SA-STUDENT

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QUESTION 1

1.1 Solve for x:

1.1.1 $x^2 - 7x + 12 = 0$ (3)

1.1.2 $x(3x + 5) = 1$ (correct to TWO decimal places) (4)

1.1.3 $x^2 < -2x + 15$ (4)

1.1.4 $\sqrt{2(1-x)} = x - 1$ (4)

1.2 Solve for x and y simultaneously:

$3^{x+y} = 27$ and $x^2 + y^2 = 17$ (6)

1.3 Determine, **without the use of a calculator**, the value of:

$$\frac{1}{\sqrt{1}+\sqrt{2}} + \frac{1}{\sqrt{2}+\sqrt{3}} + \frac{1}{\sqrt{3}+\sqrt{4}} + \ldots + \frac{1}{\sqrt{99}+\sqrt{100}}$$ (3)

[24]

QUESTION 2

2.1 Given the geometric series: $\frac{1}{5} + \frac{1}{15} + \frac{1}{45} + \ldots$

2.1.1 Is this a convergent geometric series? Justify your answer with the necessary calculations. (2)

2.1.2 Calculate the sum to infinity of this series. (2)

2.2 An arithmetic and a geometric sequence are combined to form the pattern, which is given by: $P_n = x; \frac{1}{3}; 2x; \frac{1}{9}; 3x; \frac{1}{27}; \ldots$

2.2.1 Write down the next TWO terms of the pattern. (2)

2.2.2 Determine the general term (T_n) for the odd terms of this pattern. Write down your answer in terms of x. (2)

2.2.3 Calculate the value of P_{26}. (3)

2.2.4 If $\sum_{n=1}^{21} P_n = 33{,}5$, determine the value of x. (6)

[17]

QUESTION 1

1.1 Solve for x:

1.1.1 $(3x-6)(x+2)=0$ (2)

1.1.2 $2x^2-6x+1=0$ (correct to TWO decimal places) (3)

1.1.3 $x^2-90>x$ (4)

1.1.4 $x-7\sqrt{x}=-12$ (4)

1.2 Solve for x and y simultaneously:

$$2x-y=2$$
$$xy=4$$ (5)

1.3 Show that $2.5^n-5^{n+1}+5^{n+2}$ is even for all positive integer values of n. (3)

1.4 Determine the values of x and y if: $\frac{3^{y+1}}{32}=\sqrt{96^x}$ (4)

[25]

QUESTION 2

2.1 The first term of a geometric series is 14 and the 6^{th} term is 448.

2.1.1 Calculate the value of the constant ratio, r. (2)

2.1.2 Determine the number of consecutive terms that must be added to the first 6 terms of the series in order to obtain a sum of $114\,674$. (4)

2.1.3 If the first term of another series is 448 and the 6^{th} term is 14, calculate the sum to infinity of the new series. (3)

2.2 If $\sum_{p=0}^{k}\left(\frac{1}{3}p+\frac{1}{6}\right)=20\frac{1}{6}$, determine the value of k. (5)

[14]

QUESTION 1

1.1 Solve for x:

1.1.1 $x^2+2x-15=0$ (3)

1.1.2 $5x^2-x-9=0$ (Leave your answer correct to TWO decimal places.) (3)

1.1.3 $x^2 \leq 3x$ (4)

1.2 Given: $a+\frac{64}{a}=16$

1.2.1 Solve for a. (3)

1.2.2 Hence, solve for x: $2^x+2^{6-x}=16$ (3)

1.3 **Without using a calculator**, calculate the value of $\sqrt{\frac{2^{1002}+2^{1006}}{17(2)^{998}}}$ (4)

1.4 Solve for x and y simultaneously:

$2x-y=2$ and $\frac{1}{x}-3y=1$ (6)

[26]

QUESTION 2

2.1 The first term of an arithmetic sequence is -1 and the 7^{th} term is 35.

Determine:

2.1.1 The common difference of the sequence (2)

2.1.2 The number of terms in the sequence if the last term of the sequence is 473 (3)

2.1.3 The sum of the first 40 terms in this sequence (2)

2.2 $75\ ;\ 53\ ;\ 35\ ;\ 21\ ;\ \ldots$ is a quadratic number pattern.

2.2.1 Write down the FIFTH term of the number pattern. (1)

2.2.2 Determine the n^{th} term of the number pattern. (4)

2.2.3 Determine the maximum value of the following number pattern:

$-15\ ;-\frac{53}{5}\ ;-7\ ;-\frac{21}{5}\ ;\ldots$ (4)

[16]

QUESTION 1

1.1 Solve for x:

1.1.1 $x^2 - 2x - 24 = 0$ (3)

1.1.2 $2x^2 - 3x - 3 = 0$ (correct to TWO decimal places) (3)

1.1.3 $x^2 + 5x \leq -4$ (4)

1.1.4 $\sqrt{x+28} = 2 - x$ (4)

1.2 Solve simultaneously for x and y in:

$2y = 3 + x$ and $2xy + 7 = x^2 + 4y^2$ (6)

1.3 The roots of an equation are $x = \dfrac{-n \pm \sqrt{n^2 - 4mp}}{2m}$ where m, n and p are positive real numbers. The numbers m, n and p, in that order, form a geometric sequence. Prove that x is a non-real number. (4)
[24]

QUESTION 2

Given the geometric series: $x + 90 + 81 + \ldots$

2.1 Calculate the value of x. (2)

2.2 Show that the sum of the first n terms is $S_n = 1\,000\left(1 - (0{,}9)^n\right)$. (2)

2.3 Hence, or otherwise, calculate the sum to infinity. (2)
[6]

QUESTION 1

1.1 Solve for x:

1.1.1 $x^2 - x - 20 = 0$ (3)

1.1.2 $3x^2 - 2x - 6 = 0$ (correct to TWO decimal places) (4)

1.1.3 $(x-1)^2 > 9$ (4)

1.1.4 $2\sqrt{x+6} + 2 = x$ (4)

1.2 Solve simultaneously for x and y:

$4x + y = 2$ and $4x + y^2 = 8$ (5)

1.3 If it is given that $2^x \times 3^y = 24^6$, determine the numerical value of $x - y$. (4)
[24]

QUESTION 2

2.1 Consider the quadratic sequence: 72 ; 100 ; 120 ; 132 ; ...

2.1.1 Determine T_n, the n^{th} term of the quadratic sequence. (4)

2.1.2 A term in the quadratic sequence 72 ; 100 ; 120 ; 132 ; ... is equal to the twelfth term of the sequence of first-differences. Determine the position of this term in the quadratic sequence. (5)

2.1.3 Determine the maximum value of the quadratic sequence. (3)

2.1.4 Determine the maximum value of the sequence:

–23 ; 5 ; 25 ; 37 ; ... (1)

2.2 Consider the sequence: -11 ; $2\sin 3x$; 15 ; ...
Determine the values of x in the interval $[0°; 90°]$ for which the sequence will be arithmetic. (4)
[17]

QUESTION 1

1.1 Solve for x:

1.1.1 $x^2 - 6x = 0$ (2)

1.1.2 $x^2 + 10x + 8 = 0$ (correct to TWO decimal places) (3)

1.1.3 $(1-x)(x+2) < 0$ (3)

1.1.4 $\sqrt{x+18} = x-2$ (5)

1.2 Solve simultaneously for x and y:

$x + y = 3$ and $2x^2 + 4xy - y = 15$ (6)

1.3 If n is the largest integer for which $n^{200} < 5^{300}$, determine the value of n. (3)

[22]

QUESTION 2

2.1 $7 \; ; \; x \; ; \; y \; ; \; -11 \; ; \; \ldots$ is an arithmetic sequence. Determine the values of x and y. (4)

2.2 Given the quadratic number pattern: $-3 \; ; \; 6 \; ; \; 27 \; ; \; 60 \; ; \; \ldots$

2.2.1 Determine the general term of the pattern in the form $T_n = an^2 + bn + c$. (4)

2.2.2 Calculate the value of the 50^{th} term of the pattern. (2)

2.2.3 Show that the sum of the first n first-differences of this pattern can be given by $S_n = 6n^2 + 3n$. (3)

2.2.4 How many consecutive first-differences were added to the first term of the quadratic number pattern to obtain a term in the quadratic number pattern that has a value of 21 060? (4)

[17]

QUESTION 1

1.1 Solve for x:

1.1.1 $x^2 + 5x - 6 = 0$ (3)

1.1.2 $4x^2 + 3x - 5 = 0$ (correct to TWO decimal places) (3)

1.1.3 $4x^2 - 1 < 0$ (3)

1.1.4 $\left(\sqrt{\sqrt{32} + x}\right)\left(\sqrt{\sqrt{32} - x}\right) = x$ (4)

1.2 Solve simultaneously for x and y:
$y + x = 12$ and $xy = 14 - 3x$ (5)

1.3 Consider the product $1 \times 2 \times 3 \times 4 \times \ldots \times 30$.
Determine the largest value of k such that 3^k is a factor of this product. (4)

[22]

QUESTION 2

2.1 Given the quadratic sequence: 321 ; 290 ; 261 ; 234 ; ....

2.1.1 Write down the values of the next TWO terms of the sequence. (2)

2.1.2 Determine the general term of the sequence in the form $T_n = an^2 + bn + c$. (4)

2.1.3 Which term(s) of the sequence will have a value of 74? (4)

2.1.4 Which term in the sequence has the least value? (2)

2.2 Given the geometric series: $\frac{5}{8} + \frac{5}{16} + \frac{5}{32} + \ldots = K$

2.2.1 Determine the value of K if the series has 21 terms. (3)

2.2.2 Determine the largest value of n for which $T_n > \frac{5}{8192}$ (4)

[19]

QUESTION 1

1.1 Solve for x:

1.1.1 $x^2 - 5x - 6 = 0$ (2)

1.1.2 $(3x-1)(x-4) = 16$ (correct to TWO decimal places) (4)

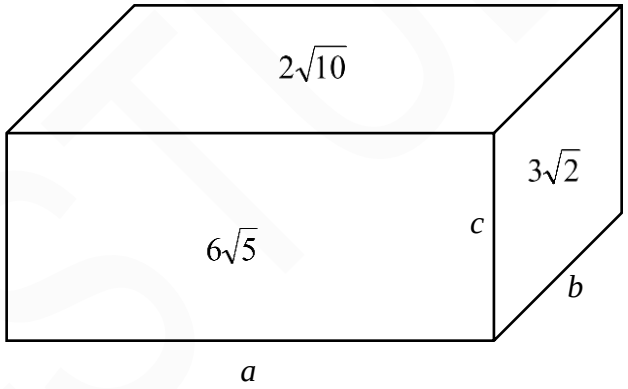
1.1.3 $4x - x^2 \geq 0$ (3)

1.1.4 $\dfrac{5^{2x} - 1}{5^x + 1} = 4$ (3)

1.2 Solve simultaneously for x and y:

$x + 3y = 2$ and $x^2 + 4xy - 5 = 0$ (5)

1.3 A rectangular box has dimensions a, b and c. The area of the surfaces are $2\sqrt{10}$; $3\sqrt{2}$ and $6\sqrt{5}$, as shown in the diagram below.



Calculate, **without using a calculator**, the volume of the rectangular box. (5)
[22]

QUESTION 2

2.1 The first FOUR terms of a quadratic pattern are: 15 ; 29 ; 41 ; 51

2.1.1 Write down the value of the 5th term. (1)

2.1.2 Determine an expression for the n^{th} term of the pattern in the form $T_n = an^2 + bn + c$. (4)

2.1.3 Determine the value of T_{27} (2)

2.2 Given a geometric sequence: 36 ; –18 ; 9 ; …

2.2.1 Determine the value of r, the common ratio. (1)

2.2.2 Calculate n if $T_n = \frac{9}{4\,096}$ (3)

2.2.3 Calculate S_∞ (2)

2.2.4 Calculate the value of $\frac{T_1 + T_3 + T_5 + T_7 + ... + T_{499}}{T_2 + T_4 + T_6 + T_8 + ... + T_{500}}$ (4)

[17]

QUESTION 3

3.1 The first three terms of an arithmetic sequence are: $2p + 3$; $p + 6$ and $p – 2$.

3.1.1 Show that $p = 11$. (2)

3.1.2 Calculate the smallest value of n for which $T_n < -55$. (3)

3.2 Given that $\sum_{k=1}^{6}(x-3k) = \sum_{k=1}^{9}(x-3k)$, prove that $\sum_{k=1}^{15}(x-3k) = 0$. (5)

[10]

QUESTION 4

Given the exponential function: $g(x) = \left(\frac{1}{2}\right)^x$

4.1 Write down the range of g. (1)

4.2 Determine the equation of g^{-1} in the form $y = …$ (2)

4.3 Is g^{-1} a function? Justify your answer. (2)

4.4 The point $M(a ; 2)$ lies on g^{-1}.

4.4.1 Calculate the value of a. (2)

4.4.2 M', the image of M, lies on g. Write down the coordinates of M'. (1)

4.5 If $h(x) = g(x + 3) + 2$, write down the coordinates of the image of M' on h. (3)

[11]

QUESTION 1

1.1 Solve for x:

1.1.1 $x^2 - 4x + 3 = 0$ (3)

1.1.2 $5x^2 - 5x + 1 = 0$ (correct to TWO decimal places) (3)

1.1.3 $x^2 - 3x - 10 > 0$ (3)

1.1.4 $3\sqrt{x} = x - 4$ (4)

1.2 Solve simultaneously for x and y:

$3x - y = 2$ and $2y + 9x^2 = -1$ (6)

1.3 If $3^{9x} = 64$ and $5^{\sqrt{p}} = 64$, calculate, WITHOUT the use of a calculator, the value of: $\dfrac{\left[3^{x-1}\right]^3}{\sqrt{5}^{\sqrt{p}}}$ (4)

[23]

QUESTION 2

2.1 Given the quadratic sequence: 2 ; 3 ; 10 ; 23 ; ...

2.1.1 Write down the next term of the sequence. (1)

2.1.2 Determine the n^{th} term of the sequence. (4)

2.1.3 Calculate the 20^{th} term of the sequence. (2)

2.2 Given the arithmetic sequence: 35 ; 28 ; 21 ; ...

Calculate which term of the sequence will have a value of -140. (3)

2.3 For which value of n will the sum of the first n terms of the arithmetic sequence in QUESTION 2.2 be equal to the n^{th} term of the quadratic sequence in QUESTION 2.1? (6)

[16]

QUESTION 1

1.1 Solve for x:

1.1.1 $x^2 - 6x - 16 = 0$ (3)

1.1.2 $2x^2 + 7x - 1 = 0$ (correct to TWO decimal places) (4)

1.2 List all the integers that are solutions to $x^2 - 25 < 0$. (4)

1.3 Solve for x and y:

$-2y + x = -1$ and $x^2 - 7 - y^2 = -y$ (6)

1.4 Evaluate: $\dfrac{3^{2018} + 3^{2016}}{3^{2017}}$ (2)

1.5 Given: $t(x) = \dfrac{\sqrt{3x-5}}{x-3}$

1.5.1 For which values of x will $\dfrac{\sqrt{3x-5}}{x-3}$ be real? (3)

1.5.2 Solve for x if $t(x) = 1$. (4)

[26]

QUESTION 2

2.1 Given the following geometric sequence: $30 \; ; \; 10 \; ; \; \dfrac{10}{3} \; ; \ldots$

2.1.1 Determine n if the n^{th} term of the sequence is equal to $\dfrac{10}{729}$. (4)

2.1.2 Calculate: $30 + 10 + \dfrac{10}{3} + \ldots$ (2)

2.2 Derive a formula for the sum of the first n terms of an arithmetic sequence if the first term of the sequence is a and the common difference is d. (4)

[10]

QUESTION 1

1.1 Solve for x:

1.1.1 $(3x-1)(x+4)=0$ (2)

1.1.2 $2x^2+9x-14=0$ (correct to TWO decimal places) (4)

1.1.3 $\sqrt{3-26x}=3x$ (4)

1.1.4 $(x-1)(x-4)>x+11$ (5)

1.2 Simplify fully:

$$\frac{\sqrt{16x^7}-\sqrt{25x^7}}{\sqrt{x}}$$ (3)

1.3 Solve simultaneously for x and y:

$xy=9$ and $x-2y-3=0$ (5)

1.4 Prove that $x^2+2xy+2y^2$ cannot be negative for $x, y \in \mathrm{R}$. (4)
[27]

QUESTION 2

2.1 Given the quadratic pattern: 5 ; 10 ; 17 ; 26 ; …

2.1.1 Write down the next TWO terms of the pattern. (2)

2.1.2 Determine the formula for the n^{th} term of the pattern. (4)

2.1.3 Which term of the pattern will have a value of 1 765? (4)

2.2 The first 24 terms of an arithmetic series are: 35 + 42 + 49 + … + 196.

Calculate the sum of ALL natural numbers from 35 to 196 that are NOT divisible by 7. (5)
[15]

QUESTION 1

1.1 Solve for x:

1.1.1 $x^2 + 9x + 14 = 0$ (3)

1.1.2 $4x^2 + 9x - 3 = 0$ (correct to TWO decimal places) (4)

1.1.3 $\sqrt{x^2 - 5} = 2\sqrt{x}$ (4)

1.2 Solve for x and y if:

$3x - y = 4$ and $x^2 + 2xy - y^2 = -2$ (6)

1.3 Given: $f(x) = x^2 + 8x + 16$

1.3.1 Solve for x if $f(x) > 0$. (3)

1.3.2 For which values of p will $f(x) = p$ have TWO unequal negative roots? (4)

[24]

QUESTION 2

2.1 Given the following quadratic number pattern: 5 ; –4 ; –19 ; –40 ; …

2.1.1 Determine the constant second difference of the sequence. (2)

2.1.2 Determine the n^{th} term (T_n) of the pattern. (4)

2.1.3 Which term of the pattern will be equal to $-25\,939$? (3)

2.2 The first three terms of an arithmetic sequence are $2k - 7$; $k + 8$ and $2k - 1$.

2.2.1 Calculate the value of the 15^{th} term of the sequence. (5)

2.2.2 Calculate the sum of the first 30 even terms of the sequence. (4)

[18]

QUESTION 3

A convergent geometric series consisting of only positive terms has first term a, constant ratio r and n^{th} term, T_n, such that $\sum_{n=3}^{\infty} T_n = \frac{1}{4}$.

3.1 If $T_1 + T_2 = 2$, write down an expression for a in terms of r. (2)

3.2 Calculate the values of a and r. (6)

[8]

QUESTION 1

1.1 Solve for x:

1.1.1 $3x^2+10x+6=0$ (correct to TWO decimal places) (3)

1.1.2 $\sqrt{6x^2-15}=x+1$ (5)

1.1.3 $x^2+2x-24\geq 0$ (3)

1.2 Solve simultaneously for x and y:

$5x+y=3$ and $3x^2-2xy=y^2-105$ (6)

1.3 1.3.1 Solve for p if $p^2-48p-49=0$ (3)

1.3.2 Hence, or otherwise, solve for x if $7^{2x}-48(7^x)-49=0$ (3)

[23]

QUESTION 2

2.1 Given the geometric sequence: $3\ ;\ 2\ ;\ k\ ;\ ...$

2.1.1 Write down the value of the common ratio. (1)

2.1.2 Calculate the value of k. (2)

2.1.3 Calculate the value of n if $T_n=\frac{128}{729}$. (4)

2.2 In a Mathematics competition, the total prize money for the finalists is R30 500. Each finalist will receive a part of the prize money according to his/her position at the end of the competition. The table below shows the position of the finalists at the end of the competition and the prize money received.

POSITION OF THE FINALIST AT THE END OF THE COMPETITION	PRIZE MONEY
Last	R100
Second from last	R250
Third from last	R400
Fourth from last	R550
. . . .	. . . .
First	Rx

2.2.1 Calculate the prize money of the finalist finishing 18th from last. (2)

2.2.2 Calculate x. (6)

[15]

QUESTION 1

1.1 Solve for x:

1.1.1 $(x-3)(x+1)=0$ (2)

1.1.2 $\sqrt{x^3}=512$ (3)

1.1.3 $x(x-4)<0$ (2)

1.2 Given: $f(x)=x^2-5x+2$

1.2.1 Solve for x if $f(x)=0$ (3)

1.2.2 For which values of c will $f(x)=c$ have no real roots? (4)

1.3 Solve for x and y:

$$x=2y+2$$
$$x^2-2xy+3y^2=4$$ (6)

1.4 Calculate the maximum value of S if $S=\frac{6}{x^2+2}$. (2)

[22]

QUESTION 2

Given the geometric sequence: $-\frac{1}{4}\ ;\ b\ ;\ -1\ ;\ .....$

2.1 Calculate the possible values of b. (3)

2.2 If $b=\frac{1}{2}$, calculate the 19^{th} term (T_{19}) of the sequence. (3)

2.3 If $b=\frac{1}{2}$, write the sum of the first 20 positive terms of the sequence in sigma notation. (4)

2.4 Is the geometric series formed in QUESTION 2.3 convergent? Give reasons for your answer. (2)

[12]

QUESTION 1

1.1 Solve for x:

1.1.1 $x(x-7)=0$ (2)

1.1.2 $x^2-6x+2=0$ (correct to TWO decimal places) (3)

1.1.3 $\sqrt{x-1}+1=x$ (5)

1.1.4 $3^{x+3}-3^{x+2}=486$ (4)

1.2 Given: $f(x)=x^2+3x-4$

1.2.1 Solve for x if $f(x)=0$ (2)

1.2.2 Solve for x if $f(x)<0$ (2)

1.2.3 Determine the values of x for which $f'(x) \geq 0$ (2)

1.3 Solve for x and y: $x=2y$ and $x^2-5xy=-24$ (4)

[24]

QUESTION 2

Given the finite arithmetic sequence: 5 ; 1 ; –3 ; ... ; –83 ; –87

2.1 Write down the fourth term (T_4) of the sequence. (1)

2.2 Calculate the number of terms in the sequence. (3)

2.3 Calculate the sum of all the negative numbers in the sequence. (3)

2.4 Consider the sequence: 5 ; 1 ; –3 ; ... ; –83 ; –87 ; ... ; –4 187
Determine the number of terms in this sequence that will be exactly divisible by 5. (4)

[11]

QUESTION 1

1.1 Solve for x:

1.1.1 $4x^2 - 25 = 0$ (3)

1.1.2 $x^2 - 5x - 2 = 0$ (correct to TWO decimal places) (3)

1.1.3 $(2-x)(x+4) \geq 0$ (3)

1.1.4 $x - 3x^{\frac{1}{2}} = 4$ (5)

1.2 Solve for x and y:

$2x - y + 1 = 0$ and $x^2 - 3x - 4 - y = y^2$ (6)

1.3 Given: $f(x) = \sqrt{2x+1}$

1.3.1 Write down the domain of f. (1)

1.3.2 Solve for x if $f(x) = 2x - 1$. (5)

[26]

QUESTION 2

2.1 Given the arithmetic series: $a + 13 + b + 27 + \ldots$

2.1.1 Show that $a = 6$ and $b = 20$ (2)

2.1.2 Calculate the sum of the first 20 terms of the series. (3)

2.1.3 Write the series in QUESTION 2.1.2 in sigma notation. (2)

2.2 Given the geometric series: $(x-2) + (x^2 - 4) + (x^3 + 2x^2 - 4x - 8) + \ldots$

2.2.1 Determine the values of x for which the series converges. (4)

2.2.2 If $x = -\frac{3}{2}$, calculate the sum to infinity of the given series. (3)

[14]

QUESTION 1

1.1 Solve for x:

1.1.1 $x^2 - x - 12 = 0$ (3)

1.1.2 $x(x+3) - 1 = 0$ (Leave your answer in simplest surd form.) (3)

1.1.3 $x(4 - x) < 0$ (3)

1.1.4 $x = \dfrac{a^2 + a - 2}{a - 1}$ if $a = 888\,888\,888\,888$ (2)

1.2 Solve the following equations simultaneously:

$y + 7 = 2x$ and $x^2 - xy + 3y^2 = 15$ (6)

1.3 Determine the range of the function $y = x + \dfrac{1}{x}$, $x \neq 0$ and x is real. (6)

[23]

QUESTION 2

2.1 Given the following quadratic sequence: $-2\ ;\ 0\ ;\ 3\ ;\ 7\ ;\ ...$

2.1.1 Write down the value of the next term of this sequence. (1)

2.1.2 Determine an expression for the n^{th} term of this sequence. (5)

2.1.3 Which term of the sequence will be equal to 322? (4)

2.2 Consider an arithmetic sequence which has the second term equal to 8 and the fifth term equal to 10.

2.2.1 Determine the common difference of this sequence. (3)

2.2.2 Write down the sum of the first 50 terms of this sequence, using sigma notation. (2)

2.2.3 Determine the sum of the first 50 terms of this sequence. (3)

[18]

QUESTION 1

1.1 Solve for x:

1.1.1 $x^2 - 9x + 20 = 0$ (3)

1.1.2 $3x^2 + 5x = 4$ (correct to TWO decimal places) (4)

1.1.3 $2x^{\frac{-5}{3}} = 64$ (4)

1.1.4 $\sqrt{2-x} = x - 2$ (4)

1.1.5 $x^2 + 7x < 0$ (3)

1.2 Given: $(3x - y)^2 + (x - 5)^2 = 0$

Solve for x and y. (4)

1.3 For which value of k will the equation $x^2 + x = k$ have no real roots? (4)
[26]

QUESTION 2

The following geometric sequence is given: 10 ; 5 ; 2,5 ; 1,25 ; ...

2.1 Calculate the value of the 5^{th} term, T_5, of this sequence. (2)

2.2 Determine the n^{th} term, T_n, in terms of n. (2)

2.3 Explain why the infinite series $10 + 5 + 2{,}5 + 1{,}25 + \ldots$ converges. (2)

2.4 Determine $S_\infty - S_n$ in the form ab^n, where S_n is the sum of the first n terms of the sequence. (4)
[10]

QUESTION 1

1.1 Solve for x:

1.1.1 $x^2 - x - 20 = 0$ (2)

1.1.2 $2x^2 - 11x + 7 = 0$ (correct to TWO decimal places) (3)

1.1.3 $5x^2 + 4 > 21x$ (5)

1.1.4 $2^{2x} - 6.2^x = 16$ (4)

1.2 Solve for x and y simultaneously:

$$y + 1 = 2x$$
$$x^2 - xy + y^2 = 7$$ (6)

1.3 The roots of a quadratic equation are given by $x = \frac{-5 \pm \sqrt{20 + 8k}}{6}$, where $k \in \{-3; -2; -1; 0; 1; 2; 3\}$.

1.3.1 Write down TWO values of k for which the roots will be rational. (2)

1.3.2 Write down ONE value of k for which the roots will be non-real. (1)

1.4 Calculate a and b if $\sqrt{\frac{7^{2014} - 7^{2012}}{12}} = a\left(7^b\right)$ and a is not a multiple of 7. (4)

[27]

QUESTION 1

1.1 Solve for x:

1.1.1 $(x-2)(4+x)=0$ (2)

1.1.2 $3x^2-2x=14$ (correct to TWO decimal places) (4)

1.1.3 $2^{x+2}+2^x=20$ (3)

1.2 Solve the following equations simultaneously:

$x=2y+3$
$3x^2-5xy=24+16y$ (6)

1.3 Solve for x: $(x-1)(x-2)<6$ (4)

1.4 The roots of a quadratic equation are: $x=\dfrac{3\pm\sqrt{-k-4}}{2}$

For which values of k are the roots real? (2)
[21]

QUESTION 2

Given the arithmetic series: $2+9+16+\ldots$ (to 251 terms).

2.1 Write down the fourth term of the series. (1)

2.2 Calculate the 251^{st} term of the series. (3)

2.3 Express the series in sigma notation. (2)

2.4 Calculate the sum of the series. (2)

2.5 How many terms in the series are divisible by 4? (4)
[12]

QUESTION 1

1.1 Solve for x in each of the following:

1.1.1 $x^2 - 2x - 35 = 0$ (3)

1.1.2 $x^2 - 16 \geq 0$ (4)

1.1.3 $9.2^{x-1} = 2.3^x$ (3)

1.2 Given: $f(x) = x^2 - 5x + c$

Determine the value of c if it is given that the solutions of $f(x) = 0$ are $\frac{5 \pm \sqrt{41}}{2}$. (3)

1.3 Solve for x and y if: $3^{x-10} = 3^{3x}$ and $y^2 + x = 20$. (5)
[18]

QUESTION 2

2.1 A geometric sequence has $T_3 = 20$ and $T_4 = 40$.

Determine:

2.1.1 The common ratio (1)

2.1.2 A formula for T_n (3)

2.2 The following sequence has the property that the sequence of numerators is arithmetic and the sequence of denominators is geometric:

$$\frac{2}{1}; \frac{-1}{5}; \frac{-4}{25}; \ldots$$

2.2.1 Write down the FOURTH term of the sequence. (1)

2.2.2 Determine a formula for the n^{th} term. (3)

2.2.3 Determine the 500^{th} term of the sequence. (2)

2.2.4 Which will be the first term of the sequence to have a NUMERATOR which is less than –59? (3)
[13]

QUESTION 1

1.1 Solve for x:

1.1.1 $3x^2 - 4x = 0$ (2)

1.1.2 $x - 6 + \frac{2}{x} = 0$; $x \neq 0$. (Leave your answer correct to TWO decimal places.) (4)

1.1.3 $x^{\frac{2}{3}} = 4$ (2)

1.1.4 $3^x(x-5) < 0$ (2)

1.2 Solve for x and y simultaneously:
$y = x^2 - x - 6$ **and** $2x - y = 2$ (6)

1.3 Simplify, without the use of a calculator:
$\sqrt{3}.\sqrt{48} - \frac{4^{x+1}}{2^{2x}}$ (3)

1.4 Given: $f(x) = 3(x-1)^2 + 5$ and $g(x) = 3$

1.4.1 Is it possible for $f(x) = g(x)$? Give a reason for your answer. (2)

1.4.2 Determine the value(s) of k for which $f(x) = g(x) + k$ has TWO unequal real roots. (2)
[23]

QUESTION 2

2.1 Given the arithmetic series: 18 + 24 + 30 + … + 300

2.1.1 Determine the number of terms in this series. (3)

2.1.2 Calculate the sum of this series. (2)

2.1.3 Calculate the sum of all the whole numbers up to and including 300 that are NOT divisible by 6. (4)

2.2 The first three terms of an infinite geometric sequence are 16, 8 and 4 respectively.

2.2.1 Determine the n^{th} term of the sequence. (2)

2.2.2 Determine all possible values of n for which the sum of the first n terms of this sequence is greater than 31. (3)

2.2.3 Calculate the sum to infinity of this sequence. (2)
[16]

QUESTION 1

1.1 Solve for x in each of the following:

1.1.1 $x^2 - x - 12 = 0$ (3)

1.1.2 (a) $2x^2 - 5x - 11 = 0$ (4)

(b) $2x^3 - 5x^2 - 11x = 0$ (2)

1.1.3 $-3(x+7)(x-5) < 0$ (4)

1.2 Given: $y + 2 = x$ and $y = x^2 - x - 10$

Solve for x and y simultaneously. (6)

1.3 Simplify: $\dfrac{3^{2015} + 3^{2013}}{9^{1006}}$ (3)
[22]

QUESTION 2

2.1 Given the geometric sequence: $7 ; x ; 63 ; ...$

Determine the possible values of x. (3)

2.2 The first term of a geometric sequence is 15. If the second term is 10, calculate:

2.2.1 T_{10} (3)

2.2.2 S_9 (2)

2.3 Given: $0 ; -\frac{1}{2} ; 0 ; \frac{1}{2} ; 0 ; \frac{3}{2} ; 0 ; \frac{5}{2} ; 0 ; \frac{7}{2} ; 0 ; ...$

Assume that this number pattern continues consistently.

2.3.1 Write down the value of the 191st term of this sequence. (1)

2.3.2 Determine the sum of the first 500 terms of this sequence. (4)

2.4 Given: $\sum_{k=2}^{20}(4x-1)^k$

2.4.1 Calculate the first term of the series $\sum_{k=2}^{20}(4x-1)^k$ if $x=1$. (2)

2.4.2 For which values of x will $\sum_{k=1}^{\infty}(4x-1)^k$ exist? (3)
[18]

QUESTION 3

3.1 Given the arithmetic sequence: $-3\ ;\ 1\ ;\ 5\ ;\ ...\ ;\ 393$

3.1.1 Determine a formula for the n^{th} term of the sequence. (2)

3.1.2 Write down the 4^{th}, 5^{th}, 6^{th} and 7^{th} terms of the sequence. (2)

3.1.3 Write down the remainders when each of the first seven terms of the sequence is divided by 3. (2)

3.1.4 Calculate the sum of the terms in the arithmetic sequence that are divisible by 3. (5)

3.2 Consider the following pattern of dots:

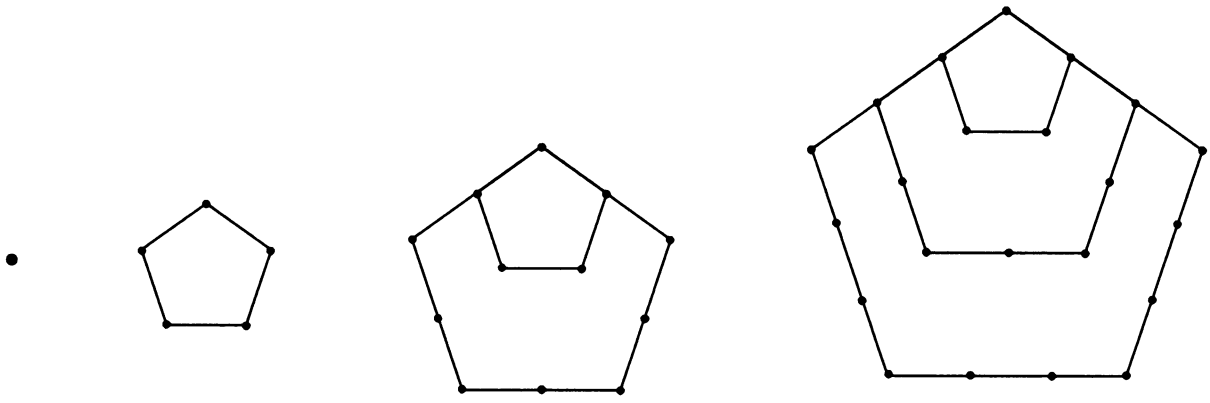


FIGURE 1 FIGURE 2 FIGURE 3 FIGURE 4

If T_n represents the total number of dots in FIGURE n, then $T_1 = 1$ and $T_2 = 5$. If the pattern continues in the same manner, determine:

3.2.1 T_5 (2)

3.2.2 T_{50} (5)
[18]

QUESTION 1

1.1 Solve for x:

1.1.1 $(x^2 - 9)(2x + 1) = 0$ (3)

1.1.2 $x^2 + x - 13 = 0$ (Leave your answer correct to TWO decimal places.) (4)

1.1.3 $2.3^x = 81 - 3^x$ (4)

1.1.4 $(x + 1)(4 - x) > 0$ (3)

1.2 Given: $2^x + 2^{x+2} = -5y + 20$

1.2.1 Express 2^x in terms of y. (2)

1.2.2 How many solutions for x will the equation have if $y = -4$? (2)

1.2.3 Solve for x if y is the largest possible integer value for which $2^x + 2^{x+2} = -5y + 20$ will have solutions. (3)

[21]

QUESTION 2

2.1 Given the geometric series: $256 + p + 64 - 32 + ...$

2.1.1 Determine the value of p. (3)

2.1.2 Calculate the sum of the first 8 terms of the series. (3)

2.1.3 Why does the sum to infinity for this series exist? (1)

2.1.4 Calculate S_∞ (3)

QUESTION 1

1.1 Solve for x in each of the following:

1.1.1 $(2x-1)(x+4)=0$ (2)

1.1.2 $3x^2 - x = 5$ (Leave your answer correct to TWO decimal places.) (4)

1.1.3 $x^2 + 7x - 8 < 0$ (4)

1.2 Given: $4y - x = 4$ and $xy = 8$

1.2.1 Solve for x and y simultaneously. (6)

1.2.2 The graph of $4y - x = 4$ is reflected across the line having equation $y = x$. What is the equation of the reflected line? (2)

1.3 The solutions of a quadratic equation are given by $x = \dfrac{-2 \pm \sqrt{2p+5}}{7}$

For which value(s) of p will this equation have:

1.3.1 Two equal solutions (2)

1.3.2 No real solutions (1)

[21]

QUESTION 2

2.1 $3x + 1$; $2x$; $3x - 7$ are the first three terms of an arithmetic sequence. Calculate the value of x. (2)

2.2 The first and second terms of an arithmetic sequence are 10 and 6 respectively.

2.2.1 Calculate the 11^{th} term of the sequence. (2)

2.2.2 The sum of the first n terms of this sequence is –560. Calculate n. (6)

[10]

QUESTION 1

1.1 Solve for x:

1.1.1 $3x^2 - 5x = 2$ (3)

1.1.2 $x - \frac{2}{x} = 5$ (4)

1.1.3 $(x+1)(x-3) > 12$ (4)

1.2 Solve simultaneously for r and p in the following set of equations:

$$6r + 5rp - 5p = 8$$
$$r + p = 2$$ (7)

1.3 The volume of a box with a rectangular base is 3 072 cm^3. The lengths of the sides are in the ratio 1 : 2 : 3. Calculate the length of the shortest side. (4)
[22]

QUESTION 2

Given the arithmetic series: $-7 - 3 + 1 + \ldots + 173$

2.1 How many terms are there in the series? (3)

2.2 Calculate the sum of the series. (3)

2.3 Write the series in sigma notation. (3)
[9]

QUESTION 3

3.1 Consider the geometric sequence: $4 ; -2 ; 1 \ldots$

3.1.1 Determine the next term of the sequence. (2)

3.1.2 Determine n if the n^{th} term is $\frac{1}{64}$. (4)

3.1.3 Calculate the sum to infinity of the series $4 - 2 + 1 \ldots$ (2)

3.2 If x is a REAL number, show that the following sequence can NOT be geometric:

$1 ; x + 1 ; x - 3 \ldots$ (4)
[12]

QUESTION 1

1.1 Solve for x:

1.1.1 $x(x+1)=6$ (3)

1.1.2 $3x^2-4x=8$ (4)

1.1.3 $4x^2+1\geq 5x$ (4)

1.2 Consider the equation: $x^2+5xy+6y^2=0$

1.2.1 Calculate the values of the ratio $\frac{x}{y}$. (3)

1.2.2 Hence, calculate the values of x and y if $x+y=8$. (5)
[19]

QUESTION 2

2.1 Given the sequence: 4 ; x ; 32

Determine the value(s) of x if the sequence is:

2.1.1 Arithmetic (2)

2.1.2 Geometric (3)

2.2 Determine the value of P if $\mathrm{P}=\sum_{k=1}^{13}3^{k-5}$ (4)

2.3 Prove that for any arithmetic sequence of which the first term is a and the constant difference is d, the sum to n terms can be expressed as $S_n=\frac{n}{2}\left(2a+(n-1)d\right)$. (4)
[13]

QUESTION 3

The following sequence is a combination of an arithmetic and a geometric sequence:

3 ; 3 ; 9 ; 6 ; 15 ; 12 ; …

3.1 Write down the next TWO terms. (2)

3.2 Calculate $T_{52}-T_{51}$. (5)

3.3 Prove that ALL the terms of this infinite sequence will be divisible by 3. (2)
[9]

QUESTION 1

1.1 Solve for x, correct to TWO decimal places, where necessary:

1.1.1 $x(x-1)=12$ (3)

1.1.2 $2x^2+3x-7=0$ (4)

1.1.3 $7x^2+18x-9>0$ (4)

1.2 Solve for x and y simultaneously:

$$2x-y=7$$
$$x^2+xy=21-y^2$$ (7)

1.3 Simplify completely, without the use of a calculator:

$$\left(\sqrt[5]{\sqrt{35}+\sqrt{3}}\right)\left(\sqrt[5]{\sqrt{35}-\sqrt{3}}\right)$$ (3)

[21]

QUESTION 2

The sequence 3 ; 9 ; 17 ; 27 ; … is a quadratic sequence.

2.1 Write down the next term. (1)

2.2 Determine an expression for the n^{th} term of the sequence. (4)

2.3 What is the value of the first term of the sequence that is greater than 269? (4)

[9]

QUESTION 3

3.1 The first two terms of an infinite geometric sequence are 8 and $\frac{8}{\sqrt{2}}$. Prove, without the use of a calculator, that the sum of the series to infinity is $16+8\sqrt{2}$. (4)

3.2 The following geometric series is given: $x = 5 + 15 + 45 + \ldots$ to 20 terms.

3.2.1 Write the series in sigma notation. (2)

3.2.2 Calculate the value of x. (3)

[9]

QUESTION 1

1.1 Solve for x:

1.1.1 $x(x-1)=30$ (3)

1.1.2 $3x^2-5x+1=0$ (Correct to ONE decimal place) (4)

1.1.3 $15x-4<9x^2$ (4)

1.2 Solve simultaneously for x and y in the following set of equations:

$$x-y=3$$
$$x^2-xy-2y^2-7=0$$ (5)

1.3 Calculate the exact value of:

$$\frac{\sqrt{10^{2009}}}{\sqrt{10^{2011}}-\sqrt{10^{2007}}}$$ (Show ALL calculations.) (3)

1.4 Simplify completely without the use of a calculator:

$$\left(1+\sqrt{2x^2}\right)^2-\sqrt{8x^2}$$ (3)

[22]

QUESTION 2

2.1 Tebogo and Matthew's teacher has asked that they use their own rule to construct a sequence of numbers, starting with 5. The sequences that they have constructed are given below.

Matthew's sequence: 5 ; 9 ; 13 ; 17 ; 21 ; …
Tebogo's sequence: 5 ; 125 ; 3 125 ; 78 125 ; 1 953 125 ; …

Write down the n^{th} term (or the rule in terms of n) of:

2.1.1 Matthew's sequence (3)

2.1.2 Tebogo's sequence (2)

2.2 Nomsa generates a sequence which is both arithmetic and geometric. The first term is 1. She claims that there is only one such sequence. Is that correct? Show ALL your workings to justify your answer. (5)

[10]

QUESTION 1

1.1 Solve for x, rounded off to TWO decimal places where necessary:

1.1.1 $x^2 = 5x - 4$ (3)

1.1.2 $x(3 - x) = -3$ (5)

1.1.3 $3 - x < 2x^2$ (5)

1.2 Determine the values of x and y if they satisfy both the following equations simultaneously:

$$2x + y = 3$$
$$x^2 + y + x = y^2$$ (8)

1.3 Given $x = 999\,999\,999\,999$, determine the exact value of $\frac{x^2 - 4}{x - 2}$.

Show ALL your calculations. (3)

1.4 Explain why the equation $\frac{x^4 + 1}{x^4} = \frac{1}{2}$ has no real roots. (2)

[26]