SA-STUDENT

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QUESTION 3

A quadratic sequence has the following properties:

- The second difference is 10.
- The first two terms are equal, i.e. $T_1 = T_2$.
- $T_1 + T_2 + T_3 = 28$

3.1 Show that the general term of the sequence is $T_n = 5n^2 - 15n + 16$. (6)

3.2 Is 216 a term in this sequence? Justify your answer with the necessary calculations. (3)
[9]

QUESTION 4

4.1 Given the function $p(x) = \left(\frac{1}{3}\right)^x$.

4.1.1 Is p an increasing or decreasing function? (1)

4.1.2 Determine p^{-1}, the inverse of p, in the form $y = \ldots$ (2)

4.1.3 Write down the domain of p^{-1}. (1)

4.1.4 Write down the equation of the asymptote of $p(x) - 5$. (1)

4.2 Given: $f(x) = \frac{4}{x-1} + 2$

4.2.1 Write down the equations of the asymptotes of f. (2)

4.2.2 Calculate the x-intercept of f. (2)

4.2.3 Sketch the graph of f, label all asymptotes and indicate the intercepts with the axes. (4)

4.2.4 Use your graph to determine the values of x for which $\frac{4}{x-1} \geq -2$. (2)

4.2.5 Determine the equation of the axis of symmetry of $f(x-2)$, that has a negative gradient. (3)
[18]

QUESTION 3

It is given that the general term of a quadratic number pattern is $T_n = n^2 + bn + 9$ and the first term of the first differences is 7.

3.1 Show that $b = 4$. (2)

3.2 Determine the value of the 60^{th} term of this number pattern. (2)

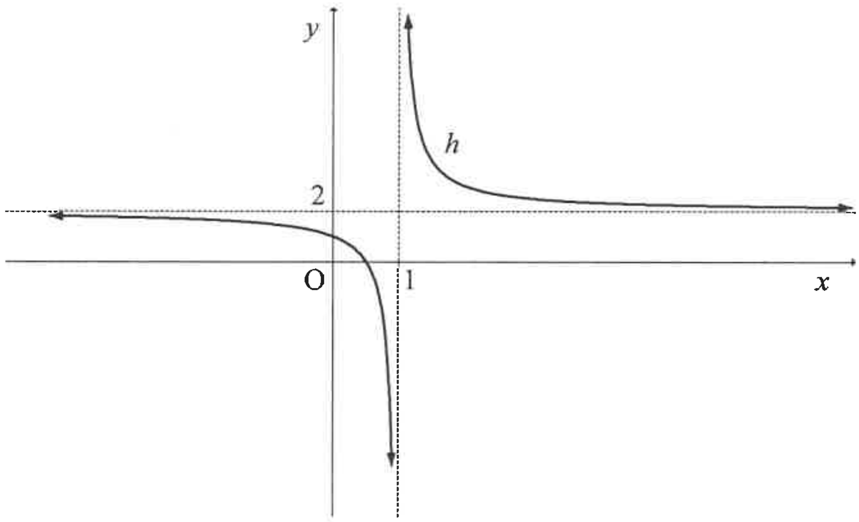
3.3 Determine the general term for the sequence of first differences of the quadratic number pattern. Write your answer in the form $T_p = mp + q$. (3)

3.4 Which TWO consecutive terms in the quadratic number pattern have a first difference of 157? (3)

[10]

QUESTION 4

4.1 Sketched below is the graph of $h(x) = \dfrac{1}{x+p} + q$. The asymptotes of h intersect at (1 ; 2).



4.1.1 Write down the values of p and q. (2)

4.1.2 Calculate the coordinates of the x-intercept of h. (2)

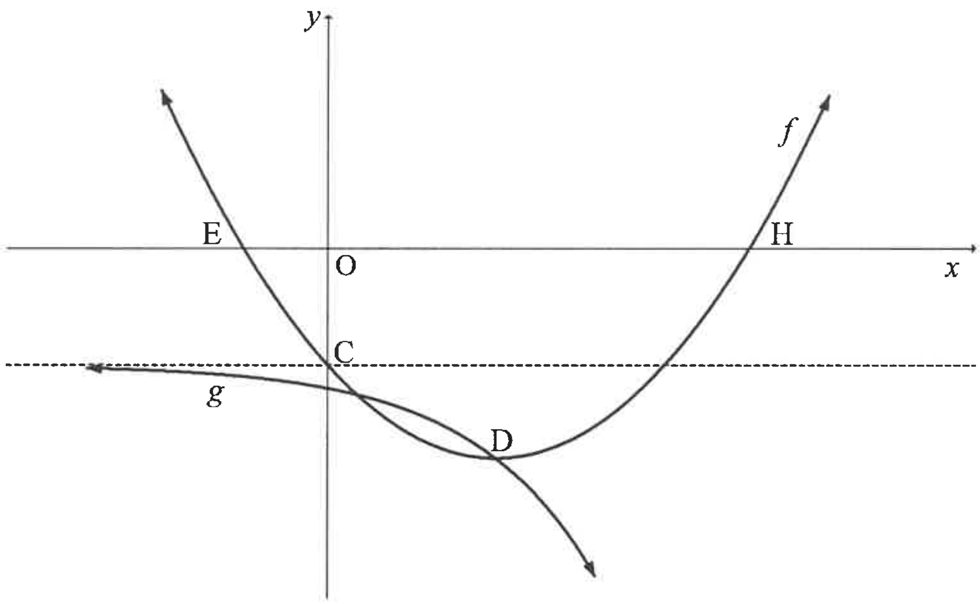
4.1.3 Write down the x-coordinate of the x-intercept of g if $g(x) = h(x+3)$. (2)

4.1.4 The equation of an axis of symmetry of h is $y = x + t$. Determine the value of t. (2)

4.1.5 Determine the values of x for which $-2 \le \dfrac{1}{x-1}$. (3)

4.2 The graphs of $f(x)=x^2-4x-5$ and $g(x)=a.2^x+q$ are sketched below.

- E and H are the x-intercepts of f.
- C is the y-intercept of f and lies on the asymptote of g.
- The two graphs intersect at D, the turning point of f.



4.2.1	Write down the y-coordinate of C.	(1)
4.2.2	Determine the coordinates of D.	(2)
4.2.3	Determine the values of a and q.	(3)
4.2.4	Write down the range of g.	(1)
4.2.5	Determine the values of k for which the value of $f(x)-k$ will always be positive.	(2)

[20]

QUESTION 3

3.1 Consider the following geometric sequence: 1 024 ; 256 ; 64 ; …

Calculate:

3.1.1 The 10^{th} term of the sequence (2)

3.1.2 $\sum_{p=0}^{8} 256\left(4^{1-p}\right)$ (4)

3.2 The first two terms of a geometric sequence are:

$-t^2 - 6t - 9$ and $\dfrac{t^3 + 9t^2 + 27t + 27}{2}$

Determine the values of t for which the sequence will converge. (5)
[11]

QUESTION 4

The graph of $g(x) = a\left(\dfrac{1}{3}\right)^x + 7$ passes through point E(–2 ; 10).

4.1 Calculate the value of a. (3)

4.2 Calculate the coordinates of the y-intercept of g. (2)

4.3 Consider: $h(x) = \left(\dfrac{1}{3}\right)^x$

4.3.1 Describe the translation from g to h. (2)

4.3.2 Determine the equation of the inverse of h, in the form $y = \ldots$ (2)
[9]

QUESTION 3

Consider the quadratic number pattern: $-145\ ;-122\ ;-101\ ;\ \ldots$

3.1 Write down the value of T_4. (1)

3.2 Show that the general term of this number pattern is $T_n = -n^2 + 26n - 170$. (3)

3.3 Between which TWO terms of the quadratic number pattern will there be a difference of -121? (4)

3.4 What value must be added to each term in the number pattern so that the value of the maximum term in the new number pattern formed will be 1? (3)
[11]

QUESTION 4

Consider the linear pattern: $5\ ;7\ ;9\ ;\ \ldots$

4.1 Determine T_{51}. (3)

4.2 Calculate the sum of the first 51 terms. (2)

4.3 Write down the expansion of $\sum_{n=1}^{5000}(2n+3)$. Show only the first 3 terms and the last term of the expansion. (1)

4.4 Hence, or otherwise, calculate $\sum_{n=1}^{5000}(2n+3)+\sum_{n=1}^{4999}(-2n-1)$.
ALL working details must be shown. (4)
[10]

QUESTION 5

Given: $f(x)=\dfrac{-1}{x-3}+2$

5.1 Write down the equations of the asymptotes of f. (2)

5.2 Write down the domain of f. (1)

5.3 Determine the coordinates of the x-intercept of f. (2)

5.4 Write down the coordinates of the y-intercept of f. (2)

5.5 Draw the graph of f. Clearly show ALL the asymptotes and intercepts with the axes. (3)
[10]

QUESTION 3

3.1 If $r = \frac{1}{5}$ and $a = 2\ 000$, determine:

3.1.1 T_n, the general term of the series (1)

3.1.2 T_7 (1)

3.1.3 Which term of the series will have a value of $\frac{16}{15\ 625}$ (3)

3.2 Consider the geometric series where $\sum_{n=1}^{\infty} T_n = 27$ and $S_3 = 26$.

Calculate the value of the constant ratio (r) of the series. (4)

[9]

QUESTION 4

The lines $y = x + 1$ and $y = -x - 7$ are the axes of symmetry of the function $f(x) = \frac{-2}{x+p} + q$.

4.1 Show that $p = 4$ and $q = -3$. (4)

4.2 Calculate the x-intercept of f. (2)

4.3 Sketch the graph of f. Clearly label ALL intercepts with the axes and the asymptotes. (4)

[10]

QUESTION 3

3.1 Prove that $\sum_{k=1}^{\infty} 4.3^{2-k}$ is a convergent geometric series. Show ALL your calculations. (3)

3.2 If $\sum_{k=p}^{\infty} 4.3^{2-k} = \frac{2}{9}$, determine the value of p. (5)

[8]

QUESTION 4

4.1 Given: $h(x) = \frac{-3}{x-1} + 2$

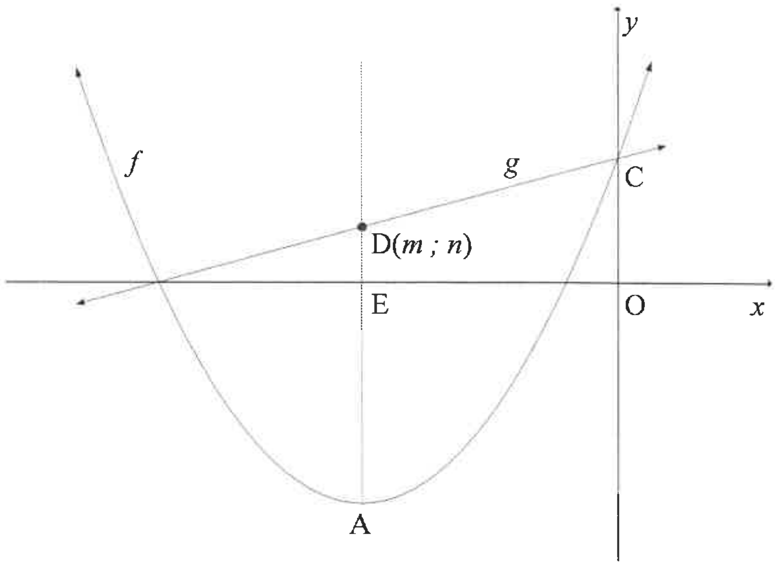
4.1.1 Write down the equations of the asymptotes of h. (2)

4.1.2 Determine the equation of the axis of symmetry of h that has a negative gradient. (2)

4.1.3 Sketch the graph of h, showing the asymptotes and the intercepts with the axes. (4)

4.2 The graphs of $f(x)=\frac{1}{2}(x+5)^2-8$ and $g(x)=\frac{1}{2}x+\frac{9}{2}$ are sketched below.

- A is the turning point of f.
- The axis of symmetry of f intersects the x-axis at E and the line g at D(m ; n).
- C is the y-intercept of f and g.



4.2.1	Write down the coordinates of A.	(2)
4.2.2	Write down the range of f.	(1)
4.2.3	Calculate the values of m and n.	(3)
4.2.4	Calculate the area of OCDE.	(3)
4.2.5	Determine the equation of g^{-1}, the inverse of g, in the form $y = \ldots$	(2)
4.2.6	If $h(x)=g^{-1}(x)+k$ is a tangent to f, determine the coordinates of the point of contact between h and f.	(4)

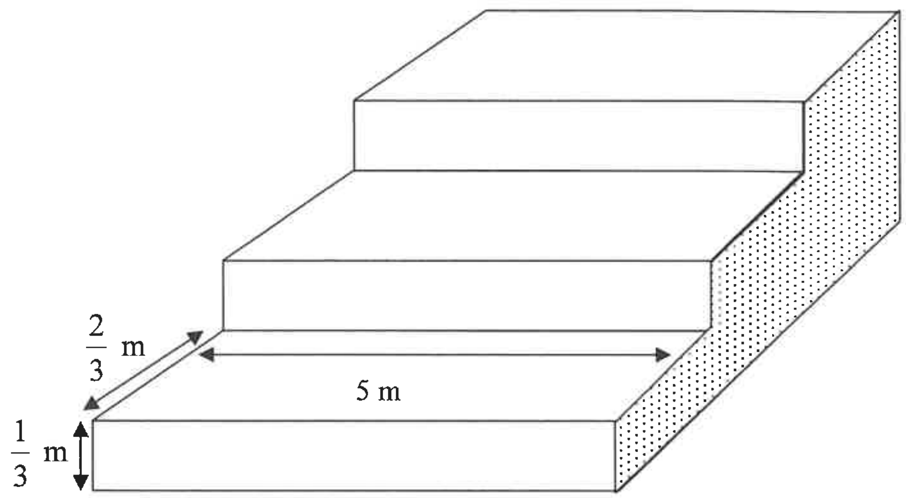
[23]

QUESTION 3

3.1 Without using a calculator, determine the value of: $\sum_{y=3}^{10} \frac{1}{y-2} - \sum_{y=3}^{10} \frac{1}{y-1}$ (3)

3.2 A steel pavilion at a sports ground comprises of a series of 12 steps, of which the first 3 are shown in the diagram below.

Each step is 5 m wide. Each step has a rise of $\frac{1}{3}$ m and has a tread of $\frac{2}{3}$ m, as shown in the diagram below.



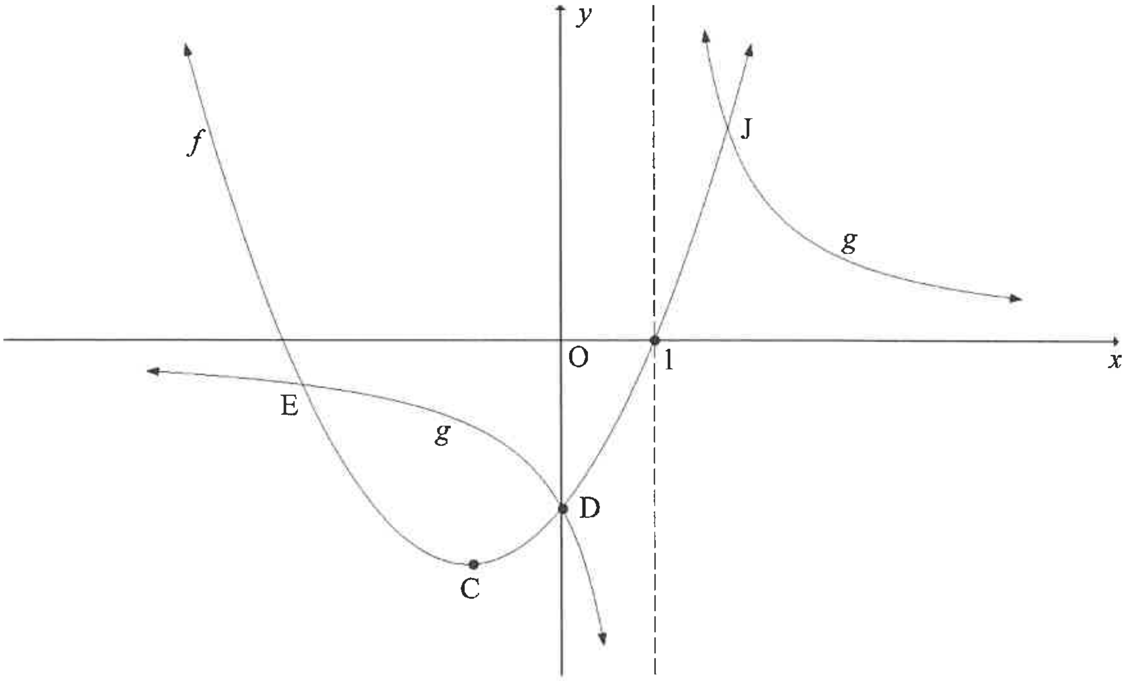
The open side (shaded on sketch) on each side of the pavilion must be covered with metal sheeting. Calculate the area (in m^2) of metal sheeting needed to cover both open sides. (6)

[9]

QUESTION 4

Below are the graphs of $f(x) = x^2 + bx - 3$ and $g(x) = \dfrac{a}{x+p}$.

- f has a turning point at C and passes through the x-axis at $(1\ ;\ 0)$.
- D is the y-intercept of both f and g. The graphs f and g also intersect each other at E and J.
- The vertical asymptote of g passes through the x-intercept of f.



4.1	Write down the value of p.	(1)
4.2	Show that $a = 3$ and $b = 2$.	(3)
4.3	Calculate the coordinates of C.	(4)
4.4	Write down the range of f.	(2)
4.5	Determine the equation of the line through C that makes an angle of $45°$ with the positive x-axis. Write your answer in the form $y = \ldots$	(3)
4.6	Is the straight line, determined in QUESTION 4.5, a tangent to f? Explain your answer.	(2)
4.7	The function $h(x) = f(m - x) + q$ has only one x-intercept at $x = 0$. Determine the values of m and q.	(4)
		[19]

2.2 Given a geometric sequence: 36 ; –18 ; 9 ; …

2.2.1 Determine the value of r, the common ratio. (1)

2.2.2 Calculate n if $T_n = \frac{9}{4\,096}$ (3)

2.2.3 Calculate S_∞ (2)

2.2.4 Calculate the value of $\frac{T_1 + T_3 + T_5 + T_7 + \ldots + T_{499}}{T_2 + T_4 + T_6 + T_8 + \ldots + T_{500}}$ (4)

[17]

QUESTION 3

3.1 The first three terms of an arithmetic sequence are: $2p + 3$; $p + 6$ and $p - 2$.

3.1.1 Show that $p = 11$. (2)

3.1.2 Calculate the smallest value of n for which $T_n < -55$. (3)

3.2 Given that $\sum_{k=1}^{6}(x - 3k) = \sum_{k=1}^{9}(x - 3k)$, prove that $\sum_{k=1}^{15}(x - 3k) = 0$. (5)

[10]

QUESTION 4

Given the exponential function: $g(x) = \left(\frac{1}{2}\right)^x$

4.1 Write down the range of g. (1)

4.2 Determine the equation of g^{-1} in the form $y = \ldots$ (2)

4.3 Is g^{-1} a function? Justify your answer. (2)

4.4 The point $M(a\,;2)$ lies on g^{-1}.

4.4.1 Calculate the value of a. (2)

4.4.2 M', the image of M, lies on g. Write down the coordinates of M'. (1)

4.5 If $h(x) = g(x + 3) + 2$, write down the coordinates of the image of M' on h. (3)

[11]

QUESTION 3

A geometric series has a constant ratio of $\frac{1}{2}$ and a sum to infinity of 6.

3.1 Calculate the first term of the series. (2)

3.2 Calculate the 8^{th} term of the series. (2)

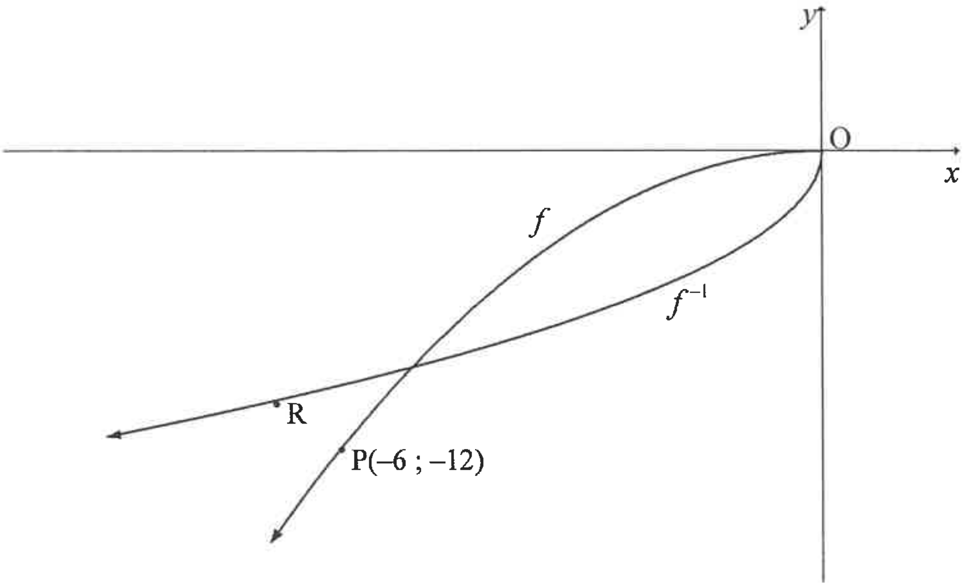
3.3 Given: $\sum_{k=1}^{n} 3(2)^{1-k} = 5{,}8125$ Calculate the value of n. (4)

3.4 If $\sum_{k=1}^{20} 3(2)^{1-k} = p$, write down $\sum_{k=1}^{20} 24(2)^{-k}$ in terms of p. (3)

[11]

QUESTION 4

In the diagram below, the graph of $f(x) = ax^2$ is drawn in the interval $x \leq 0$.
The graph of f^{-1} is also drawn. P(–6 ; –12) is a point on f and R is a point on f^{-1}.



4.1 Is f^{-1} a function? Motivate your answer. (2)

4.2 If R is the reflection of P in the line $y = x$, write down the coordinates of R. (1)

4.3 Calculate the value of a. (2)

4.4 Write down the equation of f^{-1} in the form $y = ...$ (3)

[8]

QUESTION 3

The first three terms of an arithmetic sequence are -1 ; 2 and 5.

3.1 Determine the n^{th} term, T_n, of the sequence. (2)

3.2 Calculate T_{43}. (2)

3.3 Evaluate $\sum_{k=1}^{n} T_k$ in terms of n. (3)

3.4 A quadratic sequence, with general term T_n, has the following properties:

- $T_{11} = 125$
- $T_n - T_{n-1} = 3n - 4$

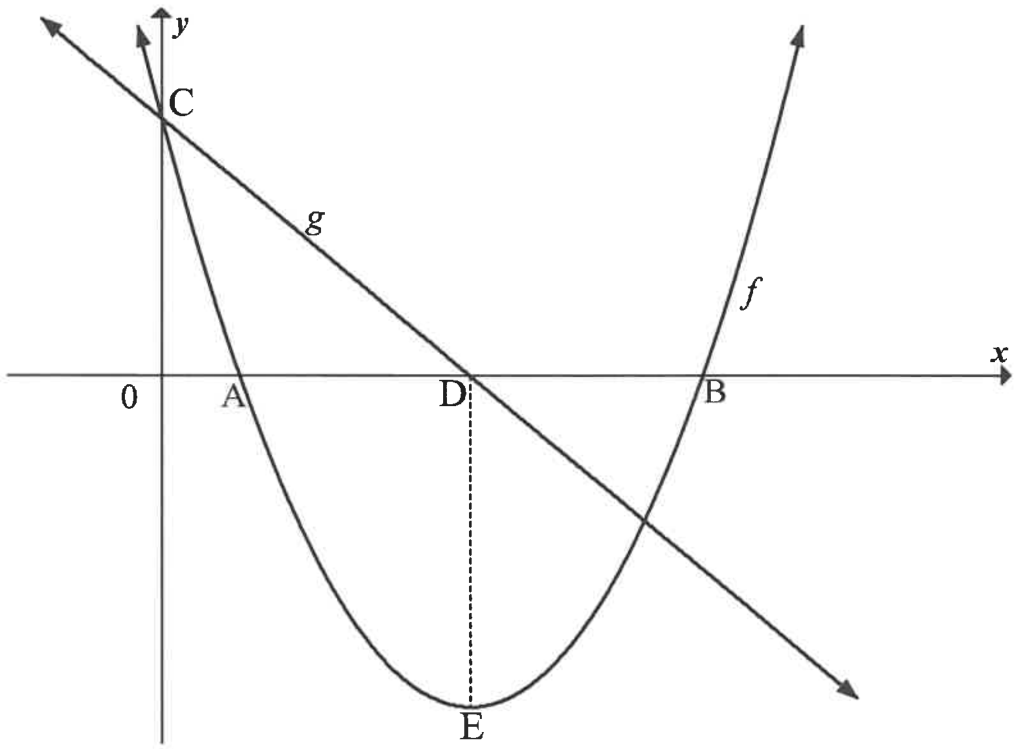
Determine the first term of the sequence. (6)

[13]

QUESTION 4

Below are the graphs of $f(x) = (x-4)^2 - 9$ and a straight line g.

- A and B are the x-intercepts of f and E is the turning point of f.
- C is the y-intercept of both f and g.
- The x-intercept of g is D. DE is parallel to the y-axis.



4.1	Write down the coordinates of E.	(2)
4.2	Calculate the coordinates of A.	(3)
4.3	M is the reflection of C in the axis of symmetry of f. Write down the coordinates of M.	(3)
4.4	Determine the equation of g in the form $y = mx + c$.	(3)
4.5	Write down the equation of g^{-1} in the form $y = \ldots$	(3)
4.6	For which values of x will $x(f(x)) \leq 0$?	(4)

[18]

QUESTION 3

Themba is planning a bicycle trip from Cape Town to Pretoria. The total distance covered during the trip will be 1 500 km. He plans to travel 100 km on the first day. For every following day he plans to cover 94% of the distance he covered the previous day.

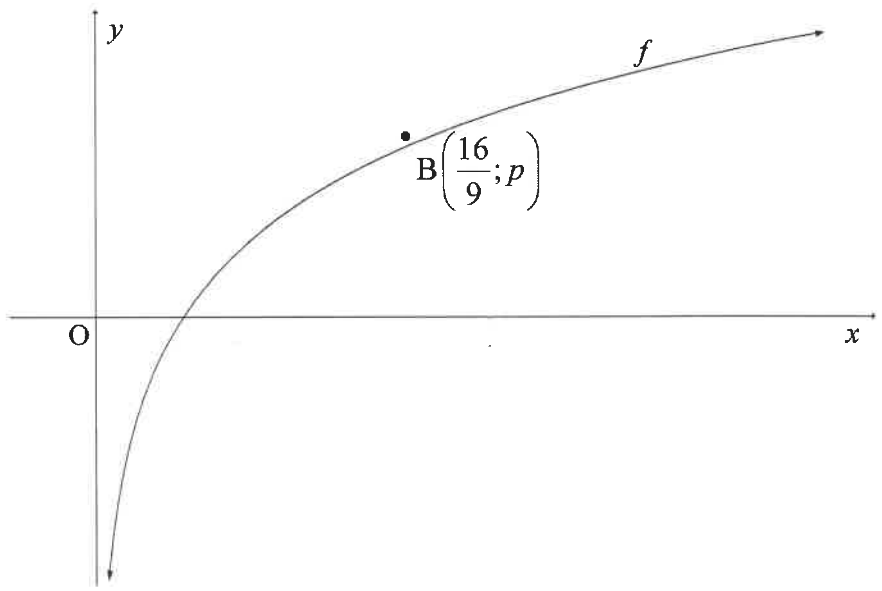
3.1 What distance will he cover on day 3 of the trip? (2)

3.2 On what day of the trip will Themba pass the halfway point? (4)

3.3 Themba must cover a certain percentage of the previous day's distance to ensure that he will eventually reach Pretoria. Calculate ALL possible value(s) of this percentage. (3)
[9]

QUESTION 4

The graph of $f(x)=\log_{\frac{4}{3}} x$ is drawn below. $B\left(\frac{16}{9};p\right)$ is a point on f.



4.1 For which value(s) of x is $\log_{\frac{4}{3}} x \le 0$? (2)

4.2 Determine the value of p, without the use of a calculator. (3)

4.3 Write down the equation of the inverse of f in the form $y = \ldots$ (2)

4.4 Write down the range of $y = f^{-1}(x)$. (2)

4.5 The function $h(x)=\left(\frac{3}{4}\right)^x$ is obtained after applying two reflections on f.

Write down the coordinates of B'', the image of B on h. (2)
[11]

QUESTION 1

1.1 Solve for x:

1.1.1 $x^2 + 9x + 14 = 0$ (3)

1.1.2 $4x^2 + 9x - 3 = 0$ (correct to TWO decimal places) (4)

1.1.3 $\sqrt{x^2 - 5} = 2\sqrt{x}$ (4)

1.2 Solve for x and y if:
$3x - y = 4$ and $x^2 + 2xy - y^2 = -2$ (6)

1.3 Given: $f(x) = x^2 + 8x + 16$

1.3.1 Solve for x if $f(x) > 0$. (3)

1.3.2 For which values of p will $f(x) = p$ have TWO unequal negative roots? (4)

[24]

QUESTION 2

2.1 Given the following quadratic number pattern: $5\ ;\ -4\ ;\ -19\ ;\ -40\ ;\ \ldots$

2.1.1 Determine the constant second difference of the sequence. (2)

2.1.2 Determine the n^{th} term (T_n) of the pattern. (4)

2.1.3 Which term of the pattern will be equal to $-25\,939$? (3)

2.2 The first three terms of an arithmetic sequence are $2k - 7\ ;\ k + 8$ and $2k - 1$.

2.2.1 Calculate the value of the 15^{th} term of the sequence. (5)

2.2.2 Calculate the sum of the first 30 even terms of the sequence. (4)

[18]

QUESTION 3

A convergent geometric series consisting of only positive terms has first term a, constant ratio r and n^{th} term, T_n, such that $\sum_{n=3}^{\infty} T_n = \frac{1}{4}$.

3.1 If $T_1 + T_2 = 2$, write down an expression for a in terms of r. (2)

3.2 Calculate the values of a and r. (6)

[8]

QUESTION 4

Given: $f(x) = -ax^2 + bx + 6$

4.1 The gradient of the tangent to the graph of f at the point $\left(-1\,;\,\frac{7}{2}\right)$ is 3.

Show that $a = \frac{1}{2}$ and $b = 2$. (5)

4.2 Calculate the x-intercepts of f. (3)

4.3 Calculate the coordinates of the turning point of f. (3)

4.4 Sketch the graph of f. Clearly indicate ALL intercepts with the axes and the turning point. (4)

4.5 Use the graph to determine the values of x for which $f(x) > 6$. (3)

4.6 Sketch the graph of $g(x) = -x - 1$ on the same set of axes as f. Clearly indicate ALL intercepts with the axes. (2)

4.7 Write down the values of x for which $f(x).g(x) \leq 0$. (3)

[23]

QUESTION 3

Given the quadratic sequence: 0; 17; 32; ...

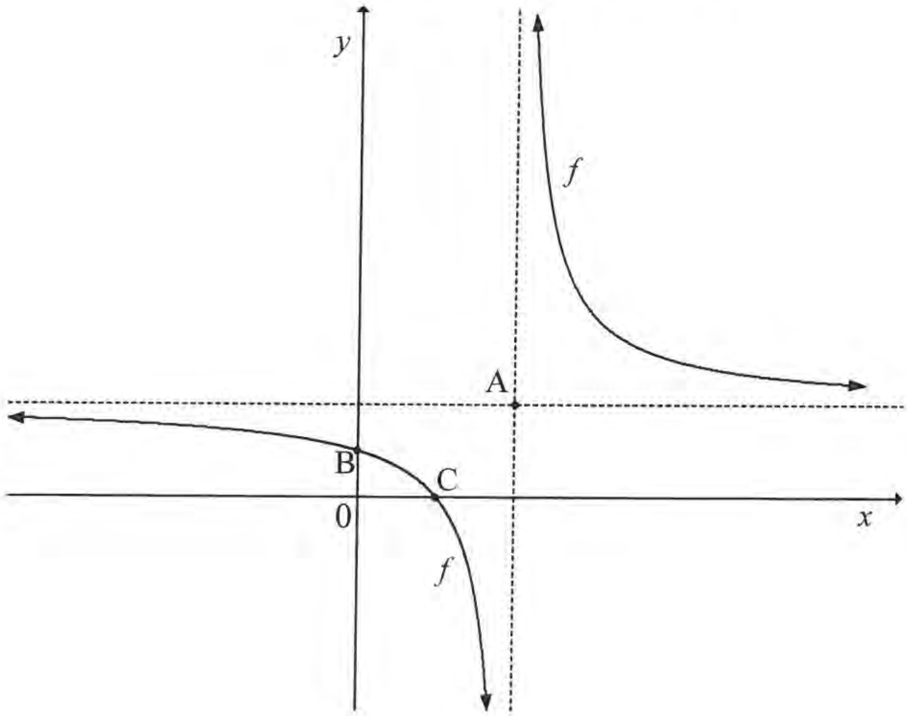
3.1 Determine an expression for the general term, T_n, of the quadratic sequence. (4)

3.2 Which terms in the quadratic sequence have a value of 56? (3)

3.3 Hence, or otherwise, calculate the value of $\sum_{n=5}^{10} T_n - \sum_{n=11}^{15} T_n$. (4)
[11]

QUESTION 4

The sketch below shows the graph of $f(x)=\frac{6}{x-4}+3$. The asymptotes of f intersect at A. The graph f intersects the x-axis and y-axis at C and B respectively.



4.1 Write down the coordinates of A. (1)

4.2 Calculate the coordinates of B. (2)

4.3 Calculate the coordinates of C. (2)

4.4 Calculate the average gradient of f between B and C. (2)

4.5 Determine the equation of a line of symmetry of f which has a positive y-intercept. (2)
[9]

QUESTION 3

3.1 6 ; 6 ; 9 ; 15 ; ... are the first four terms of a quadratic number pattern.

3.1.1 Write down the value of the fifth term (T_5) of the pattern. (1)

3.1.2 Determine a formula to represent the general term of the pattern. (4)

3.1.3 Which term of the pattern has a value of 3 249? (4)

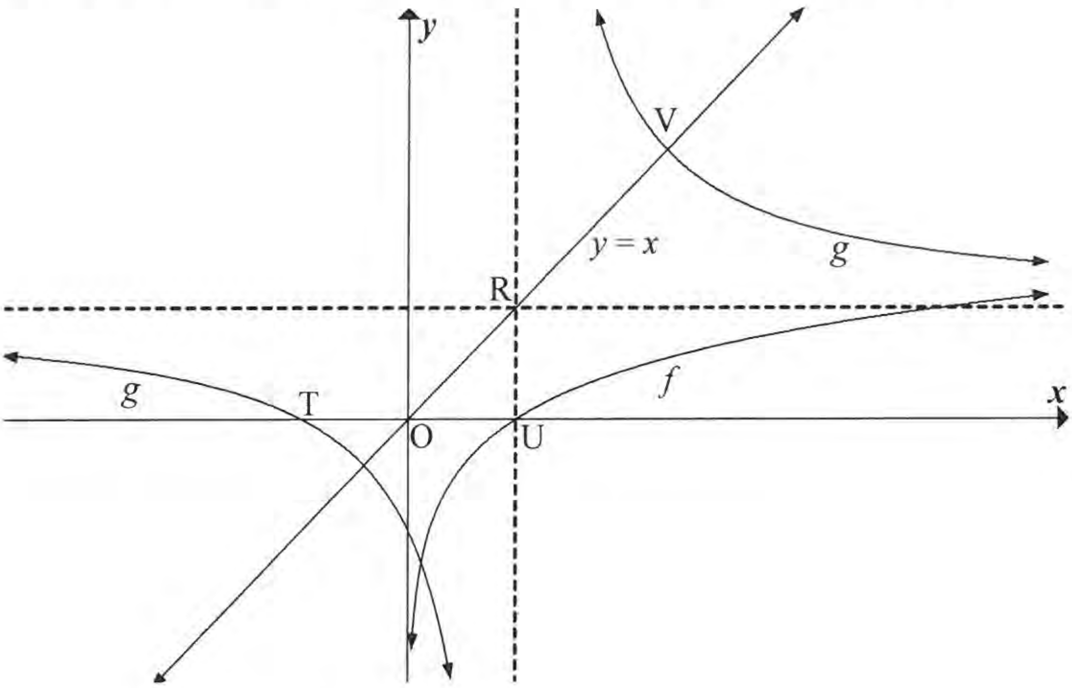
3.2 Determine the value(s) of x in the interval $x \in [0^\circ ; 90^\circ]$ for which the sequence -1 ; $2\sin 3x$; 5 ; ..... will be arithmetic. (4)

[13]

QUESTION 4

The sketch below shows the graphs of $f(x) = \log_5 x$ and $g(x) = \dfrac{2}{x-1} + 1$.

- T and U are the x-intercepts of g and f respectively.
- The line $y = x$ intersects the asymptotes of g at R, and the graph of g at V.



4.1 Write down the coordinates of U. (1)

4.2 Write down the equations of the asymptotes of g. (2)

4.3 Determine the coordinates of T. (2)

4.4 Write down the equation of h, the reflection of f in the line $y = x$, in the form $y = ...$ (2)

4.5 Write down the equation of the asymptote of $h(x-3)$. (1)

4.6 Calculate the coordinates of V. (4)

4.7 Determine the coordinates of $T^{/}$ the point which is symmetrical to T about the point R. (2)
[14]

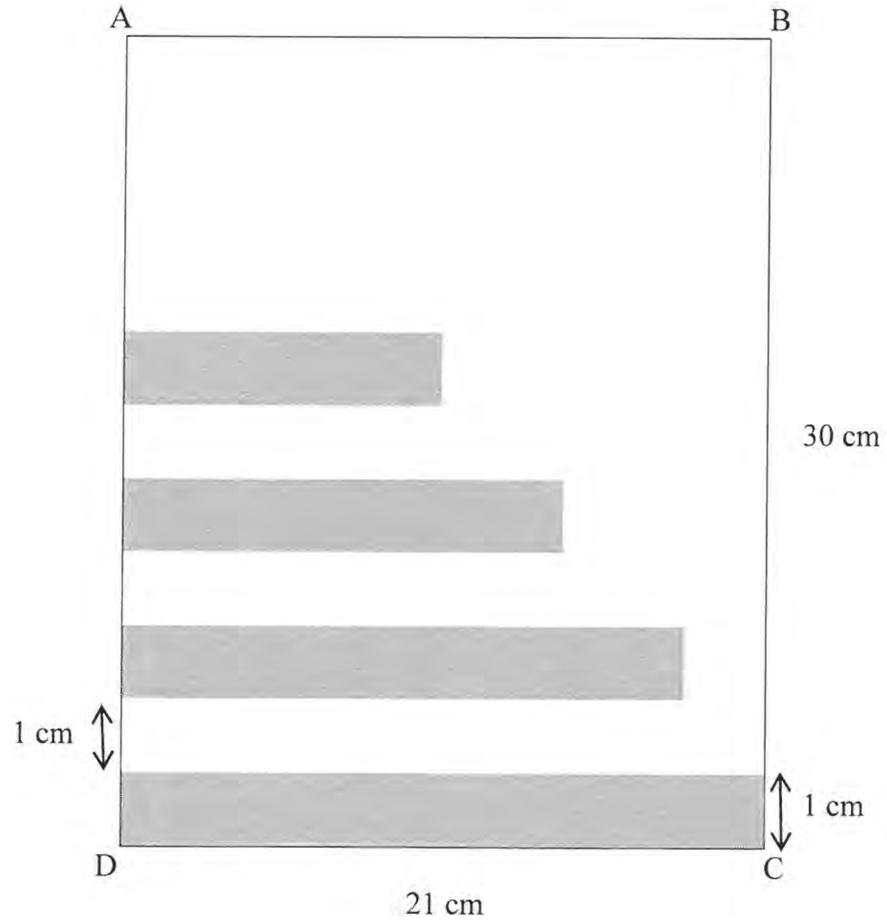
QUESTION 3

3.1 The first four terms of a quadratic number pattern are $-1\ ;\ x\ ;\ 3\ ;\ x+8$

3.1.1 Calculate the value(s) of x. (4)

3.1.2 If $x = 0$, determine the position of the first term in the quadratic number pattern for which the sum of the first n first differences will be greater than 250. (4)

3.2 Rectangles of width 1 cm are drawn from the edge of a sheet of paper that is 30 cm long such that there is a 1 cm gap between one rectangle and the next. The length of the first rectangle is 21 cm and the length of each successive rectangle is 85% of the length of the previous rectangle until there are rectangles drawn along the entire length of AD. Each rectangle is coloured grey.

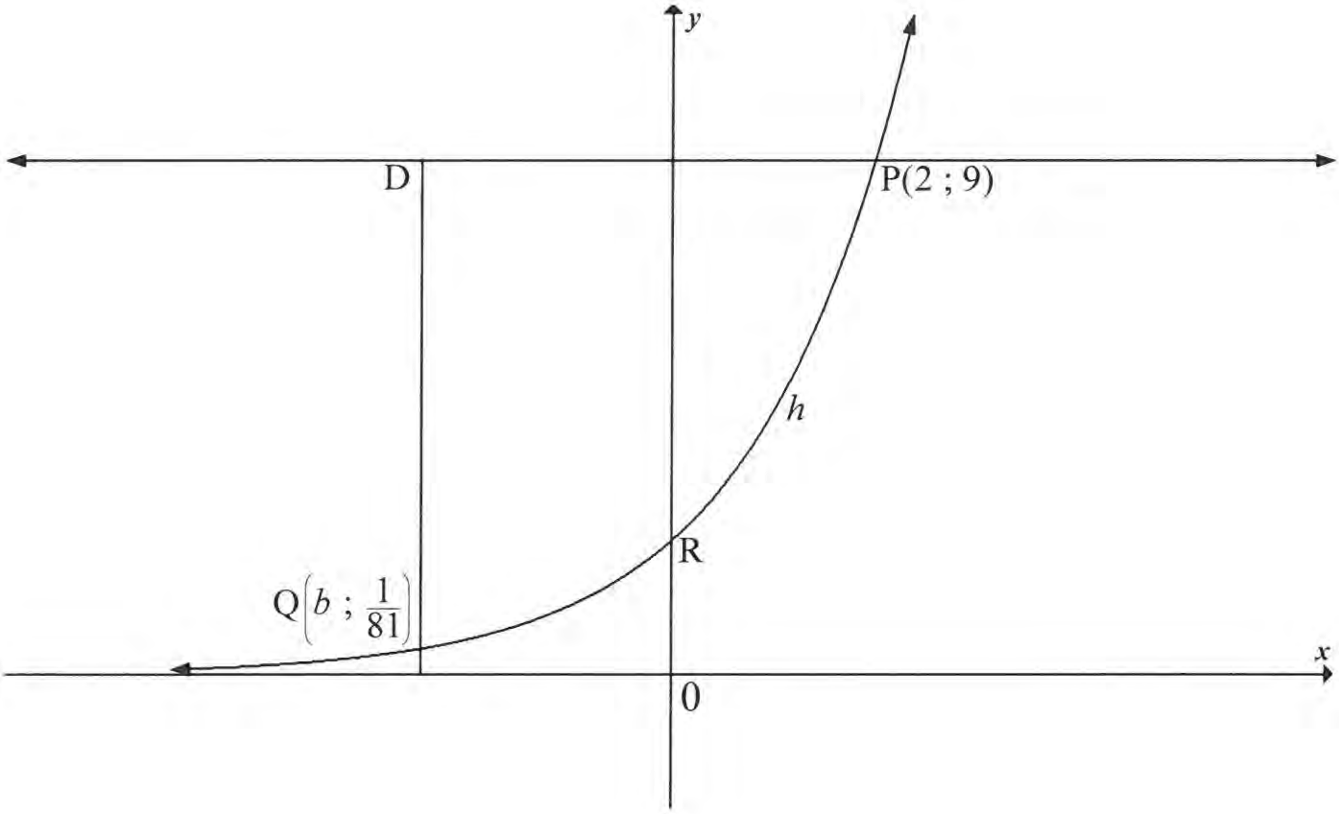


3.2.1 Calculate the length of the 10^{th} rectangle. (3)

3.2.2 Calculate the percentage of the paper that is coloured grey. (4)
[15]

QUESTION 4

Sketched below is the graph of $h(x) = a^x$, $a > 0$. R is the y-intercept of h.
The points P(2 ; 9) and $Q\left(b\ ;\ \frac{1}{81}\right)$ lie on h.



4.1	Write down the equation of the asymptote of h.	(1)
4.2	Determine the coordinates of R.	(1)
4.3	Calculate the value of a.	(2)
4.4	D is a point such that DQ $\parallel$ y-axis and DP $\parallel$ x-axis. Calculate the length of DP.	(4)
4.5	Determine the values of k for which the equation $h(x+2)+k=0$ will have a root that is less than -6.	(3)

[11]

QUESTION 3

The first four terms of a quadratic number pattern are $-1\ ;\ 2\ ;\ 9\ ;\ 20$.

3.1 Determine the general term of the quadratic number pattern. (4)

3.2 Calculate the value of the 48^{th} term of the quadratic number pattern. (2)

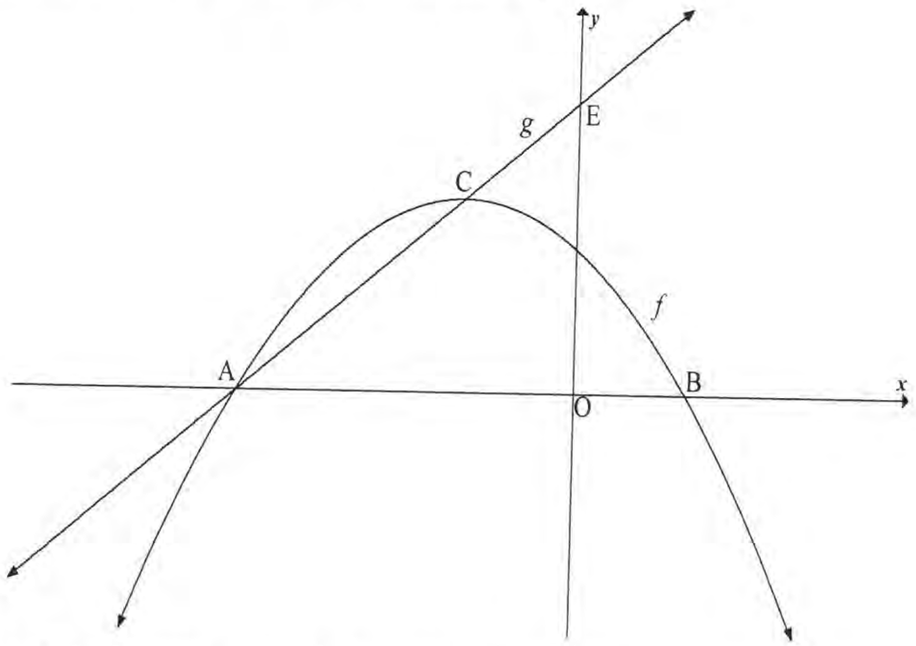
3.3 Show that the sum of the first differences of this quadratic number pattern can be given by $S_n = 2n^2 + n$ (3)

3.4 If the sum of the first 69 first differences in QUESTION 3.3 equals 9 591 (that is, $S_{69} = 9\,591$), which term of the quadratic number pattern has a value of 9 590? (2)

[11]

QUESTION 4

The sketch below shows the graphs of $f(x) = -x^2 - 2x + 3$ and $g(x) = mx + q$.
Graph f has x-intercepts at A and B(1 ; 0) and a turning point at C.
The straight line g, passing through A and C, cuts the y-axis at E.



4.1 Write down the coordinates of the y-intercept of f. (1)

4.2 Show that the coordinates of C are $(-1; 4)$. (3)

4.3 Write down the coordinates of A. (1)

4.4 Calculate the length of CE. (6)

4.5 Determine the value of k if $h(x) = 2x + k$ is a tangent to the graph of f. (5)

4.6 Determine the equation of g^{-1}, the inverse of g, in the form $y = ...$ (2)

4.7 For which value(s) of x is $g(x) \geq g^{-1}(x)$? (3)

[21]

QUESTION 3

Chris bought a bonsai (miniature tree) at a nursery. When he bought the tree, its height was 130 mm. Thereafter the height of the tree increased, as shown below.

INCREASE IN HEIGHT OF THE TREE PER YEAR		
During the first year	**During the second year**	**During the third year**
100 mm	70 mm	49 mm

3.1 Chris noted that the sequence of height increases, namely 100 ; 70 ; 49 ..., was geometric. During which year will the height of the tree increase by approximately 11,76 mm? (4)

3.2 Chris plots a graph to represent the height $h(n)$ of the tree (in mm) n years after he bought it. Determine a formula for $h(n)$. (3)

3.3 What height will the tree eventually reach? (3)

[10]

QUESTION 4

Given: $f(x) = 2^{-x} + 1$

4.1 Determine the coordinates of the y-intercept of f. (1)

4.2 Sketch the graph of f, clearly indicating ALL intercepts with the axes as well as any asymptotes. (3)

4.3 Calculate the average gradient of f between the points on the graph where $x = -2$ and $x = 1$. (3)

4.4 If $h(x) = 3f(x)$, write down an equation of the asymptote of h. (1)

[8]

QUESTION 3

Consider the series: $S_n = -3 + 5 + 13 + 21 + ...$ to n terms.

3.1 Determine the general term of the series in the form $T_k = bk + c$. (2)

3.2 Write S_n in sigma notation. (2)

3.3 Show that $S_n = 4n^2 - 7n$. (3)

3.4 Another sequence is defined as:

$Q_1 = -6$
$Q_2 = -6 - 3$
$Q_3 = -6 - 3 + 5$
$Q_4 = -6 - 3 + 5 + 13$
$Q_5 = -6 - 3 + 5 + 13 + 21$

3.4.1 Write down a numerical expression for Q_6. (2)

3.4.2 Calculate the <u>value</u> of Q_{129}. (3)

[12]

QUESTION 4

Given: $f(x) = 2^{x+1} - 8$

4.1 Write down the equation of the asymptote of *f*. (1)

4.2 Sketch the graph of *f*. Clearly indicate ALL intercepts with the axes as well as the asymptote. (4)

4.3 The graph of g is obtained by reflecting the graph of f in the y-axis. Write down the equation of g. (1)

[6]

QUESTION 2

2.1 Prove that in any arithmetic series in which the first term is a and whose constant difference is d, the sum of the first n terms is $S_n = \frac{n}{2}[2a + (n-1)d]$. (4)

2.2 Calculate the value of $\sum_{k=1}^{50}(100 - 3k)$. (4)

2.3 A quadratic sequence is defined with the following properties:

$$T_2 - T_1 = 7$$
$$T_3 - T_2 = 13$$
$$T_4 - T_3 = 19$$

2.3.1 Write down the value of:

(a) $T_5 - T_4$ (1)

(b) $T_{70} - T_{69}$ (3)

2.3.2 Calculate the value of T_{69} if $T_{89} = 23\,594$. (5)

[17]

QUESTION 3

Consider the infinite geometric series: 45 + 40,5 + 36,45 + …

3.1 Calculate the value of the TWELFTH term of the series (correct to TWO decimal places). (3)

3.2 Explain why this series converges. (1)

3.3 Calculate the sum to infinity of the series. (2)

3.4 What is the smallest value of n for which $S_\infty - S_n < 1$? (5)

[11]

QUESTION 4

Given: $g(x) = \frac{6}{x+2} - 1$

4.1 Write down the equations of the asymptotes of g. (2)

4.2 Calculate:

4.2.1 The y-intercept of g (1)

4.2.2 The x-intercept of g (2)

4.3 Draw the graph of g, showing clearly the asymptotes and the intercepts with the axes. (3)

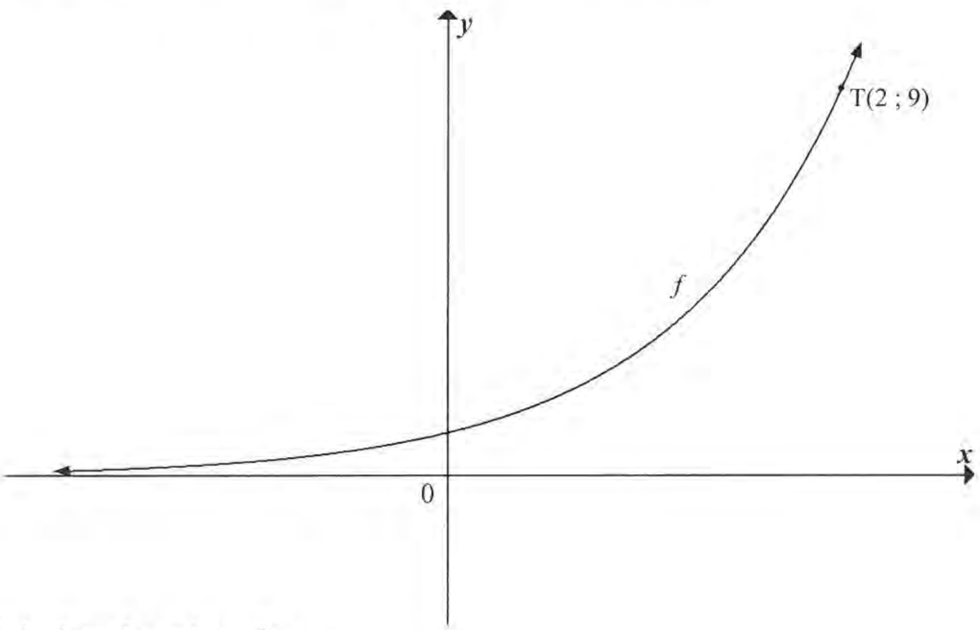
4.4 Determine the equation of the line of symmetry that has a negative gradient, in the form $y = ...$ (3)

4.5 Determine the value(s) of x for which $\frac{6}{x+2} - 1 \geq -x - 3$. (2)

[13]

QUESTION 5

The graph of $f(x) = a^x, a > 1$ is shown below. T(2 ; 9) lies on f.



5.1 Calculate the value of a. (2)

5.2 Determine the equation of $g(x)$ if $g(x) = f(-x)$. (1)

5.3 Determine the value(s) of x for which $f^{-1}(x) \geq 2$. (2)

5.4 Is the inverse of f a function? Explain your answer. (2)

[7]

QUESTION 3

3.1 Given the quadratic sequence: $-1 ; -7 ; -11 ; p ; \ldots$

3.1.1 Write down the value of p. (2)

3.1.2 Determine the n^{th} term of the sequence. (4)

3.1.3 The first difference between two consecutive terms of the sequence is 96. Calculate the values of these two terms. (4)

3.2 The first three terms of a geometric sequence are: 16 ; 4 ; 1

3.2.1 Calculate the value of the 12^{th} term. (Leave your answer in simplified exponential form.) (3)

3.2.2 Calculate the sum of the first 10 terms of the sequence. (2)

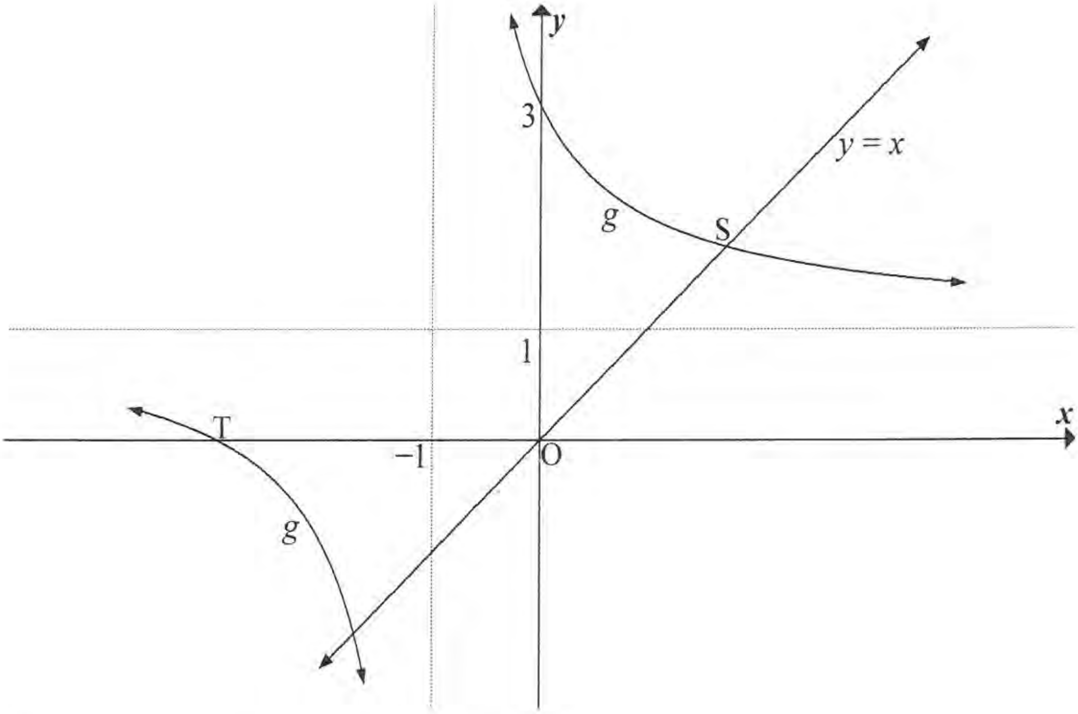
3.3 Determine the value of: $\left(1+\frac{1}{2}\right)\left(1+\frac{1}{3}\right)\left(1+\frac{1}{4}\right)\left(1+\frac{1}{5}\right) \ldots$ up to 98 factors. (4)

[19]

 Please turn over

QUESTION 4

The diagram below shows the hyperbola g defined by $g(x)=\dfrac{2}{x+p}+q$ with asymptotes $y = 1$ and $x = -1$. The graph of g intersects the x-axis at T and the y-axis at $(0 ; 3)$. The line $y = x$ intersects the hyperbola in the first quadrant at S.



4.1 Write down the values of p and q. (2)

4.2 Calculate the x-coordinate of T. (2)

4.3 Write down the equation of the vertical asymptote of the graph of h, if $h(x) = g(x+5)$ (1)

4.4 Calculate the length of OS. (5)

4.5 For which values of k will the equation $g(x) = x + k$ have two real roots that are of opposite signs? (1)
[11]

QUESTION 3

3.1 Given the arithmetic sequence: $w-3$; $2w-4$; $23-w$

3.1.1 Determine the value of w. (2)

3.1.2 Write down the common difference of this sequence. (1)

3.2 The arithmetic sequence 4 ; 10 ; 16 ; ... is the sequence of first differences of a quadratic sequence with a first term equal to 3.

Determine the 50th term of the quadratic sequence. (5)
[8]

QUESTION 4

In a geometric series, the sum of the first n terms is given by $S_n = p\left(1-\left(\frac{1}{2}\right)^n\right)$ and the sum to infinity of this series is 10.

4.1 Calculate the value of p. (4)

4.2 Calculate the second term of the series. (4)
[8]

QUESTION 5

5.1 Draw the graphs of $x^2 + y^2 = 16$ and $x + y = 4$ on the same set of axes in your ANSWER BOOK. (4)

5.2 Write down the coordinates of the points of intersection of the two graphs. (2)
[6]

QUESTION 3

3.1 A quadratic number pattern $T_n = an^2 + bn + c$ has a first term equal to 1.
The general term of the first differences is given by $4n + 6$.

3.1.1 Determine the value of a. (2)

3.1.2 Determine the formula for T_n. (4)

3.2 Given the series: $(1\times 2)+(5\times 6)+(9\times 10)+(13\times 14)+...+(81\times 82)$

Write the series in sigma notation. (It is not necessary to calculate the value of the series.) (4)

[10]

QUESTION 4

4.1 Given: $f(x)=\dfrac{2}{x+1}-3$

4.1.1 Calculate the coordinates of the y-intercept of f. (2)

4.1.2 Calculate the coordinates of the x-intercept of f. (2)

4.1.3 Sketch the graph of f in your ANSWER BOOK, showing clearly the asymptotes and the intercepts with the axes. (3)

4.1.4 One of the axes of symmetry of f is a decreasing function. Write down the equation of this axis of symmetry. (2)

4.2 The graph of an increasing exponential function with equation $f(x) = a.b^x + q$ has the following properties:

- Range: $y > -3$
- The points (0 ; – 2) and (1; – 1) lie on the graph of f.

4.2.1 Determine the equation that defines f. (4)

4.2.2 Describe the transformation from $f(x)$ to $h(x) = 2.2^x + 1$ (2)

[15]

2.4 Given: $\sum_{k=2}^{20}(4x-1)^k$

2.4.1 Calculate the first term of the series $\sum_{k=2}^{20}(4x-1)^k$ if $x=1$. (2)

2.4.2 For which values of x will $\sum_{k=1}^{\infty}(4x-1)^k$ exist? (3)
[18]

QUESTION 3

3.1 Given the arithmetic sequence: $-3\,;1\,;5\,;\ldots;393$

3.1.1 Determine a formula for the n^{th} term of the sequence. (2)

3.1.2 Write down the 4^{th}, 5^{th}, 6^{th} and 7^{th} terms of the sequence. (2)

3.1.3 Write down the remainders when each of the first seven terms of the sequence is divided by 3. (2)

3.1.4 Calculate the sum of the terms in the arithmetic sequence that are divisible by 3. (5)

3.2 Consider the following pattern of dots:

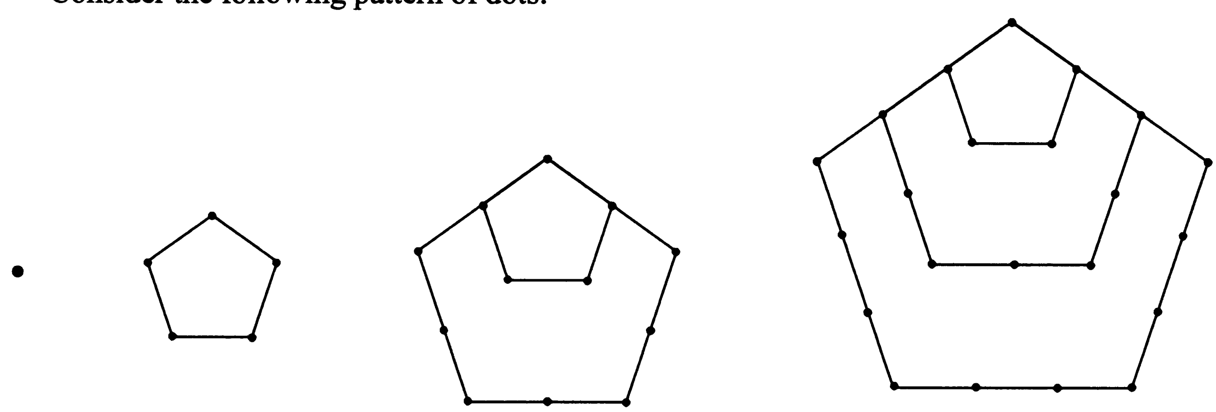


FIGURE 1 FIGURE 2 FIGURE 3 FIGURE 4

If T_n represents the total number of dots in FIGURE n, then $T_1=1$ and $T_2=5$. If the pattern continues in the same manner, determine:

3.2.1 T_5 (2)

3.2.2 T_{50} (5)
[18]

QUESTION 4

Given: $f(x) = -2x^2 - 5x + 3$

4.1 Write down the coordinates of the y-intercept of f. (1)

4.2 Determine the coordinates of the x-intercepts of f. (3)

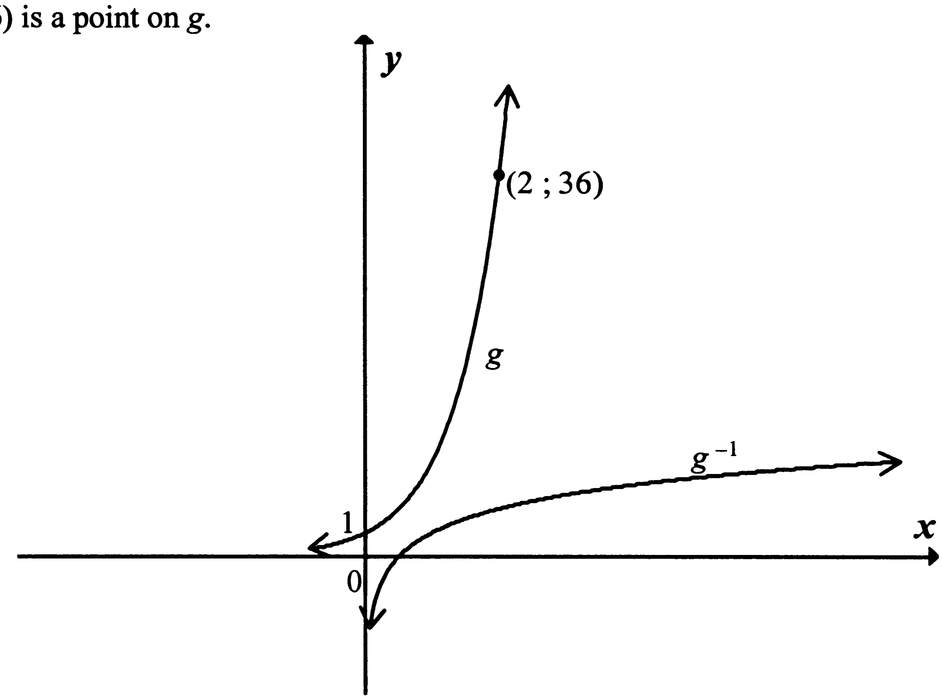
4.3 Determine the coordinates of the turning point of f. (3)

4.4 Sketch the graph of $y = f(x)$, clearly showing the coordinates of the turning points and the three intercepts with the axes. (3)

[10]

QUESTION 5

5.1 Sketched below are the graphs of $g(x) = k^x$, where $k > 0$ and $y = g^{-1}(x)$.
(2 ; 36) is a point on g.



5.1.1 Determine the value of k. (2)

5.1.2 Give the equation of g^{-1} in the form $y = ...$ (2)

5.1.3 For which value(s) of x is $g^{-1}(x) \leq 0$? (2)

5.1.4 Write down the domain of h if $h(x) = g^{-1}(x-3)$. (1)

5.2 5.2.1 Sketch the graph of the inverse of $y = 1$. (2)

5.2.2 Is the inverse of $y = 1$ a function? Motivate your answer. (2)

[11]

2.2 Consider the arithmetic sequence: $-8\ ;\ -2\ ;\ 4\ ;\ 10\ ;\ ...$

2.2.1 Write down the next term of the sequence. (1)

2.2.2 If the n^{th} term of the sequence is 148, determine the value of n. (3)

2.2.3 Calculate the smallest value of n for which the sum of the first n terms of the sequence will be greater than 10 140. (5)

2.3 Calculate $\sum_{k=1}^{30}(3k+5)$ (3)

[22]

QUESTION 3

Consider the sequence: $3\ ;\ 9\ ;\ 27\ ;\ ...$

Jacob says that the fourth term of the sequence is 81.
Vusi disagrees and says that the fourth term of the sequence is 57.

3.1 Explain why Jacob and Vusi could both be correct. (2)

3.2 Jacob and Vusi continue with their number patterns.

Determine a formula for the n^{th} term of:

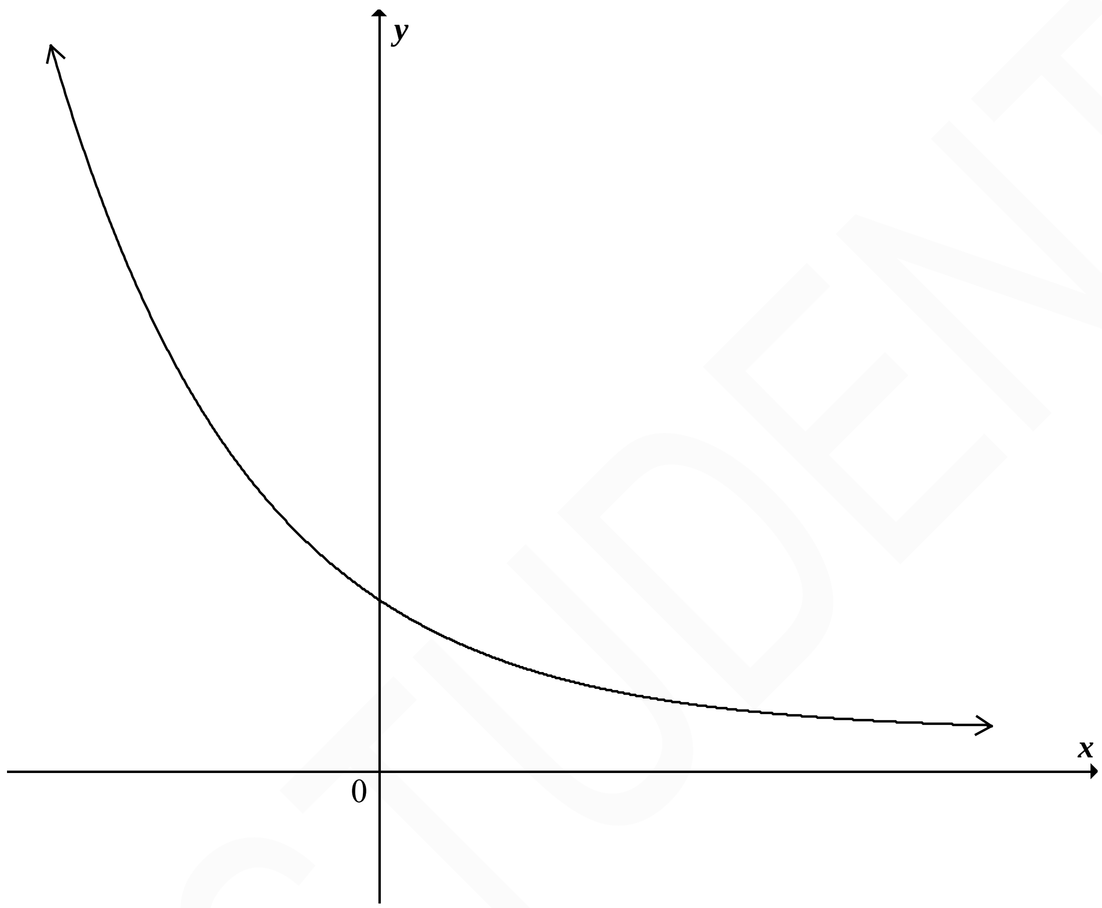
3.2.1 Jacob's sequence (1)

3.2.2 Vusi's sequence (4)

[7]

QUESTION 4

The graph of $f(x)=\left(\frac{1}{3}\right)^x$ is sketched below.



4.1 Write down the domain of f. (1)

4.2 Write down the equation of the asymptote of f. (1)

4.3 Write down the equation of f^{-1} in the form $y = ...$ (2)

4.4 Sketch the graph of f^{-1} in your ANSWER BOOK. Indicate the x-intercept and ONE other point. (3)

4.5 Write down the equation of the asymptote of $f^{-1}(x+2)$. (2)

4.6 Prove that: $[f(x)]^2 - [f(-x)]^2 = f(2x) - f(-2x)$ for all values of x. (3)

[12]

QUESTION 3

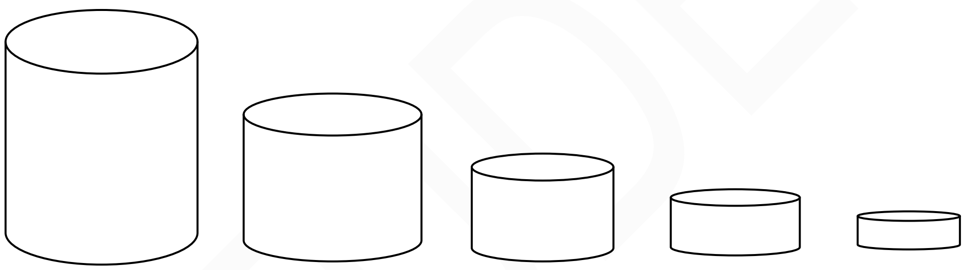
3.1 Given the geometric sequence: 27 ; 9 ; 3 …

3.1.1 Determine a formula for T_n, the n^{th} term of the sequence. (2)

3.1.2 Why does the sum to infinity for this sequence exist? (1)

3.1.3 Determine S_∞. (2)

3.2 Twenty water tanks are decreasing in size in such a way that the volume of each tank is $\frac{1}{2}$ the volume of the previous tank. The first tank is empty, but the other 19 tanks are full of water.



Would it be possible for the first water tank to hold all the water from the other 19 tanks? Motivate your answer. (4)

3.3 The n^{th} term of a sequence is given by $T_n = -2(n-5)^2 + 18$.

3.3.1 Write down the first THREE terms of the sequence. (3)

3.3.2 Which term of the sequence will have the greatest value? (1)

3.3.3 What is the second difference of this quadratic sequence? (2)

3.3.4 Determine ALL values of n for which the terms of the sequence will be less than –110. (6)

[21]

QUESTION 4

4.1 Consider the function $f(x) = 3.2^x - 6$.

4.1.1 Calculate the coordinates of the y-intercept of the graph of f. (1)

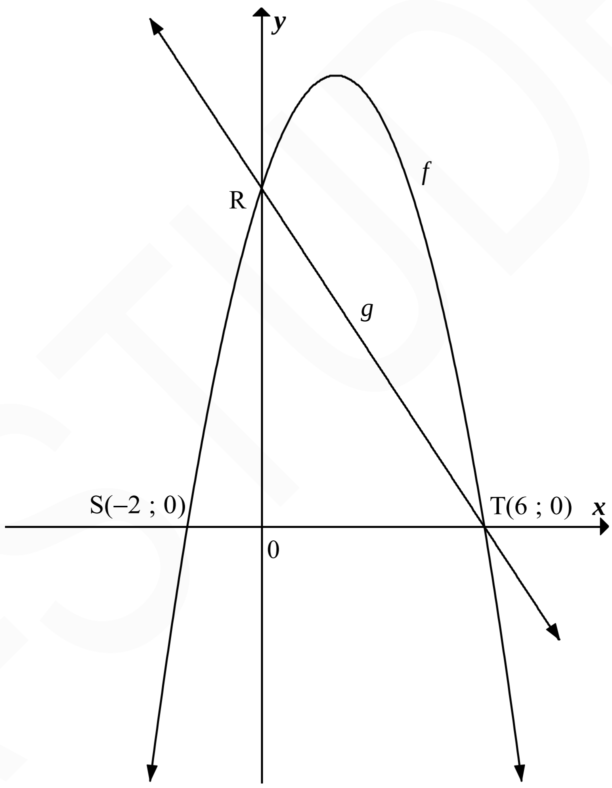
4.1.2 Calculate the coordinates of the x-intercept of the graph of f. (2)

4.1.3 Sketch the graph of f in your ANSWER BOOK.

Clearly show ALL asymptotes and intercepts with the axes. (3)

4.1.4 Write down the range of f. (1)

4.2 S(–2 ; 0) and T(6 ; 0) are the x-intercepts of the graph of $f(x) = ax^2 + bx + c$ and R is the y-intercept. The straight line through R and T represents the graph of $g(x) = -2x + d$.



4.2.1 Determine the value of d. (2)

4.2.2 Determine the equation of f in the form $f(x) = ax^2 + bx + c$. (4)

4.2.3 If $f(x) = -x^2 + 4x + 12$, calculate the coordinates of the turning point of f. (2)

4.2.4 For which values of k will $f(x) = k$ have two distinct roots? (2)

4.2.5 Determine the maximum value of $h(x) = 3^{f(x)-12}$. (3)

[20]

QUESTION 1

1.1 Solve for x:

1.1.1 $3x^2 - 5x = 2$ (3)

1.1.2 $x - \frac{2}{x} = 5$ (4)

1.1.3 $(x+1)(x-3) > 12$ (4)

1.2 Solve simultaneously for r and p in the following set of equations:

$$6r + 5rp - 5p = 8$$
$$r + p = 2$$ (7)

1.3 The volume of a box with a rectangular base is 3 072 cm^3. The lengths of the sides are in the ratio 1 : 2 : 3. Calculate the length of the shortest side. (4)
[22]

QUESTION 2

Given the arithmetic series: – 7 – 3 + 1 + … + 173

2.1 How many terms are there in the series? (3)

2.2 Calculate the sum of the series. (3)

2.3 Write the series in sigma notation. (3)
[9]

QUESTION 3

3.1 Consider the geometric sequence: 4 ; – 2 ; 1 …

3.1.1 Determine the next term of the sequence. (2)

3.1.2 Determine n if the n^{th} term is $\frac{1}{64}$. (4)

3.1.3 Calculate the sum to infinity of the series 4 – 2 + 1 … (2)

3.2 If x is a REAL number, show that the following sequence can NOT be geometric:

$$1 \; ; \; x + 1 \; ; \; x - 3 \; \ldots$$ (4)
[12]

QUESTION 4

An athlete runs along a straight road. His distance d from a fixed point P on the road is measured at different times, n, and has the form $d(n) = an^2 + bn + c$. The distances are recorded in the table below.

Time (in seconds)	1	2	3	4	5	6
Distance (in metres)	17	10	5	2	r	s

4.1 Determine the values of r and s. (3)

4.2 Determine the values of a, b and c. (4)

4.3 How far is the athlete from P when $n = 8$? (2)

4.4 Show that the athlete is moving towards P when $n < 5$, and away from P when $n > 5$. (4)
[13]

QUESTION 1

1.1 Solve for x:

1.1.1 $x(x+1)=6$ (3)

1.1.2 $3x^2-4x=8$ (4)

1.1.3 $4x^2+1\geq 5x$ (4)

1.2 Consider the equation: $x^2+5xy+6y^2=0$

1.2.1 Calculate the values of the ratio $\frac{x}{y}$. (3)

1.2.2 Hence, calculate the values of x and y if $x+y=8$. (5)

[19]

QUESTION 2

2.1 Given the sequence: $4\ ;\ x\ ;\ 32$

Determine the value(s) of x if the sequence is:

2.1.1 Arithmetic (2)

2.1.2 Geometric (3)

2.2 Determine the value of P if $\text{P}=\sum_{k=1}^{13}3^{k-5}$ (4)

2.3 Prove that for any arithmetic sequence of which the first term is a and the constant difference is d, the sum to n terms can be expressed as $S_n=\frac{n}{2}\left(2a+(n-1)d\right)$. (4)

[13]

QUESTION 3

The following sequence is a combination of an arithmetic and a geometric sequence:

$$3\ ;\ 3\ ;\ 9\ ;\ 6\ ;\ 15\ ;\ 12\ ;\ \ldots$$

3.1 Write down the next TWO terms. (2)

3.2 Calculate $T_{52}-T_{51}$. (5)

3.3 Prove that ALL the terms of this infinite sequence will be divisible by 3. (2)

[9]

QUESTION 4

A quadratic pattern has a second term equal to 1, a third term equal to – 6 and a fifth term equal to – 14.

4.1 Calculate the second difference of this quadratic pattern. (5)

4.2 Hence, or otherwise, calculate the first term of the pattern. (2)
[7]

QUESTION 5

5.1 Consider the function: $f(x) = \dfrac{-6}{x-3} - 1$

5.1.1 Calculate the coordinates of the y-intercept of f. (2)

5.1.2 Calculate the coordinates of the x-intercept of f. (3)

5.1.3 Sketch the graph of f in your ANSWER BOOK, showing clearly the asymptotes and the intercepts with the axes. (4)

5.1.4 For which values of x is $f(x) > 0$? (2)

5.1.5 Calculate the average gradient of f between $x = -2$ and $x = 0$. (4)

5.2 Draw a sketch graph of $y = ax^2 + bx + c$, where $a < 0$, $b < 0$, $c < 0$ and $ax^2 + bx + c = 0$ has only ONE solution. (4)
[19]

QUESTION 1

1.1 Solve for *x*, correct to TWO decimal places, where necessary:

1.1.1 $x(x-1)=12$ (3)

1.1.2 $2x^2+3x-7=0$ (4)

1.1.3 $7x^2+18x-9>0$ (4)

1.2 Solve for x and y simultaneously:

$2x-y=7$
$x^2+xy=21-y^2$ (7)

1.3 Simplify completely, without the use of a calculator:

$\left(\sqrt[5]{\sqrt{35}+\sqrt{3}}\right)\left(\sqrt[5]{\sqrt{35}-\sqrt{3}}\right)$ (3)

[21]

QUESTION 2

The sequence 3 ; 9 ; 17 ; 27 ; … is a quadratic sequence.

2.1 Write down the next term. (1)

2.2 Determine an expression for the n^{th} term of the sequence. (4)

2.3 What is the value of the first term of the sequence that is greater than 269? (4)

[9]

QUESTION 3

3.1 The first two terms of an infinite geometric sequence are 8 and $\frac{8}{\sqrt{2}}$. Prove, without the use of a calculator, that the sum of the series to infinity is $16+8\sqrt{2}$. (4)

3.2 The following geometric series is given: $x = 5 + 15 + 45 + \ldots$ to 20 terms.

3.2.1 Write the series in sigma notation. (2)

3.2.2 Calculate the value of *x*. (3)

[9]

QUESTION 4

4.1 The sum to n terms of a sequence of numbers is given as: $S_n = \frac{n}{2}(5n+9)$

4.1.1 Calculate the sum to 23 terms of the sequence. (2)

4.1.2 Hence calculate the 23rd term of the sequence. (3)

4.2 The first two terms of a geometric sequence and an arithmetic sequence are the same. The first term is 12. The sum of the first three terms of the geometric sequence is 3 more than the sum of the first three terms of the arithmetic sequence.

Determine TWO possible values for the common ratio, r, of the geometric sequence. (6)
[11]

QUESTION 5

Consider the function $f(x) = \frac{3}{x-1} - 2$.

5.1 Write down the equations of the asymptotes of f. (2)

5.2 Calculate the intercepts of the graph of f with the axes. (3)

5.3 Sketch the graph of f on DIAGRAM SHEET 1. (3)

5.4 Write down the range of $y = -f(x)$. (1)

5.5 Describe, in words, the transformation of f to g if $g(x) = \frac{-3}{x+1} - 2$. (2)
[11]

QUESTION 3

Given: $\sum_{t=0}^{99}(3t-1)$

3.1 Write down the first THREE terms of the series. (1)

3.2 Calculate the sum of the series. (4)
[5]

QUESTION 4

The following sequence of numbers forms a quadratic sequence:

$-3\ ;-2\ ;-3\ ;-6\ ;-11\ ;\ \ldots$

4.1 The first differences of the above sequence also form a sequence. Determine an expression for the general term of the first differences. (3)

4.2 Calculate the first difference between the 35^{th} and 36^{th} terms of the quadratic sequence. (2)

4.3 Determine an expression for the n^{th} term of the quadratic sequence. (4)

4.4 Explain why the sequence of numbers will never contain a positive term. (2)
[11]

QUESTION 5

Data regarding the growth of a certain tree has shown that the tree grows to a height of 150 cm after one year. The data further reveals that during the next year, the height increases by 18 cm. In each successive year, the height increases by $\frac{8}{9}$ of the previous year's increase in height. The table below is a summary of the growth of the tree up to the end of the fourth year.

	First year	**Second year**	**Third year**	**Fourth year**
Tree height (cm)	150	168	184	$198\frac{2}{9}$
Growth (cm)		18	16	$14\frac{2}{9}$

5.1 Determine the increase in the height of the tree during the seventeenth year. (2)

5.2 Calculate the height of the tree after 10 years. (3)

5.3 Show that the tree will never reach a height of more than 312 cm. (3)
[8]

QUESTION 2

2.1 Consider the sequence: $\frac{1}{2}; 4; \frac{1}{4}; 7; \frac{1}{8}; 10; ...$

2.1.1 If the pattern continues in the same way, write down the next TWO terms in the sequence. (2)

2.1.2 Calculate the sum of the first 50 terms of the sequence. (7)

2.2 Consider the sequence: 8 ; 18 ; 30 ; 44 ; …

2.2.1 Write down the next TWO terms of the sequence, if the pattern continues in the same way. (2)

2.2.2 Calculate the n^{th} term of the sequence. (6)

2.2.3 Which term of the sequence is 330? (4)

[21]

QUESTION 3

Given the geometric series: $8x^2 + 4x^3 + 2x^4 + ...$

3.1 Determine the n^{th} term of the series. (1)

3.2 For what value(s) of x will the series converge? (3)

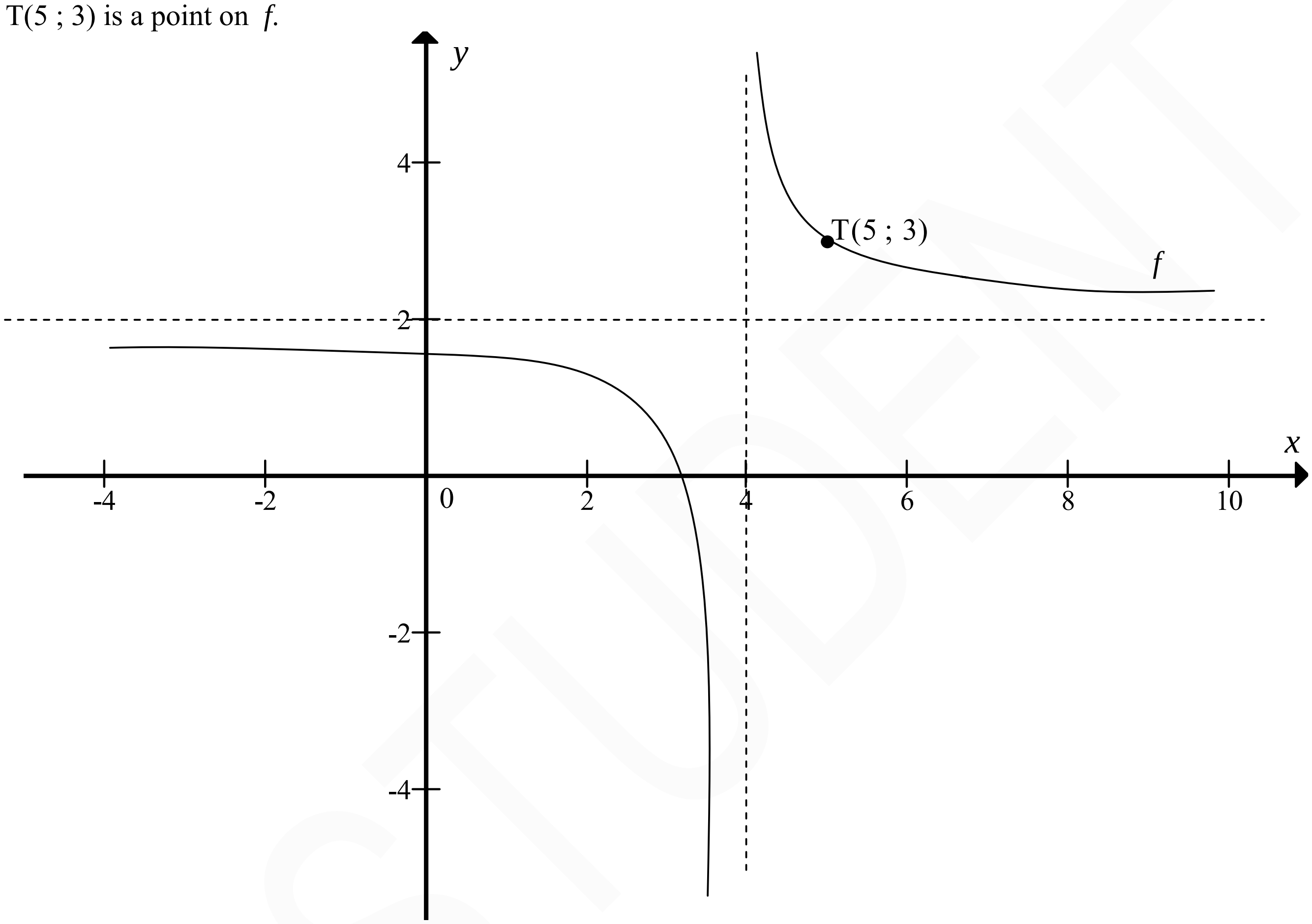
3.3 Calculate the sum of the series to infinity if $x = \frac{3}{2}$. (3)

[7]

QUESTION 4

The diagram below represents the graph of $f(x) = \dfrac{a}{x-p} + q$.

T(5 ; 3) is a point on f.



4.1 Determine the values of a, p and q. (4)

4.2 If the graph of f is reflected across the line having equation $y = -x + c$, the new graph coincides with the graph of $y = f(x)$. Determine the value of c. (3)

[7]

QUESTION 3

Given: $\sum_{t=0}^{99}(3t-1)$

3.1 Write down the first THREE terms of the series. (1)

3.2 Calculate the sum of the series. (4)
[5]

QUESTION 4

The following sequence of numbers forms a quadratic sequence:

$-3\,;-2\,;-3\,;-6\,;-11\,;\ldots$

4.1 The first differences of the above sequence also form a sequence. Determine an expression for the general term of the first differences. (3)

4.2 Calculate the first difference between the 35^{th} and 36^{th} terms of the quadratic sequence. (2)

4.3 Determine an expression for the n^{th} term of the quadratic sequence. (4)

4.4 Explain why the sequence of numbers will never contain a positive term. (2)
[11]

QUESTION 5

Data regarding the growth of a certain tree has shown that the tree grows to a height of 150 cm after one year. The data further reveals that during the next year, the height increases by 18 cm. In each successive year, the height increases by $\frac{8}{9}$ of the previous year's increase in height. The table below is a summary of the growth of the tree up to the end of the fourth year.

	First year	**Second year**	**Third year**	**Fourth year**
Tree height (cm)	150	168	184	$198\frac{2}{9}$
Growth (cm)		18	16	$14\frac{2}{9}$

5.1 Determine the increase in the height of the tree during the seventeenth year. (2)

5.2 Calculate the height of the tree after 10 years. (3)

5.3 Show that the tree will never reach a height of more than 312 cm. (3)
[8]