SA-STUDENT

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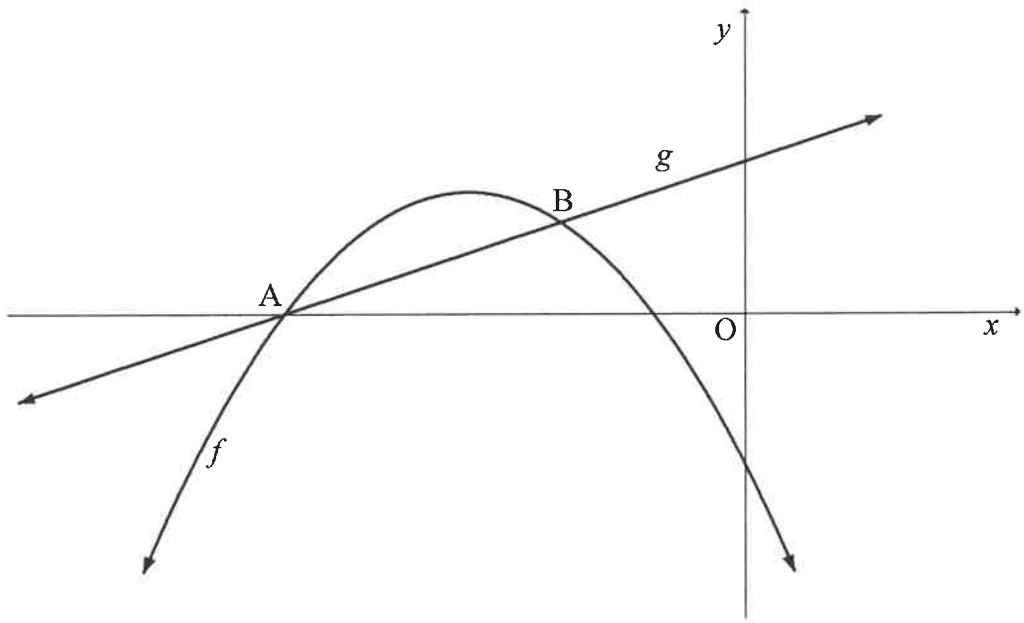
QUESTION 5

The graphs of the functions $f(x) = -(x+3)^2 + 4$ and $g(x) = x + 5$ are drawn below.
The graphs intersect at A and B.



5.1 Write down the coordinates of the turning point of f. (2)

5.2 Write down the range of f. (1)

5.3 Show that the x-coordinates of A and B are -5 and -2 respectively. (4)

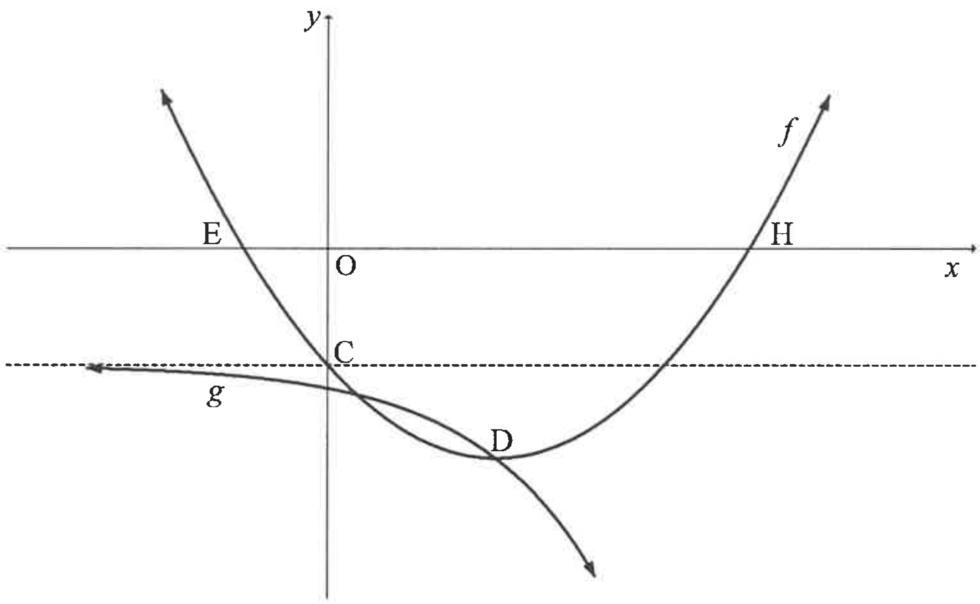
5.4 Hence, determine the values of c for which the equation $-(x+c+3)^2 + 4 = (x+c) + 5$ has ONE negative and ONE positive root. (2)

5.5 The maximum distance between f and g in the interval $x_A < x < x_B$ is k.
If $h(x) = g(x) + k$, determine the equation of h in the form $h(x) = \ldots$ (5)

[14]

4.2 The graphs of $f(x) = x^2 - 4x - 5$ and $g(x) = a.2^x + q$ are sketched below.

- E and H are the x-intercepts of f.
- C is the y-intercept of f and lies on the asymptote of g.
- The two graphs intersect at D, the turning point of f.



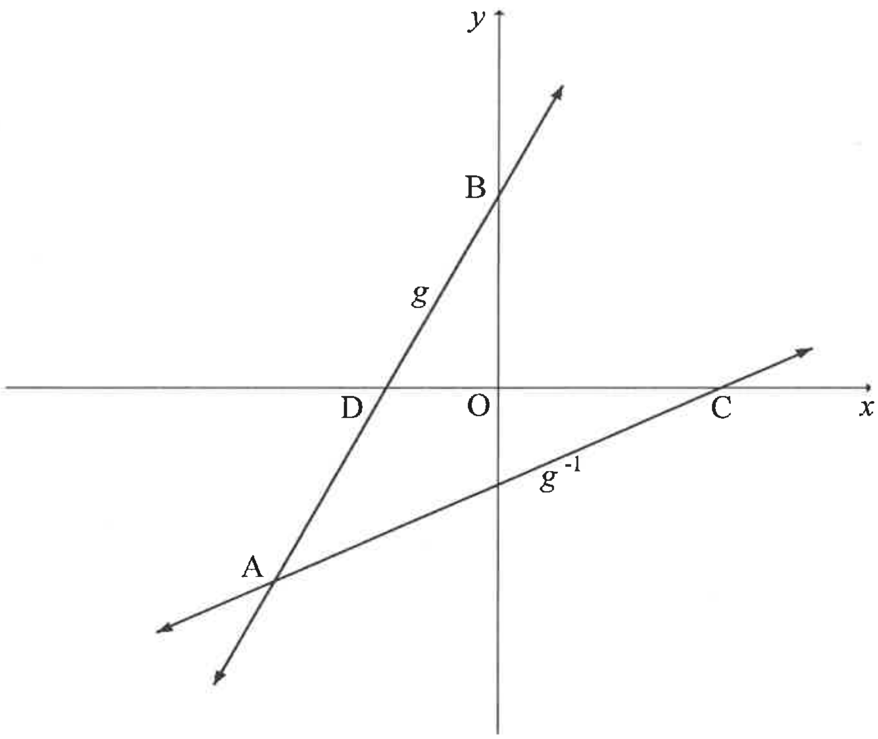
4.2.1	Write down the y-coordinate of C.	(1)
4.2.2	Determine the coordinates of D.	(2)
4.2.3	Determine the values of a and q.	(3)
4.2.4	Write down the range of g.	(1)
4.2.5	Determine the values of k for which the value of $f(x) - k$ will always be positive.	(2)

[20]

QUESTION 5

The graphs of $g(x) = 2x + 6$ and g^{-1}, the inverse of g, are shown in the diagram below.

- D and B are the x- and y-intercepts respectively of g.
- C is the x-intercept of g^{-1}.
- The graphs of g and g^{-1} intersect at A.



5.1 Write down the y-coordinate of B. (1)

5.2 Determine the equation of g^{-1} in the form $g^{-1}(x) = mx + n$. (2)

5.3 Determine the coordinates of A. (3)

5.4 Calculate the length of AB. (2)

5.5 Calculate the area of ΔABC. (5)
[13]

QUESTION 5

Consider: $g(x)=\dfrac{a}{x+p}+q$

The following information of g is given:

- Domain: $x \in \mathrm{R};\ x \neq -2$
- x-intercept at $\mathrm{K}(1\ ;\ 0)$
- y-intercept at $\mathrm{N}\left(0\ ;\ -\dfrac{1}{2}\right)$

5.1 Show that the equation of g is given by: $g(x)=\dfrac{-3}{x+2}+1$ (6)

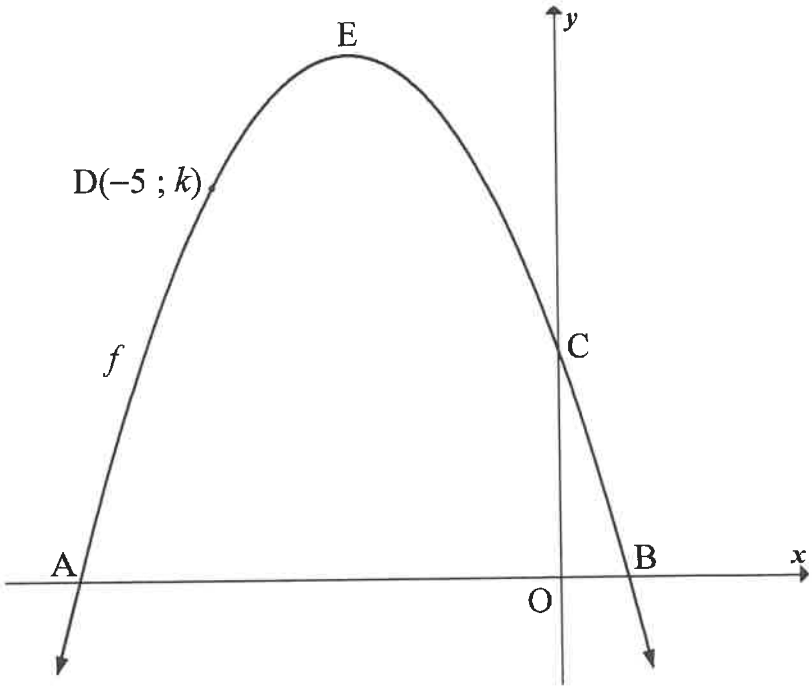
5.2 Write down the range of g. (1)

5.3 Determine the equation of h, the axis of symmetry of g, in the form $y = mx + c$, where $m > 0$. (3)

5.4 Write down the coordinates of K', the image of K reflected over h. (2)
[12]

QUESTION 6

The sketch below shows the graph of $f(x) = -x^2 - 6x + 7$.
C is the y-intercept of f.
A and B are the x-intercepts of f.
D$(-5\ ;\ k)$ is a point on f.



6.1	Calculate the coordinates of E, the turning point of f.	(3)
6.2	Write down the value of k.	(1)
6.3	Determine the equation of the straight line passing through C and D.	(4)
6.4	A tangent, parallel to CD, touches f at P. Determine the coordinates of P.	(4)
6.5	For which values of x will $f(x) - 12 > 0$?	(2) **[14]**

QUESTION 3

Consider the quadratic number pattern: $-145\ ;-122\ ;-101\ ;\ \ldots$

3.1 Write down the value of T_4. (1)

3.2 Show that the general term of this number pattern is $T_n = -n^2 + 26n - 170$. (3)

3.3 Between which TWO terms of the quadratic number pattern will there be a difference of -121? (4)

3.4 What value must be added to each term in the number pattern so that the value of the maximum term in the new number pattern formed will be 1? (3)
[11]

QUESTION 4

Consider the linear pattern: $5\ ;7\ ;9\ ;\ \ldots$

4.1 Determine T_{51}. (3)

4.2 Calculate the sum of the first 51 terms. (2)

4.3 Write down the expansion of $\sum_{n=1}^{5000}(2n+3)$. Show only the first 3 terms and the last term of the expansion. (1)

4.4 Hence, or otherwise, calculate $\sum_{n=1}^{5000}(2n+3)+\sum_{n=1}^{4999}(-2n-1)$.
ALL working details must be shown. (4)
[10]

QUESTION 5

Given: $f(x)=\dfrac{-1}{x-3}+2$

5.1 Write down the equations of the asymptotes of f. (2)

5.2 Write down the domain of f. (1)

5.3 Determine the coordinates of the x-intercept of f. (2)

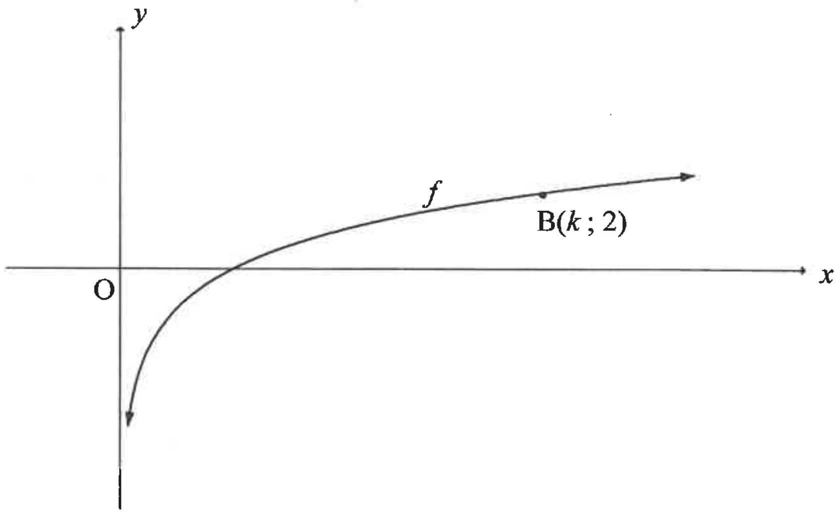
5.4 Write down the coordinates of the y-intercept of f. (2)

5.5 Draw the graph of f. Clearly show ALL the asymptotes and intercepts with the axes. (3)
[10]

QUESTION 6

The graph of $f(x) = \log_4 x$ is drawn below.

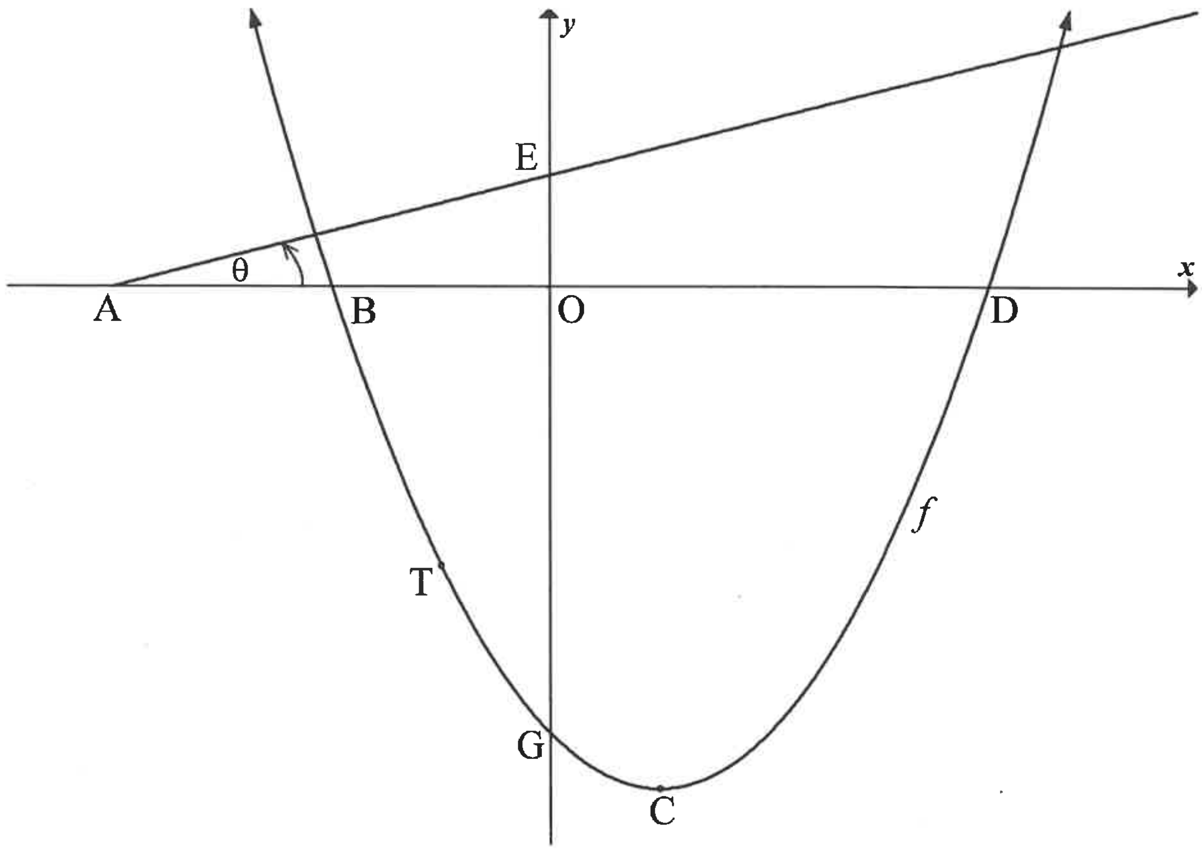
$B(k\,;2)$ is a point on f.



6.1	Calculate the value of k.	(2)
6.2	Determine the values of x for which $-1 \leq f(x) \leq 2$.	(2)
6.3	Write down the equation of f^{-1}, the inverse of f, in the form $y = \ldots$	(2)
6.4	For which values of x will $x.f^{-1}(x) < 0$?	(2)
		[8]

QUESTION 7

The graph of $f(x)=(x+4)(x-6)$ is drawn below.
The parabola cuts the x-axis at B and D and the y-axis at G.
C is the turning point of f.
Line AE has an angle of inclination of θ and cuts the x-axis and y-axis at A and E respectively.
T is a point on f between B and G.



7.1 Write down the coordinates of B and D. (2)

7.2 Calculate the coordinates of C. (2)

7.3 Write down the range of f. (1)

7.4 Given that $\theta = 14{,}04°$ and the tangent to f at T is perpendicular to AE.

7.4.1 Calculate the gradient of AE, correct to TWO decimal places. (1)

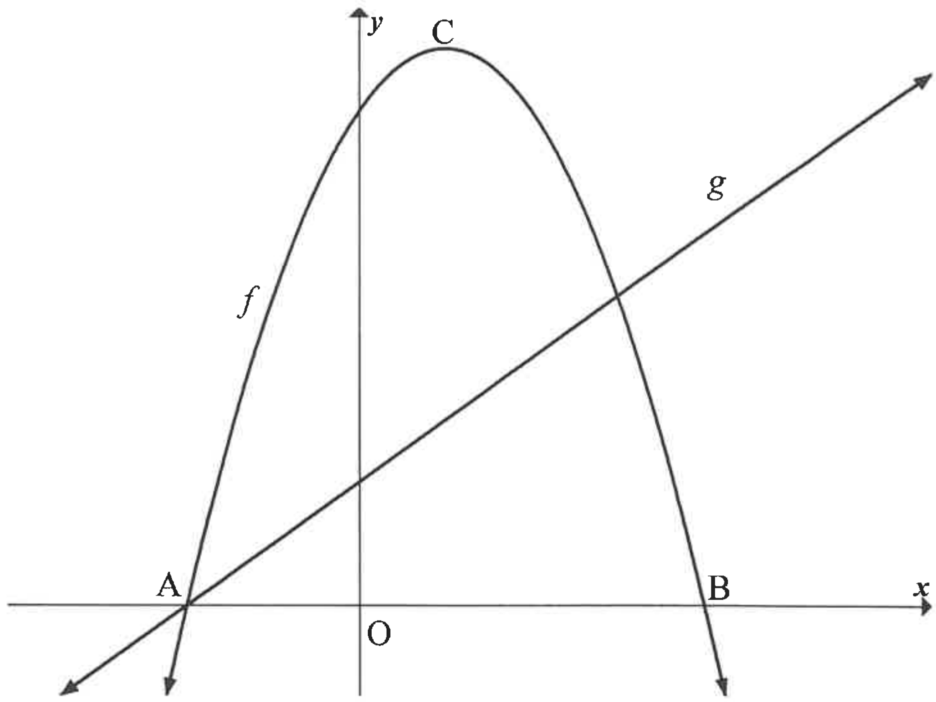
7.4.2 Calculate the coordinates of T. (5)

7.5 A straight line, g, parallel to AE, cuts f at K(–3 ; –9) and R.
Calculate the x-coordinate of R. (6)

[17]

QUESTION 5

Sketched below are the graphs of $f(x) = -2x^2 + 4x + 16$ and $g(x) = 2x + 4$.
A and B are the x-intercepts of f. C is the turning point of f.



5.1 Calculate the coordinates of A and B. (3)

5.2 Determine the coordinates of C, the turning point of f. (2)

5.3 Write down the range of f. (1)

5.4 The graph of $h(x) = f(x + p) + q$ has a maximum value of 15 at $x = 2$.
Determine the values of p and q. (3)

5.5 Determine the equation of g^{-1}, the inverse of g, in the form $y = \dots$ (2)

5.6 For which value(s) of x will $g^{-1}(x).g(x) = 0$? (2)

5.7 If $p(x) = f(x) + k$, determine the value(s) of k for which p and g will NOT intersect. (5)

[18]

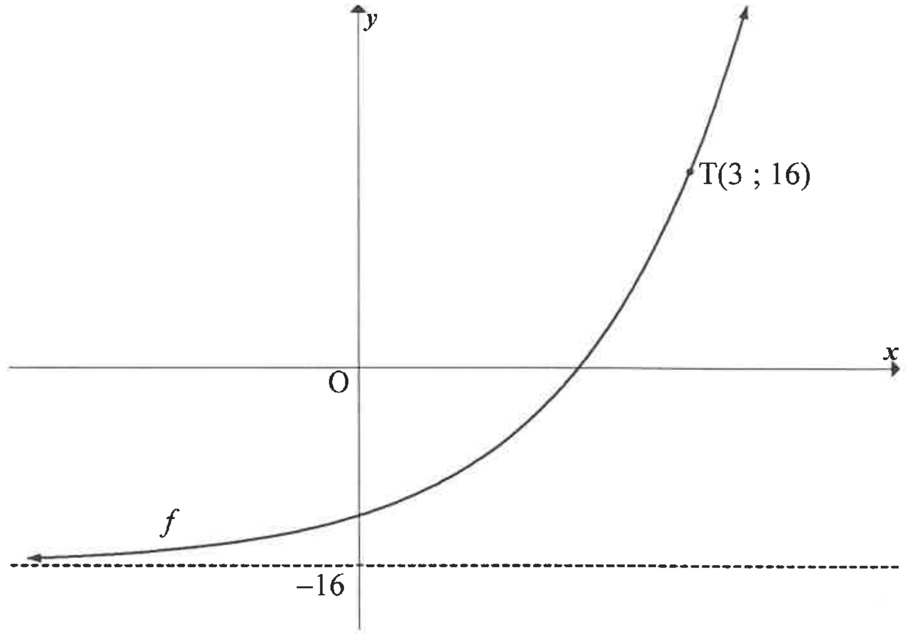
QUESTION 6

6.1 Given: $g(x) = 3^x$

6.1.1 Write down the equation of g^{-1} in the form $y = \ldots$ (2)

6.1.2 Point P(6 ; 11) lies on $h(x) = 3^{x-4} + 2$. The graph of h is translated to form g. Write down the coordinates of the image of P on g. (2)

6.2 Sketched is the graph of $f(x) = 2^{x+p} + q$. T(3 ; 16) is a point on f and the asymptote of f is $y = -16$.



Determine the values of p and q. (4)

[8]

QUESTION 7

7.1 An amount of R10 000 was invested for 4 years, earning interest at r% p.a., compounded quarterly. At the end of the 4 years, the total amount in the account was R13 080. Determine the value of r. (4)

7.2 A businesswoman deposited R9 000 into an account at the end of January 2014. She continued to make monthly deposits of R9 000 at the end of each month up to the end of December 2018. The account earned interest at a rate of 7,5% p.a., compounded monthly.

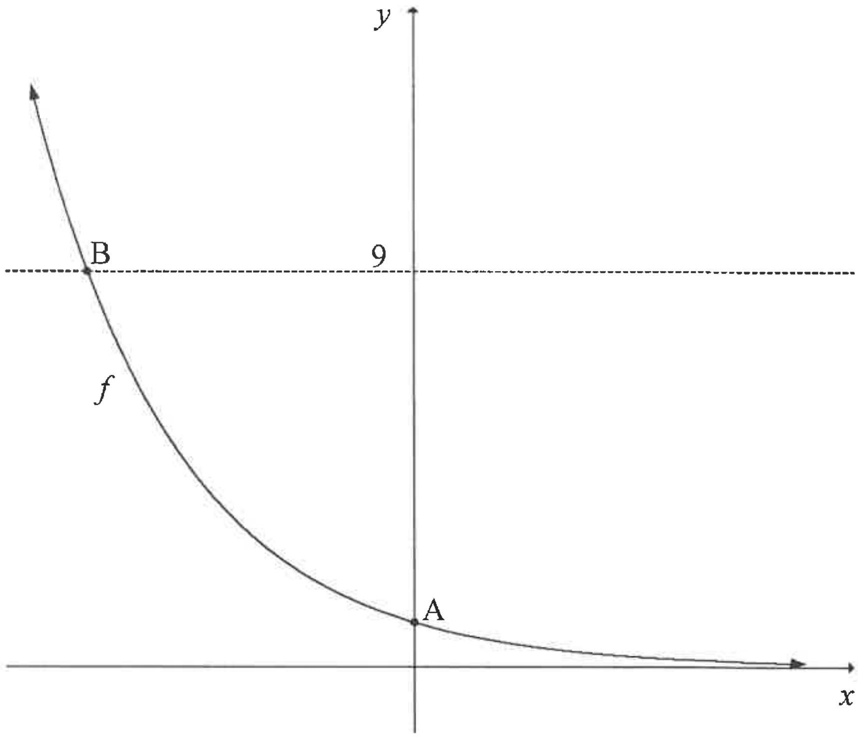
7.2.1 Calculate how much money was in the account immediately after 60 deposits had been made. (3)

7.2.2 The businesswoman left the amount calculated in QUESTION 7.2.1 for a further n months in the account. The interest rate remained unchanged and no further payments were made. The total interest earned over the entire investment period was R190 214,14. Determine the value of n. (6)

[13]

QUESTION 5

The graph of $f(x)=3^{-x}$ is sketched below. A is the y-intercept of f.
B is the point of intersection of f and the line $y=9$.



5.1 Write down the coordinates of A. (1)

5.2 Determine the coordinates of B. (3)

5.3 Write down the domain of f^{-1}. (2)

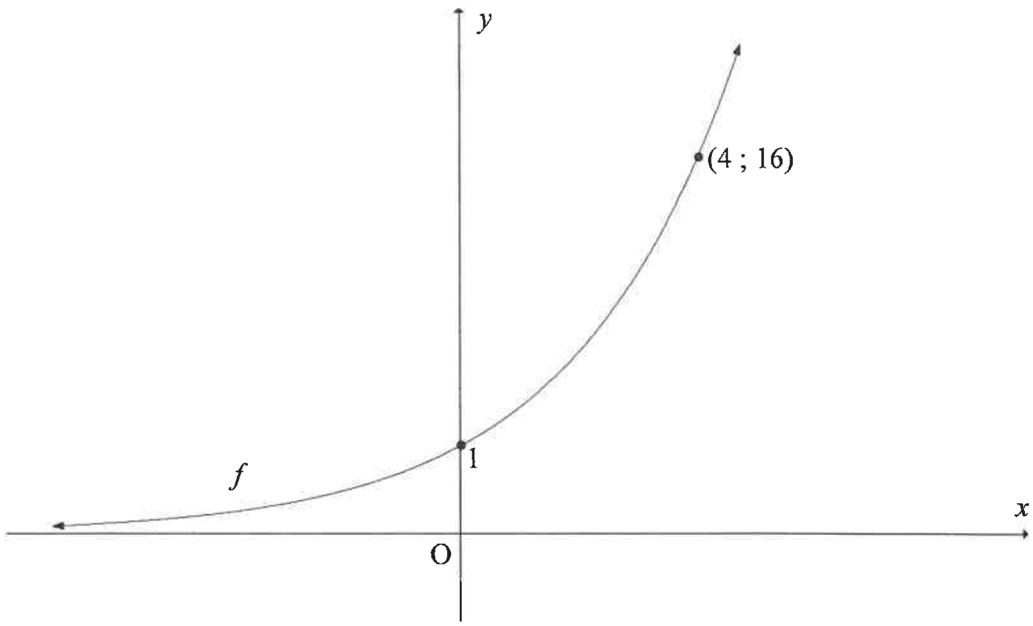
5.4 Describe the translation from f to $h(x)=\dfrac{27}{3^x}$. (3)

5.5 Determine the values of x for which $h(x)<1$. (3)

[12]

QUESTION 5

Sketched below is the graph of $f(x) = k^x$; $k > 0$. The point $(4 ; 16)$ lies on f.



5.1 Determine the value of k. (2)

5.2 Graph g is obtained by reflecting graph f about the line $y = x$. Determine the equation of g in the form $y = \ldots$ (2)

5.3 Sketch the graph g. Indicate on your graph the coordinates of two points on g. (4)

5.4 Use your graph to determine the value(s) of x for which:

5.4.1 $f(x) \times g(x) > 0$ (2)

5.4.2 $g(x) \leq -1$ (2)

5.5 If $h(x) = f(-x)$, calculate the value of x for which $\mathrm{f(x)} - \mathrm{h(x)} = \frac{15}{4}$ (4)

[16]

QUESTION 5

5.1 Given: $f(x)=\dfrac{1}{x+2}+3$

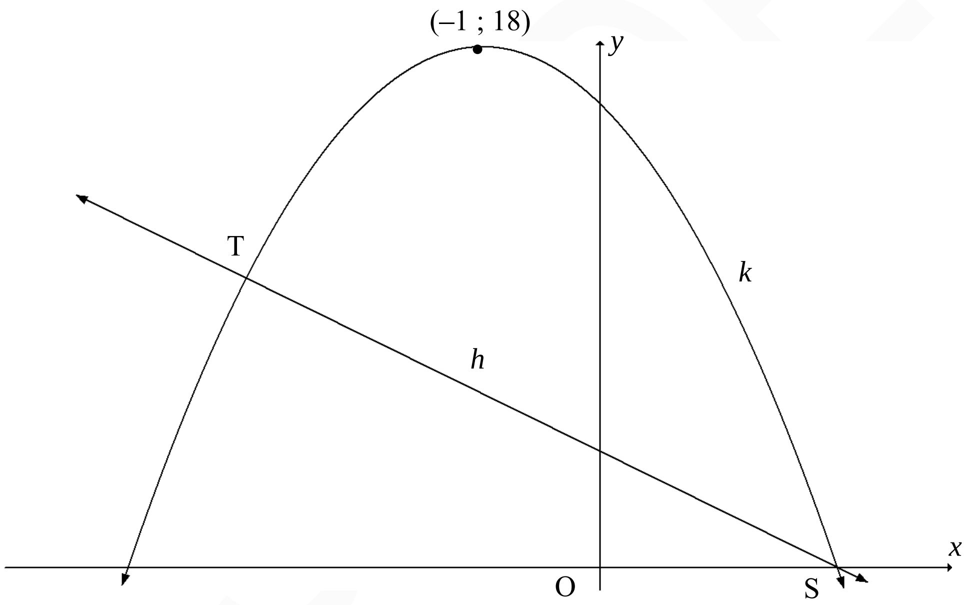
5.1.1 Determine the equations of the asymptotes of f. (2)

5.1.2 Write down the y-intercept of f. (1)

5.1.3 Calculate the x-intercept of f. (2)

5.1.4 Sketch the graph of f. Clearly label ALL intercepts with the axes and any asymptotes. (3)

5.2 Sketched below are the graphs of $k(x)=ax^2+bx+c$ and $h(x)=-2x+4$. Graph k has a turning point at (–1 ; 18). S is the x-intercept of h and k. Graphs h and k also intersect at T.



5.2.1 Calculate the coordinates of S. (2)

5.2.2 Determine the equation of k in the form $y=a(x+p)^2+q$ (3)

5.2.3 If $k(x)=-2x^2-4x+16$, determine the coordinates of T. (5)

5.2.4 Determine the value(s) of x for which $k(x)<h(x)$. (2)

5.2.5 It is further given that k is the graph of $g'(x)$.

(a) For which values of x will the graph of g be concave up? (2)

(b) Sketch the graph of g, showing clearly the x-values of the turning points and the point of inflection. (3)

[25]

QUESTION 5

Given: $f(x)=\dfrac{-1}{x-1}$

5.1 Write down the domain of f. (1)

5.2 Write down the asymptotes of f. (2)

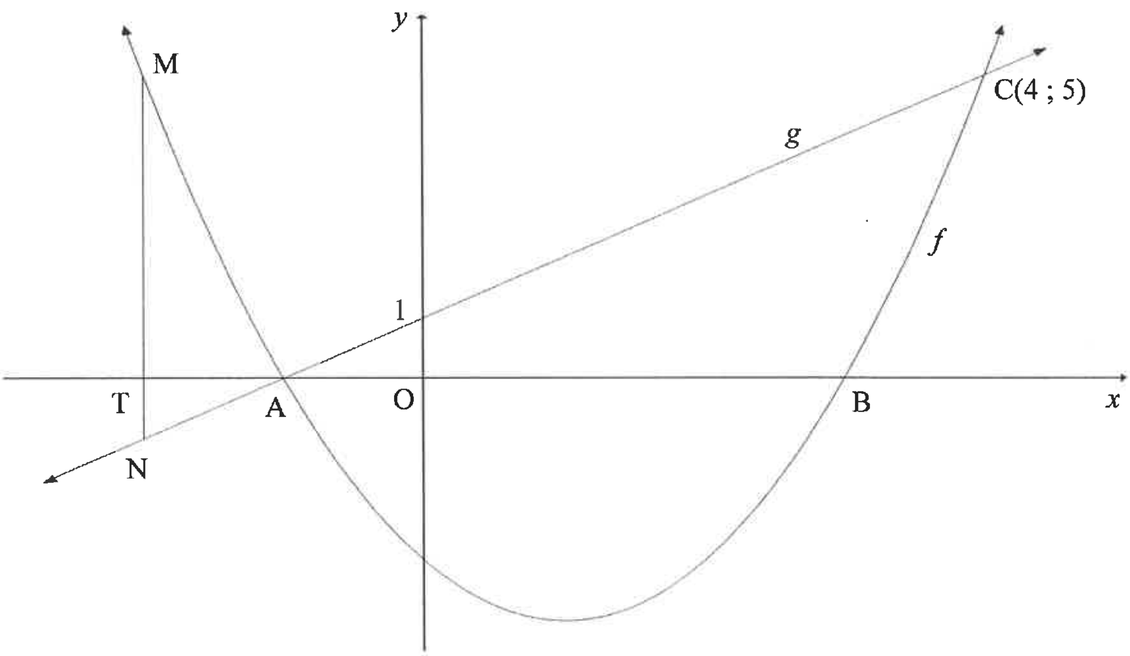
5.3 Sketch the graph of f, clearly showing all intercepts with the axes and any asymptotes. (3)

5.4 For which values of x will $x.f'(x)\geq 0$? (2)

[8]

QUESTION 6

In the diagram below, A and B are the x-intercepts of the graph of $f(x)=x^2-2x-3$.
A straight line, g, through A cuts f at C(4 ; 5) and the y-axis at (0 ; 1).
M is a point on f and N is a point on g such that MN is parallel to the y-axis.
MN cuts the x-axis at T.



6.1 Show that $g(x)=x+1$. (2)

6.2 Calculate the coordinates of A and B. (3)

6.3 Determine the range of f. (3)

6.4 If MN = 6:

6.4.1 Determine the length of OT if T lies on the negative x-axis. Show ALL your working. (4)

6.4.2 Hence, write down the coordinates of N. (2)

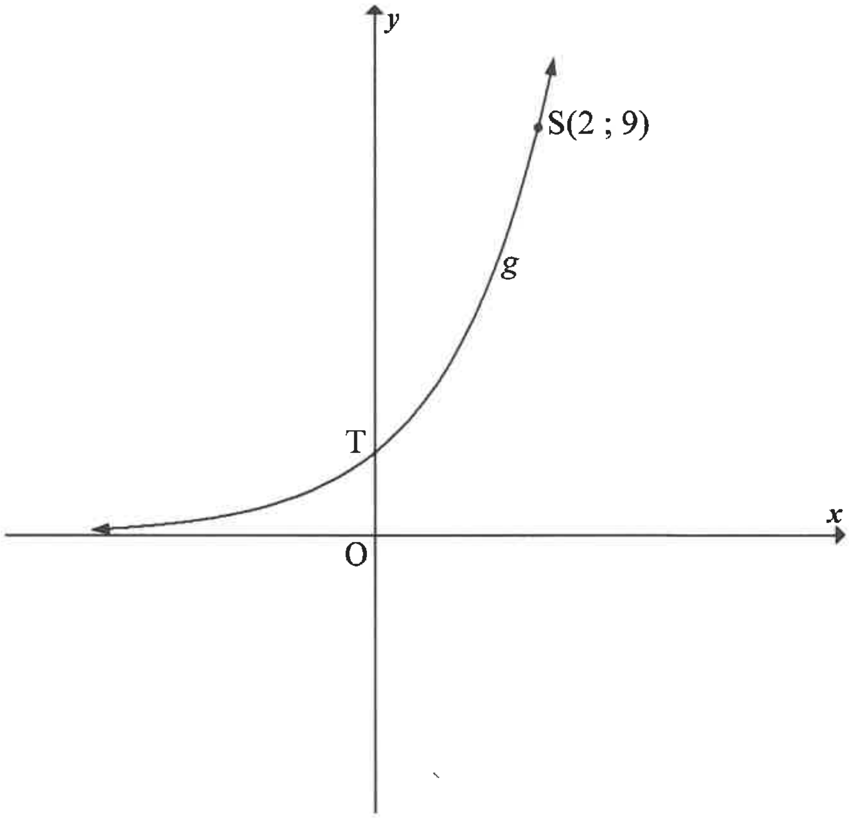
6.5 Determine the equation of the tangent to f drawn parallel to g. (5)

6.6 For which value(s) of k will $f(x)=x^2-2x-3$ and $h(x)=x+k$ NOT intersect? (1)

[20]

QUESTION 5

The graph of $g(x)=a^{x}$ is drawn in the sketch below. The point S(2 ; 9) lies on g. T is the y-intercept of g.



5.1	Write down the coordinates of T.	(2)
5.2	Calculate the value of a.	(2)
5.3	The graph h is obtained by reflecting g in the y-axis. Write down the equation of h.	(2)
5.4	Write down the values of x for which $0<\log_3 x<1$.	(2)
		[8]

QUESTION 3

Themba is planning a bicycle trip from Cape Town to Pretoria. The total distance covered during the trip will be 1 500 km. He plans to travel 100 km on the first day. For every following day he plans to cover 94% of the distance he covered the previous day.

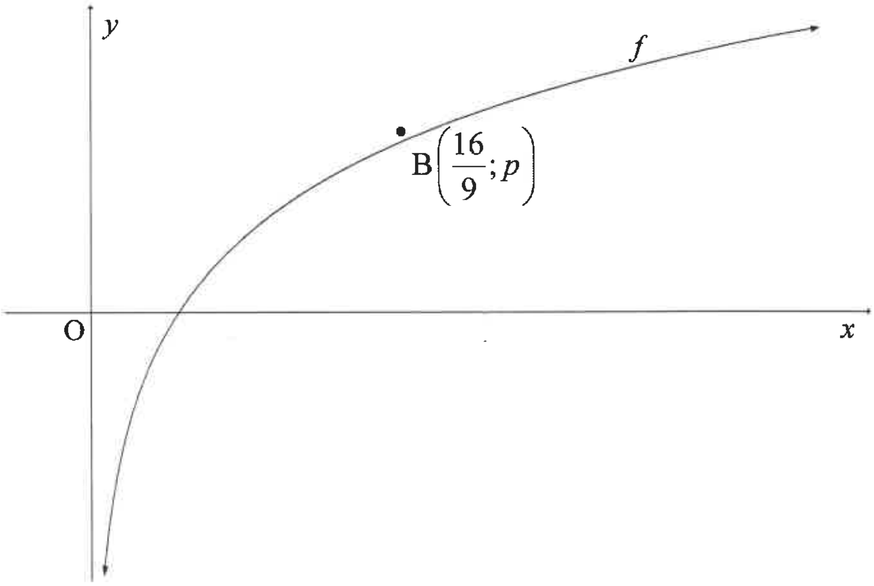
3.1 What distance will he cover on day 3 of the trip? (2)

3.2 On what day of the trip will Themba pass the halfway point? (4)

3.3 Themba must cover a certain percentage of the previous day's distance to ensure that he will eventually reach Pretoria. Calculate ALL possible value(s) of this percentage. (3)
[9]

QUESTION 4

The graph of $f(x)=\log_{\frac{4}{3}} x$ is drawn below. $B\left(\frac{16}{9};p\right)$ is a point on f.



4.1 For which value(s) of x is $\log_{\frac{4}{3}} x \le 0$? (2)

4.2 Determine the value of p, without the use of a calculator. (3)

4.3 Write down the equation of the inverse of f in the form $y = \ldots$ (2)

4.4 Write down the range of $y = f^{-1}(x)$. (2)

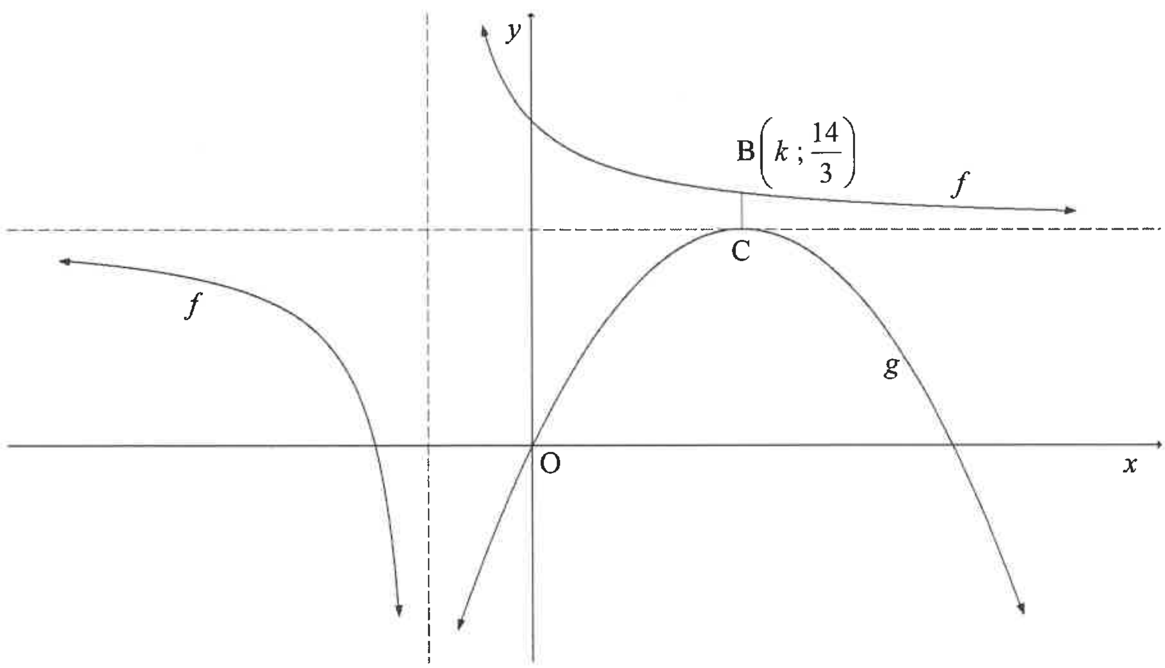
4.5 The function $h(x)=\left(\frac{3}{4}\right)^x$ is obtained after applying two reflections on f.

Write down the coordinates of B'', the image of B on h. (2)
[11]

QUESTION 5

The graphs of $f(x)=\dfrac{2}{x+1}+4$ and parabola g are drawn below.

- C, the turning point of g, lies on the horizontal asymptote of f.
- The graph of g passes through the origin.
- B$\left(k;\dfrac{14}{3}\right)$ is a point on f such that BC is parallel to the y-axis.



5.1 Write down the domain of f. (2)

5.2 Determine the x-intercept of f. (2)

5.3 Calculate the value of k. (3)

5.4 Write down the coordinates of C. (2)

5.5 Determine the equation of g in the form $y=a(x+p)^2+q$. (3)

5.6 For which value(s) of x will $\dfrac{f(x)}{g(x)}\leq 0$? (4)

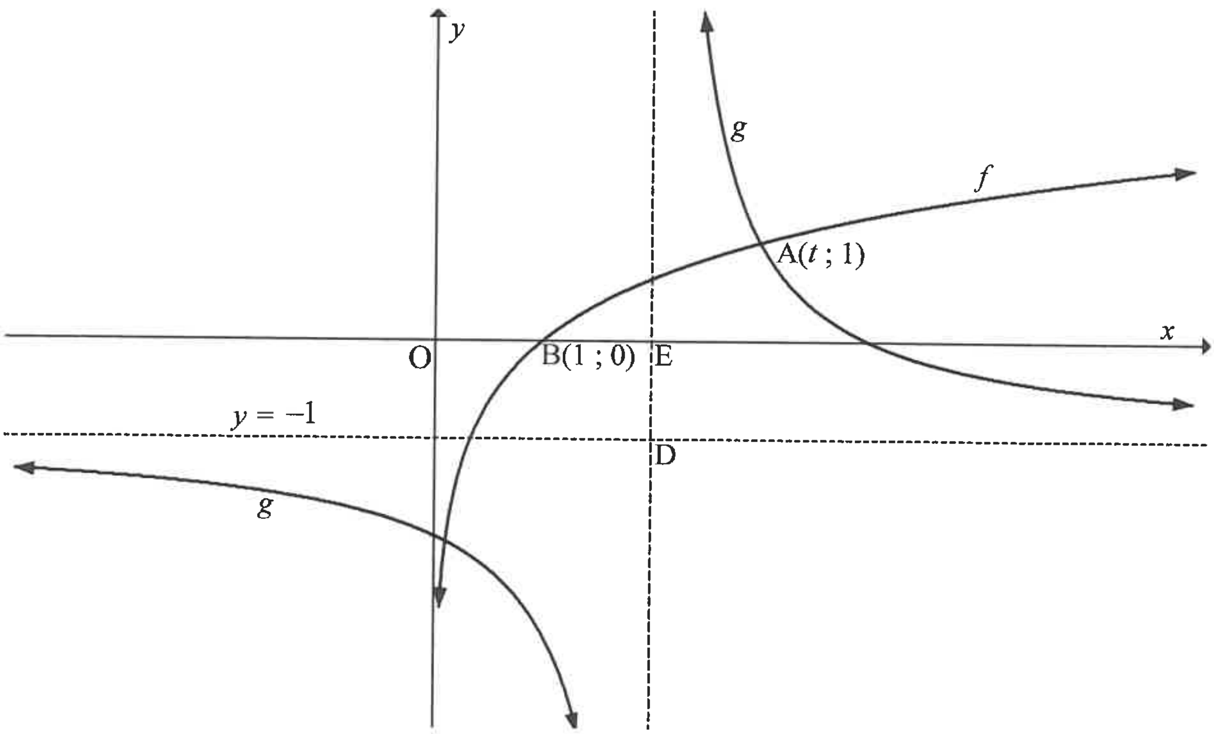
5.7 Use the graphs of f and g to determine the number of real roots of $\dfrac{2}{x}-5=-(x-3)^2-5$. Give reasons for your answer. (4)

[20]

QUESTION 5

The diagram below shows the graphs of $g(x)=\dfrac{2}{x+p}+q$ and $f(x)=\log_3 x$.

- $y=-1$ is the horizontal asymptote of g.
- B(1 ; 0) is the x-intercept of f.
- A(t ; 1) is a point of intersection between f and g.
- The vertical asymptote of g intersects the x-axis at E and the horizontal asymptote at D.
- OB = BE.



5.1 Write down the range of g. (2)

5.2 Determine the equation of g. (2)

5.3 Calculate the value of t. (3)

5.4 Write down the equation of f^{-1}, the inverse of f, in the form $y=\ldots$ (2)

5.5 For which values of x will $f^{-1}(x)<3$? (2)

5.6 Determine the point of intersection of the graphs of f and the axis of symmetry of g that has a negative gradient. (3)

[14]

QUESTION 5

Given: $f(x)=x^2-5x-14$ and $g(x)=2x-14$

5.1 On the same set of axes, sketch the graphs of f and g. Clearly indicate all intercepts with the axes and turning points. (6)

5.2 Determine the equation of the tangent to f at $x=2\frac{1}{2}$. (2)

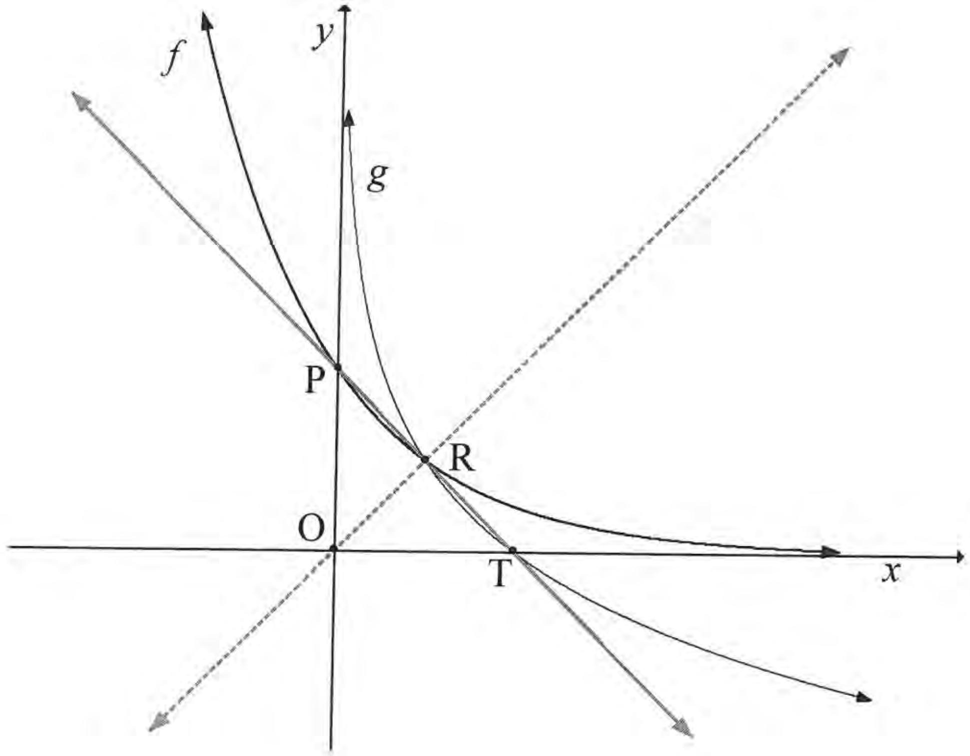
5.3 Determine the value(s) of k for which $f(x)=k$ will have two unequal positive real roots. (2)

5.4 A new graph h is obtained by first reflecting g in the x-axis and then translating it 7 units to the left. Write down the equation of h in the form $h(x)=mx+c$. (2)

[12]

QUESTION 6

In the sketch below, P is the y-intercept of the graph of $f(x)=b^x$. T is the x-intercept of graph g, the inverse of f. R is the point of intersection of f and g. Straight lines are drawn through O and R and through P and T.



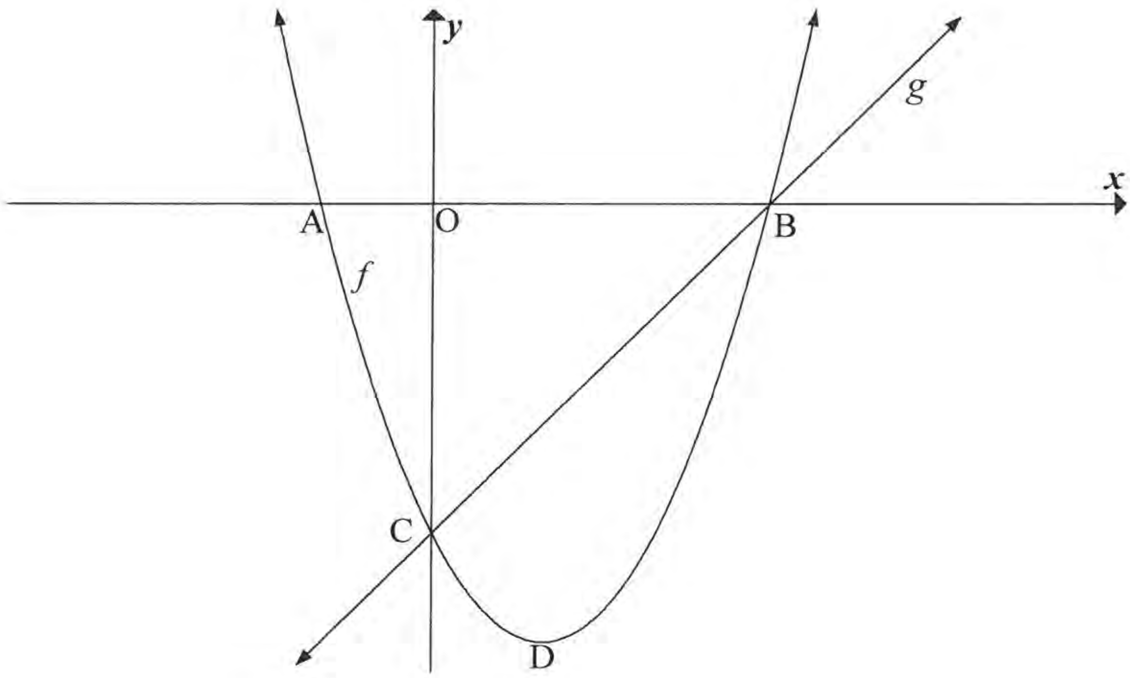
6.1	Determine the equation of g (in terms of b) in the form $y = \ldots$	(2)
6.2	Write down the equation of the line passing through O and R.	(1)
6.3	Write down the coordinates of point P.	(1)
6.4	Determine the equation of the line passing through P and T.	(2)
6.5	Calculate the value of b.	(5) **[11]**

QUESTION 5

5.1 The sketch below shows the graphs of $f(x) = x^2 - 2x - 3$ and $g(x) = x - 3$.

- A and B are the x-intercepts of f.
- The graphs of f and g intersect at C and B.

D is the turning point of f.



5.1.1 Determine the coordinates of C. (1)

5.1.2 Calculate the length of AB. (4)

5.1.3 Determine the coordinates of D. (2)

5.1.4 Calculate the average gradient of f between C and D. (2)

5.1.5 Calculate the size of $\text{O}\hat{\text{C}}\text{B}$ (2)

5.1.6 Determine the values of k for which $f(x) = k$ will have two unequal positive real roots. (3)

5.1.7 For which values of x will $f'(x) \,.\, f''(x) > 0$? (3)

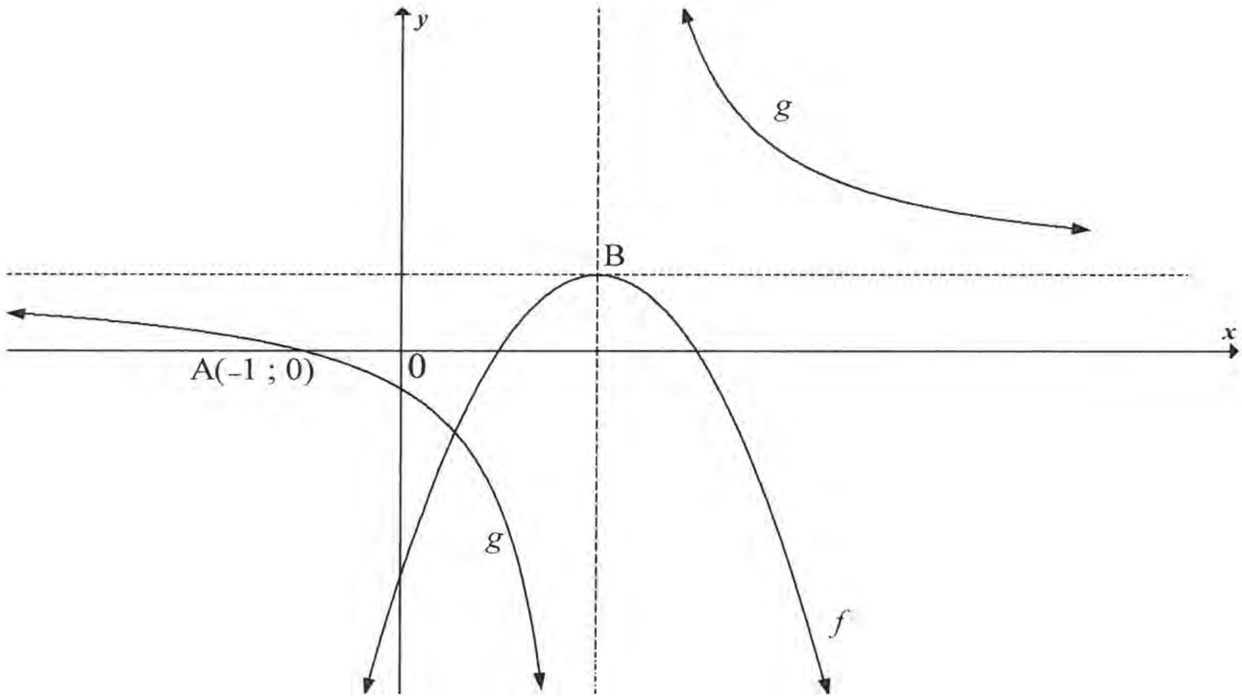
5.2 The graph of a parabola f has x-intercepts at $x = 1$ and $x = 5$. $g(x) = 4$ is a tangent to f at P, the turning point of f. Sketch the graph of f, clearly showing the intercepts with the axes and the coordinates of the turning point. (5)

[22]

QUESTION 5

Sketched below is the parabola f, with equation $f(x) = -x^2 + 4x - 3$ and a hyperbola g, with equation $(x-p)(y+t) = 3$.

- B, the turning point of f, lies at the point of intersection of the asymptotes of g.
- $A(-1\,;\,0)$ is the x-intercept of g.

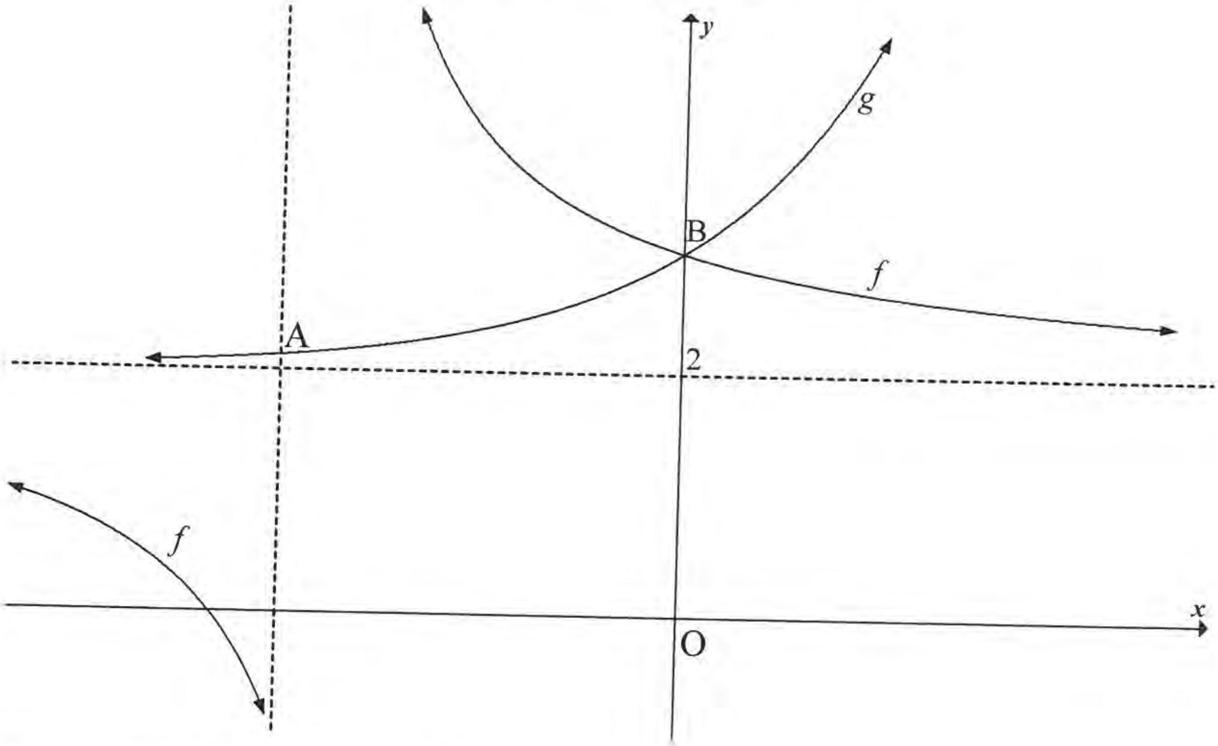


5.1	Show that the coordinates of B are $(2\,;\,1)$	(2)
5.2	Write down the range of f.	(1)
5.3	For which value(s) of x will $g(x) \geq 0$?	(2)
5.4	Determine the equation of the vertical asymptote of the graph of h if $h(x) = g(x+4)$	(1)
5.5	Determine the values of p and t.	(4)
5.6	Write down the values of x for which $f(x).g'(x) \geq 0$	(4)
		[14]

QUESTION 5

The sketch below shows the graphs of $f(x) = \dfrac{3}{x-p} + q$ and $g(x) = 2^x + r$

- g intersects the vertical asymptote of f at A.
- B is the common y-intercept of f and g.
- $y = 2$ is the common horizontal asymptote of f and g.



5.1 Write down the value of r. (1)

5.2 Determine the value of p. (4)

5.3 Determine the coordinates of A. (3)

5.4 For which value(s) of x is $f(x) - g(x) \geq 0$? (2)

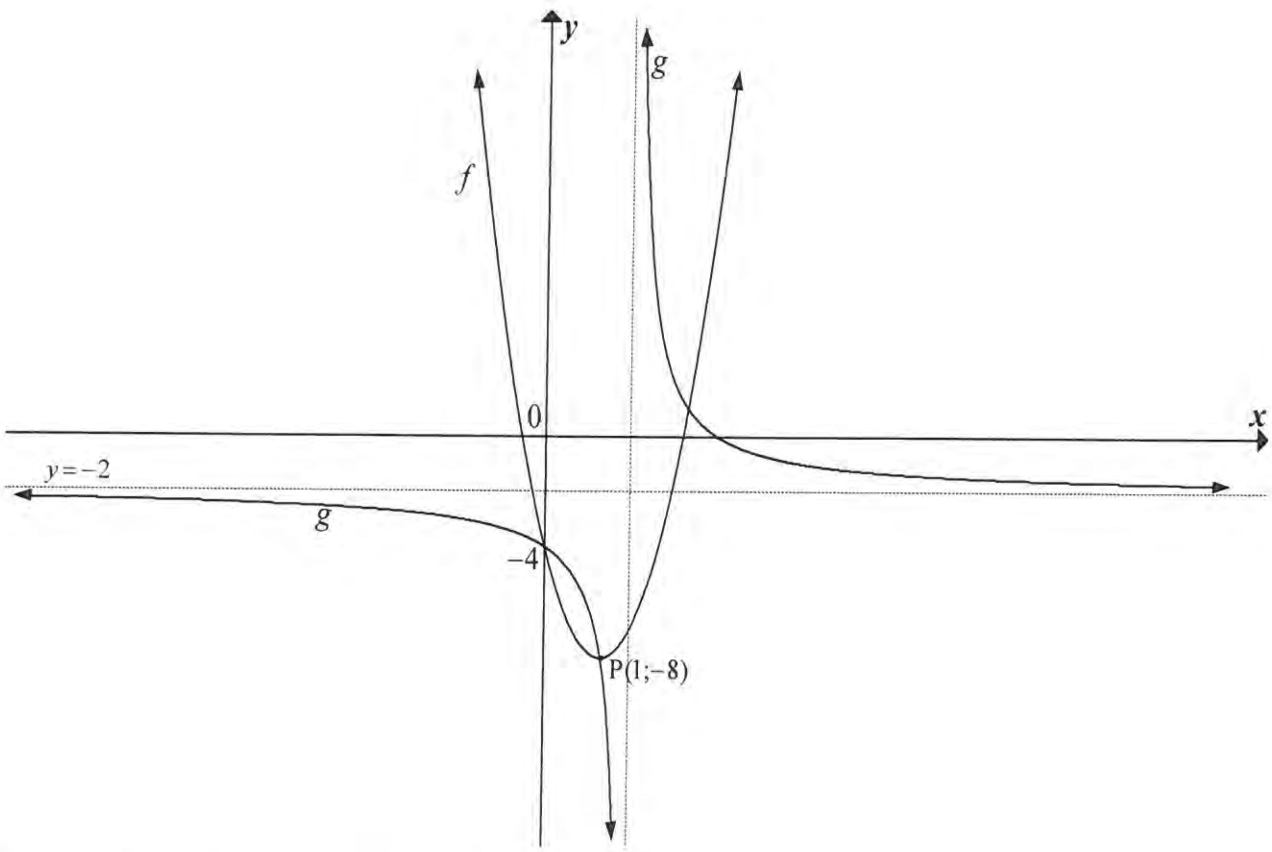
5.5 If $h(x) = f(x-2)$, write down the equation of h. (2)

[12]

QUESTION 5

The graphs of the functions $f(x) = a(x+p)^2 + q$ and $g(x) = \dfrac{k}{x+r} + d$ are sketched below.

Both graphs cut the y-axis at -4. One of the points of intersection of the graphs is P(1 ; –8), which is also the turning point of f. The horizontal asymptote of g is $y = -2$.

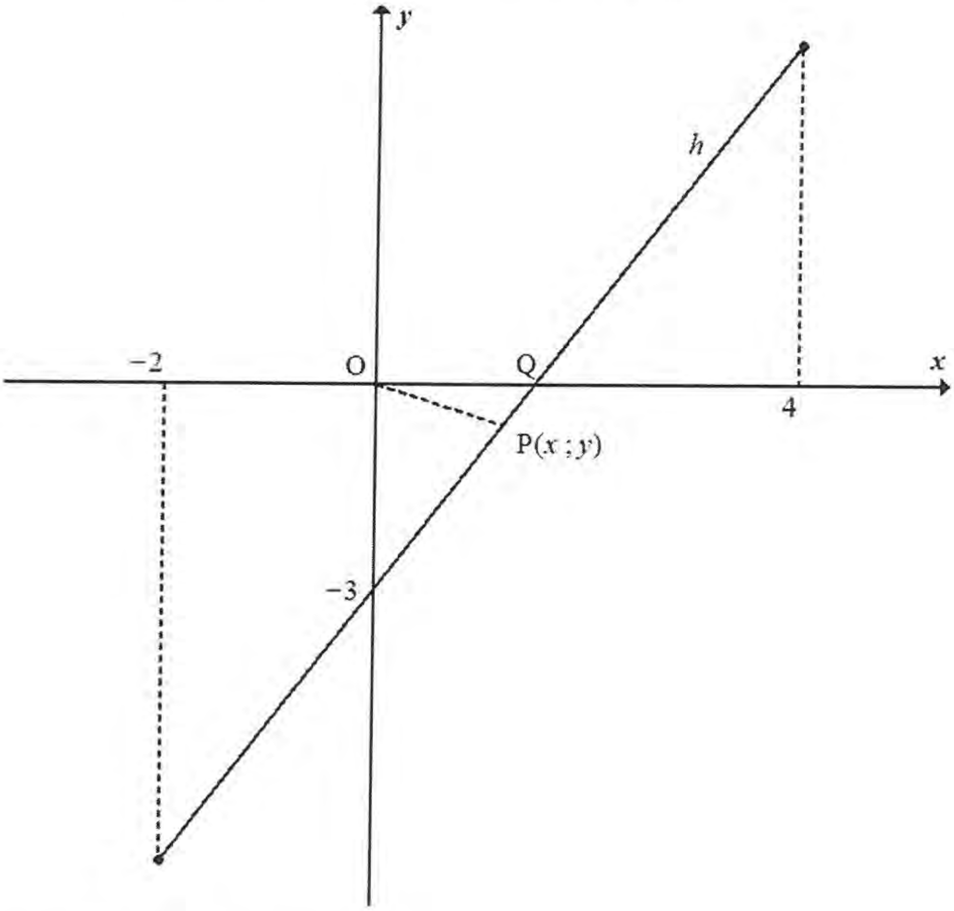


5.1	Calculate the values of a, p and q.	(4)
5.2	Calculate the values of k, r and d.	(6)
5.3	Determine the value(s) of x in the interval $x \leq 1$ for which $g(x) \geq f(x)$.	(2)
5.4	Determine the value(s) of k for which $f(x) = k$ has two, unequal positive roots.	(2)
5.5	Write down an equation for the axis of symmetry of g that has a negative gradient.	(3)
5.6	The point P is reflected in the line determined in QUESTION 5.5 to give the point Q. Write down the coordinates of Q.	(2)

[19]

QUESTION 5

Given: $h(x) = 2x - 3$ for $-2 \leq x \leq 4$. The x-intercept of h is Q.



5.1 Determine the coordinates of Q. (2)

5.2 Write down the domain of h^{-1}. (3)

5.3 Sketch the graph of h^{-1} in your ANSWER BOOK, clearly indicating the y-intercept and the end points. (3)

5.4 For which value(s) of x will $h(x) = h^{-1}(x)$? (3)

5.5 $P(x\ ;\ y)$ is the point on the graph of h that is closest to the origin. Calculate the distance OP. (5)

5.6 Given: $h(x) = f'(x)$ where f is a function defined for $-2 \leq x \leq 4$.

5.6.1 Explain why f has a local minimum. (2)

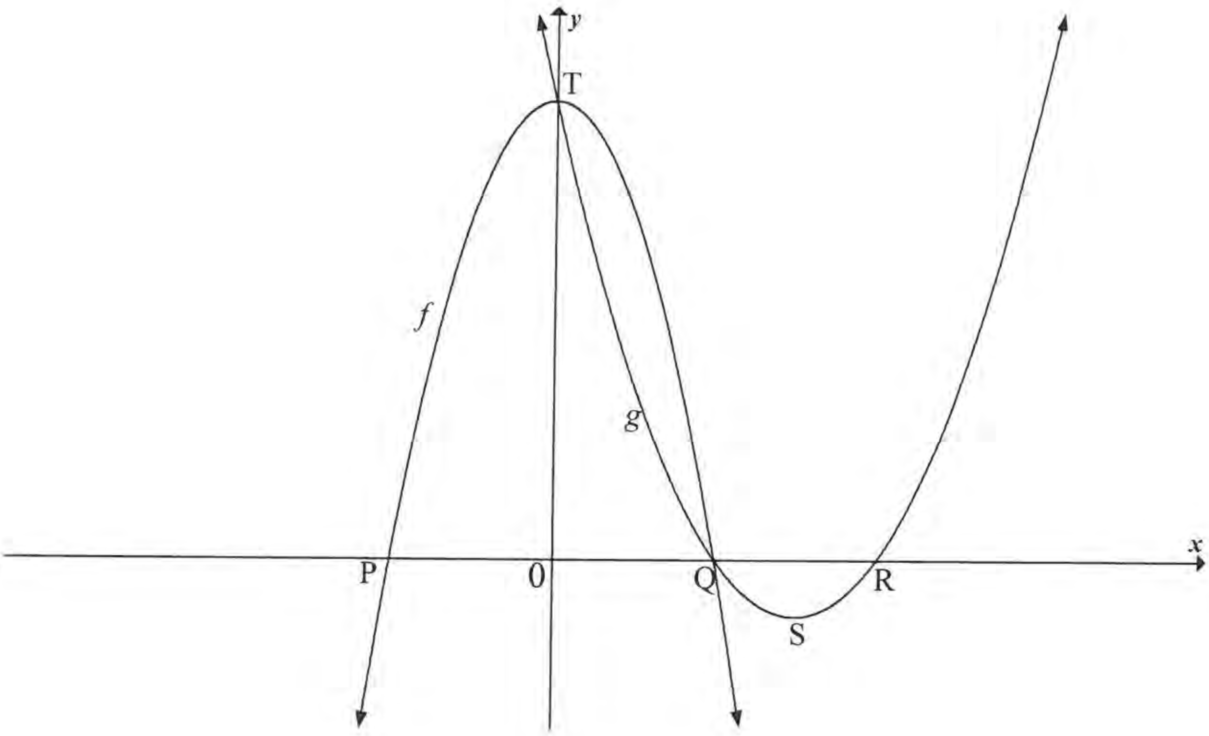
5.6.2 Write down the value of the maximum gradient of the tangent to the graph of f. (1)

[19]

QUESTION 6

6.1 The graphs of $f(x) = -2x^2 + 18$ and $g(x) = ax^2 + bx + c$ are sketched below.

Points P and Q are the x-intercepts of f. Points Q and R are the x-intercepts of g. S is the turning point of g. T is the y-intercept of both f and g.



6.1.1 Write down the coordinates of T. (1)

6.1.2 Determine the coordinates of Q. (3)

6.1.3 Given that $x = 4,5$ at S, determine the coordinates of R. (2)

6.1.4 Determine the value(s) of x for which $g''(x) > 0$. (2)

6.2 The function defined as $y = \dfrac{a}{x+p} + q$ has the following properties:

- The domain is $x \in R$, $x \neq -2$.
- $y = x + 6$ is an axis of symmetry.
- The function is increasing for all $x \in R$, $x \neq -2$.

Draw a neat sketch graph of this function. Your sketch must include the asymptotes, if any. (4)

[12]

QUESTION 4

Given: $g(x)=\dfrac{6}{x+2}-1$

4.1 Write down the equations of the asymptotes of g. (2)

4.2 Calculate:

4.2.1 The y-intercept of g (1)

4.2.2 The x-intercept of g (2)

4.3 Draw the graph of g, showing clearly the asymptotes and the intercepts with the axes. (3)

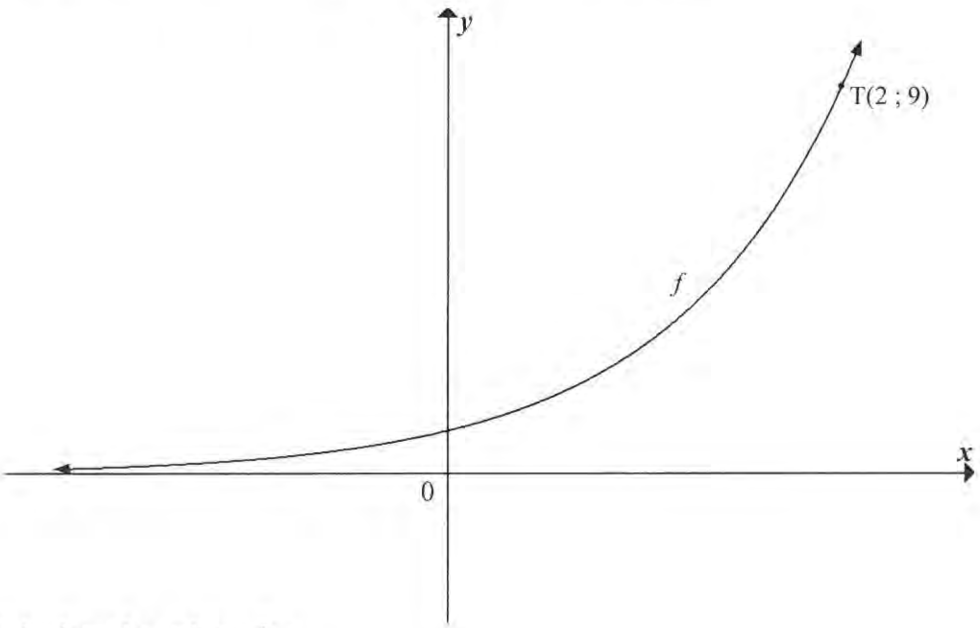
4.4 Determine the equation of the line of symmetry that has a negative gradient, in the form $y = ...$ (3)

4.5 Determine the value(s) of x for which $\dfrac{6}{x+2}-1 \geq -x-3$. (2)

[13]

QUESTION 5

The graph of $f(x)=a^x, a>1$ is shown below. T(2 ; 9) lies on f.



5.1 Calculate the value of a. (2)

5.2 Determine the equation of $g(x)$ if $g(x)=f(-x)$. (1)

5.3 Determine the value(s) of x for which $f^{-1}(x)\geq 2$. (2)

5.4 Is the inverse of f a function? Explain your answer. (2)

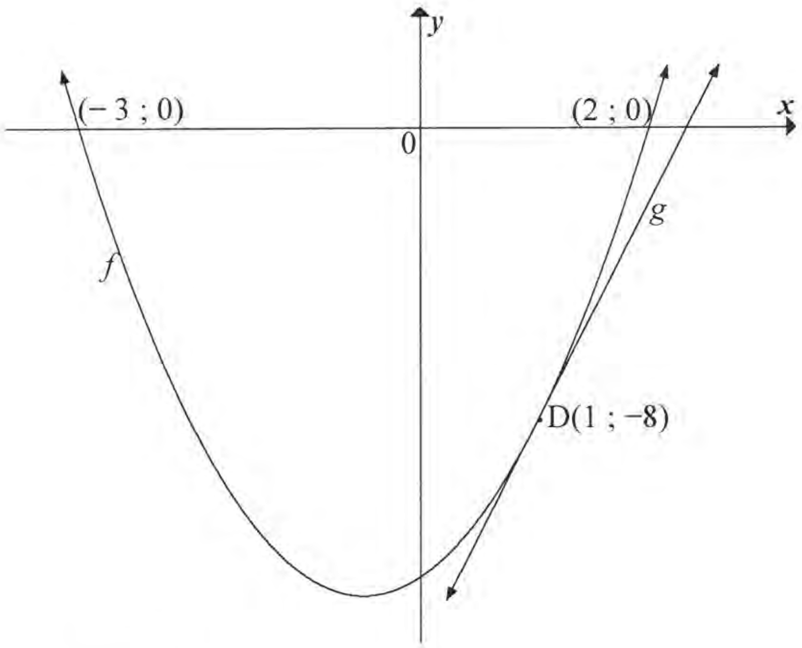
[7]

QUESTION 6

The graphs of $f(x) = ax^2 + bx + c$; $a \neq 0$ and $g(x) = mx + k$ are drawn below.

D(1 ; – 8) is a common point on f and g.

- f intersects the x-axis at (– 3 ; 0) and (2 ; 0).
- g is the tangent to f at D.

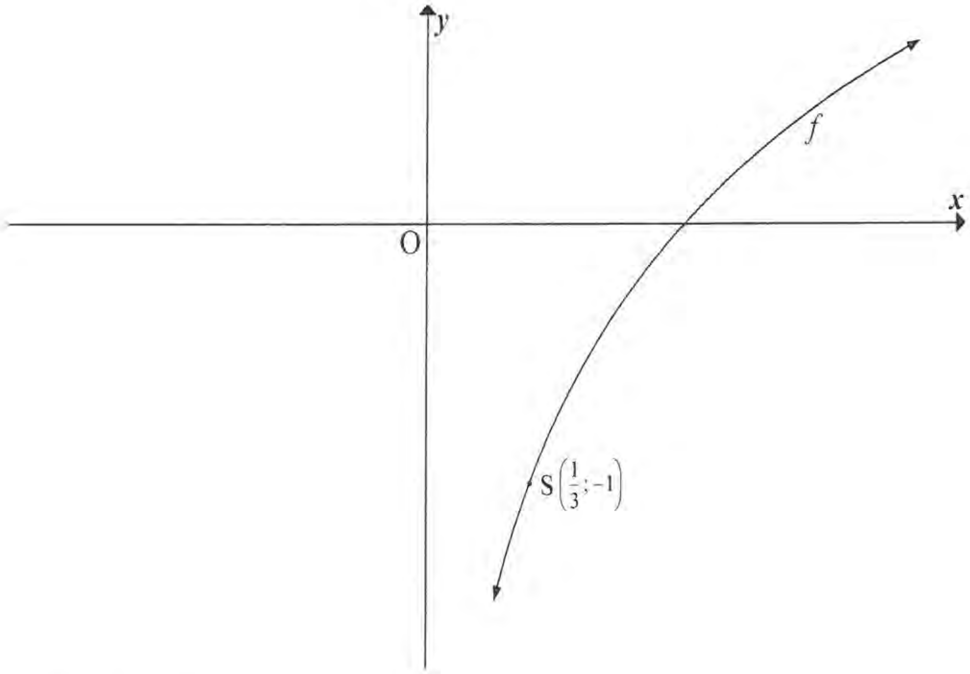


6.1	For which value(s) of x is $f(x) \leq 0$?	(2)
6.2	Determine the values of a, b and c.	(5)
6.3	Determine the coordinates of the turning point of f.	(3)
6.4	Write down the equation of the axis of symmetry of h if $h(x) = f(x-7) + 2$.	(2)
6.5	Calculate the gradient of g.	(3)

[15]

QUESTION 5

Given: $f(x) = \log_a x$ where $a > 0$. $S\left(\frac{1}{3}; -1\right)$ is a point on the graph of f.



5.1 Prove that $a = 3$. (2)

5.2 Write down the equation of h, the inverse of f, in the form $y = ...$ (2)

5.3 If $g(x) = -f(x)$, determine the equation of g. (1)

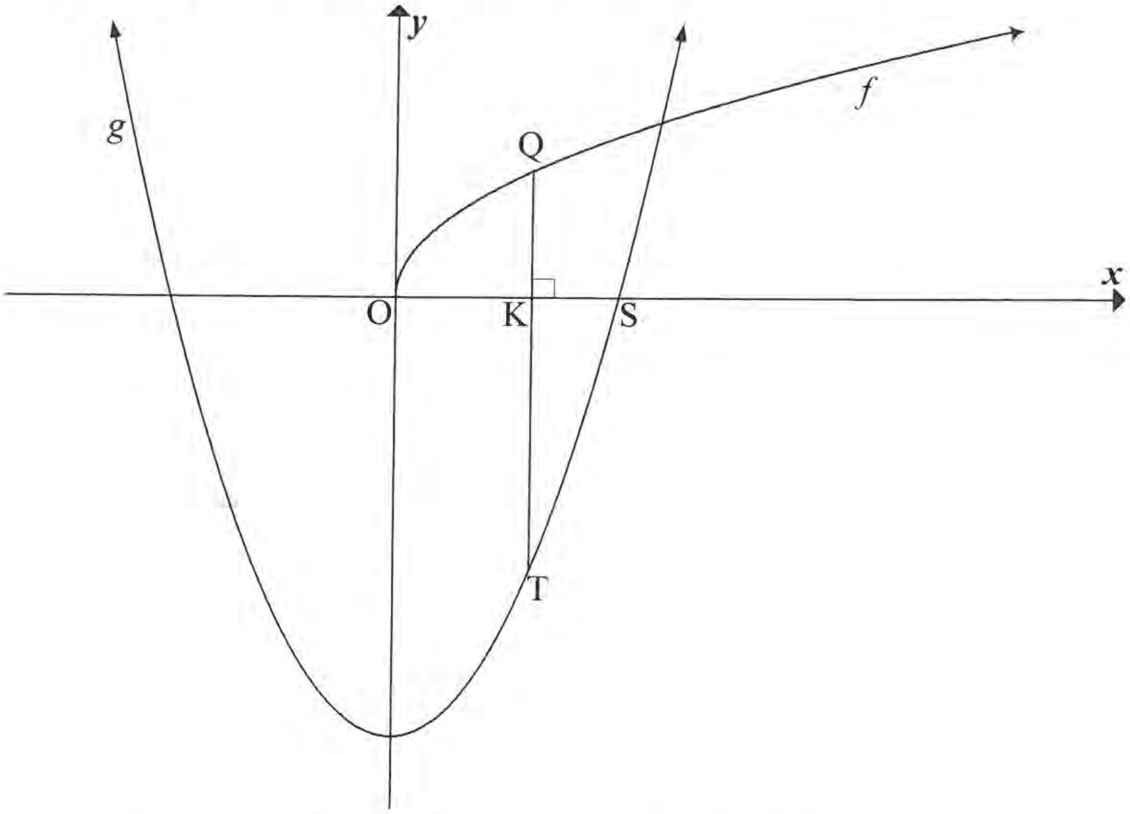
5.4 Write down the domain of g. (1)

5.5 Determine the values of x for which $f(x) \geq -3$. (3)

[9]

QUESTION 6

Given: $g(x) = 4x^2 - 6$ and $f(x) = 2\sqrt{x}$. The graphs of g and f are sketched below. S is an x-intercept of g and K is a point between O and S. The straight line QKT with Q on the graph of f and T on the graph of g, is parallel to the y-axis.



6.1 Determine the x-coordinate of S, correct to TWO decimal places. (2)

6.2 Write down the coordinates of the turning point of g. (2)

6.3 6.3.1 Write down the length of QKT in terms of x, where x is the x-coordinate of K. (3)

6.3.2 Calculate the maximum length of QT. (6)

[13]

QUESTION 3

3.1 Given the arithmetic sequence: $w-3$; $2w-4$; $23-w$

3.1.1 Determine the value of w. (2)

3.1.2 Write down the common difference of this sequence. (1)

3.2 The arithmetic sequence 4 ; 10 ; 16 ; ... is the sequence of first differences of a quadratic sequence with a first term equal to 3.

Determine the 50$^{\text{th}}$ term of the quadratic sequence. (5)
[8]

QUESTION 4

In a geometric series, the sum of the first n terms is given by $S_n = p\left(1-\left(\frac{1}{2}\right)^n\right)$ and the sum to infinity of this series is 10.

4.1 Calculate the value of p. (4)

4.2 Calculate the second term of the series. (4)
[8]

QUESTION 5

5.1 Draw the graphs of $x^2 + y^2 = 16$ and $x + y = 4$ on the same set of axes in your ANSWER BOOK. (4)

5.2 Write down the coordinates of the points of intersection of the two graphs. (2)
[6]

QUESTION 6

Consider: $f(x)=\dfrac{6}{x-2}+3$

6.1 Write down the equations of the asymptotes of the graph of f. (2)

6.2 Write down the domain of f. (1)

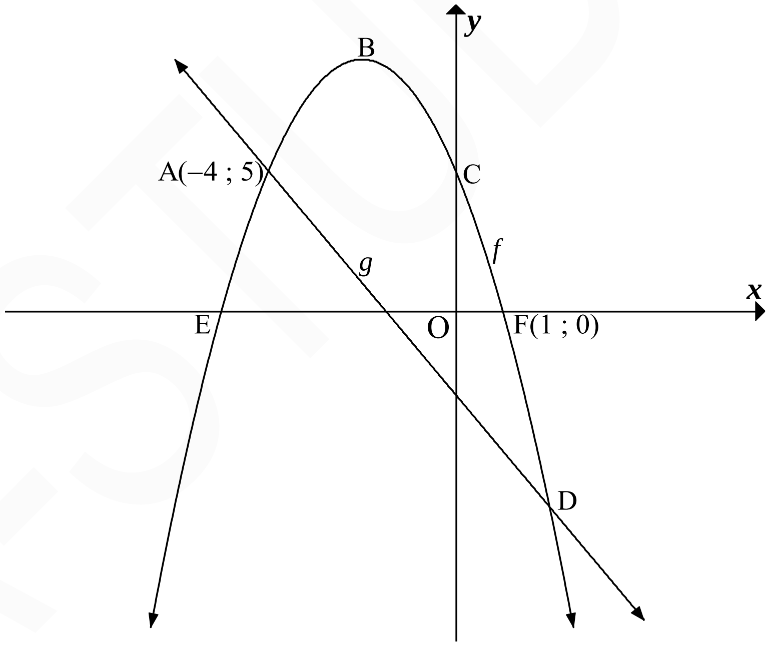
6.3 Draw a sketch graph of f in your ANSWER BOOK, indicating the intercept(s) with the axes and the asymptotes. (4)

6.4 The graph of f is translated to g. Describe the transformation in the form $(x\,;\,y)\rightarrow ...$ if the axes of symmetry of g are $y=x+3$ and $y=-x+1$. (4)

[11]

QUESTION 7

The graph of $f(x)=a(x-p)^2+q$ where a, p and q are constants, is given below.
Points E, F(1 ; 0) and C are its intercepts with the coordinates axes. A(– 4 ; 5) is the reflection of C across the axis of symmetry of f. D is a point on the graph such that the straight line through A and D has equation $g(x)=-2x-3$.



7.1 Write down the coordinates of C. (1)

7.2 Write down the equation of the axis of symmetry of f. (1)

7.3 Calculate the values of a, p and q. (6)

7.4 If $f(x)=-x^2-4x+5$, calculate the x-coordinate of D. (4)

7.5 The graph of f is reflected about the x-axis.

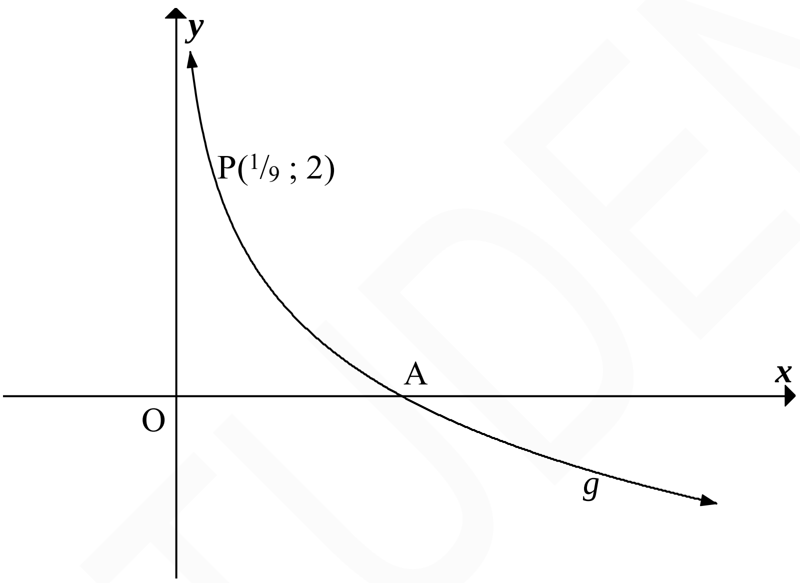
Write down the coordinates of the turning point of the new parabola. (2)

[14]

QUESTION 8

Given the graph of $g(x) = \log_{\frac{1}{3}} x$.

- A is the x-intercept of g.
- $P\left(\frac{1}{9}\,;\,2\right)$ is a point on g.



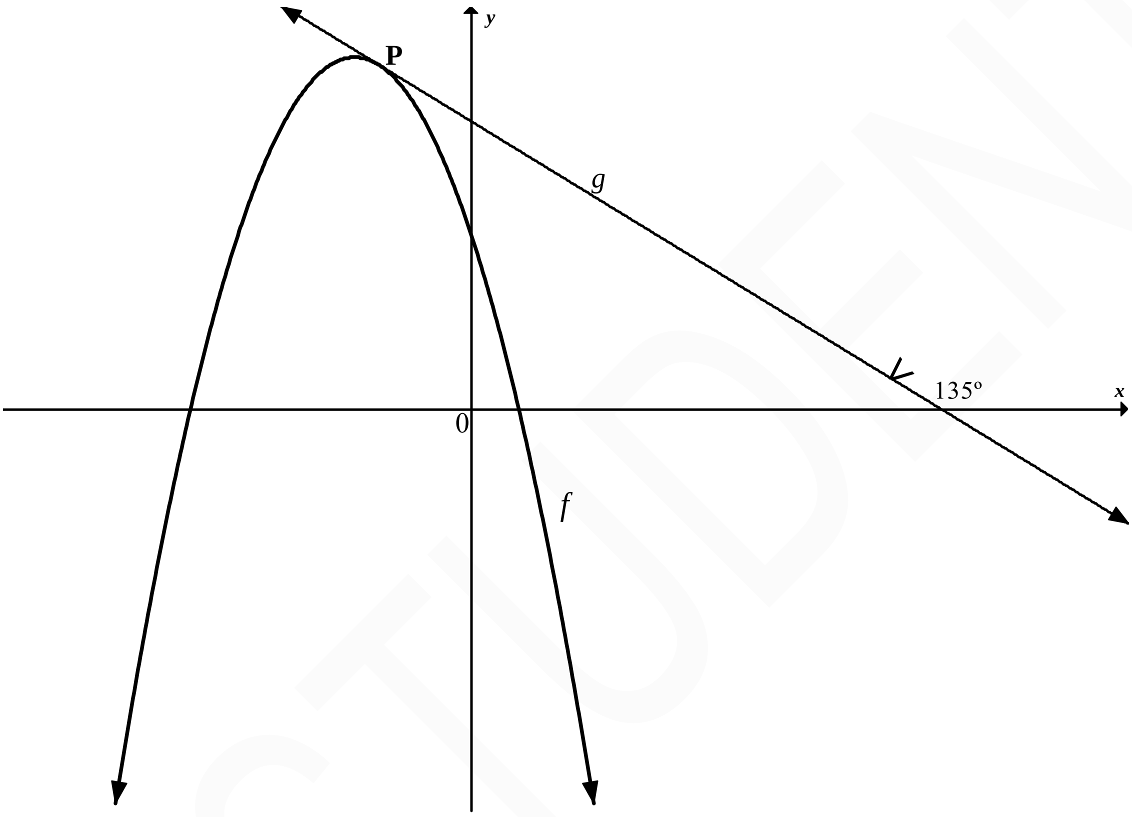
8.1 Write down the coordinates of A. (1)

8.2 Sketch the graph of g^{-1} indicating an intercept with the axes and ONE other point on the graph. (3)

8.3 Write down the domain of g^{-1}. (1)
[5]

QUESTION 5

The sketch below shows the graphs of $f(x) = -2x^2 - 5x + 3$ and $g(x) = ax + q$. The angle of inclination of graph g is $135°$ in the direction of the positive x-axis. P is the point of intersection of f and g such that g is a tangent to the graph of f at P.



5.1 Calculate the coordinates of the turning point of the graph of f. (3)

5.2 Calculate the coordinates of P, the point of contact between f and g. (4)

5.3 Hence or otherwise, determine the equation of g. (2)

5.4 Determine the values of d for which the line $k(x) = -x + d$ will not intersect the graph of f. (1)

[10]

QUESTION 4

Given: $f(x) = -2x^2 - 5x + 3$

4.1 Write down the coordinates of the y-intercept of f. (1)

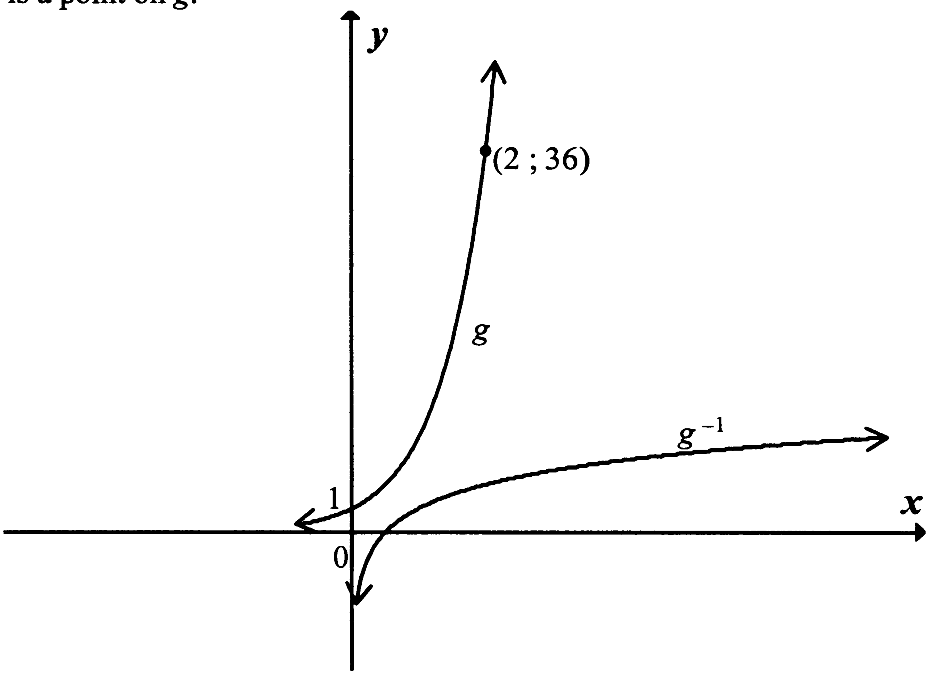
4.2 Determine the coordinates of the x-intercepts of f. (3)

4.3 Determine the coordinates of the turning point of f. (3)

4.4 Sketch the graph of $y = f(x)$, clearly showing the coordinates of the turning points and the three intercepts with the axes. (3)
[10]

QUESTION 5

5.1 Sketched below are the graphs of $g(x) = k^x$, where $k > 0$ and $y = g^{-1}(x)$.
(2 ; 36) is a point on g.



5.1.1 Determine the value of k. (2)

5.1.2 Give the equation of g^{-1} in the form $y = ...$ (2)

5.1.3 For which value(s) of x is $g^{-1}(x) \leq 0$? (2)

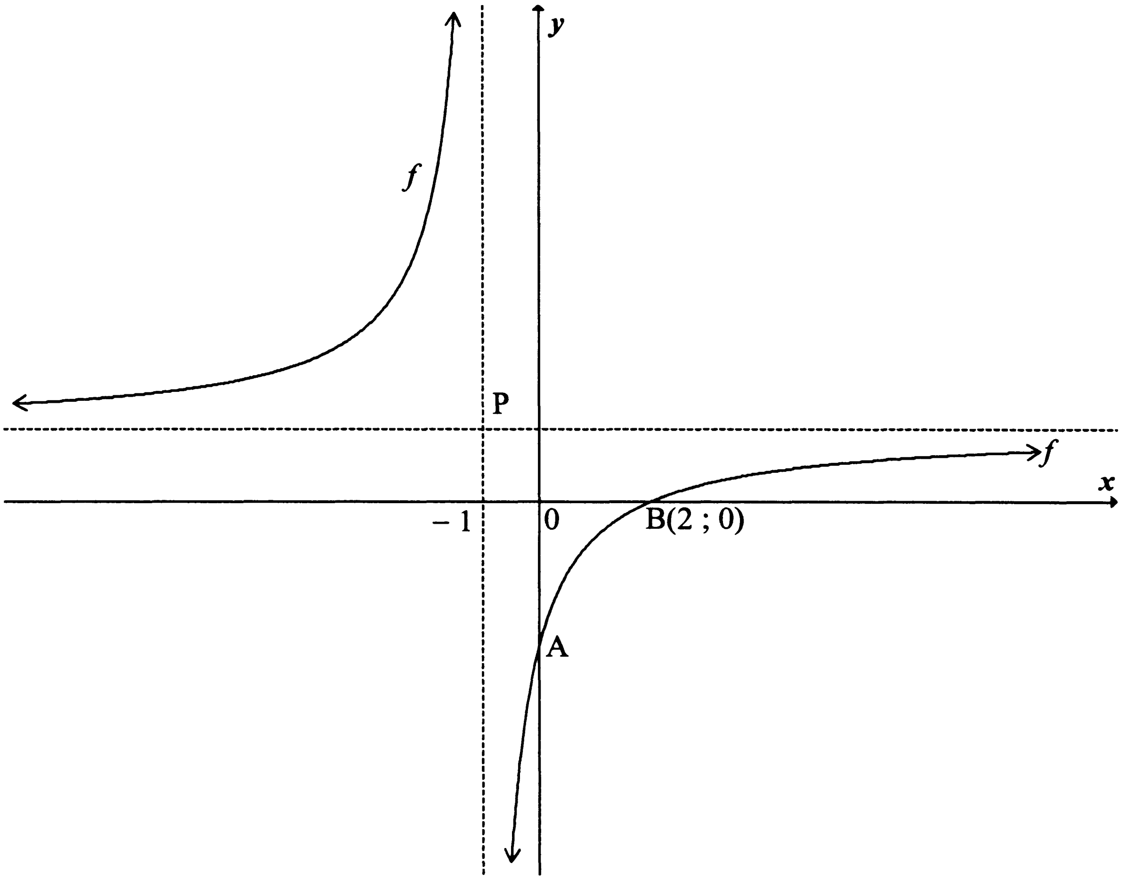
5.1.4 Write down the domain of h if $h(x) = g^{-1}(x-3)$. (1)

5.2 5.2.1 Sketch the graph of the inverse of $y = 1$. (2)

5.2.2 Is the inverse of $y = 1$ a function? Motivate your answer. (2)
[11]

QUESTION 6

A sketch of the hyperbola $f(x)=\dfrac{x-d}{x-p}$, where d and p are constants, is given below. The dotted lines are the asymptotes. The asymptotes intersect at P and B(2 ; 0) is a point on f.



6.1.1 Determine the values of d and p. (2)

6.1.2 Show that the equation of f can be written as $y=\dfrac{-3}{x+1}+1$. (2)

6.1.3 Write down the coordinates of P. (2)

6.1.4 Write down the coordinates of the image of $\mathrm{B}(2\,;0)$ if B is reflected about the axis of symmetry $y=x+2$. (2)

6.2 The exponential function, $g(x)=p.2^{x}+q$ has a horizontal asymptote at $y=1$ and passes through $(0\,;-2)$. Determine the values of p and q. (3)

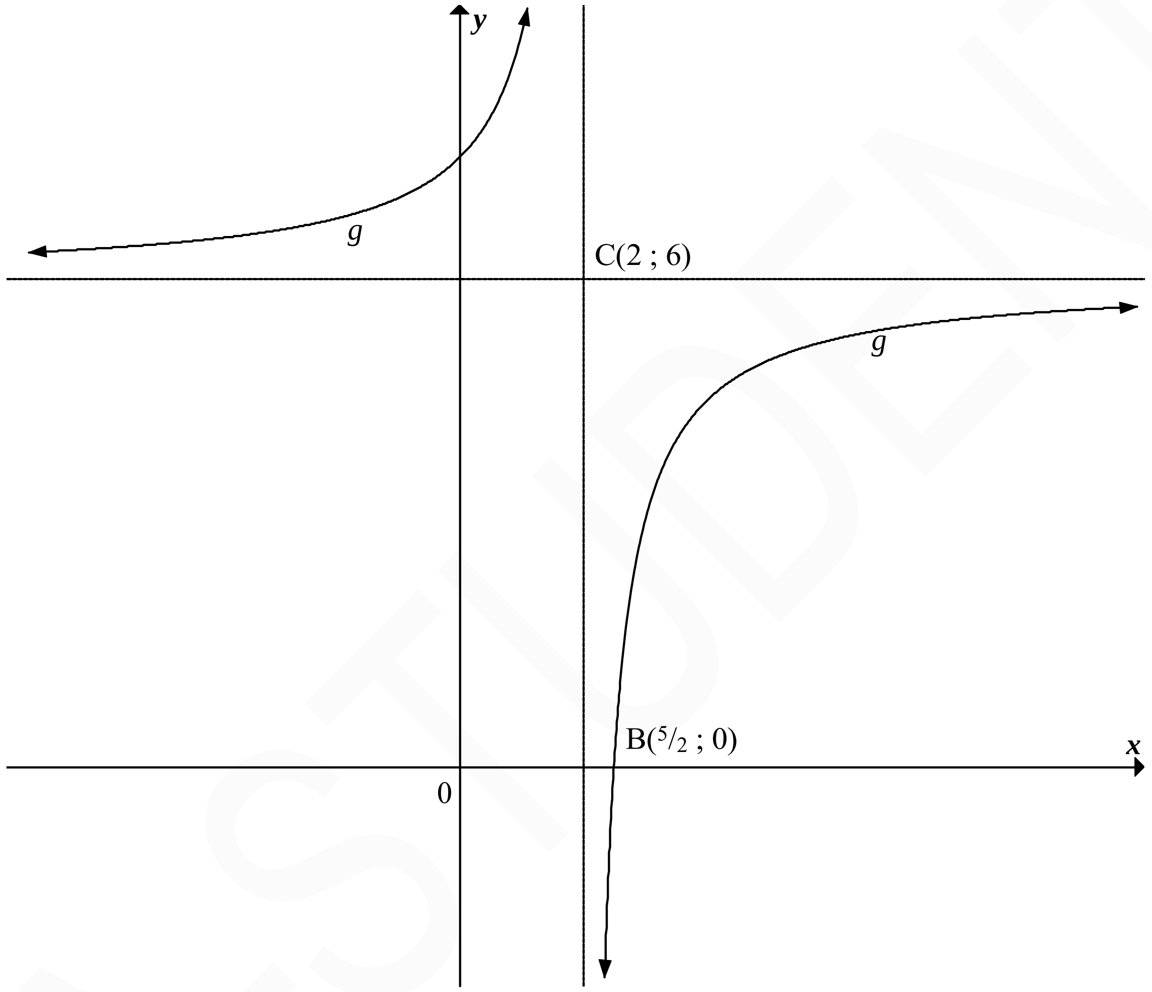
[11]

QUESTION 5

Sketched below is the graph of $g(x) = \dfrac{a}{x-p} + q$.

C(2 ; 6) is the point of intersection of the asymptotes of g.

B$\left(\dfrac{5}{2} ; 0\right)$ is the x-intercept of g.

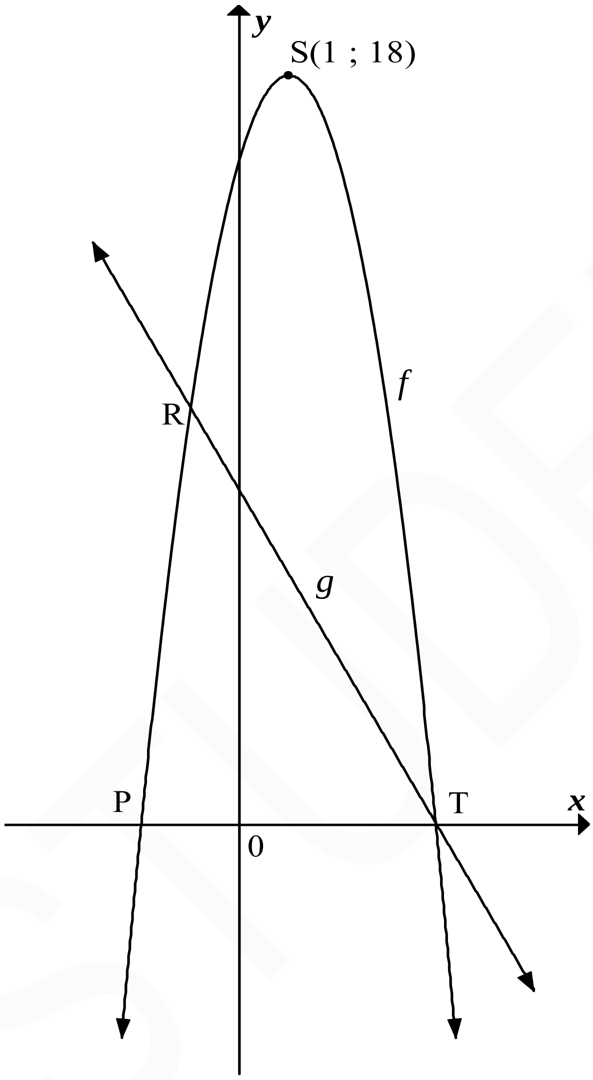


5.1 Determine the equation for g in the form $g(x) = \dfrac{a}{x-p} + q$ (4)

5.2 F is the reflection of B across C. Determine the coordinates of F. (2)
[6]

QUESTION 6

S(1 ; 18) is the turning point of the graph of $f(x) = ax^2 + bx + c$. P and T are x-intercepts of f. The graph of $g(x) = -2x + 8$ has an x-intercept at T. R is a point of intersection of f and g.



6.1 Calculate the coordinates of T. (2)

6.2 Determine the equation for f in the form $f(x) = ax^2 + bx + c$. Show ALL your working. (4)

6.3 If $f(x) = -2x^2 + 4x + 16$, calculate the coordinates of R. (4)

6.4 Use your graphs to solve for x where:

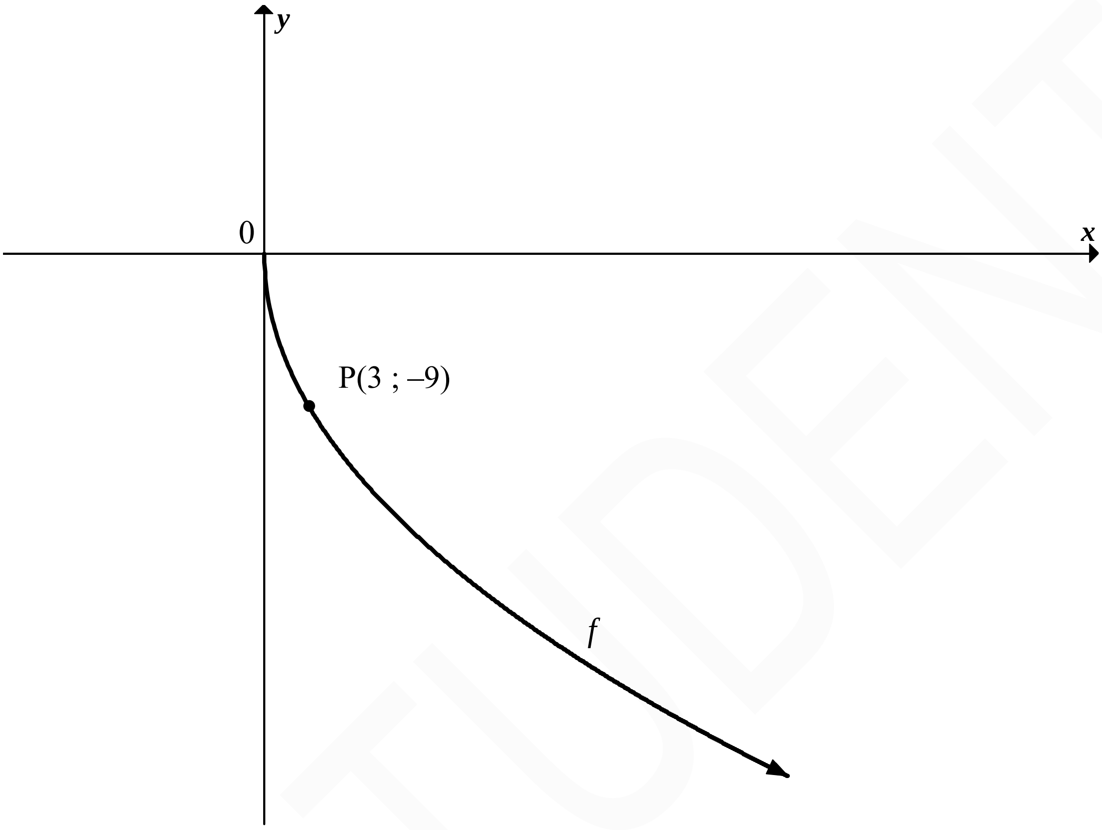
6.4.1 $f(x) \geq g(x)$ (2)

6.4.2 $-2x^2 + 4x - 2 < 0$ (4)

[16]

QUESTION 5

The graph of $f(x) = -\sqrt{27x}$ for $x \geq 0$ is sketched below.
The point P(3 ; –9) lies on the graph of f.



5.1 Use your graph to determine the values of x for which $f(x) \geq -9$. (2)

5.2 Write down the equation of f^{-1} in the form $y = ...$ Include ALL restrictions. (3)

5.3 Sketch f^{-1}, the inverse of f, in your ANSWER BOOK.
Indicate the intercept(s) with the axes and the coordinates of ONE other point. (3)

5.4 Describe the transformation from f to g if $g(x) = \sqrt{27x}$, where $x \geq 0$. (1)
[9]

QUESTION 6

The graph of a hyperbola with equation $y = f(x)$ has the following properties:

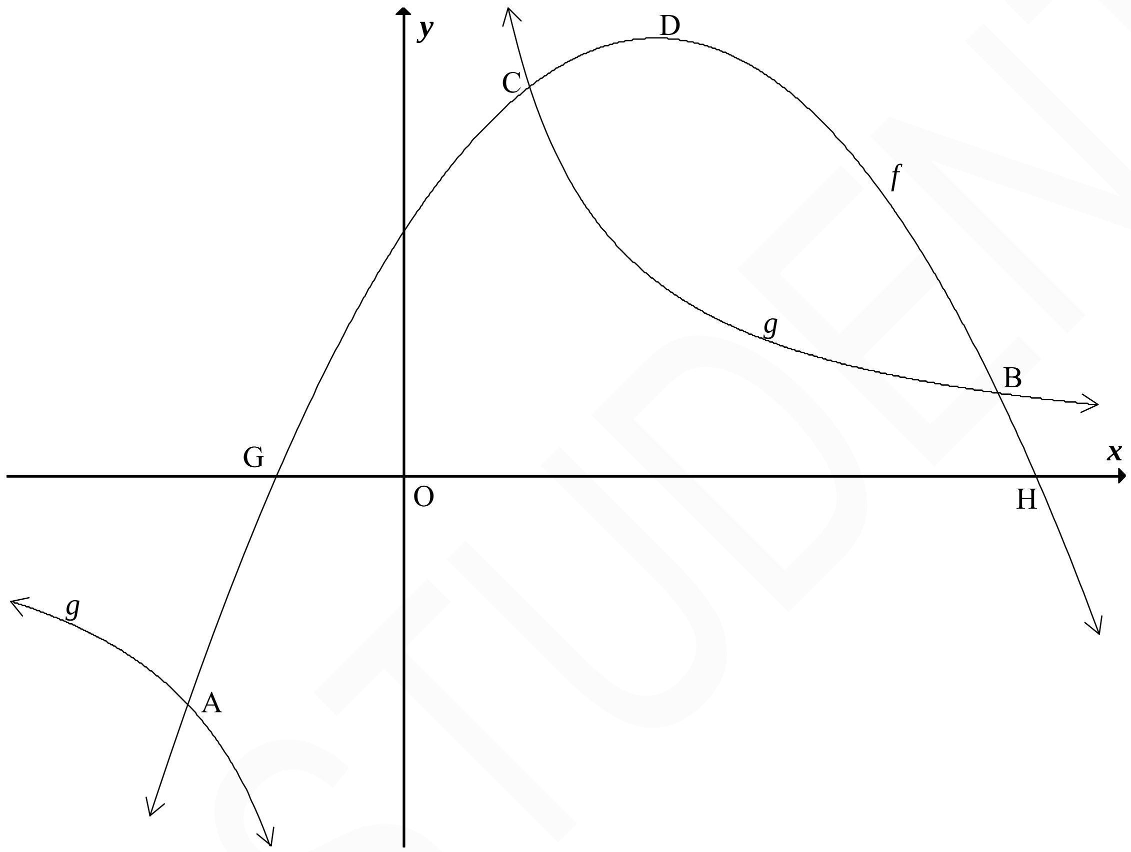
- Domain: $x \in \mathbf{R}$, $x \neq 5$
- Range: $y \in \mathbf{R}$, $y \neq 1$
- Passes through the point (2 ; 0)

Determine $f(x)$. **[4]**

QUESTION 5

The graphs of the functions $f(x) = -2x^2 + 8x + 10$ and $g(x) = \frac{16}{x}$ are sketched below.

G and H are the x-intercepts of f.
D is the turning point of f.
Points A, B and C are the points of intersection of f and g.



5.1 Write down the equations of the asymptotes of the graph of g. (2)

5.2 Determine the coordinates of H. (4)

5.3 Determine the range of f. (4)

5.4 Verify that C is the point (1 ; 16). (2)

5.5 Determine the coordinates of the turning point of p if $p(x) = f(3x)$. (3)
[15]

QUESTION 6

Given: $f(x) = 3^x$

6.1 Determine an equation for f^{-1} in the form $f^{-1}(x) = ...$ (1)

6.2 Sketch, in your ANSWER BOOK, the graphs of f and f^{-1}, showing clearly ALL intercepts with the axes. (4)

6.3 Write down the domain of f^{-1}. (2)

6.4 For which values of x will $f(x).f^{-1}(x) \leq 0$? (2)

6.5 Write down the range of $h(x) = 3^{-x} - 4$ (2)

6.6 Write down an equation for g if the graph of g is the image of the graph of f after f has been translated two units to the right and reflected about the x-axis. (2)

[13]

QUESTION 7

7.1 Lerato wants to purchase a house that costs R850 000. She is required to pay a 12% deposit and she will borrow the balance from a bank. Calculate the amount that Lerato must borrow from the bank. (2)

7.2 The bank charges interest at 9% per annum, compounded monthly on the loan amount. Lerato works out that the loan will carry an effective interest rate of 9,6% per annum. Is her calculation correct or not? Justify your answer with appropriate calculations. (4)

7.3 Lerato takes out a loan from the bank for the balance of the purchase price and agrees to pay it back over 20 years. Her repayments start one month after her loan is granted. Determine her monthly instalment if interest is charged at 9% per annum compounded monthly. (4)

7.4 Lerato can afford to repay R7 000 per month. How long will it take her to repay the loan amount if she chooses to pay R7 000 as a repayment every month? (4)

[14]

QUESTION 4

A quadratic pattern has a second term equal to 1, a third term equal to – 6 and a fifth term equal to – 14.

4.1 Calculate the second difference of this quadratic pattern. (5)

4.2 Hence, or otherwise, calculate the first term of the pattern. (2)
[7]

QUESTION 5

5.1 Consider the function: $f(x)=\dfrac{-6}{x-3}-1$

5.1.1 Calculate the coordinates of the y-intercept of f. (2)

5.1.2 Calculate the coordinates of the x-intercept of f. (3)

5.1.3 Sketch the graph of f in your ANSWER BOOK, showing clearly the asymptotes and the intercepts with the axes. (4)

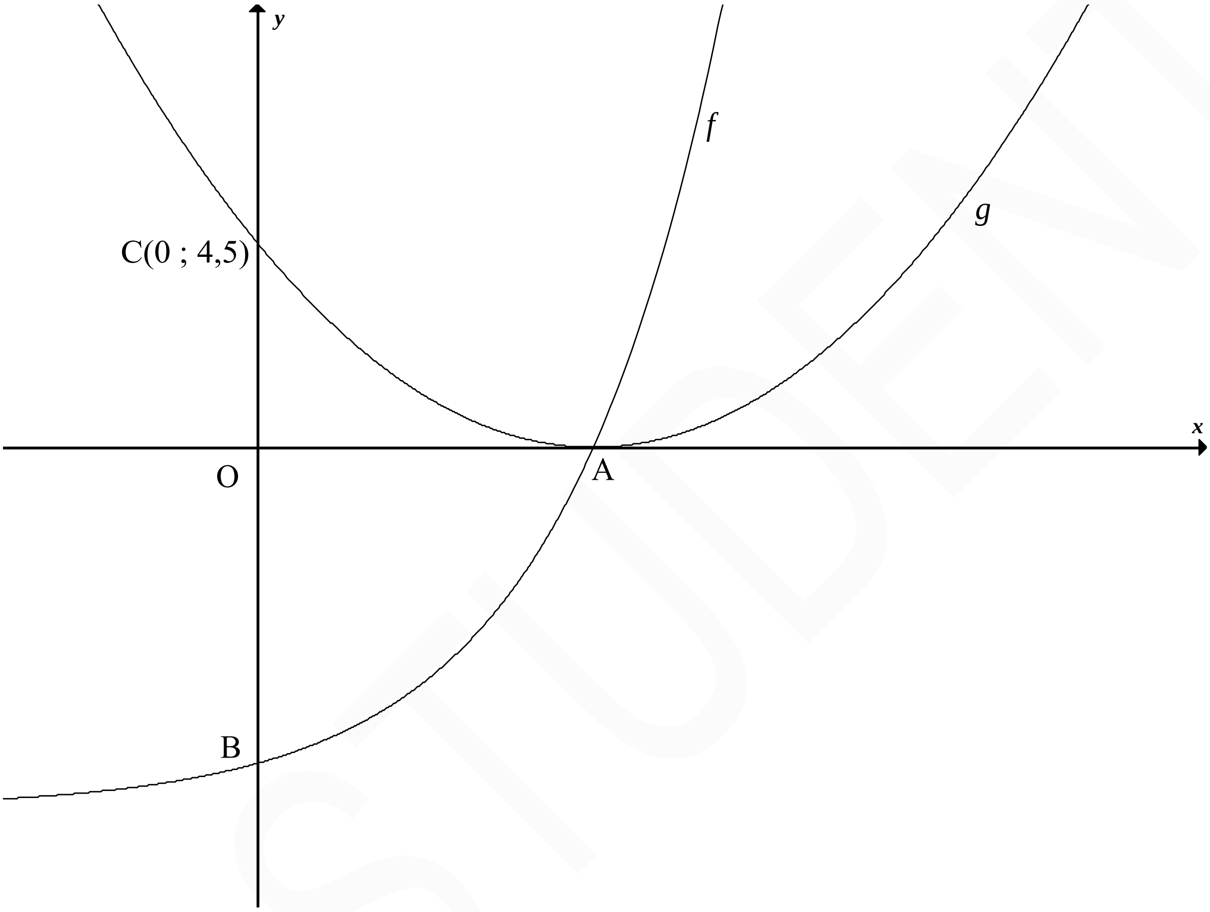
5.1.4 For which values of x is $f(x)>0$? (2)

5.1.5 Calculate the average gradient of f between $x=-2$ and $x=0$. (4)

5.2 Draw a sketch graph of $y=ax^2+bx+c$, where $a<0$, $b<0$, $c<0$ and $ax^2+bx+c=0$ has only ONE solution. (4)
[19]

QUESTION 6

The graphs of $f(x) = 2^x - 8$ and $g(x) = ax^2 + bx + c$ are sketched below.
B and C(0 ; 4,5) are the y-intercepts of the graphs of f and g respectively.
The two graphs intersect at A, which is the turning point of the graph of g and the x-intercept of the graphs of f and g.



6.1 Determine the coordinates of A and B. (4)

6.2 Write down an equation of the asymptote of the graph of f. (1)

6.3 Determine an equation of h if $h(x) = f(2x) + 8$. (2)

6.4 Determine an equation of h^{-1} in the form $y = ...$ (2)

6.5 Write down an equation of p, if p is the reflection of h^{-1} about the x-axis. (1)

6.6 Calculate $\sum_{k=0}^{3} g(k) - \sum_{k=4}^{5} g(k)$. Show ALL your working. (4)
[14]

QUESTION 4

4.1 The sum to n terms of a sequence of numbers is given as: $S_n = \frac{n}{2}(5n+9)$

4.1.1 Calculate the sum to 23 terms of the sequence. (2)

4.1.2 Hence calculate the 23rd term of the sequence. (3)

4.2 The first two terms of a geometric sequence and an arithmetic sequence are the same. The first term is 12. The sum of the first three terms of the geometric sequence is 3 more than the sum of the first three terms of the arithmetic sequence.

Determine TWO possible values for the common ratio, r, of the geometric sequence. (6)
[11]

QUESTION 5

Consider the function $f(x) = \frac{3}{x-1} - 2$.

5.1 Write down the equations of the asymptotes of f. (2)

5.2 Calculate the intercepts of the graph of f with the axes. (3)

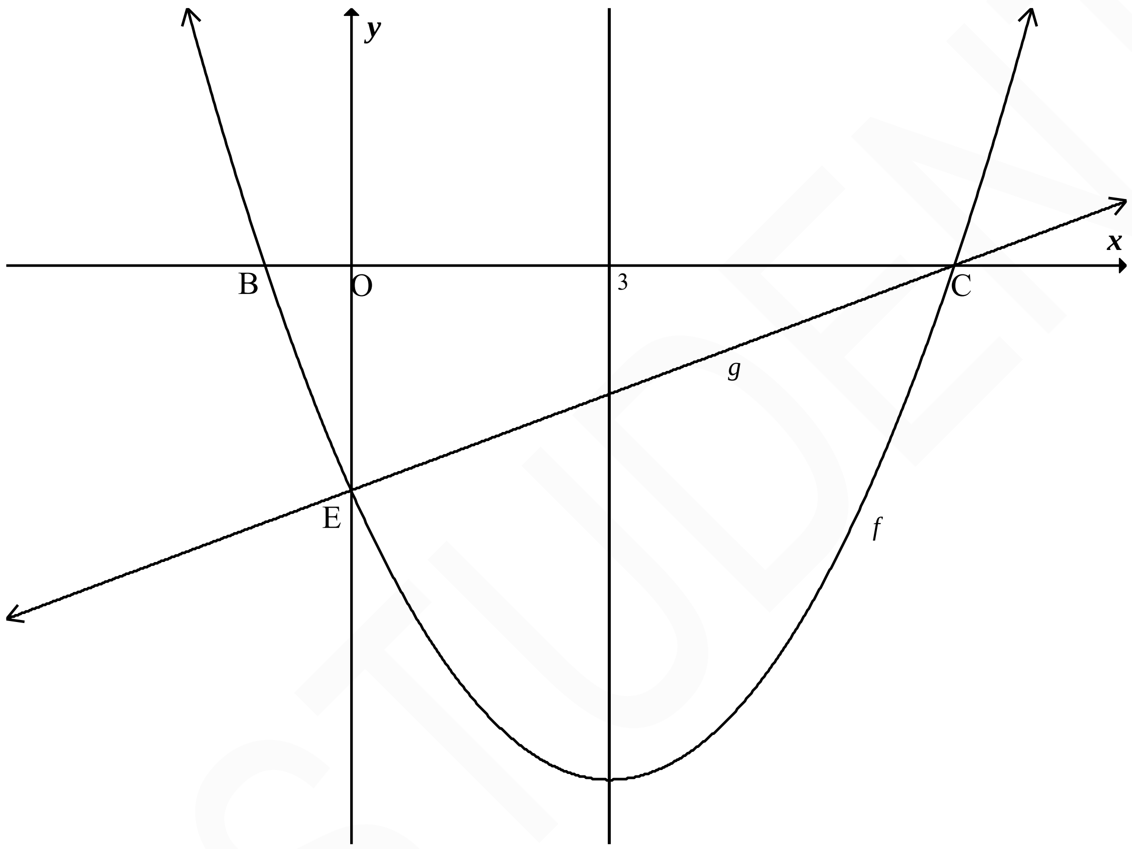
5.3 Sketch the graph of f on DIAGRAM SHEET 1. (3)

5.4 Write down the range of $y = -f(x)$. (1)

5.5 Describe, in words, the transformation of f to g if $g(x) = \frac{-3}{x+1} - 2$. (2)
[11]

QUESTION 6

A parabola f intersects the x-axis at B and C and the y-axis at E. The axis of symmetry of the parabola has equation $x = 3$. The line through E and C has equation $g(x) = \frac{x}{2} - \frac{7}{2}$.



6.1	Show that the coordinates of C are (7 ; 0).	(1)
6.2	Calculate the x-coordinate of B.	(1)
6.3	Determine the equation of f in the form $y = a(x-p)^2 + q$.	(6)
6.4	Write down the equation of the graph of h, the reflection of f in the x-axis.	(1)
6.5	Write down the maximum value of $t(x)$ if $t(x) = 1 - f(x)$.	(2)
6.6	Solve for x if $f(x^2 - 2) = 0$.	(4)

[15]

QUESTION 7

Consider the function $f(x) = \left(\frac{1}{3}\right)^x$.

7.1 Is f an increasing or decreasing function? Give a reason for your answer. (2)

7.2 Determine $f^{-1}(x)$ in the form $y = \ldots$ (2)

7.3 Write down the equation of the asymptote of $f(x) - 5$. (1)

7.4 Describe the transformation from f to g if $g(x) = \log_3 x$. (2)
[7]

QUESTION 8

8.1 R1 430,77 was invested in a fund paying i% p.a. compounded monthly. After 18 months the fund had a value of R1 711,41. Calculate i. (4)

8.2 A father decided to buy a house for his family for R800 000. He agreed to pay monthly instalments of R10 000 on a loan which incurred interest at a rate of 14% p.a. compounded monthly. The first payment was made at the end of the first month.

8.2.1 Show that the loan would be paid off in 234 months. (4)

8.2.2 Suppose the father encountered unexpected expenses and was unable to pay any instalments at the end of the 120th, 121st, 122nd and 123rd months. At the end of the 124th month he increased his payment so as to still pay off the loan in 234 months by 111 equal monthly payments.
Calculate the value of this new instalment. (7)
[15]

QUESTION 9

9.1 Use the definition to differentiate $f(x) = 1 - 3x^2$. (Use first principles.) (4)

9.2 Calculate $D_x\left[4 - \frac{4}{x^3} - \frac{1}{x^4}\right]$. (3)

9.3 Determine $\frac{dy}{dx}$ if $y = \left(1 + \sqrt{x}\right)^2$. (3)
[10]

QUESTION 3

Given: $\sum_{t=0}^{99}(3t-1)$

3.1 Write down the first THREE terms of the series. (1)

3.2 Calculate the sum of the series. (4)
[5]

QUESTION 4

The following sequence of numbers forms a quadratic sequence:

$-3\,;-2\,;-3\,;-6\,;-11\,;\ldots$

4.1 The first differences of the above sequence also form a sequence. Determine an expression for the general term of the first differences. (3)

4.2 Calculate the first difference between the 35^{th} and 36^{th} terms of the quadratic sequence. (2)

4.3 Determine an expression for the n^{th} term of the quadratic sequence. (4)

4.4 Explain why the sequence of numbers will never contain a positive term. (2)
[11]

QUESTION 5

Data regarding the growth of a certain tree has shown that the tree grows to a height of 150 cm after one year. The data further reveals that during the next year, the height increases by 18 cm. In each successive year, the height increases by $\frac{8}{9}$ of the previous year's increase in height. The table below is a summary of the growth of the tree up to the end of the fourth year.

	First year	**Second year**	**Third year**	**Fourth year**
Tree height (cm)	150	168	184	$198\frac{2}{9}$
Growth (cm)		18	16	$14\frac{2}{9}$

5.1 Determine the increase in the height of the tree during the seventeenth year. (2)

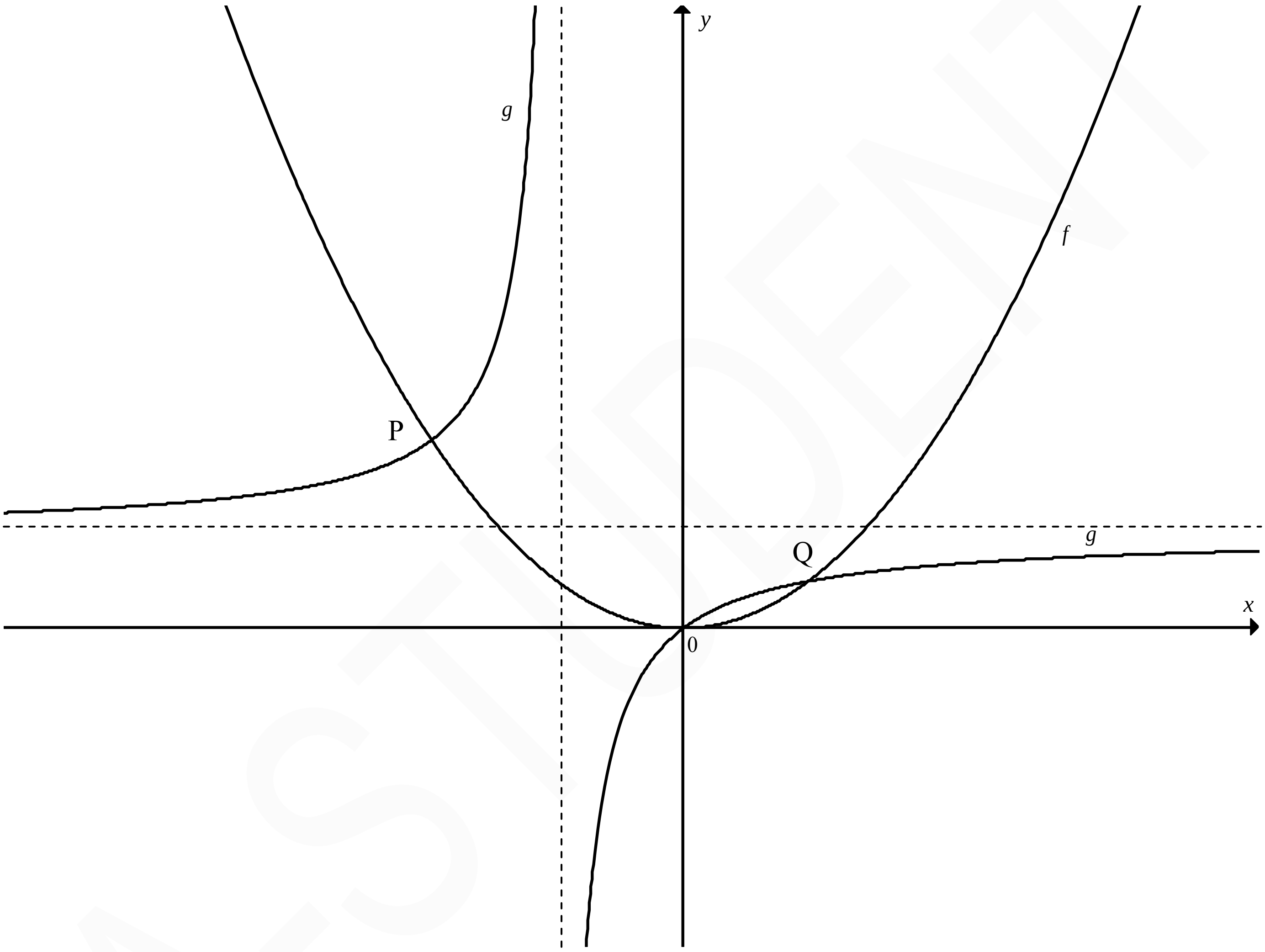
5.2 Calculate the height of the tree after 10 years. (3)

5.3 Show that the tree will never reach a height of more than 312 cm. (3)
[8]

QUESTION 6

Sketched below are the graphs of $f(x)=\frac{1}{2}x^2$ and $g(x)=-\frac{1}{x+1}+1$.

P and Q are the points of intersection of f and g.



6.1 Show that the coordinates of P and Q are P(– 2 ; 2) and Q(1 ; $\frac{1}{2}$) respectively. (6)

6.2 An axis of symmetry of the graph of g is a straight line defined as $y = mx + c$, where $m > 0$. Write down the equation of this straight line in the form $y = h(x) = \ldots$ (2)

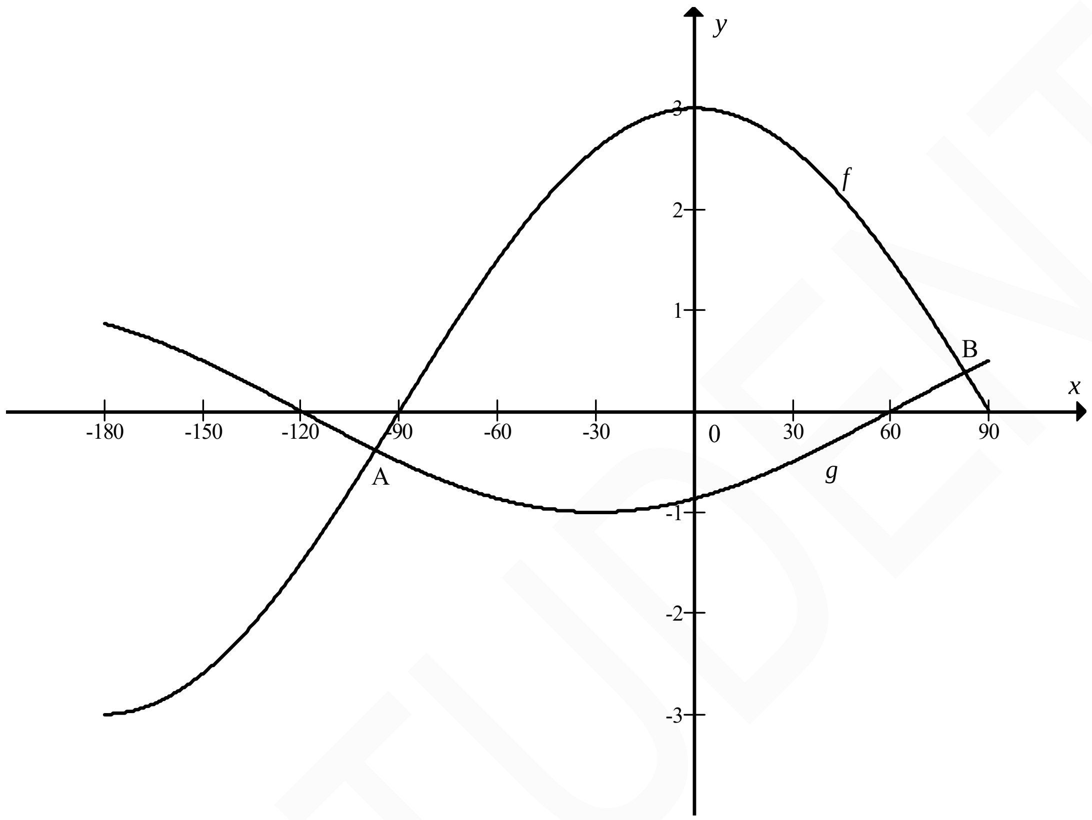
6.3 Determine the equation of h^{-1} in the form $y = \ldots$ (2)

6.4 Show algebraically that $g(x)+g\left(\frac{1}{x}\right)=g(-x).g(x-1)$. ($x \neq 0$ or $x \neq 1$) (3)

[13]

QUESTION 7

The graphs of $f(x) = 3\cos x$ and $g(x) = \sin(x - 60°)$ are sketched below for $x \in [-180° ; 90°]$.



7.1 Write down the range of f. (1)

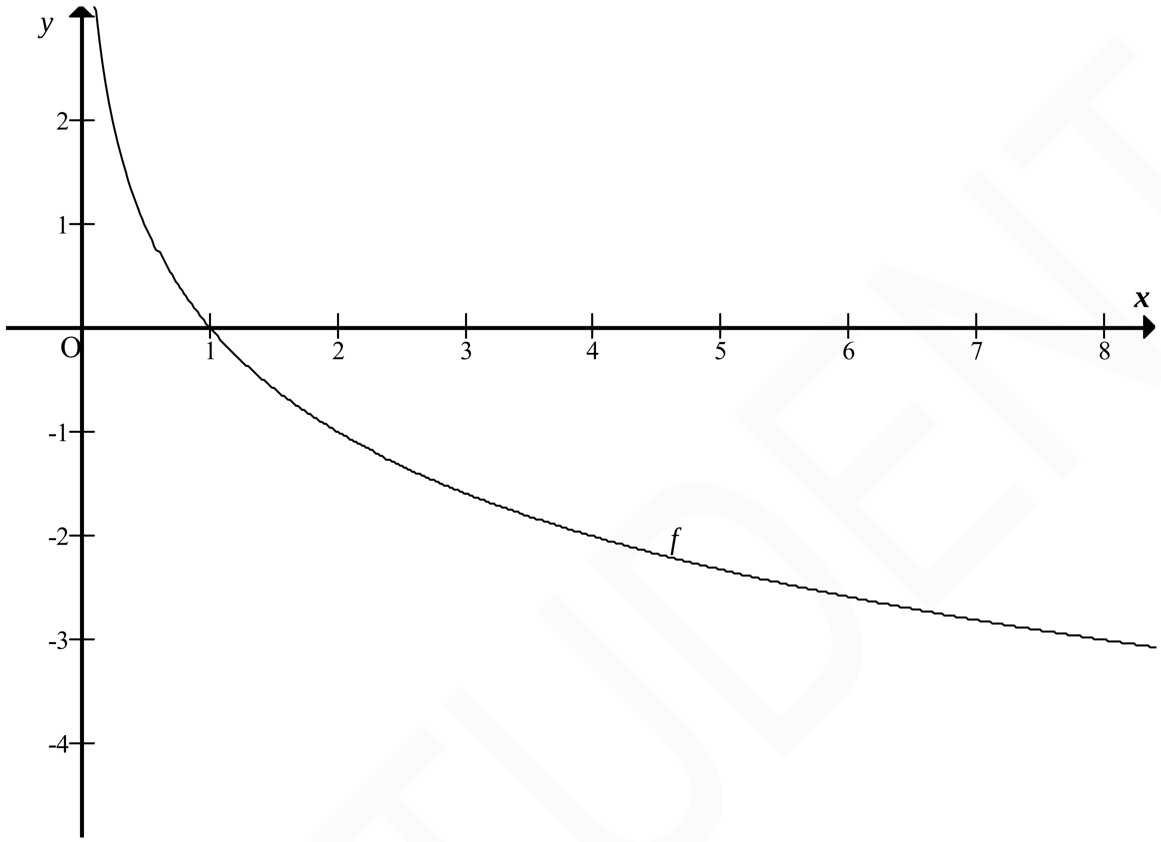
7.2 If A(– 97,37° ; – 0,38), write down the coordinates of B. (3)

7.3 Write down the period of $g(3x)$. (2)

7.4 Write down a value of x for which $g(x) - f(x)$ is a maximum. (2)
[8]

QUESTION 8

Sketched below is the graph of $f(x) = -\log_2 x$.



8.1	Write down the domain of f.	(1)
8.2	Write down the equation of f^{-1} in the form $y = \ldots$	(1)
8.3	Write down the equation of the asymptote of f^{-1}.	(1)
8.4	Explain how, using the graph of f, you would sketch the graphs of:	
	8.4.1 $g(x) = \log_2 x$	(1)
	8.4.2 $h(x) = 2^{-x} - 5$	(3)
8.5	Use the graph of f to solve for x where $\log_2 x < 3$.	(3)
		[10]

QUESTION 5

Given: $h(x) = 4^x$ and $f(x) = 2(x-1)^2 - 8$.

5.1 Sketch the graphs of h and f on the diagram sheet provided. Indicate ALL intercepts with the axes and any turning points. (8)

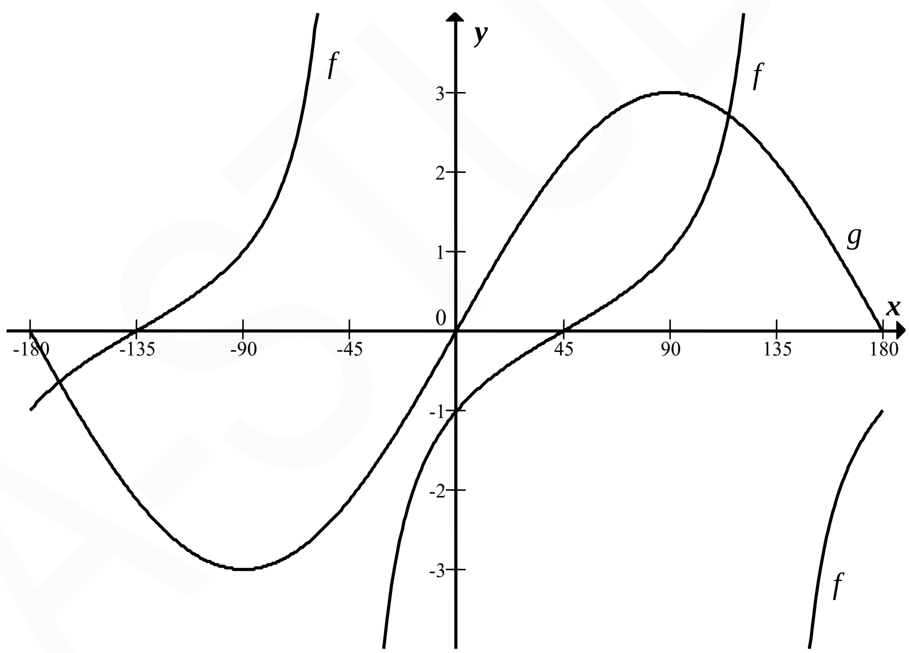
5.2 Without any further calculations, sketch the graph of $y = \log_4 x = g(x)$ on the same system of axes. (2)

5.3 The graph of f is shifted 2 units to the LEFT. Write down the equation of the new graph. (2)

5.4 Show, algebraically, that $h\left(x + \frac{1}{2}\right) = 2h(x)$. (3)
[15]

QUESTION 6

Sketched below are the graphs of the functions $f(x) = \tan(x - 45°)$ and $g(x) = 3\sin x$ for $x \in [-180° ; 180°]$.



6.1 Write down the equations of the asymptotes of $y = f(x)$ for $x \in [-90° ; 180°]$. (2)

6.2 Describe the transformation of the graph of f to h if $h(x) = \tan(45° - x)$. (2)

6.3 The period of g is reduced to 180° and the amplitude and y-intercept remain the same. Write down the equation of the resulting function. (2)
[6]