}{2}$
$= \dfrac{1}{3}$
OR/OF
$f'(x) = -3x^2 + 2x + 1$
$f''(x) = -6x + 2$
$f''(a) = -6a + 2 = 0$
$-6a = -2$
$a = \dfrac{1}{3}$</td><td>✓ answer (1)
OR/OF

✓ answer (1)</td></tr>
<tr><td>8.3.2</td><td>$b < \dfrac{4}{3}$ units</td><td>✓ $\dfrac{4}{3}$
✓ $b < \dfrac{4}{3}$ (2)</td></tr>
<tr><td></td><td></td><td>[17]</td></tr>
</table>

QUESTION9/*VRAAG* 9

9.1	Any point on f : $(x\,;\,x^2)$ $\text{distance} = \sqrt{(x-10)^2 + (x^2-2)^2}$ $= \sqrt{x^2 - 20x + 100 + x^4 - 4x^2 + 4}$ $= \sqrt{x^4 - 3x^2 - 20x + 104}$ For min distance $\frac{d}{dx}\left(x^4 - 3x^2 - 20x + 104\right) = 0$ $4x^3 - 6x - 20 = 0$ $(x-2)(4x^2 + 8x + 10) = 0$ $\Delta = 8^2 - 4(4)(10) = -96 \quad \therefore$ no roots $\therefore x = 2$ $d = \sqrt{2^4 - 3(2)^2 - 20(2) + 104} = 2\sqrt{17} = 8{,}25$	✓ $(x\,;\,x^2)$ ✓ substitution ✓ simplification ✓ answer ✓ $4x^3 - 6x - 20$ ✓ derivative = 0 ✓ $x = 2$ ✓answer (A) (8)



9.2	$m_{AB} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{x^2 - 2}{x - 10}$ $\therefore m_{\tan g} = -\frac{x-10}{x^2-2}$ $f^{/}(x) = 2x$ $\therefore 2x = -\frac{x-10}{x^2-2}$ $-2x^3 + 4x = x - 10$ $2x^3 - 3x - 10 = 0$ $x = 2$ $y = (2)^2 = 4$ $\therefore AB = \sqrt{(2-10)^2 + (4-2)^2}$ $= 2\sqrt{17} = 8{,}25$	✓ m_{AB} ✓ $m_{\tan g} = -\frac{x-10}{x^2-2}$ ✓ $f^{/}(x) = 2x$ ✓ equating ✓ standard form ✓ $x = 2$ ✓substitute into distance ✓answer (A) (8)
		[8]

QUESTION/*VRAAG* 9

9.1	$f(x)=(x+t)^2(x-3)$ $-3=(0+t)^2(0-3)$ $1=t^2$ $t=\pm1$ $\therefore t=1$ $f(x)=(x+1)^2(x-3)$ $f(x)=(x^2+2x+1)(x-3)$ $f(x)=x^3-x^2-5x-3$	✓ $f(x)=(x+t)^2(x-3)$ ✓ subs (0 ; –3) ✓ t ✓ $f(x)=(x+1)^2(x-3)$ ✓ expansion (5)
9.2	$f^{/}(x)=3x^2-2x-5$ $0=3x^2-2x-5$ $0=(x+1)(3x-5)$ $x=-1$ or $x=\frac{5}{3}$ $\text{N}\left(\frac{5}{3};-\frac{256}{27}\right)=(1{,}67;-9{,}48)$	✓ $f^{/}(x)=3x^2-2x-5$ ✓ $=0$ ✓ factors ✓ x-value ($x>0$) ✓ y-value (A) (5)
9.3.1	$x<3$; $x\neq-1$ **OR/*OF*** $x<-1$ or $-1<x<3$ **OR/*OF*** $(-\infty;-1)$ or $(-1;3)$	✓ $x<3$ ✓ $x\neq-1$ (2) **OR/*OF*** ✓ $x<-1$ ✓ $-1<x<3$ (2) **OR/*OF*** ✓ $(-\infty;-1)$ ✓ $(-1;3)$ (2)
9.3.2	$x<-1$ or $x>\frac{5}{3}$ OR/OF $x\leq-1$ or $x\geq\frac{5}{3}$ **OR/*OF*** $(-\infty;-1)$ or $\left(\frac{5}{3};\infty\right)$ OR/OF $(-\infty;-1]$ or $\left[\frac{5}{3};\infty\right)$	✓ $x<-1$ ✓ $x>\frac{5}{3}$ (2) **OR/*OF*** ✓ $(-\infty;-1)$ ✓ $\left(\frac{5}{3};\infty\right)$ (2)
9.3.3	$f^{//}(x)>0$ $6x-2>0$ $x>\frac{1}{3}$ or $\left(\frac{1}{3};\infty\right)$ ANSWER ONLY: FULL MARKS **OR/OF** $\frac{\frac{5}{3}+(-1)}{2}=\frac{1}{3}$ $x>\frac{1}{3}$ or $\left(\frac{1}{3};\infty\right)$	✓ $6x-2$ ✓ $\frac{1}{3}$ ✓ $x>\frac{1}{3}$ (3) **OR/OF** ✓ substitution ✓ $\frac{1}{3}$ ✓ $x>\frac{1}{3}$ (3)

9.4	Distance $= x^3 - x^2 - 5x - 3 - (3x^2 - 2x - 5)$ $= x^3 - 4x^2 - 3x + 2$ $\frac{d\text{Distance}}{dx} = 3x^2 - 8x - 3$ $0 = 3x^2 - 8x - 3$ $0 = (3x+1)(x-3)$ $x = 3$ or $x = -\frac{1}{3}$ Max distance $= \left(-\frac{1}{3}\right)^3 - 4\left(-\frac{1}{3}\right)^2 - 3\left(-\frac{1}{3}\right) + 2$ $= \frac{68}{27} = 2{,}52$	✓ $x^3 - 4x^2 - 3x + 2$ ✓ $\frac{d\text{Distance}}{dx} = 3x^2 - 8x - 3$ ✓ factors ✓ x-values ✓ $x = -\frac{1}{3}$ ✓ answer (6)
		[23]

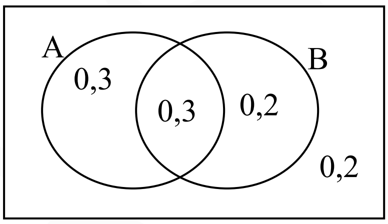
QUESTION/VRAAG 10

10.1	$h(x)=ax^3+bx^2$ $h^/(x)=3ax^2+2bx$ $h^/(4)=0$ $48a+8b=0$ $6a+b=0 \quad ...(1)$ $h(4)=32$ $64a+16b=32$ $4a+b=2 \quad ...(2)$ (1)– (2): $6a+b=0$ $4a+b=2$ $2a=-2$ $a=-1$ $4(-1)+b=2$ $b=6$	✓ $h^/(x)$ ✓ $h^/(4)=0$ ✓ $48a+8b=0$ or $6a+b=0$ ✓ $h(4)=32$ ✓ $64a+16b=32$ or $4a+b=2$ (5)
10.2	$h(x)=-x^3+6x^2$ $-x^3+6x^2=0$ $x^2(-x+6)=0$ $x=0$ or $x=6$ $\therefore$ A$(6\,;0)$	✓ $-x^3+6x^2=0$ ✓ factors ✓ A$(6\,;0)$ (3)
10.3.1	$0<x<4$ or $0\le x\le 4$ **OR/OF** $x\in(0\,;4)$ or $x\in[0\,;4]$	✓critical values ✓notation (2) **OR/OF** ✓critical values ✓notation (2)
10.3.2	$x>2$ **OR/OF** $x\in(2\,;\infty)$	✓ 2 ✓ notation (2) **OR/OF** ✓ 2 ✓ notation (2)
10.4	$f(x)=h(x-1)=-(x-1)^3+6(x-1)^2$ $f(0)=7$ $7<k<32$ or $k\in(7\,;32)$	✓ $k<32$ ✓ new y-intercept = 7 ✓ $7<k<32$ (3)
		[15]

QUESTION/VRAAG 11

11	Time = $\frac{20}{x}$ Cost = (water cost per hour × time) + (kms × R/km) $C(x)=1,6\times\left(\frac{20}{x}\right)+20\left(1,2+\frac{x}{4000}\right)$ $C(x)=\frac{32}{x}+24+\frac{x}{200}$ $C'(x)=-\frac{32}{x^2}+\frac{1}{200}=0$ $x^2=6\,400$ $x=80\,km/h$	✓ $\frac{20}{x}$ ✓ $1,6\times\left(\frac{20}{x}\right)$ ✓ $20\left(1,2+\frac{x}{4000}\right)$ ✓ $C(x)=\frac{32}{x}+24+\frac{x}{200}$ ✓ $C'(x)=-\frac{32}{x^2}+\frac{1}{200}$ ✓ $C'(x)=0$ ✓ answer (A) (7)
		[7]

QUESTION/VRAAG 12

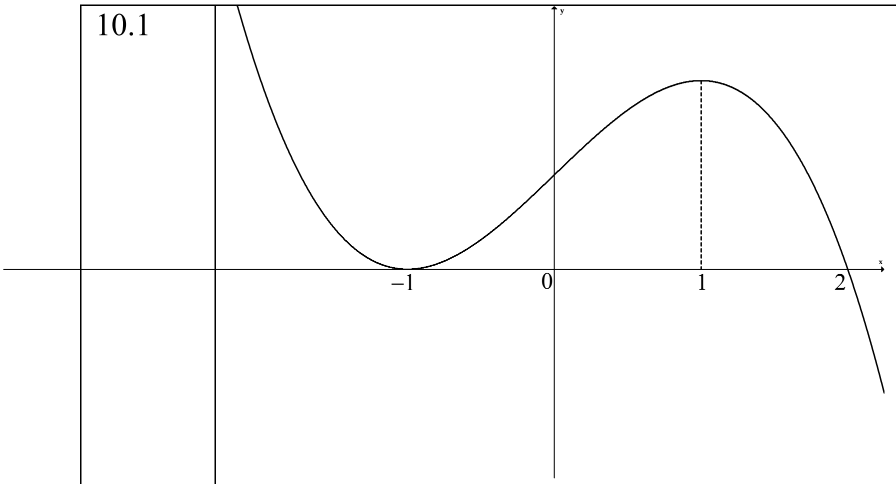
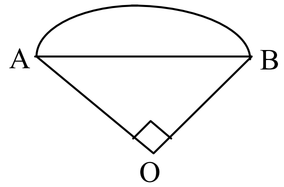
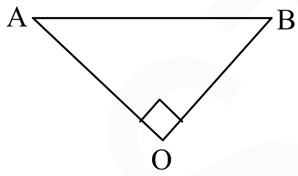
12.1.1	No, because P(A and B) $\neq 0$	✓ answer and reason (1)
12.1.2(a)	$P(\text{A and B})=0,3$ $P(\text{only B})=0,2$ $P(\text{A and B})=P(A)\times P(B)$ $0,3=P(A)\times 0,5$ $P(A)=0,6$ $P(\text{only A})=0,3$	✓ $P(\text{A and B})=P(A)\times P(B)$ ✓ 0,5 ✓ $P(A)=0,6$ ✓answer (4)
12.1.2(b)	 $P(\text{not A or not B})=0,2+0,2+0,3=0,7$ **OR/OF** $P(\text{not A or not B})=1-P(\text{A and B})=1-0,3=0,7$ **OR/OF** $P(A' \text{ or } B') = P(A') + P(B') - P(A' \text{ and } B')$ $= 0,4 + 0,5 - 0,2 = 0,7$	✓method ✓answer (2) **OR/OF** ✓method ✓answer (2) **OR/OF** ✓method ✓answer (2)

8.2.2	$g(x)=(\sqrt{x}+3)(\sqrt{x}-1)$ $g(x)=x+2x^{\frac{1}{2}}-3$ $g^{/}(x)=1+x^{-\frac{1}{2}}$	✓ x ✓ $2x^{\frac{1}{2}}$ ✓1 ✓ $x^{-\frac{1}{2}}$ (4)
		[12]

QUESTION/*VRAAG* 9

9.1	$f^{/}(x)=6x^2+6x-12$ $6x^2+6x-12=0$ $x^2+x-2=0$ $(x+2)(x-1)=0$ $x=-2$ or $x=1$ $y=20$ or $y=-7$ $\therefore$ A(–2 ; 20) and B(1 ; –7)	✓ $6x^2+6x-12$ ✓ $=0$ ✓factors ✓ x -values ✓ y -values (5)
9.2	$f^{//}(x)=12x+6$ $12x+6>0$ $12x>-6$ $x>-\frac{1}{2}$ **OR/OF** $x=\frac{-2+1}{2}=-\frac{1}{2}$ $\therefore x>-\frac{1}{2}$	✓ $12x+6$ ✓ $f^{//}(x)>0$ ✓ $x>-\frac{1}{2}$ (3) **OR/OF** ✓ $x=-\frac{1}{2}$ ✓✓ $x>-\frac{1}{2}$ (3)
9.3	$f^{/}(2)=24$ Equation of the tangent: $y-4=24(x-2)$ $y=24x-44$	✓ $f^{/}(2)$ ✓ 24 ✓ equation (3)
		[11]

QUESTION/*VRAAG* 10

10.1		✓ $x=-1$ and $x=2$ ✓ TP at $x=-1$ ✓ TP at $x=1$ ✓ shape (4)
10.2.1	 Area of segment = $\frac{1}{4}$ Area of big circle $=\frac{1}{4}\pi\ (x-x^2)^2$  Area triangle ABO counted $=$ Area $\Delta=\frac{1}{2}\left(x-x^2\right)^2$ Area of shaded region $=\frac{1}{4}\pi(x-x^2)^2-\frac{1}{2}\left(x-x^2\right)^2$ $=\frac{\pi-2}{4}\left(x-x^2\right)^2$ $=\left(\frac{\pi-2}{4}\right)\left(x^2-2x^3+x^4\right)$	✓✓ $\frac{1}{4}\pi(x-x^2)^2$ ✓ Area $\Delta=\frac{1}{2}\left(x-x^2\right)^2$ ✓ subtract areas ✓ common factor (5)

10.2.2	Area of shaded region $= \frac{(\pi - 2)}{4}(x^4 - 2x^3 + x^2)$ $\frac{dA}{dx} = \left(\frac{\pi - 2}{4}\right)(4x^3 - 6x^2 + 2x)$ $4x^3 - 6x^2 + 2x = 0$ $x(2x^2 - 3x + 1) = 0$ $x(2x - 1)(x - 1) = 0$ $x \neq 0$ or $x = \frac{1}{2}$ or $x \neq 1$	✓ $\left(\frac{\pi - 2}{4}\right)(4x^3 - 6x^2 + 2x)$ ✓ factors ✓ $x = 0; x = 1; x = \frac{1}{2}$ ✓ $x = \frac{1}{2}$ (4)
		[13]

QUESTION/*VRAAG* 11

11.1	P(A) = 1 – P(not A) = 0,6 P(A and B) = P(A) × P(B) = 0,6 × 0,3 $= \frac{9}{50}$ = 0,18	✓ 0,6 ✓ P(A and B) = P(A) × P(B) ✓ answer (A) (3)
11.2.1	$a = \frac{15}{150} = 0{,}1$	✓ $\frac{15}{150}$ (A) (1)
11.2.2	$m = 1 - 0{,}7 = 0{,}3$	✓ 0,3 (A) (1)
11.2.3	$0{,}24 + 0{,}14 + 0{,}02 + 0{,}12 + 0{,}1 + 2b = 0{,}7$ $2b = 0{,}08$ $b = 0{,}04$ $0{,}04 \times 150 = 6$	✓ addition ✓ simplification ✓ $b = 0{,}04$ ✓ 6 (4)
11.3.1	9 × 9 × 8 = 648	✓9 ✓9 × 8 (2)
11.3.2	2 × 8 × 4 = 64 2 × 8 × 5 = 80 Total number = 64 + 80 = 144	✓✓2 × 8 × 4 ✓2 × 8 × 5 ✓144 (A) (4)
		[15]

TOTAL/*TOTAAL*: 150

QUESTION/*VRAAG* 8

8.1	$-1 < x < 2$	✓✓ answer (2)
8.2	$x = \frac{-1+2}{2}$ $x = \frac{1}{2}$ **Answer Only: Full Marks**	✓ method ✓ answer (2)
8.3	From the graph $x > \frac{1}{2}$ **Answer Only: Full Marks**	✓✓ answer (2)
8.4	$g(x) = ax^3 + bx^2 + cx$ $g'(x) = 3ax^2 + 2bx + c = -6x^2 + 6x + 12$ $3a = -6.$ $2b = 6$ $c = 12$ $a = -2$ $b = 3$ $g(x) = -2x^3 + 3x^2 + 12x$	✓ $g'(x) = 3ax^2 + 2bx + c$ ✓ $a = -2$ ✓ $b = 3$ ✓ $g(x) = -2x^3 + 3x^2 + 12x$ (4)
8.5	$g'\left(\frac{1}{2}\right) = -6\left(\frac{1}{2}\right)^2 + 6\left(\frac{1}{2}\right) + 12$ $m = \frac{27}{2}$ or 13,5 $y = -2\left(\frac{1}{2}\right)^3 + 3\left(\frac{1}{2}\right)^2 + 12\left(\frac{1}{2}\right)$ $y = \frac{13}{2}$ or 6,5 $y - y_1 = m(x - x_1)$ $y - 6{,}5 = 13{,}5(x - 0{,}5)$ $y = 13{,}5x - 0{,}25$	✓max gradient at $x = \frac{1}{2}$ ✓answer ✓ y value ✓substitution ✓answer (5)
		[15]

QUESTION/*VRAAG* 9

9.1	Total surface area $= 2\ell w + 2wh + 2\ell h$ but: $\ell = 3w$ Total surface area $= 6w^2 + 2wh + 6wh$ $C = 15(6w^2) + 6(2wh + 6wh)$ $= 15(6w^2) + 6(8wh)$ $= 90w^2 + 48wh$	✓ $2\ell w + 2wh + 2\ell h$ ✓ $\ell = 3w$ ✓ $15(6w^2)$ ✓ $6(2wh + 6wh)$ (4)
9.2	$5 = 3w^2 h$ $h = \frac{5}{3w^2}$ $C = 90w^2 + 48wh$ $C(w) = 90w^2 + 48w\left(\frac{5}{3w^2}\right)$ $= 90w^2 + 80w^{-1}$ $C'(w) = 180w - 80w^{-2}$ $180w - 80w^{-2} = 0$ $180w^3 - 80 = 0$ $w^3 = \frac{80}{180}$ $w = \sqrt[3]{\frac{80}{180}}$ $w = 0{,}76$	✓ $h = \frac{5}{3w^2}$ ✓ substitution ✓ $C(w) = 90w^2 + 80w^{-1}$ ✓ derivative ✓ equating derivative to zero ✓ value of w (6)
		[10]

7.4	Substitute (2 ; b) in $y = x + \frac{12}{x}$ $b = 2 + \frac{12}{2}$ $b = 8$ $m_{\text{tangent}} = \frac{dy}{dx}$ $\frac{dy}{dx} = 1 - \frac{12}{x^2}$ $m_{\text{tangent}} = 1 - \frac{12}{2^2} = -2$ $m_{\text{perp}} = \frac{1}{2}$ Equation of perpendicular line: $y - y_1 = m(x - x_1)$ OR $y = mx + c$ $y - 8 = \frac{1}{2}(x - 2)$ $8 = \frac{1}{2}(2) + c$ $y = \frac{1}{2}x + 7$ $c = 7$ $y = \frac{1}{2}x + 7$	✓ value of b ✓ $\frac{dy}{dx} = 1 - \frac{12}{x^2}$ ✓gradient of perpendicular line ✓ equation (4)
		[14]

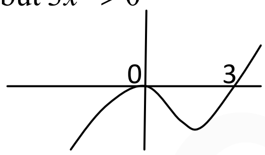
QUESTION/*VRAAG* 8

8.1	36cm	✓answer (1)
8.2	$\therefore t = 6$ $(-2t^2 + 3t - 6)$ have no real roots Insect reaches the floor only once.	✓✓✓ only once (3)
8.3	$h(t) = -2t^3 + 15t^2 - 24t + 36$ $h'(t) = -6t^2 + 30t - 24$ $-6t^2 + 30t - 24 = 0$ $t^2 - 5t + 4 = 0$ $(t-4)(t-1) = 0$ $t = 4$ or $t = 1$ Only $t = 4$ because maximum value required $h = -2(4)^3 + 15(4)^2 - 24(4) + 36 = 52\ cm$	✓ expansion ✓ $-6t^2 + 30t - 24 = 0$ ✓both values ✓answer (4)
		[8]

QUESTION/*VRAAG* 9

9.1	$f^{/}(x)=9x^2$ $3x^3=9x^2$ $3x^3-9x^2=0$ $3x^2(x-3)=0$ $x=0$ or $x=3$	✓ $f^{/}(x)=9x^2$ ✓ $x=0$ ✓ $x=3$ (3)
9.2.1	For f and $f^{/}$	✓ answer (1)
9.2.2	The point (0 ; 0) is : A point of inflection of f A turning point of $f^{/}$	✓ f: inflection point ✓ $f^{/}$: turning point (2)

9.3	$f^{//}(x)=18x$
	Distance $=f^{//}(1)-f^{/}(1)$
	$=18(1)-9(1)^2$
	$=9$
		✓ $f^{//}(x)=18x$
		
		✓substitution
		✓answer (3)

9.4	$3x^3-9x^2<0$ $3x^2(x-3)<0$ but $3x^2>0$  $\therefore x-3<0$ $\therefore x<3$, $x\neq 0$	✓ $3x^3-9x^2<0$ ✓ factors ✓ $x<3$ ✓ $x\neq 0$ (4)
		[13]

QUESTION/*VRAAG* 10

10.1	P(same day) $=\frac{4}{16}$ or $\frac{1}{4}$ or 0,25 or 25%	✓4 numerator ✓16 denominator (2)
10.2	P(2 consecutive days) = $\frac{3\times 2}{16}=\frac{3}{8}$	✓3 ✓$\times 2$ ✓ answer (3)
		[5]

7.2.1	$y = 4x^3 + 2x^{-1}$ $\frac{dy}{dx} = 12x^2 - 2x^{-2}$	✓ $+2x^{-1}$ ✓ $12x^2$ ✓ $-2x^{-2}$ (3)
7.2.2	$y = 4\sqrt[3]{x} + (3x^3)^2$ $= 4x^{\frac{1}{3}} + 9x^6$ $\frac{dy}{dx} = \frac{4}{3}x^{-\frac{2}{3}} + 54x^5$	✓ $4x^{\frac{1}{3}}$ ✓ $9x^6$ ✓ $\frac{4}{3}x^{-\frac{2}{3}}$ ✓ $54x^5$ (4)
7.3	Point of contact: (1 ; 5) $m = 2$ $y - y_1 = m(x - x_1)$ or $y = 2x + c$ $y - 5 = 2(x - 1)$ $5 = 2 + c$ $c = 3$ $y = 2x + 3$ $y = 2x + 3$	✓ $m = 2$ ✓ substitution of (1 ; 5) ✓ answer (3) **[14]**

QUESTION/*VRAAG* 8

8.1	$h(x) = -2(x + \frac{3}{2})(x-1)(x+3)$ $h(x) = -(2x+3)(x^2 + 2x - 3)$ $h(x) = -2x^3 - 7x^2 + 9$ **OR/*OF*** $h(x) = -(2x+3)(x-1)(x+3)$ $h(x) = -(2x+3)(x^2 + 2x - 3)$ $h(x) = -2x^3 - 7x^2 + 9$	✓✓ $-2(x + \frac{3}{2})(x-1)(x+3)$ ✓correct simplification (3) **OR/*OF*** ✓✓ $-(2x+3)(x-1)(x+3)$ ✓ correct simplification (3)
8.2	$h'(x) = -6x^2 - 14x$ $-6x^2 - 14x = 0$ $-2x(3x+7) = 0$ $x = 0$ or $x = -\frac{7}{3}$	✓ first derivative ✓ $= 0$ ✓ both answers (3)
8.3	$x < -\frac{7}{3}$ or $x > 0$ **OR/*OF*** $x \in \left(-\infty ; -\frac{7}{3}\right) \cup (0 ; \infty)$	✓✓ answer (2) **OR/*OF*** ✓✓ answer (2)

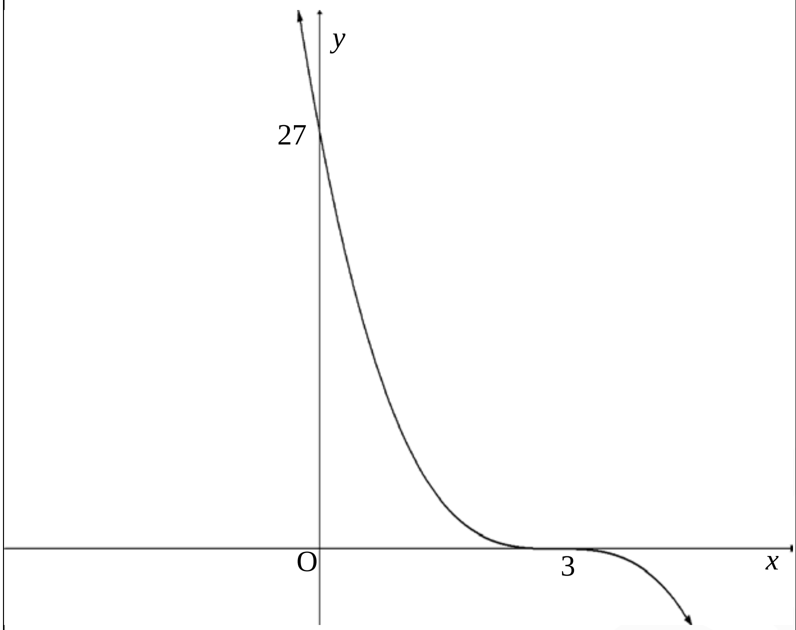
8.4	$y = 4x + 7$ $-6x^2 - 14x = 4$ $0 = 6x^2 + 14x + 4$ $0 = 3x^2 + 7x + 2$ $0 = (3x+1)(x+2)$ $x = -\frac{1}{3}$ or $x = -2$	✓ $y = 4x + 7$ ✓ $h'(x) = 4$ ✓ standard form ✓ both answers (4) **[12]**

QUESTION/*VRAAG* 9

9.1	Volume of Sphere $= \frac{4}{3}\pi(8)^3$ or $= \frac{2048\pi}{3}$ or $= 2144,66$	✓ answer (1)
9.2	$r^2 + x^2 = 8^2$ (Pythagoras) $r^2 = 64 - x^2$	✓ substitution or reason Pythagoras (1)
9.3	$V_{cone} = \frac{1}{3}\pi r^2 h$ $= \frac{1}{3}\pi(64 - x^2)(8 + x)$ $= \frac{\pi}{3}(512 + 64x - 8x^2 - x^3)$ $\frac{dV}{dx} = \frac{64\pi}{3} - \frac{16\pi}{3}x - \frac{3\pi}{3}x^2$ $0 = 64 - 16x - 3x^2$ $0 = (8 - 3x)(x + 8)$ $x = \frac{8}{3}$ $x \neq -8$ $\frac{V_{cone}}{V_{sphere}} = \frac{\frac{1}{3}\pi\left(\frac{512}{9}\right)\left(\frac{32}{3}\right)}{\frac{2048\pi}{3}}$ $= \frac{8}{27} = 0,3$	✓ $h = 8 + x$ ✓ $\frac{1}{3}\pi(64 - x^2)(8 + x)$ ✓ expansion ✓ $\frac{dV}{dx} = \frac{64\pi}{3} - \frac{16\pi}{3}x - \frac{3\pi}{3}x^2$ ✓ $x = \frac{8}{3}$ ✓ volume of the cone ✓ $\frac{8}{27}$ or 0,3 (7) **[9]**

QUESTION/*VRAAG* 9

9.1.1	$g(x)=(x+5)(x-x_1)^2$ $20=5(x_1)^2$ $x_1^{\ 2}=4$ $x_1=2$ $g(x)=(x+5)(x-2)^2$ $g(x)=(x+5)(x^2-4x+4)$ $g(x)=x^3+x^2-16x+20$	✓ $(x+5)$ ✓repeated root ✓$x_1=2$ ✓ $g(x)=(x+5)(x^2-4x+4)$ (4)
9.1.2	$g(x)=x^3+x^2-16x+20$ $g'(x)=3x^2+2x-16$ $3x^2+2x-16=0$ $(3x+8)(x-2)=0$ $x=\frac{-8}{3}$ or $x=2$ $\mathrm{R}\left(\frac{-8}{3};\frac{1372}{27}\right)$ or $\mathrm{R}(-2{,}67;50{,}81)$ $\mathrm{P}(2;0)$	✓derivative ✓equating to zero ✓factors ✓co-ordinates of R ✓ co-ordinates of P (5)
9.1.3	$g''(x)=6x+2$ $g''(0)=2$ $\therefore$ concave up **OR/OF** $g''(x)=6x+2$ $6x+2=0$ $x=-\frac{1}{3}$ is the point of inflection $\therefore$ concave up	✓ $g''(x)=6x+2$ ✓ $g''(0)=2$ ✓conclusion (3) **OR/OF** ✓ $g''(x)=6x+2$ ✓ $x=-\frac{1}{3}$ ✓conclusion (3)

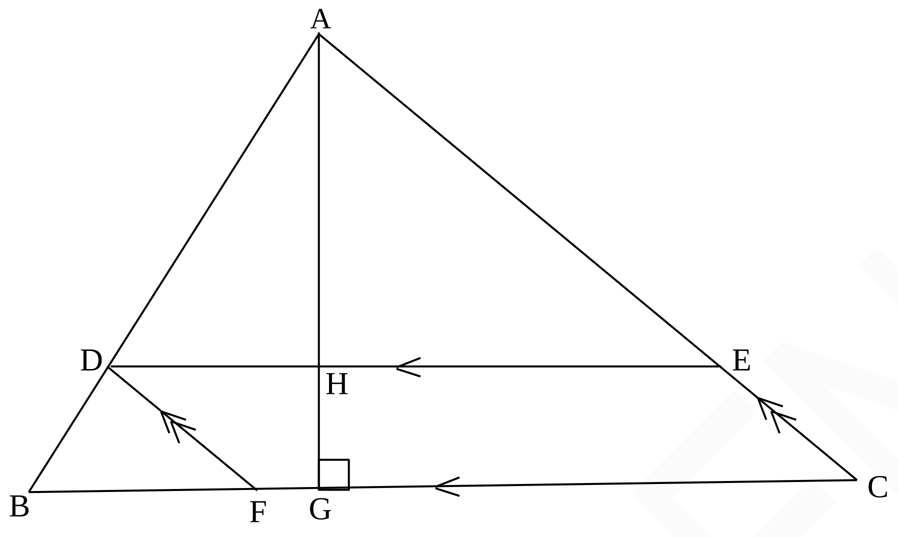
<table>
<tr><td>9.2</td><td>
</td><td>✓ y – intercept of a cubic graph

✓ point of inflection and stationary point, $x = 3$

✓ concave up for $x < 3$ and concave down for $x > 3$

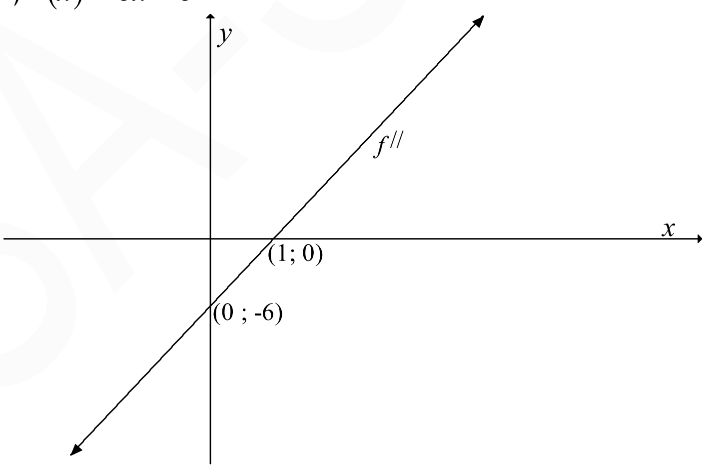
(3)</td></tr>
<tr><td></td><td></td><td>[15]</td></tr>
</table>

QUESTION/*VRAAG* 10



10.1	$\frac{\text{AH}}{\text{HG}} = \frac{3}{2}$	✓ answer (1)
10.2	Area of a parallelogram $= \text{base} \times \perp \text{height}$ Area $= \frac{3}{5}(5-t).\frac{2}{5}t$ Area $= \frac{6}{25}(5-t)t$ $A(t) = -\frac{6}{25}t^2 + \frac{6}{5}t$ $A'(t) = -\frac{12}{25}t + \frac{6}{5}$ $-\frac{12}{25}t + \frac{6}{5} = 0$ $12t - 30 = 0$ $t = \frac{30}{12}$ or $\frac{5}{2}$	✓ $\frac{2}{5}t$ ✓ $\frac{3}{5}(5-t)$ ✓ $A(t) = -\frac{6}{25}t^2 + \frac{6}{5}t$ ✓ $-\frac{12}{25}t + \frac{6}{5}$ ✓answer (5)
		[6]

QUESTION/*VRAAG* 9

9.1	$f(x)=(x+2)(x-1)(x-4)$ $=(x^2+x-2)(x-4)$ $=x^3+x^2-2x-4x^2-4x+8$ $=x^3-3x^2-6x+8$ $b=-3\ ;\ c=-6\ ;\ d=8$	✓✓ $f(x)=(x+2)(x-1)(x-4)$ ✓ expansion ✓ x^3-3x^2-6x+8 (4)
9.2	$f(x)=x^3-3x^2-6x+8$ $f^{/}(x)=0$ $3x^2-6x-6=0$ $x^2-2x-2=0$ $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$ $=\dfrac{2\pm\sqrt{(2)^2-4(1)(-2)}}{2(1)}$ $=\dfrac{2\pm\sqrt{12}}{2}$ $x=-0{,}73$	✓ $f^{/}(x)=0$ ✓ $3x^2-6x-6$ ✓ substitution into correct formula ✓ $x=-0{,}73$ (4)
9.3	$f(x)=x^3-3x^2-6x+8$ $f(-1)=(-1)^3-3(-1)^2-6(-1)+8$ or $f(-1)=(1)(-2)(-5)$ $=10$ $=10$ $f^{/}(-1)=3(-1)^2-6(-1)-6$ $=3$ $y-10=3(x+1)$ $y=3x+13$	✓ $f(-1)=10$ ✓ $f^{/}(-1)=3$ ✓ substitution ✓ $y=3x+13$ (4)
9.4	$f^{//}(x)=6x-6$ 	✓ $f^{//}(x)=6x-6$ ✓x- intercept ✓y- intercept (3)

9.5	f concave upwards $f''(x) > 0$ $6x - 6 > 0$ $x > 1$ NOTE: Answer only 2 / 2	✓ $f''(x) > 0$ ✓ $x > 1$ (2) **[17]**

QUESTION/*VRAAG* 10

	. $f(x) = -3x^3 + x$ $-9x^2 + 1 = 0$ $x = \frac{1}{3}$ or $x = -\frac{1}{3}$ Maximum of f will be at $x = \frac{1}{3}$ $f\left(\frac{1}{3}\right) = -3\left(\frac{1}{3}\right)^3 + \left(\frac{1}{3}\right)$ $= \frac{2}{9}$ Maximum of $f(x) + q$ will also be at $x = \frac{1}{3}$ $f\left(\frac{1}{3}\right) + q = \frac{8}{9}$ $\frac{2}{9} + q = \frac{8}{9}$ $q = \frac{6}{9}$ $= \frac{2}{3}$ For $f(x) + q$ to have a maximum of $\frac{8}{9}$ the value of q has to be $\frac{2}{3}$.	✓ $-9x^2 + 1 = 0$ ✓ $x = \frac{1}{3}$ *or* $x = -\frac{1}{3}$ ✓ Maximum at $x = \frac{1}{3}$ ✓ $f\left(\frac{1}{3}\right) = \frac{2}{9}$ ✓ $\frac{2}{9} + q = \frac{8}{9}$ ✓ $q = \frac{2}{3}$ **[6]**

QUESTION/*VRAAG* 8

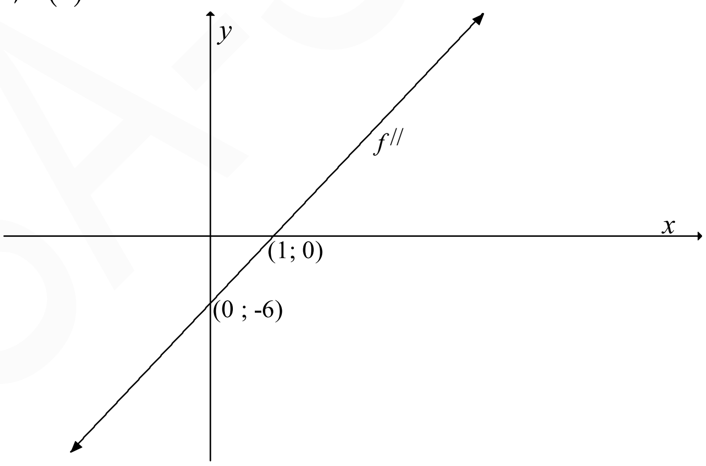
8.1	$C(0;12)$	✓ $C(0;12)$ (1)
8.2	$-x^3+13x+12=0$ $x^3-13x-12=0$ $(x+1)(x^2-x-12)=0$ $(x+1)(x-4)(x+3)=0$ $A(-3;0)$ $B(4;0)$	✓ $f(x)=0$ ✓ $(x+1)$ ✓ (x^2-x-12) ✓ $x=-3$ or 4 ✓ clearly indicating A and B (5)
8.3	$f'(x)=-3x^2+13$ $f''(x)=-6x$ $-6x=0$ $x=0$ For $f(x)$, point of inflection will be at (0 ; 12). *Vir* $f(x)$, *sal buigpunt wees by* (0 ; 12) For $g(x)$, point of inflection will be at (0 ; –12). Vir $g(x)$, *sal buigpunt wees by* (0 ; –12). **OR/OF** $g(x)=x^3-13x-12$ $g'(x)=3x^2-13$ $g''(x)=6x$ $6x=0$ $x=0$ $(0;-12)$ **OR/*OF*** $f'(x)=-3x^2+13$ TP's where $-3x^2+13=0$ $x^2=\frac{13}{3}$ $x=\pm\sqrt{\frac{13}{3}}$ $=\pm 2{,}08$ x-value of point of inflection: $\frac{-2{,}08+2{,}08}{2}=0$ For $f(x)$, point of inflection will be at (0 ; 12). *Vir* $f(x)$, *sal buigpunt wees by* (0 ; 12) For $g(x)$, point of inflection will be at (0 ; –12). Vir $g(x)$, *sal buigpunt wees by* (0 ; –12).	✓ $f'(x)=-3x^2+13$ ✓ $f''(x)=-6x$ ✓ equating to zero ✓ $(0;-12)$ (4) **OF/OR** ✓ $g'(x)=3x^2-13$ ✓ $g''(x)=6x$ ✓ equating to zero ✓ $(0;-12)$ (4) **OR/*OF*** ✓ $f'(x)=-3x^2+13$ ✓ $-3x^2+13=0$ ✓ x-values of TPs ✓ $(0;-12)$ (4)

8.4	$f'(x) = -3x^2 + 13$ $-3x^2 + 13 = -14$ $-3x^2 = -27$ $x^2 = 9$ $x = 3$ or $x = -3$	✓ equating derivative to – 14 ✓ simplification ✓✓ answers (4) **[14]**

QUESTION/*VRAAG* 9

9.1.1	AC = $t-30$	✓answer (1)
9.1.2	$30^2=(t-30)^2+p^2$ [Pythagoras] $p^2=900-(t-30)^2$ $p^2=900-(t^2-60t+900)$ $p^2=900-t^2+60t-900$ $p^2=60t-t^2$	✓ $p^2=900-(t-30)^2$ ✓ $(t^2-60t+900)$ ✓ $p^2=60t-t^2$ (3)
9.2	$V(t)=\frac{1}{3}\pi r^2 t$ $=\frac{1}{3}\pi(60t-t^2)t$ $=20\pi t^2-\frac{1}{3}\pi t^3$	✓ substitution (1)
9.3	$V(t)=20\pi t^2-\frac{1}{3}\pi t^3$ $V'(t)=40\pi t-\pi t^2$ $40\pi t-\pi t^2=0$ $t(40\pi-t\pi)=0$ $t=0$ OR $t=40$ cm N/A	✓ $40\pi t$ ✓ $-\pi t^2$ ✓ answer with selection (3)
9.4	Volume of cone/keël $=20(\pi)(40)^2-\frac{1}{3}\pi(40)^3$ $=10\ 666{,}67\pi$ or $33510{,}33211$ Volume of sphere/sfeer $=\frac{4}{3}\pi r^3$ $=\frac{4}{3}\pi(30)^3$ $=36000\pi$ or $113097{,}3355$ $\frac{10666{,}67\pi}{36000\pi}$ = 0,296296 ≈ 29,63%	✓ volume of cone ✓ volume of sphere ✓ $\frac{10666{,}67\pi}{36000\pi}$ ✓ % cut out (4) **[12]**

QUESTION/*VRAAG* 9

9.1	$f(x)=(x+2)(x-1)(x-4)$ $=(x^2+x-2)(x-4)$ $=x^3+x^2-2x-4x^2-4x+8$ $=x^3-3x^2-6x+8$ $b=-3$; $c=-6$; $d=8$	✓✓ $f(x)=(x+2)(x-1)(x-4)$ ✓ expansion ✓ x^3-3x^2-6x+8 (4)
9.2	$f(x)=x^3-3x^2-6x+8$ $f'(x)=0$ $3x^2-6x-6=0$ $x^2-2x-2=0$ $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ $=\frac{2\pm\sqrt{(2)^2-4(1)(-2)}}{2(1)}$ $=\frac{2\pm\sqrt{12}}{2}$ $x=-0{,}73$	✓ $f'(x)=0$ ✓ $3x^2-6x-6$ ✓ substitution into correct formula ✓ $x=-0{,}73$ (4)
9.3	$f(x)=x^3-3x^2-6x+8$ $f(-1)=(-1)^3-3(-1)^2-6(-1)+8$ or $f(-1)=(1)(-2)(-5)$ $=10$ $=10$ $f'(-1)=3(-1)^2-6(-1)-6$ $=3$ $y-10=3(x+1)$ $y=3x+13$	✓ $f(-1)=10$ ✓ $f'(-1)=3$ ✓ substitution ✓ $y=3x+13$ (4)
9.4	$f''(x)=6x-6$ 	✓ $f''(x)=6x-6$ ✓x- intercept ✓y- intercept (3)

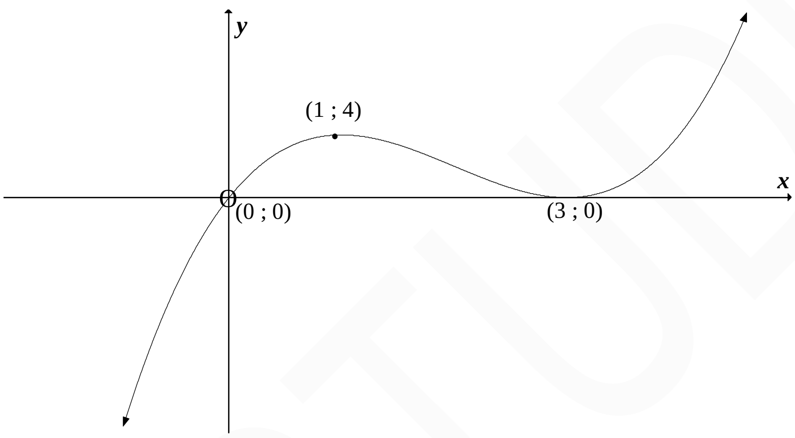
9.5	f concave upwards $f''(x) > 0$ $6x - 6 > 0$ $x > 1$ NOTE: Answer only 2 / 2	✓ $f''(x) > 0$ ✓ $x > 1$ (2) **[17]**

QUESTION/*VRAAG* 10

	. $f(x) = -3x^3 + x$ $-9x^2 + 1 = 0$ $x = \frac{1}{3}$ or $x = -\frac{1}{3}$ Maximum of f will be at $x = \frac{1}{3}$ $f\left(\frac{1}{3}\right) = -3\left(\frac{1}{3}\right)^3 + \left(\frac{1}{3}\right)$ $= \frac{2}{9}$ Maximum of $f(x) + q$ will also be at $x = \frac{1}{3}$ $f\left(\frac{1}{3}\right) + q = \frac{8}{9}$ $\frac{2}{9} + q = \frac{8}{9}$ $q = \frac{6}{9}$ $= \frac{2}{3}$ For $f(x) + q$ to have a maximum of $\frac{8}{9}$ the value of q has to be $\frac{2}{3}$.	✓ $-9x^2 + 1 = 0$ ✓ $x = \frac{1}{3}$ *or* $x = -\frac{1}{3}$ ✓ Maximum at $x = \frac{1}{3}$ ✓ $f\left(\frac{1}{3}\right) = \frac{2}{9}$ ✓ $\frac{2}{9} + q = \frac{8}{9}$ ✓ $q = \frac{2}{3}$ **[6]**

QUESTION/*VRAAG* 8

8.1	$f(x)=x^3-6x^2+9x$
	$f^{/}(x)=3x^2-12x+9$
	$f^{//}(x)=6x-12=0$
	$x=2$
	$f^{//}(0)=6(0)-12$
	$=-12$
	$f^{//}(3)=6(3)-12$
	$=6$
	$f^{//}(x)<$ | $f^{//}(x)>0$; 2
	Point of inflection at $x=2$
		✓ x^3-6x^2+9x
		✓ $3x^2-12x+9$
		✓ $6x-12$
		✓ $6x-12=0$
		✓ explanation
		(5)

8.2	 	✓shape ✓ (0 ; 0) ✓ (3 ; 0) as TP ✓(1 ; 4) (4)
8.3	f concave up for $x>2$ $y=-f(x)$ will be concave down for $x>2$	✓✓ $x>2$ (2)
8.4.1	$(3;7)$	✓ 3 ✓7 (2)
8.4.2	Do not agree with Claire as her statement is incorrect. Between $x=1$ and $x=3$ the graph of f is decreasing. Therefore at $x=2$ the gradient will have a negative value. *Stem nie saam met Claire nie, want haar stelling in verkeerd.* *Die grafiek van f is dalend/afnemend tussen* $x=1$ *en* $x=3$. *By $x=2$ moet die gradiënt dus 'n negatiewe waarde hê.* **OR/*OF*** $f^{/}(2)=3(2)^2-12(2)+9$ $=-3$ $\neq 1$	✓no ✓justification (2) **[15]**

QUESTION/*VRAAG* 9

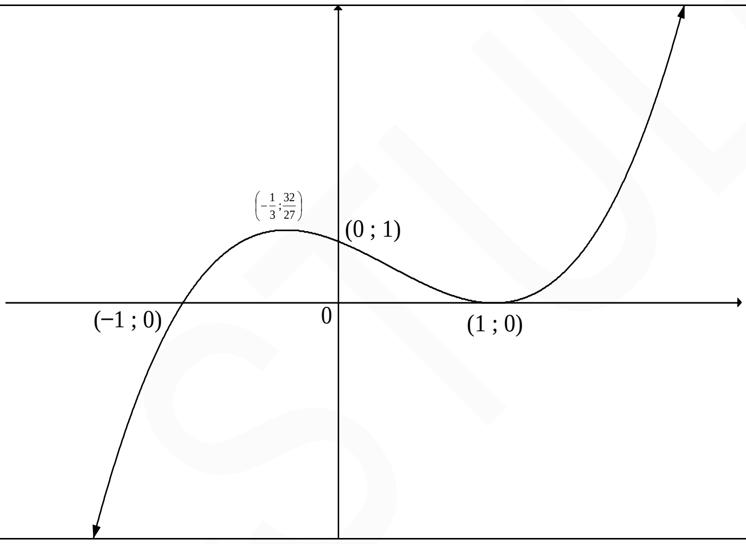
	$y = x^2 + 2$ $P(x\,;x^2+2)$ $B(0\,;3)$ $PB^2 = (x-0)^2 + (x^2+2-3)^2$ $= x^2 + x^4 - 2x^2 + 1$ $= x^4 - x^2 + 1$ PB will be a minimum if PB^2 is a minimum $\frac{d(PB^2)}{dx} = 4x^3 - 2x$ $4x^3 - 2x = 0$ $x(2x^2-1) = 0$ $x = 0$ or $x^2 = \frac{1}{2}$ $x = \frac{1}{\sqrt{2}}$ $PB^2 = \left(\frac{1}{\sqrt{2}}\right)^4 - \left(\frac{1}{\sqrt{2}}\right)^2 + 1$ $= \frac{1}{4} - \frac{1}{2} + 1$ $= \frac{3}{4}$ $PB = \frac{\sqrt{3}}{2} = 0{,}87$ **OR/*OF***	✓ $(x-0)^2 + (x^2+2-3)^2$ ✓ $x^4 - x^2 + 1$ ✓ $4x^3 - 2x$ ✓ $\frac{d(PB^2)}{dx} = 0$ ✓ $x = \frac{1}{\sqrt{2}}$ ✓ $PB^2 = \left(\frac{1}{\sqrt{2}}\right)^4 - \left(\frac{1}{\sqrt{2}}\right)^2 + 1$ ✓ answer **OR/*OF***

	Gradient of tangent to curve $=2x$ Gradient of line joining B and the curve $=\dfrac{x^2+2-3}{x-0}$ $=\dfrac{x^2-1}{x}$ Shortest distance will be where tangent to curve is perpendicular to the line joining P and the curve. $\dfrac{x^2-1}{x}=-\dfrac{1}{2x}$ $2x(x^2-1)=-x$ $2x^3-2x=0$ $x(2x^2-1)=0$ $x=0$ or $x^2=\dfrac{1}{2}$ $x=\dfrac{1}{\sqrt{2}}$ $PB^2=\left(\dfrac{1}{\sqrt{2}}\right)^4-\left(\dfrac{1}{\sqrt{2}}\right)^2+1$ $=\dfrac{1}{4}-\dfrac{1}{2}+1$ $=\dfrac{3}{4}$ $PB=\dfrac{\sqrt{3}}{2}=0,87$	✓ $=2x$ ✓ $=\dfrac{x^2-1}{x}$ ✓ $\dfrac{x^2-1}{x}=-\dfrac{1}{2x}$ ✓ $2x^3-2x=0$ ✓ $x=\dfrac{1}{\sqrt{2}}$ ✓ $PB^2=\left(\dfrac{1}{\sqrt{2}}\right)^4-\left(\dfrac{1}{\sqrt{2}}\right)^2+1$ ✓ answer
	OR/*OF*	**OR/*OF***
	$P(k;k^2+2)$ and B(0; 3) $BP\perp$ tangent passing through $y=x^2+2$ at P. $m_{\text{tangent at P}}=2k$ $m_{BP}=-\dfrac{1}{2k}$ Equation of BP: $y=\left(-\dfrac{1}{2k}\right)x+3$ $y_P=\left(-\dfrac{1}{2k}\right)(k)+3=2,5$ $\Rightarrow k^2+2=2,5$ and so $k=\sqrt{0,5}$ and $P(\sqrt{0,5};\ 2,5)$ $BP=\sqrt{\left(\sqrt{0,5}-0\right)^2+(2,5-3)^2}=\sqrt{\dfrac{3}{4}}=\dfrac{\sqrt{3}}{2}=0,87$	✓ $P(k;k^2+2)$ ✓ $m_{\text{tangent at P}}=2k$ ✓ $m_{BP}=-\dfrac{1}{2k}$ ✓ $y=\left(-\dfrac{1}{2k}\right)x+3$ ✓ value of y at P ✓ value of k ✓ answer **[7]**

8.3	$y = x^3 + bx^2 + cx - 4$ $y' = 3x^2 + 2bx + c$ $y'' = 6x + 2b$ At point of inflection: $y'' = 6x + 2b = 0$ Substitute $x = 2$: $6(2) + 2b = 0$ $2b = -12$ $b = -6$ $y = x^3 - 6x^2 + cx - 4$ Substitute $(2; 4)$: $4 = 2^3 - 6(2)^2 + c(2) - 4$ $2c = 24$ $c = 12$ $y = x^3 - 6x^2 + 12x - 4$	✓ $y' = 3x^2 + 2bx + c$ ✓ $y'' = 6x + 2b$ ✓ $y'' = 0$ ✓ sub $x = 2$ into $y'' = 0$ ✓ value of b ✓ substitute $(2;4)$ ✓ value of c (7) **[16]**

QUESTION/*VRAAG* 9

9.1	(0 ; 1)	✓ answer (1)
9.2	$f(x) = x^3 - x^2 - x + 1$ $f(x) = x^2(x-1) - (x-1)$ $f(x) = (x-1)(x^2-1)$ $f(x) = (x-1)(x-1)(x+1)$ $f(x) = 0$ $(x-1)(x-1)(x+1) = 0$ x-intercepts: (–1; 0); (1; 0) **OR** $f(x) = x^3 - x^2 - x + 1$ $f(x) = (x-1)(x^2-1)$ $f(x) = (x-1)(x-1)(x+1)$ $f(x) = 0$ $(x-1)(x-1)(x+1) = 0$ x-intercepts: (–1; 0); (1; 0) **OR**	✓ $(x-1)$ ✓ (x^2-1) ✓ $(x-1)(x-1)(x+1)$ ✓(–1; 0) ✓(1; 0) (5) ✓ $(x-1)$ ✓ (x^2-1) ✓ $(x-1)(x-1)(x+1)$ ✓(–1; 0) ✓(1; 0) (5)

	$f(x) = x^3 - x^2 - x + 1$ $f(x) = (x+1)(x^2 - 2x + 1)$ $f(x) = (x+1)(x-1)(x-1)$ $f(x) = 0$ $(x-1)(x-1)(x+1) = 0$ x-intercepts: (–1; 0); (1; 0)	✓ $(x + 1)$ ✓ $(x^2 - 2x + 1)$ ✓ $(x-1)(x-1)(x+1)$ ✓(–1; 0) ✓(1; 0) (5)
9.3	$f(x) = x^3 - x^2 - x + 1$ $f'(x) = 3x^2 - 2x - 1$ $f'(x) = 0$ $(3x+1)(x-1) = 0$ $x = -\frac{1}{3}$ or $x = 1$ $y = \frac{32}{27}$ $y = 0$ $\left(-\frac{1}{3}; \frac{32}{27}\right)$ $(1; 0)$	✓ $f'(x) = 3x^2 - 2x - 1$ ✓ $f'(x) = 0$ ✓factorisation ✓x value ✓x value ✓$y = \frac{32}{27}$ (6)
9.4		✓y– and x–intercepts ✓shape ✓turning points (3)
9.5	$f'(x) < 0$ $-\frac{1}{3} < x < 1$ **OR/*OF*** $(-\frac{1}{3}; 1)$	✓ $x > -\frac{1}{3}$ ✓ $x < 1$ (2) ✓ $(-\frac{1}{3}$ ✓ $1)$ (2) **[17]**

QUESTION/*VRAAG* 10

10.1	$60 = 2b + 2r + \frac{1}{2}(2\pi r)$ $2b = 60 - 2r - \pi r$ $b = 30 - r - \frac{1}{2}\pi r$	✓ $60 = 2b + 2r + \frac{1}{2}(2\pi r)$ ✓ $b = 30 - r - \frac{1}{2}\pi r$ (2)
10.2	Area = area of rectangle + area of semicircle $A(r) = \text{length} \times \text{breadth} + \frac{1}{2}(\text{area of circle})$ $= (2r)\left(30 - r - \frac{1}{2}\pi r\right) + \frac{1}{2}(\pi r^2)$ $= 60r - 2r^2 - \pi r^2 + \frac{1}{2}\pi r^2$ $= 60r - 2r^2 - \frac{1}{2}\pi r^2$ $= 60r - \left(2 + \frac{1}{2}\pi\right)r^2$ For a maximum, $A^{/}(r) = 0$ $60 - 2\left(2 + \frac{1}{2}\pi\right)r = 0$ $60 - (4 + \pi)r = 0$ $r = \frac{60}{4 + \pi}$ $= 8{,}40$ m	✓ $(2r)\left(30 - r - \frac{1}{2}\pi r\right)$ ✓ $\frac{1}{2}(\pi r^2)$ ✓ $60r - 2r^2 - \frac{1}{2}\pi r^2$ ✓ $A^{/}(r) = 0$ ✓ $60 - 2\left(2 + \frac{1}{2}\pi\right)r$ ✓answer (6) **[8]**

QUESTION/*VRAAG* 8

Penalise candidates a maximum of one mark (overall) for notation error in 8.1 and 8.2

8.1	$\underbrace{f'(x) = \lim_{h\to 0}}_{\text{Notation}} \underbrace{\frac{f(x+h)-f(x)}{h}}_{\text{formula}}$ Formula can be implied
8.2	• If function and derivative is mixed but splitting of fractions is evident max ¾ • If they start with differentiation – breakdown 0/4
8.3	$y'' = 0$ can be implied

QUESTION/*VRAAG* 9

9.2	No working is shown(calculator used) • If the cubic becomes a quadratic 2/5 • If three brackets 5/5
9.3	$f'(x) = 0$ cannot be implied $f'(x) = 3x^2 - 2x - 1$ $x = -\left(\frac{-2}{2(3)}\right)$ **BE CAREFUL** 1/6 for derivative $= \frac{1}{3}$
9.4	If dots only indicated on the graph 1/3 – x and y-intercepts
9.5	Only CA from a cubic graph Each answer gets evaluated independently

QUESTION/*VRAAG* 10

10.2	Derivative equal to zero is an independent mark $A'(r) = 0$ can be implied if working is correct

QUESTION/*VRAAG* 11

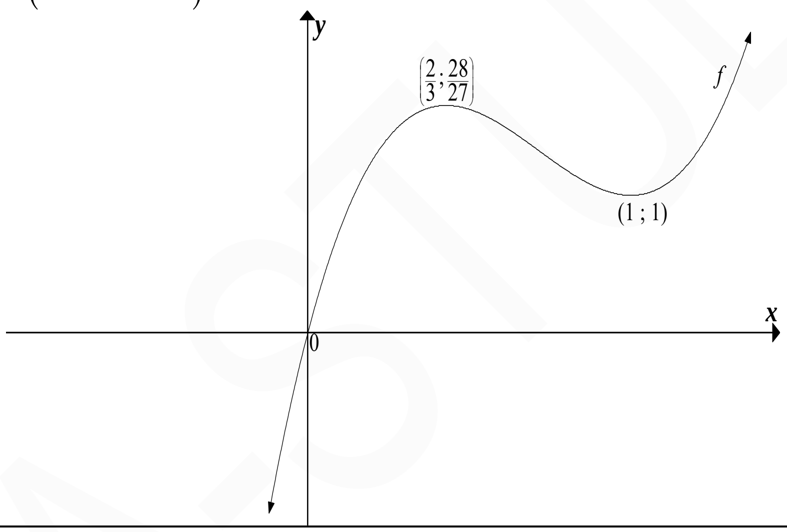
If percentages are used – penalize once per question

11.1	Answer only 2/2 2 or 0 marks
11.3.2	Do not penalize rounding
11.3.3	Do not penalize rounding

7.3	$h(x) = ax^2,\ x > 0$ $h^{-1}:\ x = ay^2 \quad y > 0$ $y = \sqrt{\frac{x}{a}}$ $h^{-1}(8) = \sqrt{\frac{8}{a}}$ $h^{/}(x) = 2ax$ $h^{/}(4) = 2a(4)$ $= 8a$ $\sqrt{\frac{8}{a}} = 8a$ $64a^2 = \frac{8}{a}$ $a^3 = \frac{1}{8}$ $a = \frac{1}{2}$	✓ $y = \sqrt{\frac{x}{a}}$ ✓ $\sqrt{\frac{8}{a}}$ ✓ $h^{/}(4) = 8a$ ✓ $\sqrt{\frac{8}{a}} = 8a$ ✓ $a^3 = \frac{1}{8}$ ✓ $a = \frac{1}{2}$ (6)
		[14]

QUESTION/*VRAAG* 8

8.1	$f^{/}(x) = 0$ $6x^2 - 10x + 4 = 0$ $3x^2 - 5x + 2 = 0$ $(3x-2)(x-1) = 0$ $x = \frac{2}{3}$ or $x = 1$ $y = 2\left(\frac{2}{3}\right)^3 - 5\left(\frac{2}{3}\right)^2 + 4\left(\frac{2}{3}\right)$ $\quad y = 2(1)^3 - 5(1)^2 + 4(1)$ $y = \frac{28}{27}$ or $y = 1$ Turning points are $\left(\frac{2}{3}\,;\frac{28}{27}\right)$ and $(1\,;1)$	✓ derivative ✓ derivative $= 0$ ✓ factors ✓ x-values ✓ y-values (5)

8.2	$2x^3 - 5x^2 + 4x = 0$ $x(2x^2 - 5x + 4) = 0$ $x = 0 \quad \text{or} \quad x = \dfrac{5 \pm \sqrt{25 - 4(2)(4)}}{4}$ $= \dfrac{5 \pm \sqrt{-7}}{4}$ No real roots / *Geen reële wortels* **OR / OF** $2x^3 - 5x^2 + 4x = 0$ $x(2x^2 - 5x + 4) = 0$ $x = 0 \quad \text{or} \quad b^2 - 4ac = 25 - 4(2)(4)$ $= -7 < 0$ No real roots / *Geen reële wortels*	✓ $x(2x^2 - 5x + 4) = 0$ ✓ $x = 0$ ✓ $\dfrac{5 \pm \sqrt{-7}}{4}$ (3) ✓ $x(2x^2 - 5x + 4) = 0$ ✓ $x = 0$ ✓ $b^2 - 4ac < 0$ (3)
8.3	$f(x) = 2x^3 - 5x^2 + 4x$ $x(2x^2 - 5x + 4) = 0$  	✓(0 ; 0) ✓turning points ✓shape (3)

8.4	$f(x)=2x^3-5x^2+4x$
	$f^{/}(x)=6x^2-10x+4$
	$f^{//}(x)=12x-10$
	$f^{//}(x)>0$
	$12x-10>0$
	$x>\frac{5}{6}$
	
	OR
	Point of inflection: $x=-\frac{b}{3a}$
	$x=-\frac{(-5)}{3(2)}$
	$x=\frac{5}{6}$
	The function is concave up for $x>\frac{5}{6}$ since $a>0$
	
	OR
	Point of inflection: $x=\frac{\frac{2}{3}+1}{2}$
	$x=\frac{5}{6}$
	The function is concave up for $x>\frac{5}{6}$ since $a>0$
		✓ $12x-10$
		✓ $f^{//}(x)>0$
		✓ answer
		(3)
		
		✓ $x=-\frac{(-5)}{3(2)}$
		✓ $x=\frac{5}{6}$
		✓ $f^{//}(x)>0$
		(3)
		
		✓ $x=\frac{\frac{2}{3}+1}{2}$
		✓ $x=\frac{5}{6}$
		✓ $f^{//}(x)>0$
		(3)
		[14]

QUESTION/*VRAAG* 9

9.	Length of one side of the square / *lengte van sy van vierkant* $=\frac{x}{4}$ Length of the rectangle / *lengte van die reghoek*: $2l+x+\frac{x}{4}=6$ $l=\frac{6-\frac{5x}{4}}{2}$ $=\frac{24-5x}{8}$ $A=\left(\frac{x}{4}\right)^2+\frac{x}{4}\left(\frac{24-5x}{8}\right)$ $=\frac{x^2}{16}+\frac{24x-5x^2}{32}$ $=\frac{24x-3x^2}{32}$ $A=\frac{24x-3x^2}{32}.$ For minimum area / *Vir minimum oppervlakte* $\frac{dA}{dx}=0$ $\frac{dA}{dx}=\frac{24-6x}{32}$ $6x=24$ $x=4$	✓ $\frac{x}{4}$ ✓ $\frac{6-\frac{5x}{4}}{2}$ or $\frac{24-5x}{8}$ ✓ $\left(\frac{x}{4}\right)^2$ ✓ $\frac{x}{4}\left(\frac{24-5x}{8}\right)$ ✓ $\frac{dA}{dx}=0$ ✓ $\frac{24-6x}{32}$ ✓ $x=4$ (7)
		[7]

QUESTION/*VRAAG* 10

10.1.1	$P(S \text{ and } T) = P(S) \times P(T)$ $\frac{1}{6} = \left(\frac{1}{4}\right) \times P(T)$ $P(T) = \frac{2}{3}$	✓ $P(S \text{ and } T) = P(S) \times P(T)$ ✓ $P(T) = \frac{2}{3}$ (2)
10.1.2	$P(S \text{ or } T) = P(S) + P(T) - P(S \text{ and } T)$ $= \left(\frac{1}{4}\right) + \left(\frac{2}{3}\right) - \frac{1}{6}$ $= \frac{3}{4}$	✓ $\left(\frac{1}{4}\right) + \left(\frac{2}{3}\right) - \frac{1}{6}$ ✓ $\frac{3}{4}$ (2)
10.2.1	$5!$ $= 120$	✓ 5 ✓ $5!$ or 120 (2)
10.2.2	5^5 $= 3125$	✓ 5^5 or 3 125 (1)
10.3	$n(E) = 5! \times 2! \times 2!$ $n(S) = 7!$ $P(E) = \frac{5! \times 2! \times 2!}{7!}$ $= \frac{2}{21}$	✓ $5!$ ✓ $2! \times 2!$ ✓ $\frac{5! \times 2! \times 2!}{7!}$ ✓ $\frac{2}{21}$ (4)
		[11]

QUESTION/*VRAAG* 9

9.1	$f'(x)=3x^2+8x-3=0$ $(3x-1)(x+3)=0$ $x=\frac{1}{3}$ *or* $x=-3$	✓ equating derivative to zero ✓ factors ✓ x – values (3)
9.2	$f''(x)=6x+8$ $6x+8<0$ $x<-\frac{4}{3}$ **OR** $x=\dfrac{\frac{1}{3}-3}{2}$ $=\frac{4}{3}$ $\therefore x<-\frac{4}{3}$	✓ $6x+8$ ✓✓ $x<-\frac{4}{3}$ (3) ✓ $\dfrac{\frac{1}{3}-3}{2}$ ✓✓ $x<-\frac{4}{3}$ (3)
9.3	$x\le -3$ or $x\ge \frac{1}{3}$ **OR/OF** $[-\infty;-3]\cup\left[\frac{1}{3};\infty\right]$	✓ $x\le -3$ ✓ $x\ge\frac{1}{3}$ (2) ✓ $[-\infty;-3]$ ✓ $\left[\frac{1}{3};\infty\right]$ (2)
9.4	$f(0)=-18$ $d=-18$ $f(x)=ax^3+bx^2+cx-18$ $f'(x)=3ax^2+2bx+c$ $f'(x)=3x^2+8x-3$ $3a=3$ $2b=8$ $a=1$ $b=4$ $c=-3$ $f(x)=x^3+4x^2-3x-18$ **OR/OF** $f'(x)=3x^2+8x-3$ By integration/*Deur integrasie* $f(x)=x^3+4x^2-3x+d$ $f(0)=d=-18$ $a=1$ $b=4$ $c=-3$	✓ $d=-18$ ✓ $f'(x)=3ax^2+2bx+c$ ✓ $a=1$ ✓ $b=4$ ✓ $c=-3$ (5) ✓ $f(x)=x^3+4x^2-3x+d$ ✓ $d=-18$ ✓ $a=1$ ✓ $b=4$ ✓ $c=-3$ (5)
		[13]

QUESTION/*VRAAG* 10

10.1	$M(t) = -t^3 + 3t^2 + 72t$ $M(3) = -(3)^3 + 3(3)^2 + 72(3)$ $= 216$ 216 molecules/*molekules*	✓ $M(3) = -(3)^3 + 3(3)^2 + 72(3)$ ✓ 216 (2)
10.2	$M(t) = -t^3 + 3t^2 + 72t$ $M^{/}(t) = -3t^2 + 6t + 72$ $M^{/}(2) = -3(2)^2 + 6(2) + 72$ $= 72$ 72 molecules per hour/*molekules per uur*	✓ $M^{/}(t) = -3t^2 + 6t + 72$ ✓ $M^{/}(2)$ ✓ 72 (3)

10.3	$M(t) = -t^3 + 3t^2 + 72t$
	$M^{/}(t) = -3t^2 + 6t + 72$
	$M^{//}(t) = 0$
	$-6t + 6 = 0$
	$t = 1$
	Maximum rate of change of the number of molecules of the drug in the bloodstream is after 1 hour./*Maksimum tempo van verandering van die getal molekules in die bloedstroom is na 1 uur*
		✓ $M^{//}(t)$
		✓ $M^{//}(t) = 0$
		
		✓ answer
		
		(3)
		[8]

QUESTION/*VRAAG* 8

8.1	$y = 12$	✓answer (1)
8.2	$12 = (0-2)^2(0-k)$ $k = -3$ $(x-2)^2(x+3) = 0$ $x = -3$ **OR/OF** $y = 0$ $(x-2)^2(x-k) = 0$ $(x^2 - 4x + 4)(x-k) = 0$ $x^3 - kx^2 - 4x^2 + 4kx + 4x - 4k = 0$ But $-4k$ is the y-intercept *Maar* $-4k$ *is die y-afsnit* $-4k = 12$ $k = -3$ $x = -3$	✓ substituting (0;12) ✓ $k = -3$ ✓ $x = -3$ (3) ✓$-4k$ ✓ $-4k = 12$ or $k = -3$ ✓ $x = -3$ (3)
8.3	$f(x) = x^3 + 3x^2 - 4x^2 - 12x + 4x + 12$ $f(x) = x^3 - x^2 - 8x + 12$ $f'(x) = 3x^2 - 2x - 8$ $3x^2 - 2x - 8 = 0$ $(3x+4)(x-2) = 0$ $x = -\frac{4}{3}$ or $x = 2$ $y = \frac{500}{27}$ or $18{,}52$ or $18\frac{14}{27}$ $C\left(-\frac{4}{3}; 18{,}52\right)$	✓ $f(x) = x^3 - x^2 - 8x + 12$ ✓derivative ✓derivative equal to 0 ✓factors or formula ✓ $x = -\frac{4}{3}$ ✓ $y = \frac{500}{27}$ or 18,52 or $18\frac{14}{27}$ (6)

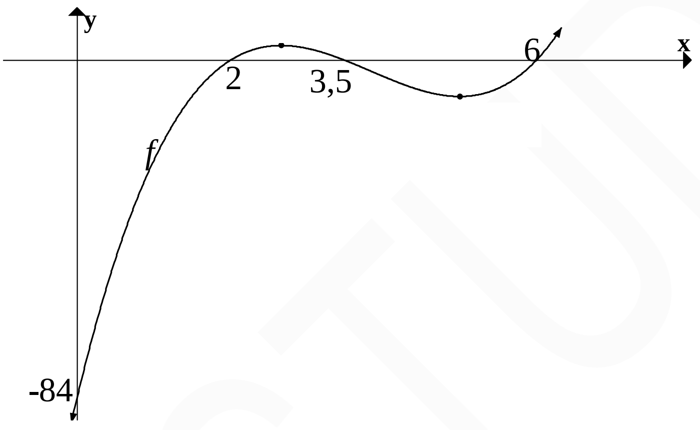
8.4	$f''(x) = 6x - 2$ $6x - 2 < 0$ $x < \frac{1}{3}$ f is concave down when $x < \frac{1}{3}$ *f is konkaaf na onder vir $x < \frac{1}{3}$*	✓ $6x - 2$ ✓✓ $x < \frac{1}{3}$ (3)
	OR/OF	
	$f''(x) = 6x - 2$ $6x - 2 = 0$ $x = \frac{1}{3}$ f is concave down when $x < \frac{1}{3}$ *f is konkaaf na onder vir $x < \frac{1}{3}$*	✓ $6x - 2$ ✓✓ $x < \frac{1}{3}$ (3)
	OR/OF	
	$x = \frac{x_c + x_d}{2}$ $= \frac{-\frac{4}{3} + 2}{2}$ $= \frac{1}{3}$ or/of $x = -\frac{b}{3a}$ $= -\frac{-1}{3(1)}$ $= \frac{1}{3}$ f is concave down when $x < \frac{1}{3}$ *f is konkaaf na onder vir $x < \frac{1}{3}$*	✓ $\frac{-\frac{4}{3} + 2}{2}$ or $-\frac{-1}{3(1)}$ ✓✓ $x < \frac{1}{3}$ (3) **[13]**

QUESTION/*VRAAG* 9

9.1	$V = \pi r^2 h$ $\pi r^2 h = 340$ $h = \dfrac{340}{\pi r^2}$	✓formula ✓equating to 340 ✓ $h = \dfrac{340}{\pi r^2}$ (3)
9.2	$A = 2\pi r^2 + 2\pi rh$ $= 2\pi r^2 + 2\pi r\left(\dfrac{340}{\pi r^2}\right)$ $= 2\pi r^2 + \dfrac{680}{r}$ $A'(r) = 4\pi r - \dfrac{680}{r^2}$ $A'(r) = 0$ for minimum surface area/ *vir minimum buite-oppervlakte* $4\pi r - \dfrac{680}{r^2} = 0$ $r^3 = \dfrac{680}{4\pi} = \dfrac{170}{\pi}$ $= 54{,}11268$ $r = 3{,}78$ cm	✓ $2\pi r^2 + 2\pi rh$ ✓substituting h ✓ $4\pi r - \dfrac{680}{r^2}$ ✓ $A'(r) = 0$ ✓ $r^3 = \dfrac{680}{4\pi}$ ✓answer (6) **[9]**

QUESTION/*VRAAG* 8

8.1	$f(x+h) = -(x+h)^2 + 4 = -(x^2 + 2xh + h^2) + 4$ $= -x^2 - 2xh - h^2 + 4$ $f(x+h) - f(x) = -2xh - h^2$ $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ $= \lim_{h \to 0} \frac{-2xh - h^2}{h}$ $= \lim_{h \to 0} \frac{h(-2x - h)}{h}$ $= \lim_{h \to 0}(-2x - h)$ $= -2x$ **OR/*OF*** $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ $= \lim_{h \to 0} \frac{-(x+h)^2 + 4 - (-x^2 + 4)}{h}$ $= \lim_{h \to 0} \frac{-x^2 - 2xh - h^2 + 4 + x^2 - 4}{h}$ $= \lim_{h \to 0} \frac{-2xh - h^2}{h}$ $= \lim_{h \to 0} \frac{h(-2x - h)}{h}$ $= \lim_{h \to 0}(-2x - h)$ $= -2x$	✓ finding $f(x+h)$ ✓ $-2xh - h^2$ ✓formula ✓factorisation ✓answer (5) ✓formula ✓ finding $f(x+h)$ ✓ $-2xh - h^2$ ✓factorisation ✓answer (5)
8.2.1	$y = 3x^2 + 10x$ $\frac{dy}{dx} = 6x + 10$	✓ $6x$ ✓ 10 (2)
8.2.2	$f(x) = \left(x - \frac{3}{x}\right)^2$ $= x^2 - 6 + \frac{9}{x^2}$ $= x^2 - 6 + 9x^{-2}$ $f'(x) = 2x - 18x^{-3}$	✓ $x^2 - 6 + \frac{9}{x^2}$ ✓ $9x^{-2}$ ✓ $2x - 18x^{-3}$ (3)

8.3.1	$f(2)=2(2)^3-23(2)^2+80(2)-84$ $=0$ $\therefore (x-2)$ is a factor	✓substitution of 2 into *f* ✓value of 0 (2)
8.3.2	$f(x)=2x^3-23x^2+80x-84$ $=(x-2)(2x^2-19x+42)$ $=(x-2)(2x-7)(x-6)$	✓ $2x^2-19x+42$ ✓ $(x-2)(2x-7)(x-6)$ (2)
8.3.3	$f'(x)=6x^2-46x+80$ $6x^2-46x+80=0$ $3x^2-23x+40=0$ $(3x-8)(x-5)=0$ $x=\frac{8}{3}$ or $x=5$	✓ $f'(x)=6x^2-46x+80$ ✓ $f'(x)=0$ ✓factors ✓*x*-values (4)
8.3.4		✓*x*-intercepts ✓*y*-intercept ✓shape (3)
8.3.5	$6x^2-46x+80=40$ $6x^2-46x+40=0$ $3x^2-23x+20=0$ $(3x-20)(x-1)=0$ $x=\frac{20}{3}$ or $x=1$ But *x* must be an integer, so $x=1$ at the point where tangent touches *f* /*x moet heelgetal wees so x = 1 by punt waar die raaklyn f raak:* $y=f(1)=2(1)^3-23(1)^2+80(1)-84=-25$ $y=mx+c$ $-25=40(1)+c$ $-65=c$ $(0;-65)$	✓ $6x^2-46x+80=40$ ✓factors ✓$x=1$ ✓ *y*-value ✓ $-25=40(1)+c$ ✓answer (6) **[27]**

QUESTION/*VRAAG* 9

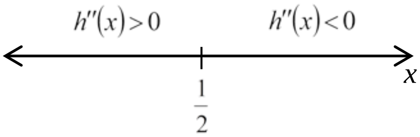
9.1	$340 = \pi r^2 h$ $\therefore h = \dfrac{340}{\pi r^2}$	✓substitution into volume formula ✓answer (2)
9.2	$A = 2\pi r^2 + 2\pi rh$ $= 2\pi r^2 + 2\pi r\left(\dfrac{340}{\pi r^2}\right)$ $= 2\pi r^2 + 680r^{-1}$	✓formula ✓substitution of *h* (2)
9.3	$A(r) = 2\pi r^2 + 680r^{-1}$ $A'(r) = 4\pi r - 680r^{-2}$ $4\pi r - 680r^{-2} = 0$ $4\pi r = \dfrac{680}{r^2}$ $r^3 = \dfrac{680}{4\pi}$ $r = \sqrt[3]{\dfrac{680}{4\pi}}$ cm or $3{,}78$ cm	✓ $4\pi r$ ✓ $-680r^{-2}$ ✓ $r^3 = \dfrac{680}{4\pi}$ ✓answer (4) **[8]**

QUESTION/*VRAAG* 10

10.1.1	160	✓answer (1)
10.1.2	$P(M) = \dfrac{60}{160}$ $= \dfrac{3}{8}$ $= 0{,}375$	✓60 ✓answer (2)
10.1.3	$P(Male) \times P(Coffee) = P(Male\ and\ Coffee)$ $P(Manlik) \times P(Koffie) = P(Manlik\ en\ Koffie)$ $\dfrac{3}{8} \times \dfrac{80}{160} = \dfrac{b}{160}$ $\dfrac{3}{16} = \dfrac{b}{160}$ $16b = 480$ $b = 30$	✓formula ✓ $\dfrac{80}{160}$ ✓ $\dfrac{b}{160}$ ✓ answer (4)

QUESTION/*VRAAG* 9

9.1	Substitute Q(2; 10) into $h(x) = -x^3 + ax^2 + bx$ $-2^3 + a(2^2) + b(2) = 10$ $-8 + 4a + 2b = 10$ $2a + b = 9$ ....line 1 $h'(x) = -3x^2 + 2ax + b$ At Q: $h'(2) = 0$ $-3(2)^2 + 2a(2) + b = 0$ $-12 + 4a + b = 0$ $4a + b = 12$ ....line 2 line 2 – line 1: $2a = 3$ $a = \frac{3}{2}$ Substitute in line 1: $b = 6$	✓substitute Q into *h* ✓finding derivative ✓ $h'(2)$ ✓equating derivative to 0 ✓solving simultaneously for *a* and *b* (5)
9.2	$f(-1) = -(-1)^3 + \frac{3}{2}(-1)^2 + 6(-1)$ $= -3{,}5$ Average gradient/*Gemiddelde gradiënt* $= \frac{f(x_Q) - f(x_P)}{x_Q - x_P}$ Average gradient/*Gemiddelde gradiënt* $= \frac{10 - (-3{,}5)}{2 - (-1)}$ $= 4{,}5$	✓ $f(-1) = -3{,}5$ ✓formula ✓substitution ✓answer (4)

9.3	$h'(x) = -3x^2 + 3x + 6$ $h''(x) = -6x + 3$ $= -3(2x - 1)$  For $x < \frac{1}{2}$, h is concave up and for $x > \frac{1}{2}$, h is concave down *Vir* $x < \frac{1}{2}$,*is* h *konkaaf na bo en vir* $x > \frac{1}{2}$,*is* h *konkaaf na onder* $\therefore$ concavity changes at $x = \frac{1}{2}$ / $\therefore$ *konkwiteit verander by* $x = \frac{1}{2}$	✓ $h'(x) = -3x^2 + 3x + 6$ ✓ $h''(x) = -6x + 3$ ✓explanation using $h''(x)$ (3)
9.4	The graph of h has a point of inflection at $x = \frac{1}{2}$ / *Die grafiek van h het 'n buigpunt by* $x = \frac{1}{2}$. **OR/*OF*** The graph of h changes from concave up to concave down at $x = \frac{1}{2}$ / *Die grafiek van h verander by* $x = \frac{1}{2}$ *van konkaaf op na konkaaf af*	✓answer (1) ✓answer (1)
9.5	Gradient of g is -12/*Gradiënt van g is –12* Gradient of tangent is/*Gradiënt van die raaklyn is*: $h'(x) = -3x^2 + 3x + 6$ $h'(x) = -12$ $-3x^2 + 3x + 6 = -12$ $3x^2 - 3x + 18 = 0$ $x^2 - x + 6 = 0$ $(x - 3)(x + 2) = 0$ $x = -2$ only	✓ $h'(x) = -3x^2 + 3x + 6$ ✓ $h'(x) = -12$ ✓factors ✓selection of x-value (4) **[17]**

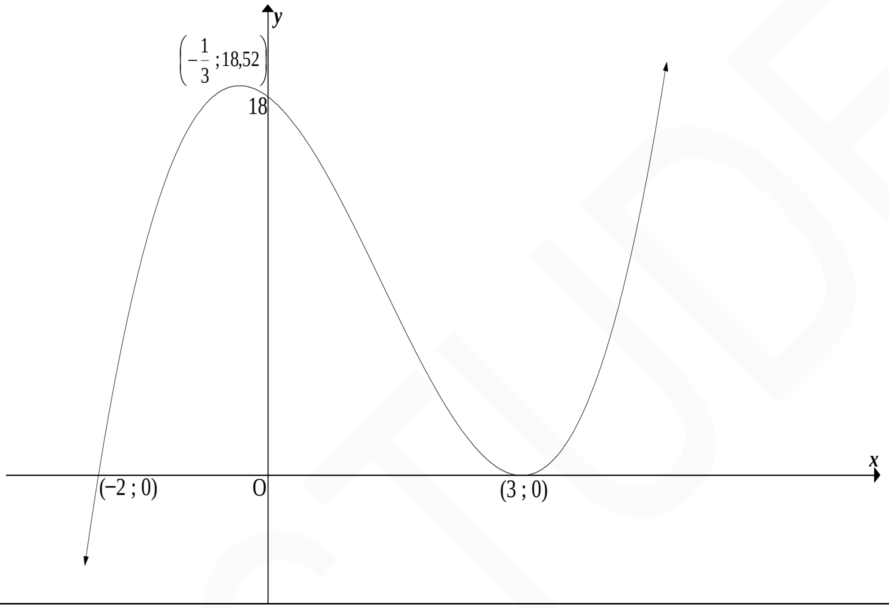
QUESTION/*VRAAG* 10

10.1	$\frac{h}{r} = \tan 60^0$ $r = \frac{h}{\tan 60^0}$ $\therefore r = \frac{h}{\sqrt{3}}$	✓ $\frac{h}{r} = \tan 60^0$ ✓answer (2)
10.2	$V_{cone} = \frac{1}{3}\pi r^2 h$ $= \frac{1}{3}\pi\left(\frac{h}{\sqrt{3}}\right)^2 h$ $= \frac{1}{9}\pi h^3$ $\frac{dV}{dh} = \frac{1}{3}\pi h^2$ $\left.\frac{dV}{dh}\right\|_{h=9} = \frac{1}{3}\pi (9)^2$ $= 27\pi$ or $84{,}82 \text{ cm}^3/\text{cm}$	✓formula ✓substitution of the value of r in terms of h ✓simplified volume answer ✓derivative ✓answer (5) **[7]**

QUESTION/*VRAAG* 9

9.1	$\pi r^2 h = 6$ $h = \dfrac{6}{\pi r^2}$	✓ $h = \dfrac{6}{\pi r^2}$ (1)
9.2	$S = 10\left(2\pi r^2 + 2\pi rh + 4\pi r^2\right)$ $= 10\left[2\pi rh + 6\pi r^2\right]$ $= 20\pi rh + 60\pi r^2$ $= 20\pi r\left(\dfrac{6}{\pi r^2}\right) + 60\pi r^2$ $= 60\pi r^2 + \dfrac{120}{r}$	✓✓ $10\left(2\pi r^2 + 2\pi rh + 4\pi r^2\right)$ ✓ $20\pi rh + 60\pi r^2$ ✓ substitution/*substitusie* (4)
	OR/*OF* Area of/*van* 10 spheres/*sfere* $= 10 \times 4 \times \pi \times r^2 = 40\pi r^2$ Area of/*van* 10 cylinders/*silinders* $= 10(2\pi r^2 + 2\pi rh)$ $= 10(2\pi r^2 + 2\pi r \dfrac{6}{\pi r^2})$ $= 20\pi r^2 + \dfrac{120}{r}$ Total area/*Totale area* $= 40\pi r^2 + 20\pi r^2 + \dfrac{120}{r}$ $= 60\pi r^2 + \dfrac{120}{r}$	✓area of 10 spheres/ *area van 10 sfere* ✓ area of 10 cylinders/ *area van 10 silinders* ✓ substitution/*substitusie* ✓simplification/*vereen-voudiging* (4)
9.3	$S^{/} = 120\pi r - 120r^{-2} = 0$ $120\pi r - \dfrac{120}{r^2} = 0$ $120\pi r^3 - 120 = 0$ $r^3 = \dfrac{120}{120\pi}$ $\therefore r = \dfrac{1}{\pi^{\frac{1}{3}}} = 0{,}68$ cm	✓ $120\pi r - 120r^{-2}$ ✓ $= 0$ ✓ $r^3 = \dfrac{120}{120\pi}$ ✓answer/*antwoord* (4) **[9]**

QUESTION/*VRAAG* 9

9.1	$f'(x) = 3x^2 - 8x - 3 = 0$ $(3x+1)(x-3) = 0$ $x = -\frac{1}{3}$ or $x = 3$ $y = \frac{500}{27}$ $\left(\text{or } y = 18\frac{14}{27} \text{ or } 18{,}52\right)$ $y = 0$ Turning points are/*Draaipunte is* $\left(-\frac{1}{3};\frac{500}{27}\right)$ and $(3;0)$	✓derivative/*afgeleide* ✓derivative/ *afgeleide* = 0 ✓factors/*faktore* ✓x-values/*waardes* ✓✓each y-values/*elke y-waarde* (6)
9.2		✓x-intercepts/*afsnitte* ✓y-intercept/*afsnit* ✓turning points/ *draaipunte* ✓shape/*vorm* (4)
9.3	$x < \frac{-1}{3}$ or $0 < x < 3$ **OR** $(-\infty\,;-\frac{1}{3}) \cup (0;3)$	✓ $x < \frac{-1}{3}$ ✓both critical points/ *beide kritieke-punte* ✓notation/*notasie* (3)

QUESTION/*VRAAG* 10

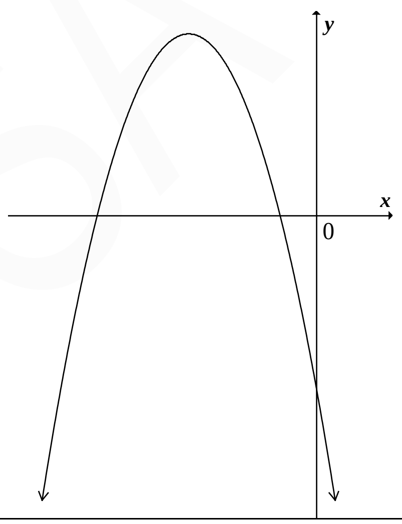
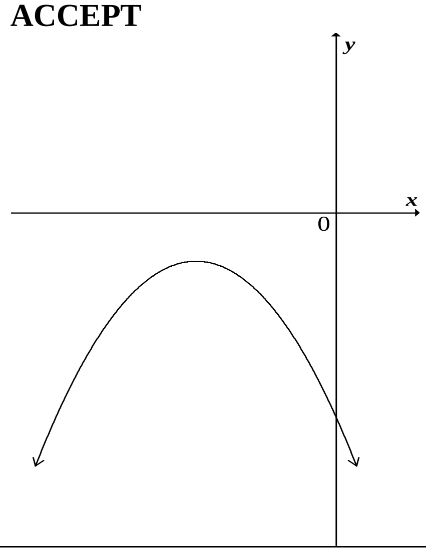
10.1	$l+2h=40$ $l=40-2h$	✓answer (1)
10.2	$2b+2h=100$ $b=50-h$ $V=lbh$ $V=h(40-2h)(50-h)$	✓ $2b+2h=100$ ✓ $b=50-h$ ✓volume formula (3)
10.3	$V=(50h-h^2)(40-2h)$ $V=2h^3-140h^2+2000h$ $V'=6h^2-280h+2000=0$ $h=\dfrac{280\pm\sqrt{(-280)^2-4(6)(2000)}}{2(6)}$ $h\neq 37{,}86$ or $h=8{,}80$ $\therefore$ for a box as large as possible, $h=8{,}80\text{ cm}$ *vir die grootste moontlike boks = 8,80 cm*	✓simplifying/*vereenvoudig* ✓derivative / *afgeleide* ✓✓*h*-values in any form / *h-waardes in enige vorm* ✓answer/*antwoord* (5) **[9]**

QUESTION/*VRAAG* 11

11.1.1	P(male/*manlik*) = $\dfrac{83}{180}$ or 0,46 or 46,11%	✓answer/*antwoord* (1)
11.1.2	P(not game park/*nie wildreservaat*) = 1 – P(game park/*wildreservaat*) $=1-\dfrac{62}{180}$ $=\dfrac{59}{90}$ or 0,66 or 65,56% **OR/*OF*** P(not game park/*nie wildreservaat*) $=\dfrac{98}{180}+\dfrac{20}{180}$ $=\dfrac{118}{180}$ $=\dfrac{59}{90}$ or 0,66 or 65,56%	✓ $1-\dfrac{62}{180}$ ✓answer/*antwoord* (2) ✓ $\dfrac{98}{180}+\dfrac{20}{180}$ ✓answer/*antwoord* (2)

QUESTION/*VRAAG* 10

10.1.1	$f(x) = -\frac{2}{x}$ $f(x+h) = -\frac{2}{(x+h)}$ $f(x+h) - f(x) = -\frac{2}{(x+h)} - \left(-\frac{2}{x}\right)$ $= \frac{-2x + 2(x+h)}{x(x+h)}$ $= \frac{-2x + 2x + 2h}{x(x+h)}$ $= \frac{2h}{x(x+h)}$ $f'(x) = \lim_{h\to 0} \frac{\frac{2h}{x(x+h)}}{h}$ $= \lim_{h\to 0}\left(\frac{2}{x^2 + xh}\right)$ $= \frac{2}{x^2}$	✓ substitution ✓ simplification ✓ formula ✓ common factor ✓ answer (5)
	OR $f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$ $= \lim_{h\to 0} \frac{\left[-\frac{2}{(x+h)}\right] - \left(-\frac{2}{x}\right)}{h}$ $= \lim_{h\to 0} \frac{\frac{-2x + 2(x+h)}{x(x+h)}}{h}$ $= \lim_{h\to 0} \frac{\frac{-2x + 2x + 2h}{x(x+h)}}{h}$ $= \lim_{h\to 0} \frac{\frac{2h}{x(x+h)}}{h}$ $= \lim_{h\to 0}\left(\frac{2}{x^2 + xh}\right)$ $= \frac{2}{x^2}$	✓ formula ✓ substitution ✓ simplification ✓ common factor ✓ answer (5)

10.1.2	$f'(x)=\frac{2}{x^2}$ $x^2 \geq 0$ for $x \in R$ $f'(x) > 0$ for $x \in R;\ x \neq 0$	✓ $x^2 \geq 0$ or $\frac{2}{x^2} \geq 0$ for $x \in R$ ✓ $f'(x) > 0$ for $x \in R;\ x \neq 0$ (2)
10.2	$y=\frac{1}{4}x^2-2x$ $\frac{dy}{dx}=\frac{1}{2}x-2$	✓ $\frac{1}{2}x$ ✓ -2 (2)
10.3	$y=4\left(\sqrt[3]{x^2}\right)$ $y=4x^{\frac{2}{3}}$ and $x=w^{-3}$ $y=4(w^{-3})^{\frac{2}{3}}$ $=4w^{-2}$ $\frac{dy}{dw}=-8w^{-3}$ $=-\frac{8}{w^3}$	✓ $y=4x^{\frac{2}{3}}$ ✓ subs: $4(w^{-3})^{\frac{2}{3}}$ ✓simplification ✓ answer (4)
10.4	$f'(x)=3ax^2+2bx+c$ $a<0$ shape (max TP) $c<0$ y - intercept is negative $b<0$ axis of symmetry on LHS of y - axis **ACCEPT**   	✓ $f'(x)=3ax^2+2$ ✓ shape (max TP) ✓ axis of symmetry on LHS if y-axis ✓y – intercept is below x-axis (4) **[17]**

QUESTION/*VRAAG* 11

11.1	$f(x) = -(x-1)(x-2)(x-4)$ $f(x) = -(x^2-3x+2)(x-4)$ $f(x) = -x^3+7x^2-14x+8$	✓ $-(x-1)(x-2)(x-4)$ ✓ $a=7$ ✓ $b=-14$ ✓ $c=8$ (4)
11.2	$f(x) = -x^3+7x^2-14x+8$ $f'(x)=0$ $-3x^2+14x-14=0$ $3x^2-14x+14=0$ $x = \dfrac{14 \pm \sqrt{14^2-4(3)(14)}}{2(3)}$ $= \dfrac{14 \pm \sqrt{28}}{6}$ $= \dfrac{7 \pm \sqrt{7}}{3}$ $x = 1{,}45$ or $x = 3{,}22$	✓ $f'(x)=0$ ✓ $-3x^2+14x-14=0$ ✓subs into formula ✓x –value ✓x – value (5)
11.3	$x < 1{,}45$ or $x > 3{,}22$	✓critical values ✓✓notation (3) **[12]**

QUESTION/*VRAAG* 12

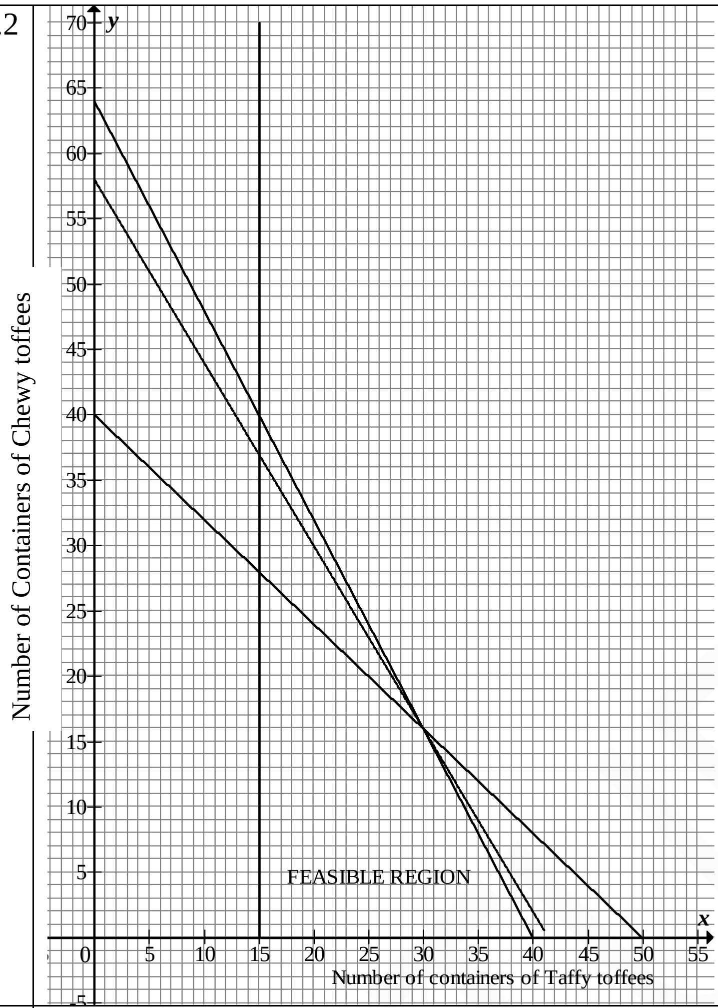
12.1	$40 - x$	✓answer (1)
12.2	$P(x) = (40-x)(144+4x)$ $= 4(40-x)(36+x)$ $= 5\,760 + 16x - 4x^2$	✓concept of multiplication ✓ $(144+4x)$ ✓ answer (3)
12.3	$P'(x) = 16 - 8x$ $P'(x) = 0$ $16 - 8x = 0$ $8x = 16$ $x = 2$ Cost = 144 + 4 (2) = R 152 **OR** Max at $x = \dfrac{40-36}{2} = 2$ Cost = 144 + 4 (2) = R 152	✓ $P'(x) = 16-8x$ ✓ $P'(x)=0$ ✓ $x = 2$ ✓answer (4) ✓ x = 40 & 36 are solutions to $P(x) = 0$ ✓✓ $x = \dfrac{40-36}{2} = 2$ ✓ answer (4)

	OR	
		Number of watches	Cost	Income	
	Year 0: 40, 144, 5 760 Year 1: 39, 148, 5 772 Year 2: 38, 152, 5 776 Year 3: 37, 156, 5 772	✓✓ explanation
	Max Income at $x = 2$ Max cost = R 152	✓ $x = 2$ ✓ R 152 (4) **[8]**

Year	Number of watches	Cost	Income
Year 0:	40	144	5 760
Year 1:	39	148	5 772
Year 2:	38	152	5 776
Year 3:	37	156	5 772

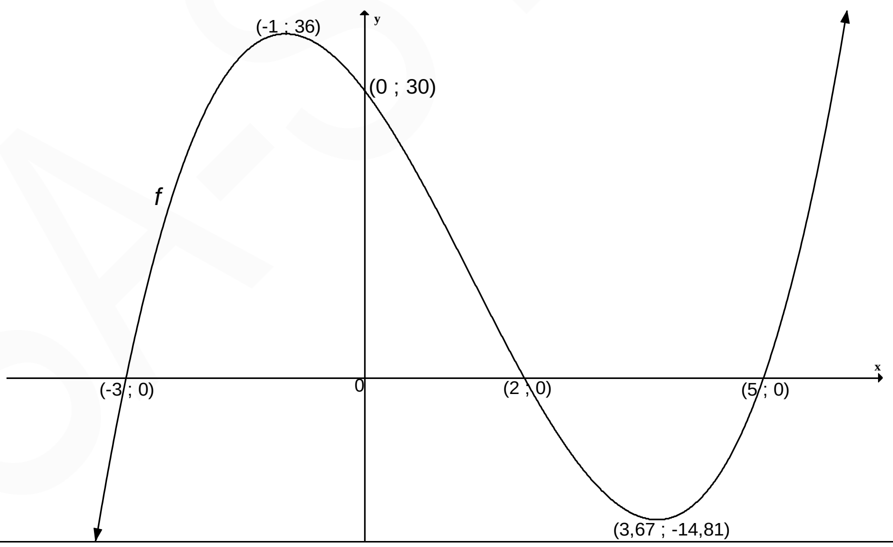
QUESTION/*VRAAG* 13

13.1	$40x + 50y \leq 2000$ $64x + 40y \leq 2560$ $x \geq 15$	✓✓ $40x + 50y \leq 2000$ ✓✓ $64x + 40y \leq 2560$ ✓ $x \geq 15$ (5)
	OR $y \leq -\frac{4}{5}x + 40$ $y \leq -\frac{8}{5}x + 64$ $x \geq 15$	✓✓ $y \leq -\frac{4}{5}x + 40$ ✓✓ $y \leq -\frac{8}{5}x + 64$ ✓ $x \geq 15$ (5)
	OR $\frac{y}{40} + \frac{x}{50} \leq 1$ $\frac{y}{64} + \frac{x}{40} \leq 1$ $x \geq 15$	✓✓ $\frac{y}{40} + \frac{x}{50} \leq 1$ ✓✓ $\frac{y}{64} + \frac{x}{40} \leq 1$ ✓ $x \geq 15$ (5)

13.2		✓ feasible region ✓ $40x + 50y \leq 2000$ ✓ $64x + 40y \leq 2560$ ✓ $x \geq 15$ (4)
13.3	40 containers	✓✓ answer (2)
13.4	$P = 1400x + 1000y$ $m = -\frac{14}{10}$ $m = -\frac{7}{5}$ Using the search line : Maximum achieved at (30; 16)	✓ $P = 1400x + 1000y$ ✓ search line ✓Max at (30 ; 16) (3) **[14]**

TOTAL/*TOTAAL*: 150

QUESTION/VRAAG 9

9.1	$(x-2)$ is a factor of *f / is 'n faktor van f.*	✓ answer (1)
9.2	$f(x)=x^3-4x^2-11x+30$ $=(x-2)(x^2-2x-15)$ $=(x-2)(x+3)(x-5)$ $f(x)=0$ $(x+3)(x-2)(x-5)=0$ $x=-3$ or $x=2$ or $x=5$ x-intercepts: (–3; 0); (2;0); (5;0)	✓ $(x^2-2x-15)$ ✓(–3; 0) ✓(2; 0) ✓(5; 0) (4)
9.3	$f(x)=x^3-4x^2-11x+30$ $f'(x)=3x^2-8x-11$ At turning points $f'(x)=0$ $(3x-11)(x+1)=0$ $x=-1$ or $x=\frac{11}{3}$ $y=36$ $y=-\frac{400}{27}$ $(-14{,}81)$ TP's are $(-1;36)$ and $\left(\frac{11}{3};-14{,}81\right)$	✓ $f'(x)=3x^2-8x-11$ ✓ $f'(x)=0$ ✓x - value ✓x - value ✓y - values (5)
9.4		✓y and x - intercepts ✓shape ✓turning points (3)

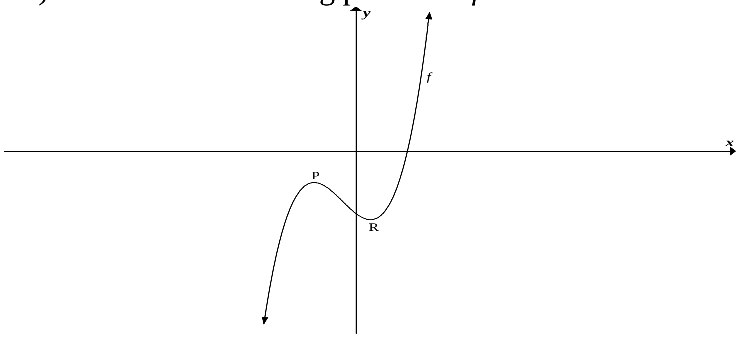
9.5	$f'(x) < 0$ if $-1 < x < 3{,}67$ **OR** (-1 ; 3,67)	✓extreme values ✓notation (2) ✓extreme values ✓notation (2) **[15]**

QUESTION/VRAAG 10

10.1	After t hours: $BF = 30t$ km and $CD = 40t$ km $\therefore BC = 100 - 40t$ $FC = \sqrt{(30t)^2 + (100-40t)^2}$ $= \sqrt{900t^2 + 10000 - 8000t + 1600t^2}$ $= \sqrt{2500t^2 - 8000t + 10000}$	✓ $BF = 30t$ ✓ $BC = 100 - 40t$ ✓Pythagoras ✓answer (4)
10.2	FC is a minimum when FC^2 is a minimum. $FC^2 = 2500t^2 - 8000t + 10000$ $\frac{dFC^2}{dt} = 5000t - 8000 = 0$ $t = \frac{8000}{5000} = 1{,}6\text{hrs}$ (96 minutes)	✓$FC^2 = 2500t^2 - 8000t + 10000$ ✓ $\frac{dFC^2}{dt} = 5000t - 8000$ ✓ $\frac{dFC^2}{dt} = 0$ ✓answer (4)
10.3	$FC = \sqrt{2500t^2 - 8000t + 10000}$ $= \sqrt{2500(1.6)^2 - 8000(1.6) + 10000}$ $= 60$ They will be 60km apart.	✓subs into equation ✓answer (2) **[10]**

QUESTION/*VRAAG* 9

Given: $f(x)=x^3+ax^2+bx-2$
P(–1 ; – 1) and R are the turning points of *f*.

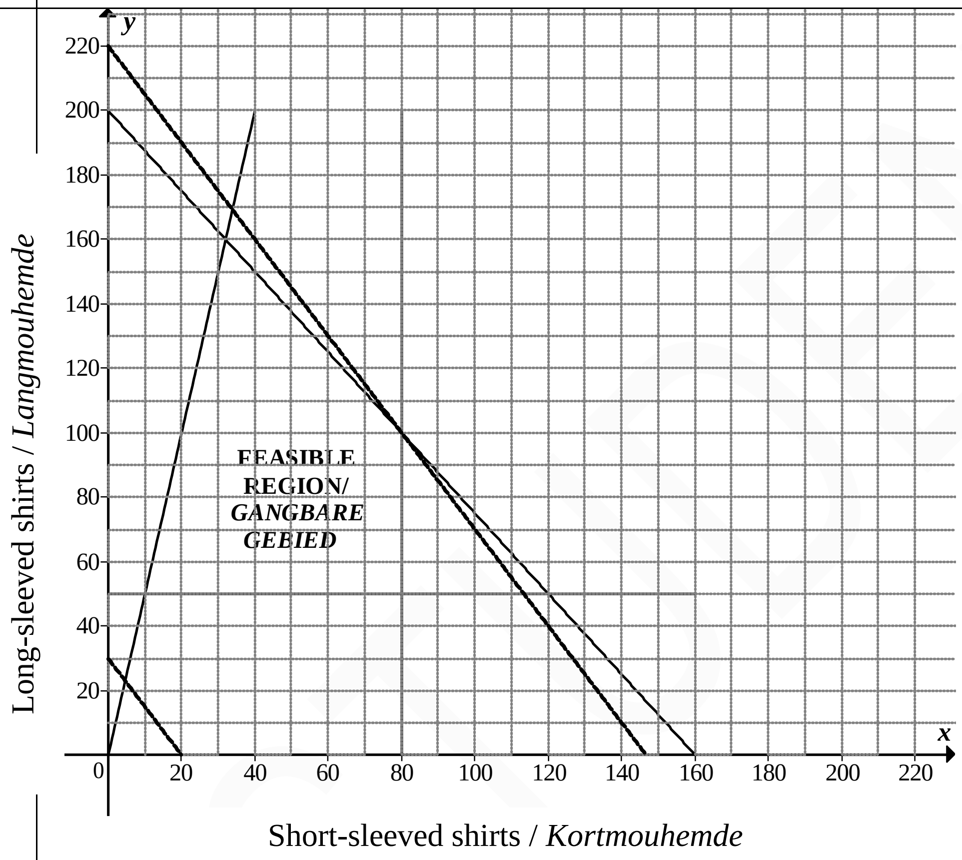


9.1	$f(x)=x^3+ax^2+bx-2$ $-1=(-1)^3+a(-1)^2+b(-1)-2$ $2=a-b \qquad ...(1)$ $f'(x)=3x^2+2ax+b$ $0=3(-1)^2+2a(-1)+b$ $-3=-2a+b \qquad ...(2)$ $-1=-a \qquad ...(1)+(2)$ $a=1$ $b=-1$ **NOTE:** • The = 0 must be explicitly stated • If the candidate starts with what is given and then prove (1 ; 1) is a TP: 0/6 marks	✓ $-1=(-1)^3+a(-1)^2+b(-1)-2$ ✓ $2=a-b$ ✓ $f'(x)=3x^2+2ax+b$ ✓ $f'(-1)=0$ ✓ $-3=-2a+b$ ✓ method (6)
9.2	R is a turning point of *f*, hence at R, $f'(x)=0$ i.e. $3x^2+2x-1=0$ $(3x-1)(x+1)=0$ $x=\frac{1}{3}$ or -1 $\therefore x=\frac{1}{3}$ **NOTE:** Answer only: 1/3 marks	✓ $f'(x)=0$ ✓ $f'(x)=3x^2+2x-1$ ✓ selection of $x=\frac{1}{3}$ (3)
9.3	$(-1;2f(-1)-4)$ $=(-1;-6)$ **OR** $\left(\frac{1}{3};2f\left(\frac{1}{3}\right)-4\right)$ $=\left(\frac{1}{3};-\frac{226}{27}\right)$ or $(0{,}33;-8{,}37)$	✓ *x*-coordinate ✓ *y*-coordinate (2) ✓ *x*-coordinate ✓ *y*-coordinate (2) **[11]**

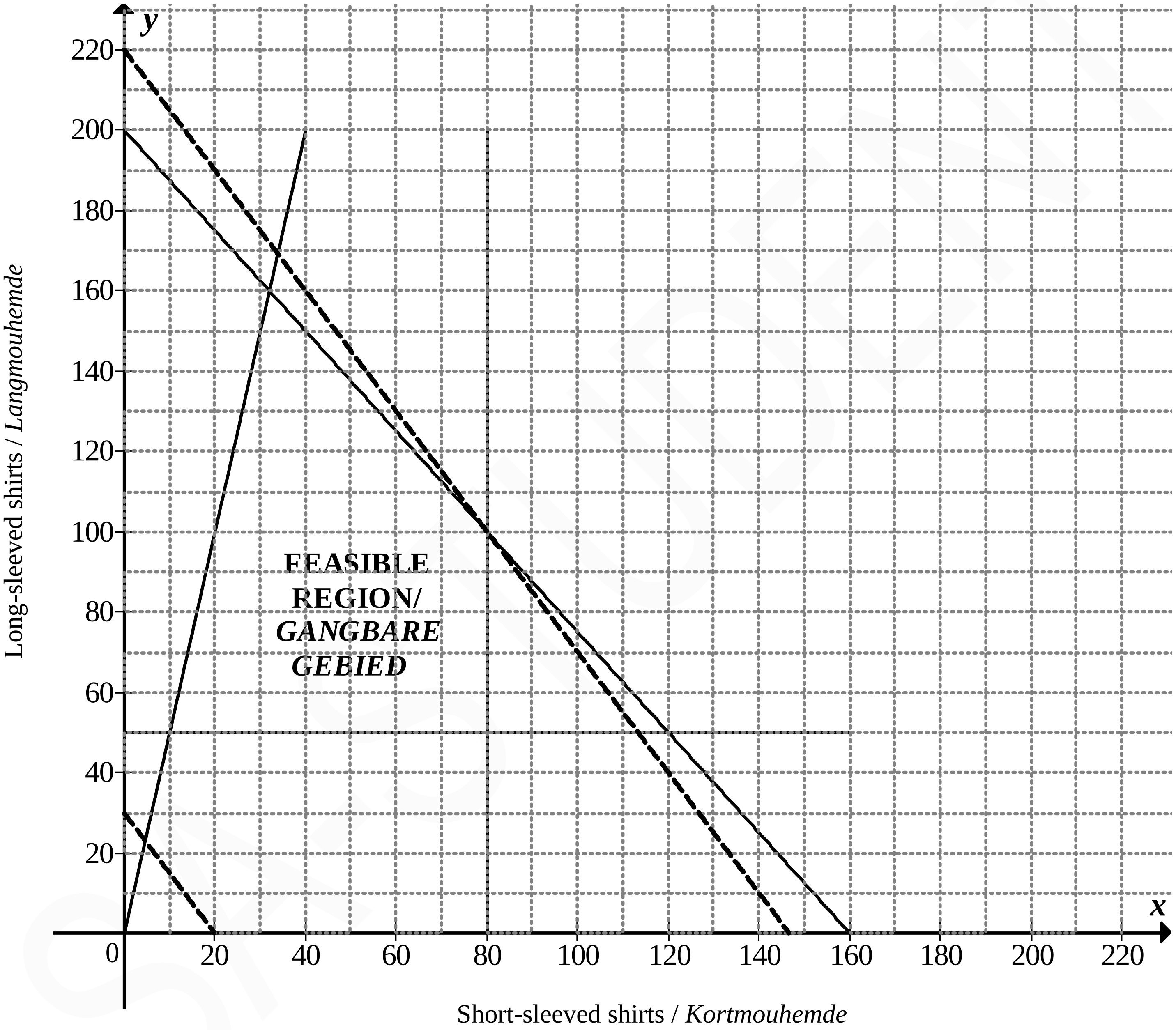
QUESTION/*VRAAG* 10

10.1	$\frac{dr}{dt} = -0{,}4t + 10$ $0 = -0{,}4t + 10$ $0{,}4t = 10$ $t = \frac{10}{0{,}4}$ $= 25$ seconds	✓ $\frac{dr}{dt} = -0{,}4t + 10$ ✓ $0 = -0{,}4t + 10$ ✓ t value (3)
	OR $t = -\frac{b}{2a}$ $= -\frac{10}{2(-0{,}2)}$ $= 25$ seconds	✓ $t = -\frac{b}{2a}$ ✓ substitution ✓ t value (3)
	OR $r = \frac{1}{5}t(50 - t)$ $0 = \frac{t}{5}(50 - t)$ $t = 0$ *or* $t = 50$ Fastest at $t = \frac{0 + 50}{2}$ $t = 25$ seconds	✓ $t = 0$ or $t = 50$ ✓ $t = \frac{0+50}{2}$ ✓ t value (3)
10.2	$-0{,}2t^2 + 10t = 0$ $t(-0{,}2t + 10) = 0$ $-0{,}2t + 10 = 0$ or $t = 0$ $t = \frac{-10}{-0{,}2}$ $= 50$ sec Hence the water stops flowing 50 seconds after it started. **NOTE:** Answer only: 3/3 marks	✓ $-0{,}2t^2 + 10t = 0$ ✓ factors ✓ answer (3)
	OR $-0{,}2t^2 + 10t = 0$ $t^2 - 50t = 0$ $t(t - 50) = 0$ $t = 0$ or $t = 50$ Hence the water stops flowing 50 seconds after it started.	✓ $-0{,}2t^2 + 10t = 0$ ✓ factors ✓ answer (3) **[6]**

QUESTION/*VRAAG* 11

11.1	$y \geq 50$ $x \leq 80$ $y \leq 5x$ **OR** $\frac{y}{5} \leq x$ $y \leq -\frac{5}{4}x + 200$ **OR** $5x + 4y \leq 800$	✓ $y \geq 50$ ✓ $x \leq 80$ ✓ $y \leq 5x$ ✓ $5x + 4y \leq 800$ (4)
11.2	 	✓ $y = 50$ ✓ $x = 80$ ✓ $5x + 4y = 800$ ✓ $y = 5x$ ✓feasible region / *gangbare gebied* (5) **NOTE:** Maximum of 2 marks for vertical and horizontal lines
11.3.1	$P = 30x + 20y$	✓ $P = 30x + 20y$ (1)
11.3.2	$y = -\frac{3}{2}x + \frac{P}{20}$ 80 short-sleeved shirts and 100 long-sleeved shirts (Point (80 ; 100))	✓80 short-sleeved shirts ✓100 long-sleeved shirts (2)
11.4	$P = ax + by$ $m = -\frac{a}{b}$ $-\frac{5}{4} = -\frac{a}{b}$ $\frac{a}{b} = \frac{5}{4}$ **NOTE:** Answer only: 2/2 marks	✓ $-\frac{5}{4} = -\frac{a}{b}$ ✓ answer (2) **[14]**

TOTAL/*TOTAAL*: 150


y
x
0
20
40
60
80
100
120
140
160
180
200
220
FEASIBLE REGION/ *GANGBARE GEBIED*
Long-sleeved shirts / *Langmouhemde*
Short-sleeved shirts / *Kortmouhemde*

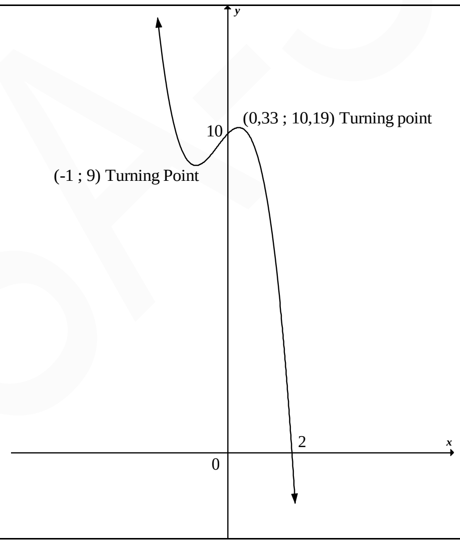
QUESTION 9

9.1	$f(x) = 2x^3$ $f(x+h) = 2(x+h)^3$ $= 2(x^3 + 3x^2h + 3xh^2 + h^3)$ $= 2x^3 + 6x^2h + 6xh^2 + 2h^3$ $f(x+h) - f(x) = 2x^3 + 6x^2h + 6xh^2 + 2h^3 - 2x^3$ $= 6x^2h + 6xh^2 + 2h^3$ $f'(x) = \lim_{h \to 0} \frac{6x^2h + 6xh^2 + 2h^3}{h}$ $= \lim_{h \to 0} \frac{h(6x^2 + 6xh + 2h^2)}{h}$ $= \lim_{h \to 0}(6x^2 + 6xh + 2h^2)$ $f'(x) = 6x^2$	✓ substitution ✓ expansion ✓ formula ✓ $6x^2 + 6xh + 2h^2$ ✓ answer (5)
	OR $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ $= \lim_{h \to 0} \frac{2(x+h)^3 - 2x^3}{h}$ $= \lim_{h \to 0} \frac{2(x^3 + 3x^2h + 3xh^2 + h^3) - 2x^3}{h}$ $= \lim_{h \to 0} \frac{6x^2h + 6xh^2 + 2h^3}{h}$ $= \lim_{h \to 0} \frac{h(6x^2 + 6xh + 2h^2)}{h}$ $= \lim_{h \to 0}(6x^2 + 6xh + 2h^2)$ $f'(x) = 6x^2$	✓ formula ✓ substitution ✓ expansion ✓ $6x^2 + 6xh + 2h^2$ ✓ answer (5)
9.2	$y = \frac{2\sqrt{x} + 1}{x^2}$ $= 2x^{-\frac{3}{2}} + x^{-2}$ $\frac{dy}{dx} = -3x^{-\frac{5}{2}} - 2x^{-3}$	✓ $2x^{-\frac{3}{2}}$ ✓ x^{-2} ✓ $-3x^{-\frac{5}{2}}$ ✓ $-2x^{-3}$ (4)

9.3	$f'(-1) = -7$ $f'(x) = 2ax + b$ $-7 = -2a + b$ $f(-1) = -7(-1) + 3$ $= 10$ $\therefore\ a - b + 5 = 10$ $a - b = 5$............[1] $-2a + b = -7$.........[2] $-a = -2$.........[1] + [2] $a = 2$ $b = -3$	✓ $f'(x) = 2ax + b$ ✓ substitution of $x = -1$ ✓ $-7 = -2a + b$ ✓ $f(-1) = 10$ ✓ $a = 2$ ✓ $b = -3$ (6) **[15]**

QUESTION 10

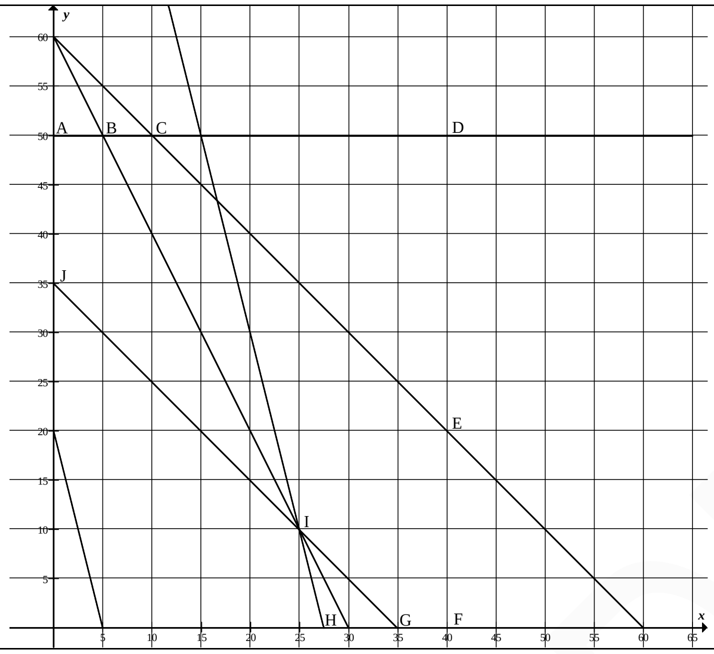
$$f(x) = -x^3 - x^2 + x + 10$$

10.1	$(0;10)$	✓ $(0;10)$ (1)
10.2	$0 = -x^3 - x^2 + x + 10$ $0 = -(x-2)(x^2+3x+5)$ $x - 2 = 0$ or $x^2 + 3x + 5 = 0$ $x = 2$ $x = \dfrac{-3 \pm \sqrt{3^2 - 4(1)(5)}}{2(1)}$ $= \dfrac{-3 \pm \sqrt{-11}}{2}$ which has no solution Therefore the only x-intercept of f is $(2;0)$	✓ $(x-2)$ ✓ (x^2+3x+5) ✓ $x = \dfrac{-3 \pm \sqrt{-11}}{2}$ ✓ no solution (4)
10.3	$f'(x) = -3x^2 - 2x + 1$ $0 = -3x^2 - 2x + 1$ $0 = (3x-1)(x+1)$ $x = \dfrac{1}{3}$ or $x = -1$ $y = -\left(\dfrac{1}{3}\right)^3 - \left(\dfrac{1}{3}\right)^2 + \left(\dfrac{1}{3}\right) + 10$ or $y = -(-1)^3 - (-1)^2 + (-1) + 10$ $= \dfrac{275}{27}$ $= 9$ $\left(\dfrac{1}{3}; 10\dfrac{5}{27}\right)$ $(-1; 9)$	✓ $f'(x) = -3x^2 - 2x + 1$ ✓ $f'(x) = 0$ ✓ factors ✓ x-values ✓ $\left(\dfrac{1}{3}; 10\dfrac{5}{27}\right)$ ✓ $(-1; 9)$ (6)
10.4		✓ shape ✓ intercepts ✓ turning points (3) **[14]**

QUESTION 11

11.1	Length of box = $3x$ Volume $= l \times b \times h$ $9 = 3x \cdot x \cdot h$ $9 = 3x^2 h$ $h = \dfrac{3}{x^2}$	✓ length of box = $3x$ ✓ $9 = 3x \cdot x \cdot h$ ✓ $h = \dfrac{3}{x^2}$ (3)
11.2	$C = \left(2(3xh) + 2xh\right) \times 50 + \left(2 \times 3x^2\right) \times 100$ $= 8x\left(\dfrac{3}{x^2}\right) \times 50 + 600x^2$ $= \dfrac{1200}{x} + 600x^2$ **OR** $C = (h \times 8x) \times 50 + \left(2 \times 3x^2\right) \times 100$ $= 8x\left(\dfrac{3}{x^2}\right) \times 50 + 600x^2$ $= \dfrac{1200}{x} + 600x^2$	✓ $\left(2(3xh) + 2xh\right) \times 50$ ✓ $\left(2 \times 3x^2\right) \times 100$ ✓ substitution of $h = \dfrac{3}{x^2}$ (3) ✓ $(h \times 8x) \times 50$ ✓ $\left(2 \times 3x^2\right) \times 100$ ✓ substitution of $h = \dfrac{3}{x^2}$ (3)
11.3	$C = 1200x^{-1} + 600x^2$ $\dfrac{dC}{dx} = -1200x^{-2} + 1200x$ $0 = -1200x^{-2} + 1200x$ $1200x^3 = 1200$ $x^3 = 1$ $x = 1$ Therefore the width of the box is 1 metre.	✓ $\dfrac{dC}{dx} = -1200x^{-2} + 1200x$ ✓ $\dfrac{dC}{dx} = 0$ ✓ $x^3 = 1$ ✓ $x = 1$ (4) **[10]**

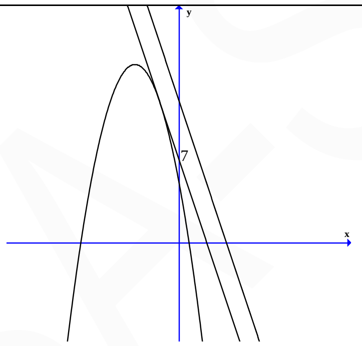
QUESTION 12

12.1	 	✓✓ region ABIJ shaded (2) **NOTE:** If region BCEFGI is shaded: award ONE mark If any other region is shaded: award 0 marks
12.2	$x \leq 40$ $x + y \leq 60$ $y \geq 0$	✓ $x \leq 40$ ✓ $x + y \leq 60$ ✓ $y \geq 0$ (3)
12.3	$x = 25$	✓answer (1)
12.4	At I(25 ; 10), $P = 4(25) + 10 = 110$ Maximum value of P is 110 when x = 25 and y = 10	✓ $x = 25$ ✓ $y = 10$ ✓substitution ✓ maximum value of P is 110 (4)
12.5	$C = kx + y$ $y = -kx + C$ $-k < -1$ $k > 1$	✓ $y = -kx + C$ ✓ $k > 1$ (2) **NOTE:** Answer only: award TWO marks **[12]**

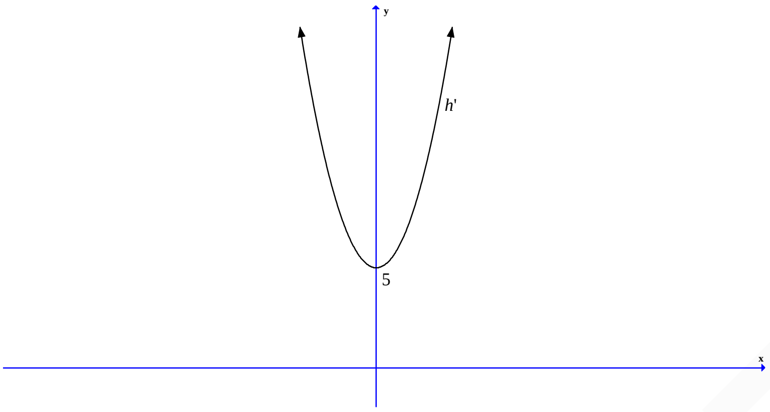
TOTAL: 150

QUESTION 9

9.1.1	$f(x) = -x^3 - x^2 + 16x + 16$ $f'(x) = -3x^2 - 2x + 16$ $0 = -3x^2 - 2x + 16$ $3x^2 + 2x - 16 = 0$ $(3x+8)(x-2) = 0$ $x = -\frac{8}{3}$ or $x = 2$ **Note:** if neither $f'(x) = 0$ nor $0 = -3x^2 - 2x + 16$ explicitly stated, award maximum 3/4 marks **OR** $f(x) = -x^3 - x^2 + 16x + 16$ $f'(x) = -3x^2 - 2x + 16$ $0 = -3x^2 - 2x + 16$ $0 = 3x^2 + 2x - 16$ $x = \frac{-2 \pm \sqrt{2^2 - 4(3)(-16)}}{2(3)}$ $x = -\frac{8}{3}$ or $x = 2$	✓ $f'(x) = -3x^2 - 2x + 16$ ✓ $f'(x) = 0$ or $0 = -3x^2 - 2x + 16$ ✓ factors ✓ x values (4) ✓ $f'(x) = -3x^2 - 2x + 16$ ✓ $f'(x) = 0$ or $0 = -3x^2 - 2x + 16$ ✓ subs into formula ✓ x values (4)
9.1.2	$f''(x) = 0$ $-6x - 2 = 0$ $x = -\frac{1}{3}$ **OR** $x = \frac{-\frac{8}{3} + 2}{2}$ $x = -\frac{1}{3}$ **OR** $f'(x) = -3x^2 - 2x + 16$ $x = \frac{-(-2)}{2(-3)}$ $= -\frac{1}{3}$ **OR**	✓ $f''(x) = -6x - 2$ ✓ $-6x - 2 = 0$ ✓ answer (3) ✓ $x = \frac{-\frac{8}{3} + 2}{2}$ ✓✓ answer (3) ✓✓ $x = \frac{-(-2)}{2(-3)}$ ✓ answer (3)

	$f(x) = -x^3 - x^2 + 16x + 16$ $x = \frac{-(-1)}{3(-1)}$ $= -\frac{1}{3}$	✓✓ $x = \frac{-(-1)}{3(-1)}$ ✓ answer (3)
9.2.1	$g(x) = -2x^2 - 9x + 5$ $g(-1) = -2(-1)^2 - 9(-1) + 5$ $= 12$ $g'(x) = -4x - 9$ $m_{\tan} = -4(-1) - 9$ $= -5$ $y = -5x + c$ $12 = -5(-1) + c$ $c = 7$ $y = -5x + 7$ **OR** $g(x) = -2x^2 - 9x + 5$ $g(-1) = -2(-1)^2 - 9(-1) + 5$ $= 12$ $g'(x) = -4x - 9$ $m_{\tan} = -4(-1) - 9$ $= -5$ $y - 12 = -5(x + 1)$ $y = -5x + 7$	✓ $g(-1) = 12$ ✓ $g'(x) = -4x - 9$ ✓ $m_{\tan} = -5$ ✓ answer (4) ✓ $g(-1) = 12$ ✓ $g'(x) = -4x - 9$ ✓ $m_{\tan} = -5$ ✓ answer (4)
9.2.2	 $q > 7$ **OR** $y = -5x + q$ and $y = -2x^2 - 9x + 5$ $-5x + q = -2x^2 - 9x + 5$ $q = -2(x+1)^2 + 7$ $\therefore q > 7$	✓ sketch ✓ 7 ✓ correct inequality (3) ✓ method ✓ 7 ✓ correct inequality (3)

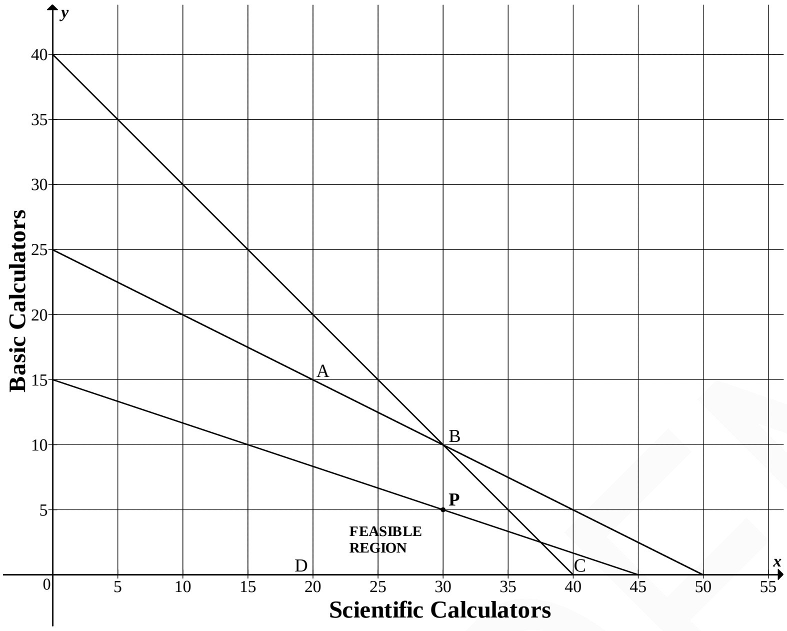
	OR $y = -5x + q$ and $y = -2x^2 - 9x + 5$ $-5x + q = -2x^2 - 9x + 5$ $2x^2 + 4x + q - 5 = 0$ $x = \frac{-4 \pm \sqrt{16 - 4(2)(q-5)}}{2(2)}$ $x = \frac{-4 \pm \sqrt{56 - 8q}}{4}$ $56 - 8q < 0$ $q > 7$ **OR** Since $g(-1) = 12$ and at $x = -1$, tangent equation is $y = -5x + 7$, $y = -5x + q$ not intersecting g $\Rightarrow$ $12 < -5(-1) + q$ $12 - 5 < q$ $7 < q$	✓ method ✓ 7 ✓correct inequality (3) ✓ method ✓ 7 ✓correct inequality (3)
9.3	$h'(x) = 12x^2 + 5$ For all values of x: $x^2 \geq 0$ $12x^2 \geq 0$ $12x^2 + 5 \geq 5$ $12x^2 + 5 > 0$ For all values of x: $h'(x) > 0$ All tangents drawn to h will have a positive gradient. It will never be possible to draw a tangent with a negative gradient to the graph of h. **OR** $h'(x) = 12x^2 + 5$ Suppose $h'(x) < 0$ and try to solve for x : $12x^2 + 5 < 0$ $x^2 < -\frac{5}{12}$ but x^2 is always positive $\therefore$ no solution for x $\therefore h'(x) \geq 0$ for all $x \in R$ i.e. there are no tangents with negative slopes	✓ $h'(x) = 12x^2 + 5$ ✓ clearly argues that $h'(x) > 0$ ✓ conclusion (3) ✓ $h'(x) = 12x^2 + 5$ ✓ clearly argues that $h'(x) < 0$ is impossible ✓ conclusion (3)

	OR $h'(x) = 12x^2 + 5$  Since clearly $h'(x) > 0$ for all $x \in R$, it will never be possible to draw a tangent with a negative gradient to the graph of *h*.	✓ $h'(x) = 12x^2 + 5$ ✓ argues $h'(x) > 0$ by drawing a sketch ✓ conclusion (3) **[17]**

QUESTION 10

10.1	$s(t) = 2t^2 - 18t + 45$ $s'(t) = 4t - 18$ $s'(0) = 4(0) - 18$ $= -18\ m/s$ **Note:** answer only award 0/3 marks	✓ $s'(t)$ ✓ subs t = 0 into $s'(t)$ formula ✓ answer (3)
10.2	$s''(t) = 4\,\text{m/s}^2$	✓ answer (1)
10.3	$4t - 18 = 0$ $4t = 18$ $t = \frac{9}{2}$ seconds or 4,5 seconds	✓ $s'(t) = 0$ ✓ answer (2)
	OR $s(t) = 2\left(t - \frac{9}{2}\right)^2 + \frac{9}{2}$ $t = \frac{9}{2}$ seconds or 4,5 seconds	✓ $s(t) = 2\left(t - \frac{9}{2}\right)^2 + \frac{9}{2}$ ✓ answer (2)
	OR $s(t) = 2t^2 - 18t + 45$ $t = -\frac{-18}{2(2)}$ $t = \frac{9}{2}$ seconds or 4,5 seconds	✓ $t = -\frac{-18}{2(2)}$ ✓ answer (2) **[6]**

QUESTION 11

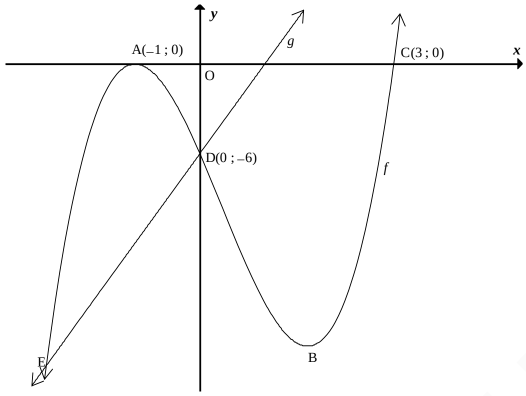


11.1	No, because (15 ; 5) does not lie within the feasible region. **OR** No, because according to the constraints, the *x*-value (number of scientific calculators) must be at least 20.	✓ answer (<u>with</u> motivation) (1)
11.2	$x \geq 20$ $x + 2y \leq 50$ $x + y \leq 40$ $y \geq 0$ **OR** $x \geq 20$ $y \leq -\frac{1}{2}x + 25$ $y \leq -x + 40$ $y \geq 0$ **OR** $x \geq 20$ $\frac{y}{25} + \frac{x}{50} \leq 1$ $\frac{y}{40} + \frac{x}{40} \leq 1$ $y \geq 0$ **OR** $x \geq 20$ $40x + 40y \leq 1600$ $25x + 50y \leq 1250$ $y \geq 0$	✓ $x \geq 20$ ✓✓ $x + 2y \leq 50$ ✓✓ $x + y \leq 40$ ✓ $y \geq 0$ (6)
11.3.1	A	✓ answer (1)
11.3.2	All points on the search line yield the same profit. Hence no such point exists. **OR** If such an $(x ; y)$ exists, Q = $x + 3y$ and $y = -\frac{1}{3}x + 15$ so $45 = 3y + x = Q$ Q = 4 500 Hence, there is no such point.	✓✓ No point exists (2) ✓✓ No point exists (2)

11.3.3	$Q = ax + by$ $y = -\frac{a}{b}x + \frac{Q}{b}$ $-1 \leq -\frac{a}{b} \leq -\frac{1}{2}$ $\frac{1}{2} \leq \frac{a}{b} \leq 1$ The maximum value of $\frac{a}{b}$ is 1.	✓ $y = -\frac{a}{b}x + \frac{Q}{b}$ ✓ $-1 \leq -\frac{a}{b}$ ✓ $\frac{a}{b} \leq 1$ ✓ answer (4) **[14]**

TOTAL: 150

QUESTION 9

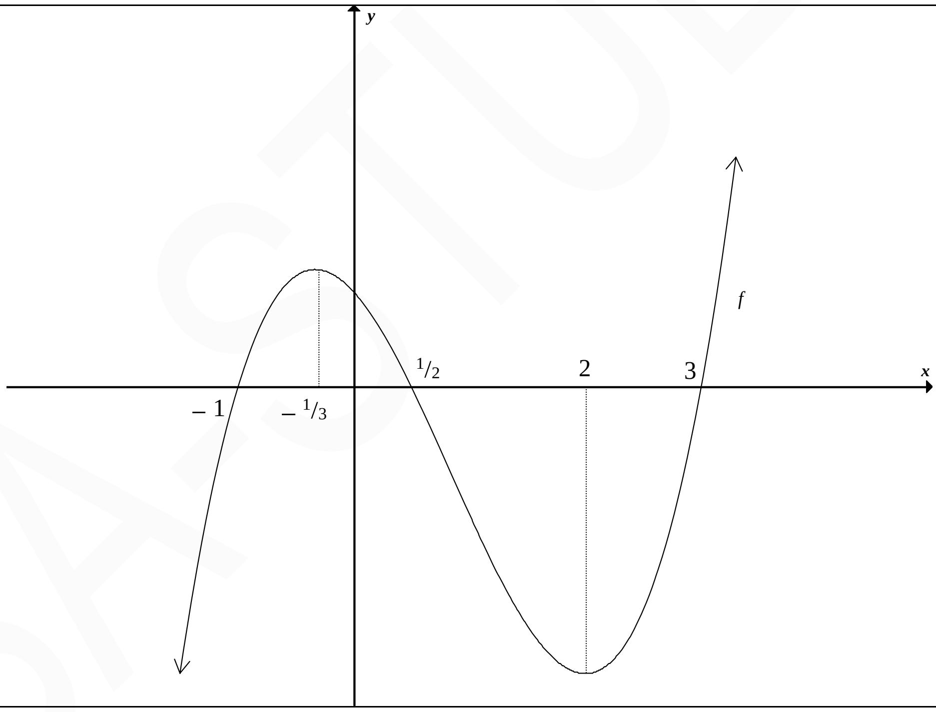


9.1	$f(x) = a(x+1)^2(x-3)$ $-6 = a(0+1)^2(0-3)$ $-6 = -3a$ $a = 2$ $f(x) = 2(x^2+2x+1)(x-3)$ $= 2x^3 - 2x^2 - 10x - 6$	✓✓ substitution of x-values ✓ subs (0 ; – 6) ✓ $a = 2$ ✓ simplification (5)
9.2	$f'(x) = 6x^2 - 4x - 10$ $6x^2 - 4x - 10 = 0$ $3x^2 - 2x - 5 = 0$ $(3x-5)(x+1) = 0$ $x = \frac{5}{3}$ or $x = -1$ $B\left(\frac{5}{3} ; -\frac{512}{27}\right)$ **OR** $B(1,67 ; -18,96)$	✓ $f'(x) = 6x^2 - 4x - 6$ ✓ $f'(x) = 0$ ✓ factors ✓ x-value ✓ y-value (5)
9.3	$h(x) = 2x^3 - 2x^2 - 10x - 6 - (6x - 6)$ $= 2x^3 - 2x^2 - 16x$ $h'(x) = 6x^2 - 4x - 16$ $0 = 3x^2 - 2x - 8$ $0 = (3x+4)(x-2)$ $x = -\frac{4}{3}$ or $x = 2$ $\therefore x = -\frac{4}{3}$	✓ $h(x) = 2x^3 - 2x^2 - 16x$ ✓ $h'(x) = 6x^2 - 4x - 16$ ✓ $h'(x) = 0$ ✓ factors ✓ correct x-value (5) **[15]**

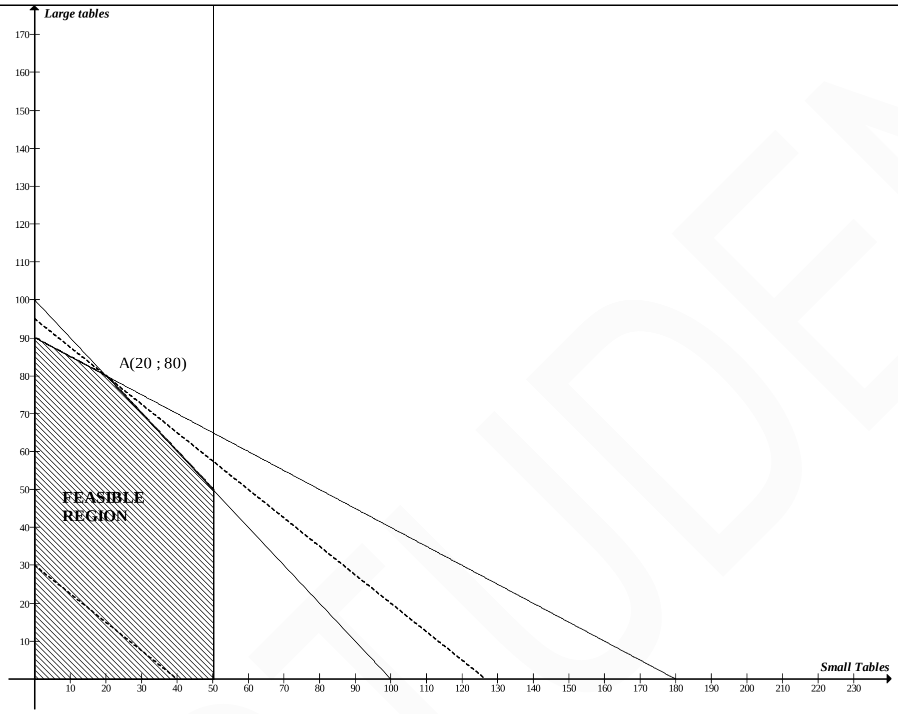
QUESTION 10

10.1	$y = 5(1) - 8$ $= -3$ Point of contact is (1 ; – 3)	✓ subs 1 (1)
10.2	$-3 = 2(1)^3 + p(1)^2 + q(1) - 7$ $2 = p + q$ $g'(x) = 6x^2 + 2px + q$ $g'(1) = 5$ $5 = 6(1)^2 + 2p(1) + q$ $-1 = 2p + q$ $p = -3$ $q = 5$	✓ subs (1 ; – 3) ✓ $g'(x) = 6x^2 + 2px + q$ ✓ subs x = 1 and y = 5 ✓ simplification ✓ p-value ✓ q-value (6) **[7]**

QUESTION 11

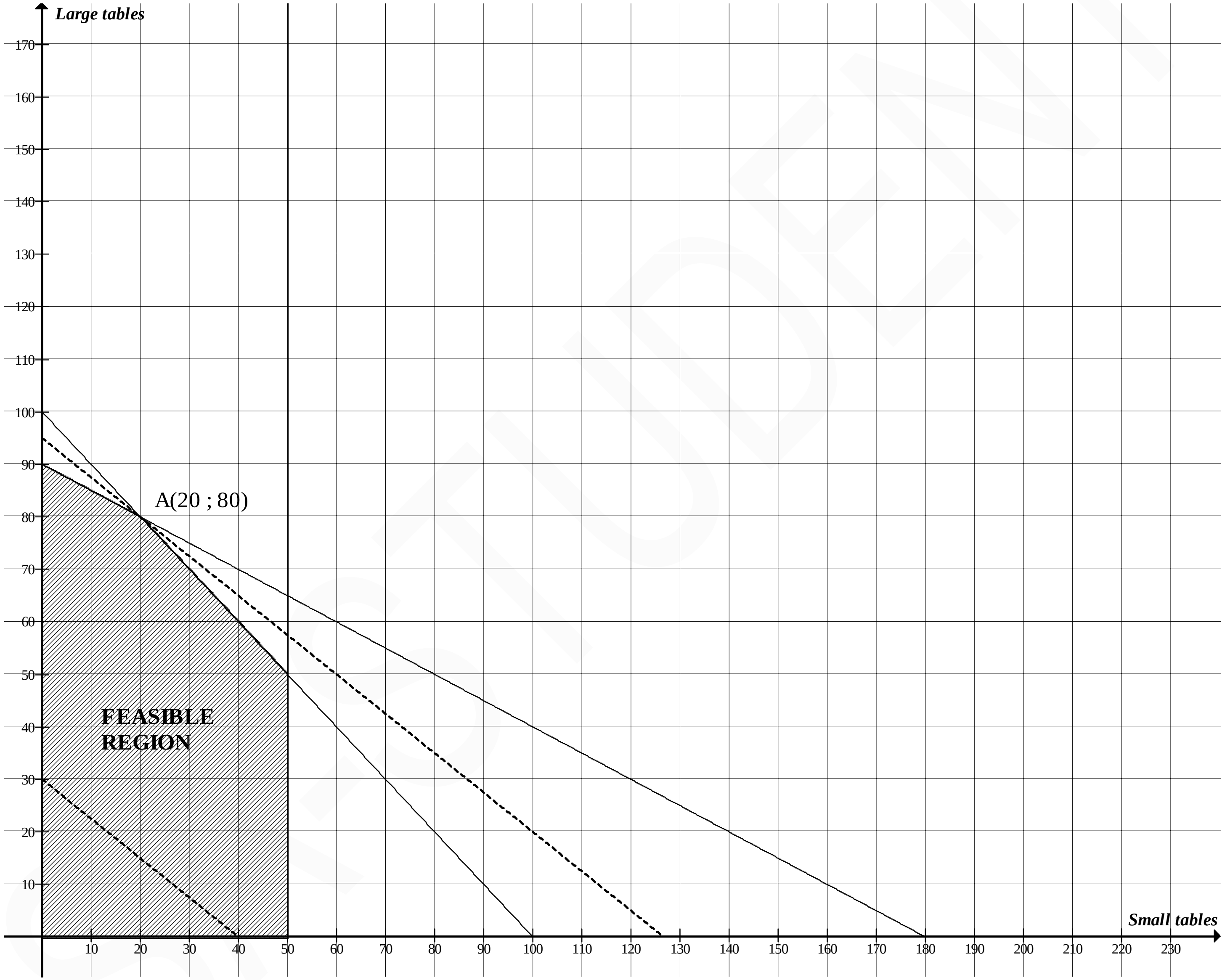
		✓ x-intercepts ✓ turning point ✓✓ shape **[4]**

QUESTION 12

12.1	$x, y \in N_0$ $x + y \leq 100$ **OR** $y \leq -x + 100$ $x \leq 50$ **OR** $x \leq 50$ $x + 2y \leq 180$ **OR** $y \leq -\frac{1}{2}x + 90$	✓✓ $x + y \leq 100$ ✓✓ $x + 2y \leq 180$ ✓ $x \leq 50$ (5)
12.2		✓✓✓ each constraint ✓ feasible region (4)
12.3	90 tables	✓ answer (1)
12.4	$P = 300x + 400y$	✓ answer (1)
12.5	Maximum at A (20 ; 80) 20 small tables and 80 large tables.	✓✓ answer (2)
12.6	$P = qx + 400y$ $m = -\frac{q}{400}$ $-1 \leq -\frac{q}{400} \leq -\frac{1}{2}$ $200 \leq q \leq 400$	✓ $m = -\frac{q}{400}$ ✓ $200 \leq q \leq 400$ (2) **[15]**

TOTAL: 150

QUESTION 12.2



QUESTION 9

9.1

$f(x) = -2x^3 + ax^2 + bx + c$

$f'(x) = -6x^2 + 2ax + b$

$= -6(x-5)(x-2)$

$= -6(x^2 - 7x + 10)$

$= -6x^2 + 42x - 60$

$2a = 42$

$a = 21$

$b = -60$

> **Note:**
> A candidate who substitutes the values of a, b and c and then checks (by substitution) that $T(2;-9)$ and $S(5;18)$ lie on the curve:
> award max 2/7 marks

$f(5) = -2(5)^3 + 21(5)^2 - 60(5) + c$

$18 = -25 + c$

$c = 43$

OR

$f(2) = -2(2)^3 + 21(2)^2 - 60(2) + c$

$-9 = -52 + c$

$c = 43$

$a = 21$; $b = -60$; $c = 43$

Marks:
✓ $f'(x) = -6x^2 + 2ax + b$
✓✓ $-6(x-5)(x-2)$
✓ $b = -60$
✓ $2a = 42$
✓ subs (5 ; 18) or (2 ; -9)
✓ $c = 43$
(7)

> **Note:**
> A candidate who substitutes the values of a, b and c into the function i.e. gets $f(x) = -2x^3 - 21x^2 - 60x + 43$ and then shows by substitution that $T(2;-9)$ and $S(5;18)$ are on the curve **and** works out the derivative i.e. gets $f'(x) = -6x^2 - 42x - 60$ **and** shows (by substitution into the derivative) that the turning points are at $x = 2$ and $x = 5$ (assuming what s/he sets out to prove and proving what is given): **award max 4/7 marks** as follows:
> ✓ $x = 2$ from $f'(x) = 0$ OR subs $x = 2$ into the derivative and gets 0
> ✓ $x = 5$ from $f'(x) = 0$ OR subs $x = 5$ into the derivative and gets 0
> ✓substitution of $x = 2$ in f and gets -9
> ✓substitution of $x = 5$ in f and gets 18

OR

$f'(x) = -6x^2 + 2ax + b$

$f'(2) = -6(2)^2 + 2a(2) + b$

$0 = -24 + 4a + b$

$b = 24 - 4a$

$f'(5) = -6(5)^2 + 2a(5) + b$

$0 = -150 + 10a + b$

$0 = -150 + 10a + (24 - 4a)$

$0 = -126 + 6a$

$6a = 126$

$a = 21$

$b = -60$

> **Note:**
> If derivative equal to zero is not written: penalize once only

$f(5) = -2(5)^3 + 21(5)^2 - 60(5) + c$

$18 = -25 + c$

$c = 43$

OR

$f(2) = -2(2)^3 + 21(2)^2 - 60(2) + c$

$-9 = -52 + c$

$c = 43$

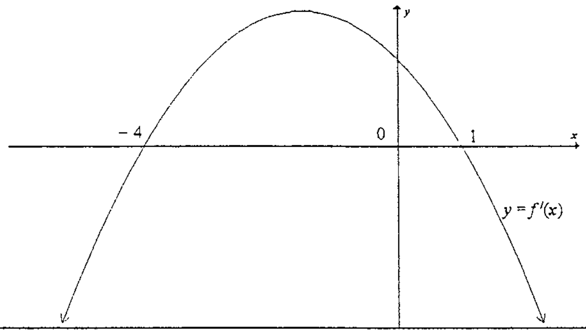
$a = 21$; $b = -60$; $c = 43$

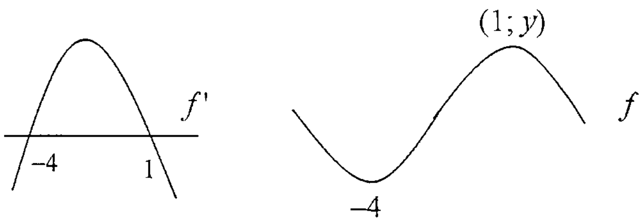
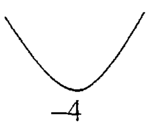
Marks:
✓ $f'(x) = -6x^2 + 2ax + b$
✓ $f'(2) = 0$
✓ $f'(5) = 0$
✓ $6a = 126$
✓ $b = -60$
✓ subs (5 ; 18) or (2 ; -9)
✓ $c = 43$
(7)

	OR $f(2)=-9$ i.e. $-16+4a+2b+c=-9$ $4a+2b+c=7$ $f(5)=18$ i.e. $-250+25a+5b+c=18$ $25a+5b+c=268$ $21a+3b=261$ $f'(x)=-6x^2+2ax+b$ and $f'(2)=0$ OR $f'(5)=0$ $4a+b=24$ $\quad$ $10a+b=150$ $12a+3b=72$ $\quad$ OR $\quad$ $30a+3b=450$ $9a=189$ $\quad$ $9a=189$ $a=\frac{189}{9}$ $\quad$ $a=\frac{189}{9}$ $a=21$ $\quad$ $a=21$ $12(21)+3b=72$ $3b=-180$ $b=-60$ $4a+2b+c=7$ $\quad$ OR $\quad$ $25a+5b+c=268$ $4(21)+2(-60)+c=7$ $\quad$ $25(21)+5(-60)+c=268$ $c=43$ $\quad$ $c=43$	✓ $-16+4a+2b+c=-9$ and $-250+25a+5b+c=18$ ✓ $f'(x)=-6x^2+2ax+b$ ✓ $f'(2)=0$ or $f'(5)=0$ ✓ $9a=189$ ✓ $b=-60$ ✓ subs (5 ; 18) or (2 ; -9) ✓ $c=43$ (7)
9.2	$f'(x)=-6x^2+42x-60$ $m_{\tan}=-6(1)^2+42(1)-60$ $=-24$ $f(1)=-2(1)^3+21(1)^2-60(1)+43$ $=2$ Point of contact is (1 ; 2) $y-2=-24(x-1)$ $y=-24x+26$ **OR** $y=-24x+c$ $2=-24(1)+c$ $c=26$ $y=-24x+26$	✓ $f'(x)=-6x^2+42x-60$ ✓ subs $f'(1)$ ✓ $m_{\tan}=-24$ ✓ $f(1)=2$ ✓ $y-2=-24(x-1)$ OR $y=-24x+26$ (5)
9.3	$f'(x)=-6x^2+42x-60$ $f''(x)=-12x+42$ $0=-12x+42$ $x=\frac{7}{2}$ **OR**	✓ $f''(x)=-12x+42$ ✓ $x=\frac{7}{2}$ (2) ✓ $x=\frac{2+5}{2}$

	$x = \dfrac{2+5}{2}$ $x = \dfrac{7}{2}$ **OR** $x = \dfrac{-21}{3(-2)}$ $= \dfrac{7}{2}$	✓ $x = \dfrac{7}{2}$ (2) ✓ $x = \dfrac{-21}{3(-2)}$ ✓ $x = \dfrac{7}{2}$ (2) **[14]**

QUESTION 10



10.1	x-value of turning point: $x = \dfrac{-4+1}{2}$ $= -\dfrac{3}{2}$ $\therefore x > -\dfrac{3}{2}$ OR $\therefore x \in \left(-\dfrac{3}{2}; \infty\right)$	✓ $x > -\dfrac{3}{2}$ OR $\left(-\dfrac{3}{2}; \infty\right)$ (1)
10.2	f has a local minimum at $x = -4$ because:  **OR** $f'(x) < 0$ for $x < -4$, so f is decreasing for $x < -4$. $f'(x) > 0$ for $-4 < x < 1$, so f is increasing for $-4 < x < 1$. i.e.  $\therefore f$ has a local minimum at $x = -4$ **OR**	✓ $x = -4$ ✓✓ graph (3) ✓ $x = -4$ ✓ $f'(x) < 0$ for $x < -4$ ✓ $f'(x) > 0$ for $-4 < x < 1$ (3)

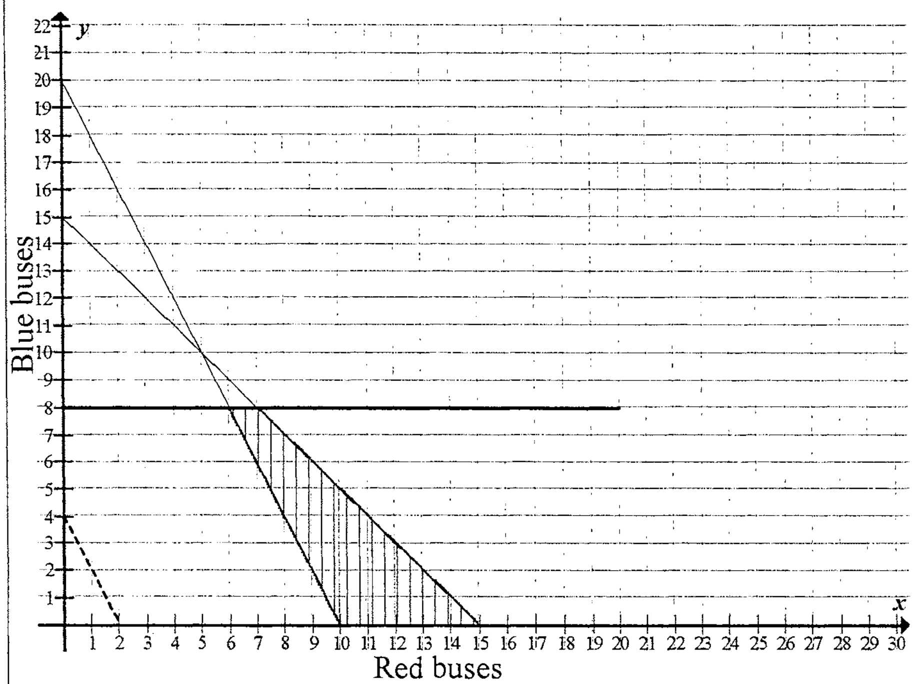
	OR Gradient of f changes from negative to positive at $x = -4$	✓ $x = -4$ ✓gradient negative for $x < -4$ ✓ gradient positive for $-4 < x < 1$ (3)
	OR $f'(-4) = 0$ $f''(-4) > 0$ so graph is concave up at $x = -4$, so f has a local minimum at $x = -4$.	✓ $f'(-4) = 0$ ✓ $f''(-4) > 0$ ✓ $x = -4$ (3) **[4]**

QUESTION 11

11.1	$V(0) = 100 - 4(0)$ $= 100$ litres	✓ answer (1)
11.2	Rate in – rate out $= 5 - k \quad l/\text{min}$ $V'(t) = -4 \quad l/\text{min}$	✓ $5 - k$ ✓ -4 ✓ units stated once (3)
11.3	$5 - k = -4$ $k = 9 \quad l/\text{min}$ **Note:** Answer only: award 2/2 marks **OR** Volume at any time t = initial volume + incoming total – outgoing total $100 + 5t - kt = 100 - 4t$ $5t - kt = -4t$ $9t - kt = 0$ $t(9 - k) = 0$ At 1 minute from start, $t = 1$, $9 - k = 0$, so $k = 9$ **OR** Since $\frac{dV}{dt} = -4$, the volume of water in the tank is decreasing by 4 litres every minute. So k is greater than 5 by 4, that is, $k = 9$.	✓ $5 - k = -4$ ✓ $k = 9$ (2) ✓ $100 + 5t - kt = 100 - 4t$ ✓ $k = 9$ (2) ✓✓ $k = 9$ (2) **[6]**

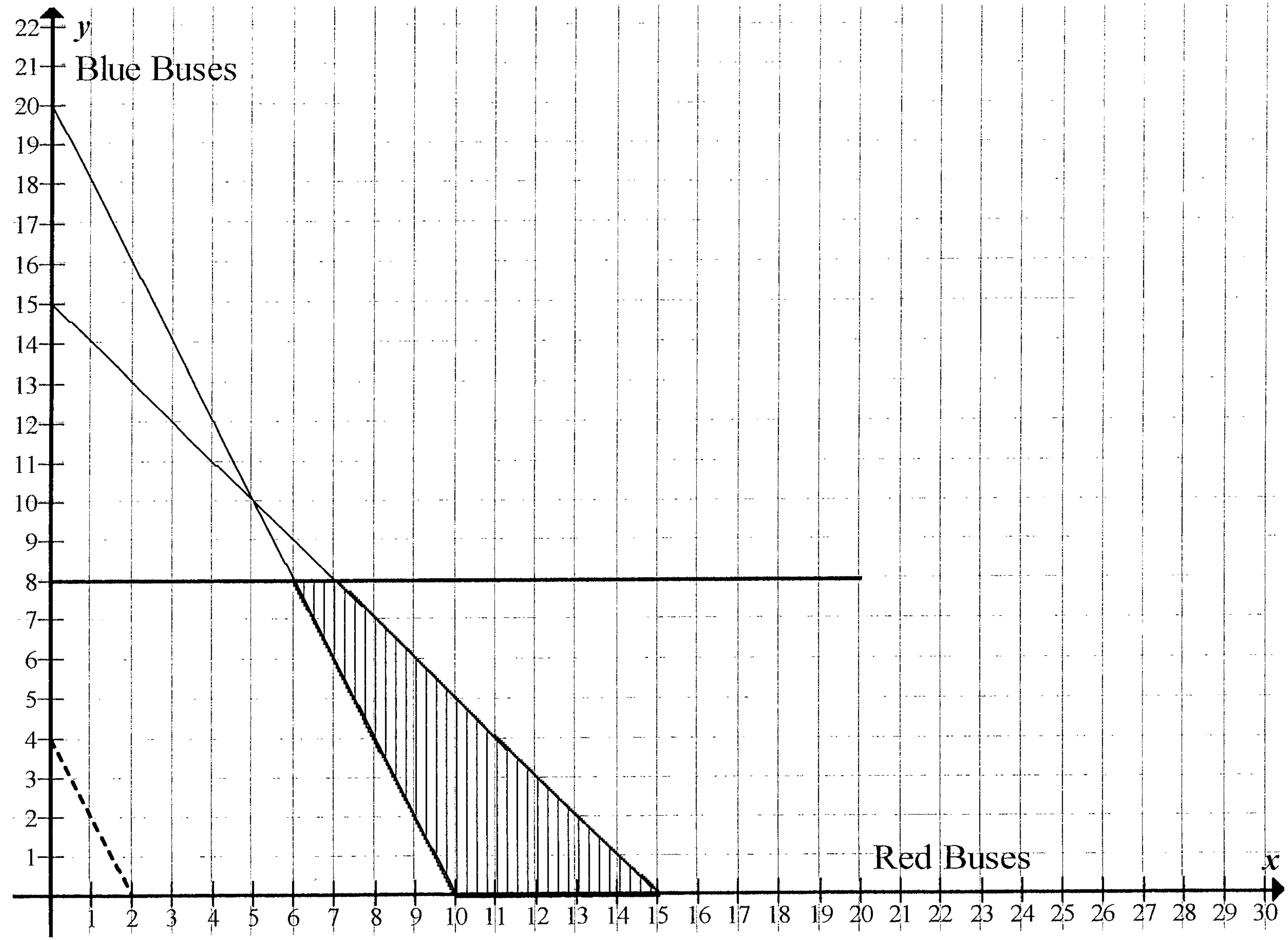
QUESTION 12

Note: If the wrong inequality $50x + 25y \leq 500$ is used, candidate wrongly says that there are more learners than available seats. Maximum of 10 marks.

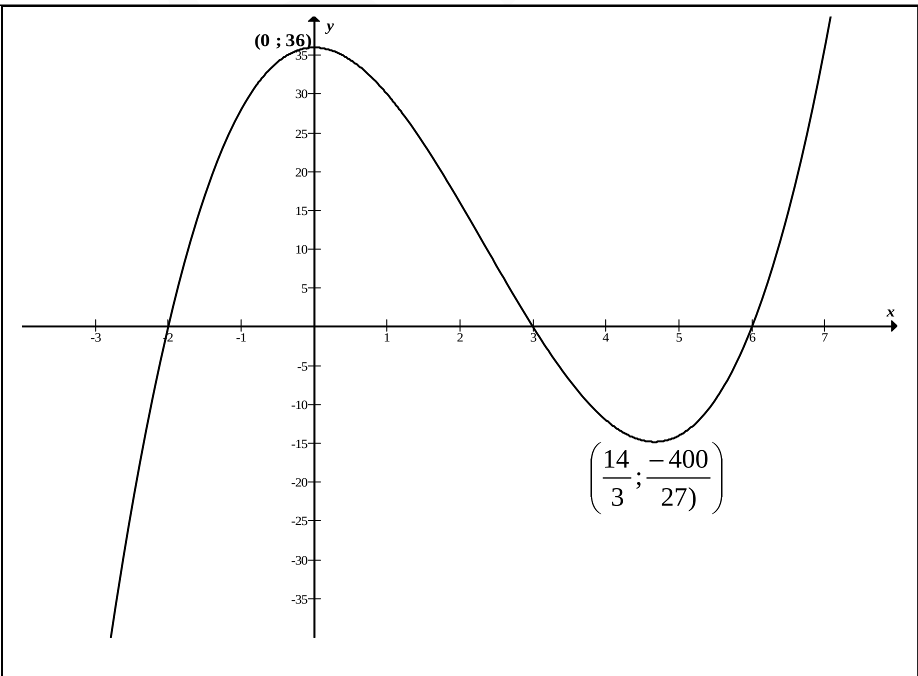
12.1	$x, y \in \mathbf{N}$ $x + y \leq 15$ **OR** $y \leq -x + 15$ $50x + 25y \geq 500$ **OR** $y \geq -2x + 20$ $y \leq 8$ **OR** $y \leq 8$ **Note:** If candidate gives $50x + 25y = 500$: max 5/6 marks **Note:** for the inequality's marks to be awarded, the LHS and the RHS must be correct	✓✓ $x + y \leq 15$ ✓✓ $y \leq 8$ ✓✓ $50x + 25y \geq 500$ (6)
12.2		✓ $x + y \leq 15$ ✓ $50x + 25y \geq 500$ ✓ $y \leq 8$ ✓ feasible region (4)
12.3	$C = 600x + 300y$	✓ answer (1)
12.4.1	(6 ; 8) ; (7 ; 6) ; (8 ; 4) ; (9 ; 2) and (10 ; 0) **NOTE:** The gradient of the search line is $m = -\frac{2}{1}$	3 marks for all correct solutions 2 marks if only 3 or 4 correct solutions 1 mark if only 1 or 2 correct solutions (3)
12.4.2	$C = 6(600) + 8(300) = R6\,000$ or $C = 7(600) + 6(300) = R6\,000$ or $C = 8(600) + 4(300) = R6\,000$ or $C = 9(600) + 2(300) = R6\,000$ or $C = 10(600) + 0(300) = R6\,000$	✓ subs ✓ answer (2)
12.5	8 red ; 4 blue	✓ answer (1) **[17]**

TOTAL: 150

QUESTION 12.2

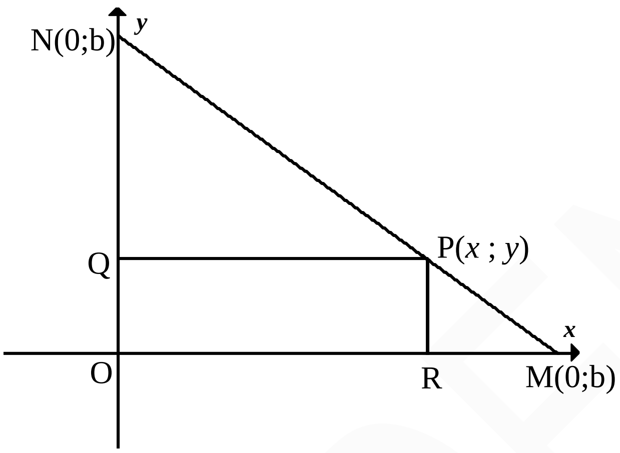


QUESTION 10

10.1	$(-6)(-3)(+2) = 36$ y-intercept is 36 **OR** $g(x) = (x-6)(x^2 - x - 6)$ $g(x) = x^3 - 7x^2 + 36$ y-intercept : (0;36)	✓ $(-6)(-3)(+2)$ ✓ y-intercept is 36 (1) ✓ trinomial ✓ 36 (1)
10.2	$g(x) = 0$ $x = 6$ or $x = 3$ or $x = -2$ intercepts are (6 ; 0) and (3 ; 0) and (– 2; 0)	✓ $g(x) = 0$ ✓ all x-intercepts (2)
10.3	$g(x) = (x-6)(x^2 - x - 6)$ $= x^3 - 7x^2 + 36$ $g'(x) = 3x^2 - 14x$ $0 = x(3x - 14)$ $x = 0$ or $x = \frac{14}{3}$ Turning points are (0 ; 36) and $\left(\frac{14}{3} ; -\frac{400}{27}\right)$	✓ $x^3 - 7x + 36$ ✓ $g'(x) = 3x^2 - 14x$ ✓ $g'(x) = 0$ ✓ answers ✓✓ points (6)
10.4	 	✓ x-intercepts ✓✓ turning points ✓ shape (4)

10.5	$g(x).g'(x<0$ $x<-2$ or $0<x<3$ or $\frac{14}{3}<x<6$	1 mark for each inequality (3) **[16]**

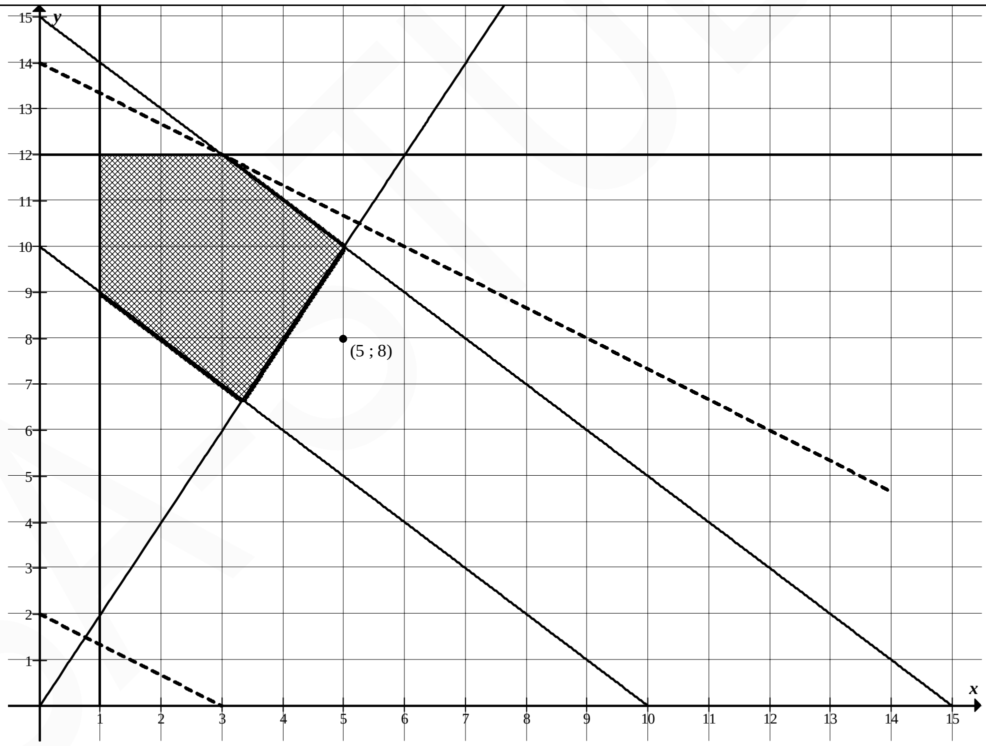
QUESTION 11



11.1	$m=-\frac{b}{a}$ $y-b=\frac{-b}{a}(x-0)$ $y=\frac{-b}{a}x+b$ OR $y=mx+b$ $0=ma+b$ $m=\frac{-b}{a}$ $y=-\frac{b}{a}x+b$ OR $\frac{x}{a}+\frac{y}{b}=1$	✓ $m=-\frac{b}{a}$ ✓ answer (2)
11.2	A = xy $A=x\left(\frac{-bx}{a}+b\right)$ $=-\frac{b}{a}x^2+bx$ $\frac{dA}{dx}=-\frac{2b}{a}x+b$ $0=-\frac{2b}{a}x+b$ $-ba=-2bx$ $x=\frac{a}{2}$ $y=-\frac{b}{a}\left(\frac{a}{2}\right)+b$ $=\frac{b}{2}$ P$\left(\frac{a}{2};\frac{b}{2}\right)$ which is the midpoint of MN **OR**	✓ area formula ✓ substitution ✓ $\frac{dA}{dx}=-\frac{2b}{a}x+b$ ✓ $\frac{dA}{dx}=0$ ✓ x-value ✓ y-value (6)

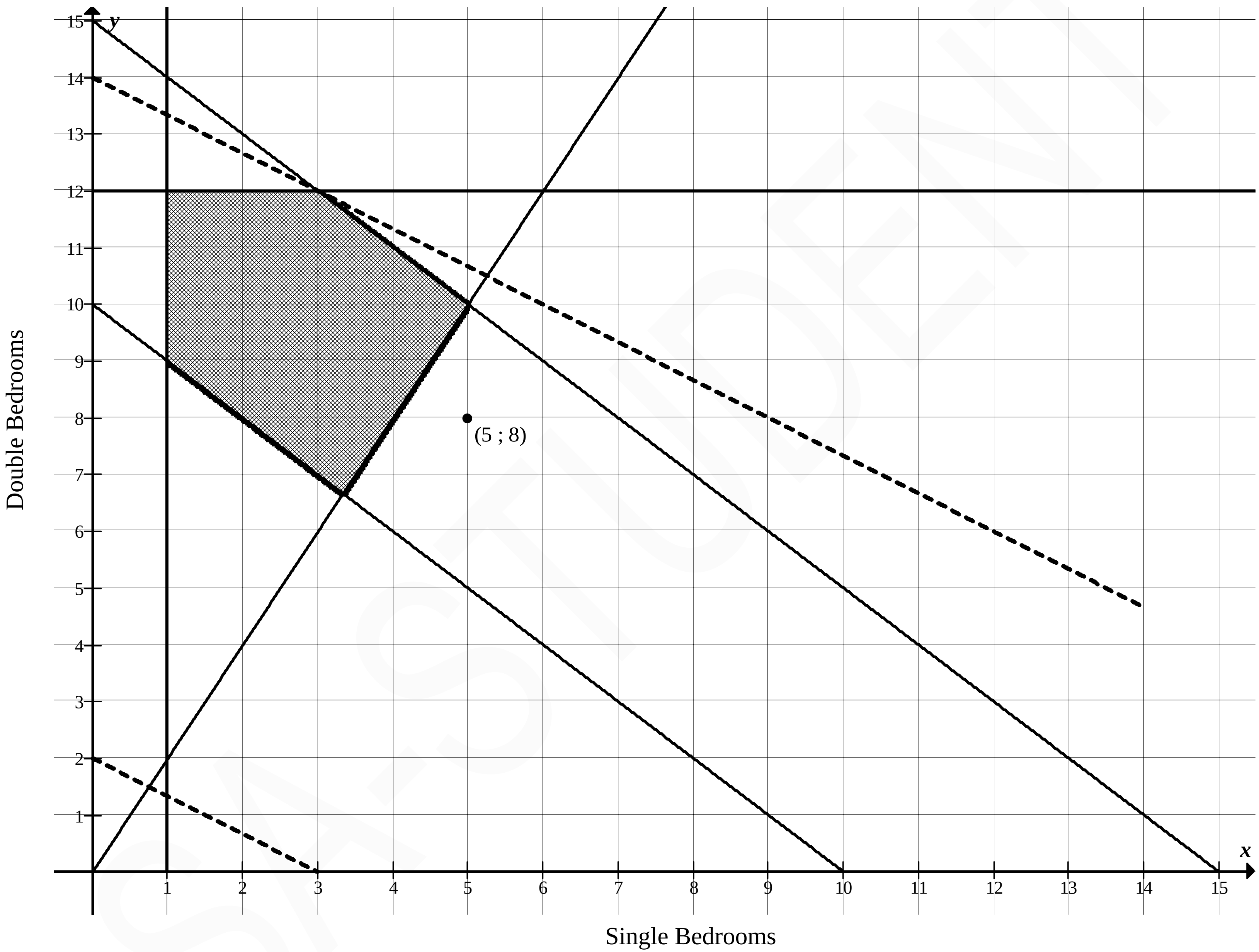
	$\frac{x}{a}+\frac{y}{b}=1$ $\frac{y}{b}=1-\frac{x}{a}$ To maximise xy, we maximise $\frac{xy}{ab}=\frac{x}{a}\left(\frac{y}{b}\right)=\frac{x}{a}\left(1-\frac{x}{a}\right)$ This is a maximum when $\frac{x}{a}=\frac{1}{2}$ i.e. $x=\frac{a}{2}$ By the midpoint theorem, P is then the midpoint of MN.	**(6)** **[8]**

QUESTION 12

12.1	$x \geq 1$ $y \leq 12$ $x+y \geq 10$ $x+y \leq 15$ $y \geq 2x$ $x, y \in N_0$	✓ $x \geq 1$ ✓ $y \leq 12$ ✓ $x+y \geq 10$ ✓ $x+y \leq 15$ ✓✓ $y \geq 2x$ (6)
12.2		✓ $x \geq 1$; ✓ $y \leq 12$ ✓ $x+y \leq 15$ ✓ $x+y \geq 10$ ✓ $y \geq 2x$ ✓ ✓feasible region (7)
12.3	No. The point (5 ; 8) lies outside the feasible region **OR** 8 is not greater than 2(5) =10	✓ No ✓ Reason (2)

12.4	I = 600x + 900y $y = -\frac{2}{3}x + \frac{I}{900}$ Maximum Income at(3 ; 12) 3 single bedrooms and 12 double bedrooms **OR** To optimise profit, the group must build as many rooms as possible and then, as many double rooms as possible. So 15 rooms, 12 double rooms, hence 3 single rooms.	✓ objective function ✓ search line ✓ answer (3) **[18]**

TOTAL: 150

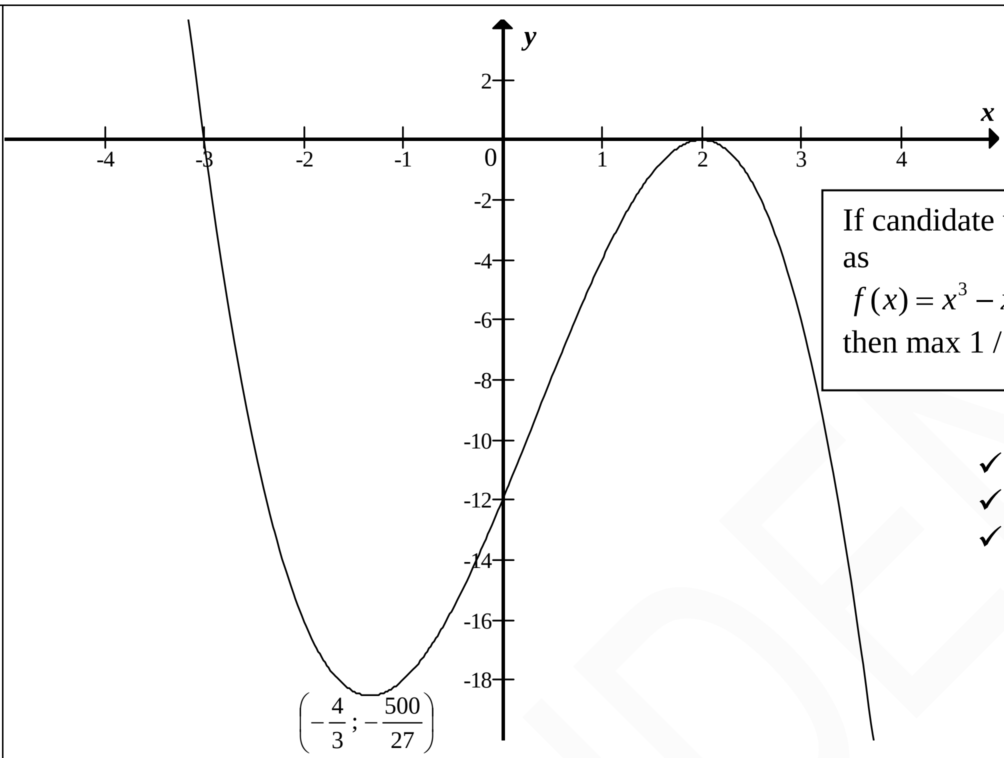

y
x
15
14
13
12
11
10
9
8
7
6
5
4
3
2
1
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15
(5 ; 8)
Double Bedrooms
Single Bedrooms

- Consistent Accuracy will apply as a general rule.
- If a candidate does a question twice and does not delete either, mark the FIRST attempt.
- If a candidate does a question, crosses it out and does not re-do it, mark the deleted attempt.

QUESTION 11

11.1	$0=-x^3+x^2+8x-12$ $x^3-x^2-8x+12=0$ $(x-2)(x^2+x-6)=0$ $(x-2)(x-2)(x+3)=0$ $x=2$ or $x=-3$ x-intercepts are $(2\ ;\ 0)$ and $(-3\ ;\ 0)$ **OR** $0=-x^3+x^2+8x-12$ $x^3-x^2-8x+12=0$ $(x+3)(x^2-4x+4)=0$ $(x+3)(x-2)(x-2)=0$ $x=2$ or $x=-3$ x-intercepts are $(2\ ;\ 0)$ and $(-3\ ;\ 0)$	✓ any one of factors ✓ quadratic factor ✓ linear factors ✓✓ x-answers (5)
11.2	$f'(x)=-3x^2+2x+8$ $0=3x^2-2x-8$ $0=(x-2)(3x+4)$ $x=2$ or $x=-\frac{4}{3}$ turning points are $(2\ ;\ 0)$ and $\left(-\frac{4}{3}\ ;-\frac{500}{27}\right)$ OR $(2\ ;\ 0)$ and $(-1{,}33\ ;-18{,}52)$ **NOTE:** If = 0 is omitted in 11.2: penalty 1 mark If not in coordinate form but coordinates implied: OK	✓ $f'(x)=0$ ✓ $-3x^2+2x+8=0$ or $3x^2-2x-8=0$ ✓ factors ✓ x-values ✓ y-values (5)

- Consistent Accuracy will apply as a general rule.
- If a candidate does a question twice and does not delete either, mark the FIRST attempt.
- If a candidate does a question, crosses it out and does not re-do it, mark the deleted attempt.

11.3	 If candidate used function as $f(x)=x^3-x^2-8x+12$ then max 1 / 3	✓ shape ✓ y-intercept ✓ turning pts (3)
11.4	$f''(x)=0$ $6x-2=0$ $x=\frac{1}{3}$ or $f''(x)=0$ $-6x+2=0$ $x=\frac{1}{3}$ **OR** $x=\dfrac{2-\frac{4}{3}}{2}$ $x=\frac{1}{3}$ **Note:** If write down $f''(x)=6x-2$ or $f''(x)=-6x+2$ then 1 / 2	✓ method ✓ answer Answer only: Full marks (2)
11.5	$(2\ ;\ -3)$ and $\left(-\frac{4}{3};-\frac{581}{27}\right)$ **OR** $(2\ ;\ -3)$ and $(-1{,}33\ ;-21{,}52)$	✓✓ each answer (2) **[17]**

- Consistent Accuracy will apply as a general rule.
- If a candidate does a question twice and does not delete either, mark the FIRST attempt.
- If a candidate does a question, crosses it out and does not re-do it, mark the deleted attempt.

QUESTION 12

12.1	$s(0) = 5(0)^3 - 65(0)^2 + 200(0) + 100$ $= 100$ metres **NOTE:** If subs $t = 8$, then answer = 100: 0 / 2	✓ $t = 0$ ✓ answer (2) Answer only: full marks
12.2	$s(t) = 5t^3 - 65t^2 + 200t + 100$ $s'(t) = 15t^2 - 130t + 200$ $s'(4) = 15(4)^2 - 130(4) + 200$ $= -80$ metres per minute **NOTE:** If used average rate of change between $t = 0$ and $t = 4$: 0 / 3 If subs $t = 4$ into $s(t)$: 0 / 3	✓ $s'(t) = 15t^2 - 130t + 200$ ✓ substitution $t = 4$ ✓ answer (– 80) (3)
12.3	The height of the car above sea level is decreasing at 80 metres per minute and the car is travelling downwards hence it is a negative rate of change. **OR** The **vertical** velocity of the car at $t = 4$ is 80 metres per minute. **NOTE:** Mark this CA even if answer to QUESTION 12.2 is completely inaccurate.	✓ speed 80 metres per minute ✓ downwards (2)
12.4	$s'(t) = 15t^2 - 130t + 200$ $s''(t) = 30t - 130$ $130 = 30t$ $t = 4{,}3\dot{3}$ minutes **OR** $t = \frac{-(-130)}{2(15)}$ $t = 4{,}3\dot{3}$ minutes	✓ $s''(t) = 30t - 130$ ✓ $s''(t) = 0$ ✓ answer (3) **[10]**

- Consistent Accuracy will apply as a general rule.
- If a candidate does a question twice and does not delete either, mark the FIRST attempt.
- If a candidate does a question, crosses it out and does not re-do it, mark the deleted attempt.

QUESTION 13

<table>
<tr><td>13.1</td><td>$x+3y\le 18$
$x+y\le 8$
$2x+y\le 14$
$x,y\ge 0$

OR
$6x+18y\le 108$
$8x+8y\le 64$
$14x+7y\le 98$
$x,y\ge 0$

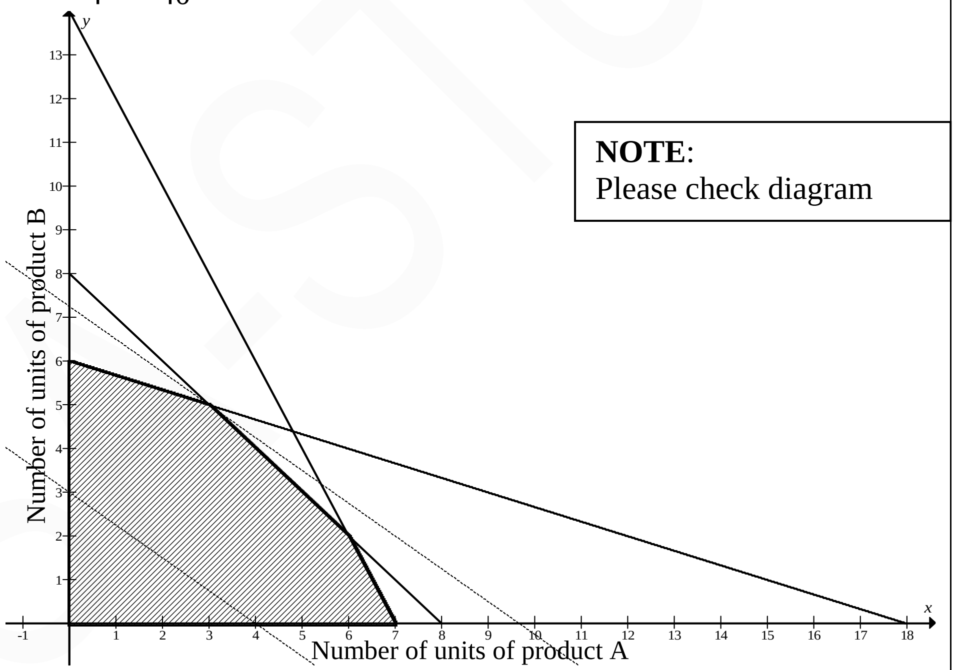
OR
$y\le -\frac{1}{3}x+6$
$y\le -x+8$
$y\le -2x+14$
$x,y\ge 0$

NOTE:
If written as equations (inequality omitted): max 6 / 7

If inequalities sign the wrong way round: max 6 / 7

One should note that x and y should be counting numbers</td><td>✓✓ answer
✓✓ answer
✓✓ answer
✓ answer
(7)</td></tr>
<tr><td>13.2</td><td>P = 30x + 40y

NOTE:
If P = 30A + 40B then 1 / 2</td><td>✓✓ answer
(2)</td></tr>
<tr><td>13.3</td><td>$y=-\frac{3}{4}x+\frac{P}{40}$



NOTE:
Please check diagram

Maximum at (3 ; 5)</td><td>✓✓ answer
(2)

Answer only : Full marks</td></tr>
<tr><td>13.4</td><td>$-2<m<-1$

NOTE:
accept 1 < m < 2: 2 / 2.
If ≤ signs used then max 1 / 2</td><td>✓✓answer
(2)

[13]</td></tr>
</table>