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YOU EITHER SUFFER THE PAIN **OF DISCIPLINE** OR THE PAIN OF REGRET. — JIM RohN —

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QUESTION 6

6.1   A company bought a photocopier for R150 000 on 1 July 2022. They will use the old photocopier as a trade-in when they replace it with a similar new photocopier in 5 years' time on 30 June 2027.

      6.1.1   The average rate of inflation over the next 5 years will be 6,5% p.a. Determine the price of a similar new photocopier in 5 years' time.   (2)

      6.1.2   Calculate the trade-in value of the old photocopier after 5 years, if it depreciates at a rate of 9% p.a. on a straight-line method.   (2)

      6.1.3   The company set up a sinking fund to cover the replacement cost of the new photocopier. The fund earns interest at the rate of 7,85% p.a., compounded monthly. The company made its first monthly deposit on 31 July 2022 and will continue to do so until 31 May 2027, one month prior to the new photocopier being bought. How much should be deposited at the end of each month so that the company will be able to buy the new photocopier?   (4)

6.2   Today, Andrew borrowed R200 000 from a bank. The bank charges interest at 5,25% p.a., compounded quarterly. Andrew will make repayments of R6 000 at the end of every 3 months. His first repayment will be made in 3 months from now. How long, in years, will it take Andrew to settle the loan?   (5)

      [13]

QUESTION 7

7.1   Determine $f'(x)$ from first principles if $f(x) = -2x^2 - 1$.   (5)

7.2   Determine:

      7.2.1 $f'(x)$, if it is given that $f(x) = -2x^3 + 3x^2$ (2)

      7.2.2 $\dfrac{dy}{dx}$ if $y = 2x + \dfrac{1}{\sqrt{4x}}$ (4)

7.3   The graph $y = f'(x)$ has a minimum turning point at $(1\,;-3)$. Determine the values of x for which f is concave down.   (2)

      [13]

QUESTION 6

6.1   R12 000 was invested in a fund that paid interest at $m\%$ p.a., compounded quarterly.
        After 24 months, the value of the investment was R13 459.

        Determine the value of m.   (4)

6.2   On 31 January 2022, Tino deposited R1 000 in an account that paid interest at 7,5%
        p.a., compounded monthly. He continued depositing R1 000 on the last day of every
        month. He will make the last deposit on 31 December 2022.

        Will Tino have sufficient funds in the account on 1 January 2023 to buy a computer
        that costs R13 000? Justify your answer by means of an appropriate calculation.   (4)

6.3   Thabo plans to buy a car that costs R250 000. He will pay a deposit of 15% and take
        out a loan for the balance. The interest on the loan is 13% p.a., compounded monthly.

        6.3.1   Calculate the value of the loan.   (1)

        6.3.2   The first repayment will be made 6 months after the loan has been
                  granted. The loan will be repaid over a period of 6 years after it has been
                  granted. Calculate the MONTHLY instalment.   (5)
                                                                                                    [14]

QUESTION 7

7.1   Determine $f'(x)$ from first principles if $f(x) = x^2 + x$.   (5)

7.2   Determine $f'(x)$ if $f(x) = 2x^5 - 3x^4 + 8x$.   (3)

7.3   The tangent to $g(x) = ax^3 + 3x^2 + bx + c$ has a minimum gradient at the
        point $(-1\,;-7)$. For which values of x will g be concave up?   (4)
                                                                                                    [12]

QUESTION 7

7.1   How many years will it take for an investment to double in value, if it earns interest at a rate of 8,5% p.a., compounded quarterly?   (4)

7.2   A company purchased machinery for R500 000. After 5 years, the machinery was sold for R180 000 and new machinery was bought.

7.2.1   Calculate the rate of depreciation of the old machinery over the 5 years, using the reducing-balance method.   (4)

7.2.2   The rate of inflation for the cost of the new machinery is 6,3% p.a. over the 5 years. What will the new machinery cost at the end of 5 years?   (2)

7.2.3   The company set up a sinking fund and made the first payment into this fund on the day the old machinery was bought. The last payment was made three months before the new machinery was purchased at the end of the 5 years. The interest earned on the sinking fund was 10,25% p.a., compounded monthly. The money from the sinking fund and the R180 000 from the sale of the old machinery was used to pay for the new machinery.

Calculate the monthly payment into the sinking fund.   (5)
    [15]

QUESTION 8

8.1   Determine $f'(x)$ from first principles if it is given that $f(x) = -x^2$.   (5)

8.2   Determine:

8.2.1 $f'(x)$, if it is given that $f(x) = 4x^3 - 5x^2$ (2)

8.2.2 $D_x \left[\dfrac{-6 \sqrt[3]{x} + 2}{x^4} \right]$ (4)
    [11]

QUESTION 8

8.1   A farmer bought a tractor for R980 000. The value of the tractor depreciates annually at a rate of 9,2% p.a. on the reducing-balance method. Calculate the book value of the tractor after 7 years.   (3)

8.2   How many years will it take for an amount of R75 000 to accrue to R116 253,50 in an account earning interest of 6,8% p.a., compounded quarterly?   (4)

8.3   Thabo wanted to save R450 000 as a deposit to buy a house on 30 June 2018.

        8.3.1   He deposited a fixed amount of money at the end of every month into an account earning interest of 8,35% p.a., compounded monthly. His first deposit was made on 31 July 2013 and his 60th deposit on 30 June 2018. Calculate the amount he deposited monthly.   (3)

        8.3.2   Thabo bought a house costing R1 500 000 and used his savings as the deposit. He obtained a home loan for the balance of the purchase price at an interest of 12% p.a., compounded monthly over 25 years. He made his first monthly instalment of R11 058,85 towards the loan on 31 July 2018.

                  (a)   What will the balance outstanding on the loan be on 30 June 2039, 21 years after the loan was granted?   (3)

                  (a)   Calculate the interest Thabo will have paid over the first 21 years of the loan.   (3)

                                                                                                        [16]

QUESTION 9

9.1   Determine $f'(x)$ from first principles if it is given that $f(x) = 2x^2 - 3x$.   (5)

9.2   Determine:

        9.2.1 $\dfrac{dy}{dx}$ if $y = 4x^5 - 6x^4 + 3x$ (3)

        9.2.2 $D_x\left[-\dfrac{\sqrt[3]{x}}{2} + \left(\dfrac{1}{3x}\right)^2 \right]$ (4)

                                                                                                        [12]

QUESTION 8

8.1   Determine $f'(x)$ from first principles if it is given that $f(x) = 3x^2$.   (5)
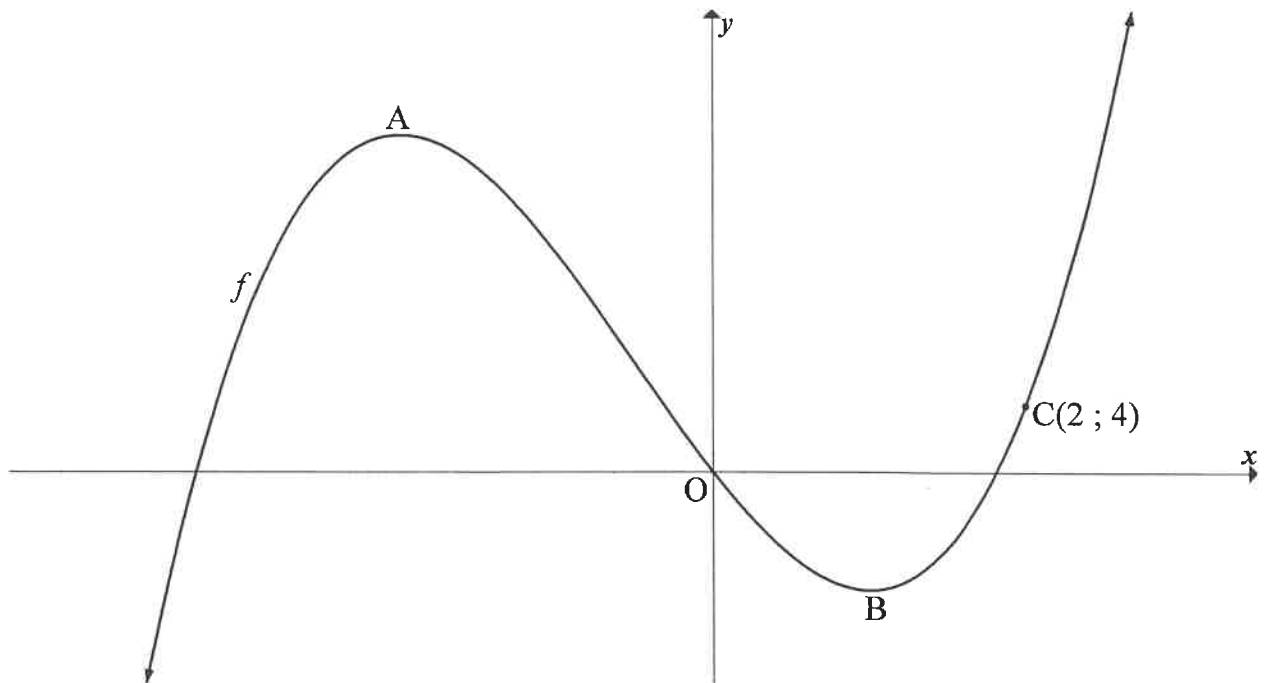
8.2   Determine:

      8.2.1 $f'(x)$ if $f(x) = x^2 - 3 + \dfrac{9}{x^2}$ (3)

      8.2.2 $g'(x)$ if $g(x) = \left(\sqrt{x} + 3\right)\left(\sqrt{x} - 1\right)$ (4)

                                                             [12]

QUESTION 9

The graph of $f(x) = 2x^3 + 3x^2 - 12x$ is sketched below.
A and B are the turning points of f. C(2 ; 4) is a point on f.

9.1   Determine the coordinates of A and B.   (5)

9.2   For which values of x will f be concave up?   (3)

9.3   Determine the equation of the tangent to f at C(2 ; 4).   (3)

                                                               [11]

QUESTION 6

6.1 On 31 January 2020, Tshepo made the first of his monthly deposits of R1 000 into a savings account. He continues to make monthly deposits of R1 000 at the end of each month up until 31 January 2032. The interest rate was fixed at 7,5% p.a., compounded monthly.

6.1.1 What will the investment be worth immediately after the last deposit? (4)

6.1.2 If he makes no further payments but leaves the money in the account, how much money will be in the account on 31 January 2033? (2)

6.2 Jim bought a new car for R250 000. The value of the car depreciated at a rate of 22% p.a. annually according to the reducing-balance method. After how many years will its book value be R92 537,64? (3)

6.3 Mpho is granted a loan under the following conditions:

- The interest rate is 11,3% p.a., compounded monthly.
- The period of the loan is 6 years.
- The monthly repayment on the loan is R1 500.
- Her first repayment is made one month after the loan is granted.

6.3.1 Calculate the value of the loan. (3)

6.3.2 How much interest will Mpho pay in total over the first 5 years? (4)

[16]

QUESTION 7

7.1 Determine $f'(x)$ from first principles if $f(x) = 2x^2 - 1$. (5)

7.2 Determine:

7.2.1 $\dfrac{d}{dx}\left(\sqrt[5]{x^2} + x^3\right)$ (3)

7.2.2 $f'(x)$ if $f(x) = \dfrac{4x^2 - 9}{4x + 6}$; $x \neq -\dfrac{3}{2}$ (4)

[12]

QUESTION 6

6.1　Two friends, Kuda and Thabo, each want to invest R5 000 for four years. Kuda invests his money in an account that pays simple interest at 8,3% per annum. At the end of four years, he will receive a bonus of exactly 4% of the accumulated amount. Thabo invests his money in an account that pays interest at 8,1% p.a., compounded monthly.

Whose investment will yield a better return at the end of four years? Justify your answer with appropriate calculations.　(5)

6.2　Nine years ago, a bank granted Mandy a home loan of R525 000. This loan was to be repaid over 20 years at an interest rate of 10% p.a., compounded monthly. Mandy's monthly repayments commenced exactly one month after the loan was granted.

6.2.1　Mandy decided to make monthly repayments of R6 000 instead of the required R5 066,36. How many payments will she make to settle the loan?　(5)

6.2.2　After making monthly repayments of R6 000 for nine years, Mandy required money to fund her daughter's university fees. She approached the bank for another loan. Instead, the bank advised Mandy that the extra amount repaid every month could be regarded as an investment and that she could withdraw this full amount to fund her daughter's studies. Calculate the maximum amount that Mandy may withdraw from the loan account.　(4)

[14]

QUESTION 7

7.1　Determine $f'(x)$ from first principles if it is given that $f(x) = 4 - 7x$.　(4)

7.2　Determine $\dfrac{dy}{dx}$ if $y = 4x^8 + \sqrt{x^3}$　(3)

7.3　Given: $y = ax^2 + a$

Determine:

7.3.1　$\dfrac{dy}{dx}$　(1)

7.3.2　$\dfrac{dy}{da}$　(2)

7.4　The curve with equation $y = x + \dfrac{12}{x}$ passes through the point A(2 ; b). Determine the equation of the line perpendicular to the tangent to the curve at A.　(4)

[14]

QUESTION 6

6.1   Sandile bought a car for R180 000. The value of the car depreciated at 15% per annum according to the reducing-balance method. The book value of Sandile's car is currently R79 866,96.

      6.1.1   How many years ago did Sandile buy the car?   (3)

      6.1.2   At exactly the same time that Sandile bought the car, Anil deposited R49 000 into a savings account at an interest rate of 10% p.a., compounded quarterly. Has Anil accumulated enough money in his savings account to buy Sandile's car now?   (3)

6.2   Exactly 10 months ago, a bank granted Jane a loan of R800 000 at an interest rate of 10,25% p.a., compounded monthly.
The bank stipulated that the loan:
- Must be repaid over 20 years
- Must be repaid by means of monthly repayments of R7 853,15, starting one month after the loan was granted

      6.2.1   How much did Jane owe immediately after making her 6$^{\text{th}}$ repayment?   (4)

      6.2.2   Due to financial difficulties, Jane missed the 7$^{\text{th}}$, 8$^{\text{th}}$ and 9$^{\text{th}}$ payments. She was able to make payments from the end of the 10$^{\text{th}}$ month onwards. Calculate Jane's increased monthly payment in order to settle the loan in the original 20 years.   (5)

[15]

QUESTION 7

7.1   Given $f(x) = x^2 + 2$.

    Determine $f^{/}(x)$ from first principles.   (4)

7.2   Determine $\dfrac{dy}{dx}$ if:

      7.2.1 $y = 4x^3 + \dfrac{2}{x}$ (3)

      7.2.2 $y = 4.\sqrt[3]{x} + (3x^3)^2$ (4)

7.3   If g is a linear function with $g(1) = 5$ and $g^{/}(3) = 2$, determine the equation of g in the form $y = \dots$.   (3)

[14]

QUESTION 7

7.1   Selby decided today that he will save R15 000 per quarter over the next four years. He will make the first deposit into a savings account in three months' time and he will make his last deposit at the end of four years from now.

   7.1.1   How much will Selby have at the end of four years if interest is earned at 8,8% per annum, compounded quarterly?   (3)

   7.1.2   If Selby decides to withdraw R100 000 from the account at the end of three years from now, how much will he have in the account at the end of four years from now?   (3)

7.2   Tshepo takes out a home loan over 20 years to buy a house that costs R1 500 000.

   7.2.1   Calculate the monthly instalment if interest is charged at 10,5% p.a., compounded monthly.   (4)

   7.2.2   Calculate the outstanding balance immediately after the 144th payment was made.   (5)
                                                                                           [15]

QUESTION 8

8.1   Determine $f^{\prime}(x)$ from first principles if it is given $f(x) = x^2 - 5$.   (5)

8.2   Determine $\dfrac{dy}{dx}$ if:

   8.2.1 $y = 3x^3 + 6x^2 + x - 4$ (3)

   8.2.2 $yx - y = 2x^2 - 2x \quad ; \quad x \neq 1$ (4)
                                                                                           [12]

QUESTION 8

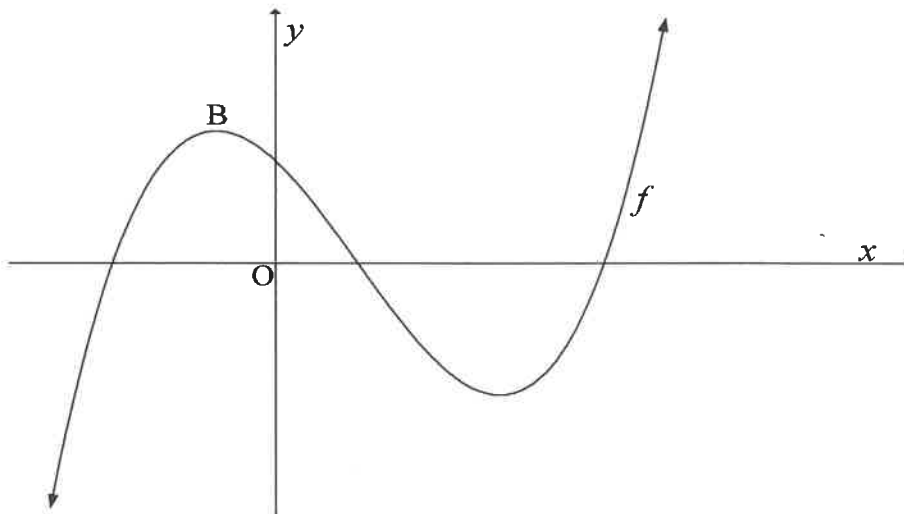8.1   Determine $f'(x)$ from first principles if $f(x) = 4x^2$.   (5)

8.2   Determine:

8.2.1 $D_x \left[\dfrac{x^2 - 2x - 3}{x + 1} \right]$ (3)

8.2.2 $f''(x)$ if $f(x) = \sqrt{x}$ (3)

[11]

QUESTION 9

The sketch below represents the curve of $f(x) = x^3 + bx^2 + cx + d$. The solutions of the equation $f(x) = 0$ are -2 ; 1 and 4.

9.1   Calculate the values of b, c and d.   (4)

9.2   Calculate the x-coordinate of B, the maximum turning point of f.   (4)

9.3   Determine an equation for the tangent to the graph of f at $x = -1$.   (4)

9.4   In the ANSWER BOOK, sketch the graph of $f''(x)$. Clearly indicate the x- and y-intercepts on your sketch.   (3)

9.5   For which value(s) of x is $f(x)$ concave upwards?   (2)

[17]

QUESTION 6

6.1 Calculate how many years it will take for the value of a truck to decrease to 50% of its original value if depreciation is calculated at 15% per annum using the reducing-balance method. (4)

6.2 Every month Tshepo deposited R1 500 for his retirement into an account that paid interest at a rate of 9,2% per annum, compounded monthly. Tshepo made his first instalment on his 23rd birthday and the last instalment one month before his 55th birthday. Calculate how much money he had in the account on his 55th birthday. (5)

6.3 Abram has R150 000 to invest in two separate accounts. One account pays interest at a rate of 8,4% per annum, compounded quarterly, and the other account at a rate of 9,6% per annum, compounded monthly. How much money should he invest in each account so that he will collect the same amount from each account at the end of 12 years? (6)

[15]

QUESTION 7

7.1 Given: $f(x) = 2 - 3x^2$

Determine $f'(x)$ from first principles. (5)

7.2 Determine:

7.2.1 $D_x\left[(4x+5)^2\right]$ (3)

7.2.2 $\dfrac{dy}{dx}$ if $y = \sqrt[4]{x} + \dfrac{x^2 - 8}{x^2}$ (4)

[12]

QUESTION 6

6.1 Mbali invested R10 000 for 3 years at an interest rate of r % p.a., compounded monthly. At the end of this period, she received R12 146,72. Calculate r, correct to ONE decimal place. (5)

6.2 Piet takes a loan from a bank to buy a car for R235 000. He agrees to repay the loan over a period of 54 months. The first instalment will be paid one month after the loan is granted. The bank charges interest at 11% p.a., compounded monthly.

    6.2.1 Calculate Piet's monthly instalment. (4)

    6.2.2 Calculate the total amount of interest that Piet will pay during the first year of the repayment of the loan. (6)

[15]

QUESTION 7

7.1 Given: $f(x) = 2x^2 - x$

Determine $f'(x)$ from first principles. (6)

7.2 Determine:

    7.2.1 $D_x[(x+1)(3x-7)]$ (2)

    7.2.2 $\dfrac{dy}{dx}$ if $y = \sqrt{x^3} - \dfrac{5}{x} + \dfrac{1}{2}\pi$ (4)

[12]

QUESTION 7

7.1   A company bought a new machine for R500 000. After 3 years, the machine has a book value of R331 527. Calculate the yearly rate of depreciation if the machine was depreciated according to the reducing-balance method.   (3)

7.2   Musa takes a personal loan from a bank to buy a motorcycle that costs R46 000. The bank charges interest at 24% per annum, compounded monthly.

         How many months will it take Musa to repay the loan, if the monthly instalment is R1 900?   (4)

7.3   Neil set up an investment fund. Exactly 3 months later and every 3 months thereafter he deposited R3 500 into the fund. The fund pays interest at 7,5% p.a., compounded quarterly. He continued to make quarterly deposits into the fund for 6½ years from the time that he originally set up the fund.

         Neil made no further deposits into the fund, but left the money in the same fund at the same rate of interest. Calculate how much he will have in the fund 10 years after he originally set it up.   (6)

                                                                                [13]

QUESTION 8

8.1   Given $f(x) = 3 - 2x^2$. Determine $f'(x)$, using first principles.   (5)

8.2   Determine $\dfrac{dy}{dx}$ if $y = \dfrac{12x^2 + 2x + 1}{6x}$.   (4)

8.3   The function $f(x) = x^3 + bx^2 + cx - 4$ has a point of inflection at (2 ; 4). Calculate the values of b and c.   (7)

                                                                                [16]

QUESTION 9

Given: $f(x) = x^3 - x^2 - x + 1$

9.1   Write down the coordinates of the y-intercept of f.   (1)

9.2   Calculate the coordinates of the x-intercepts of f.   (5)

9.3   Calculate the coordinates of the turning points of f.   (6)

9.4   Sketch the graph of f in your ANSWER BOOK. Clearly indicate all intercepts with the axes and the turning points.   (3)

9.5   Write down the values of x for which $f'(x) < 0$.   (2)

                                                                                [17]

QUESTION 6

6.1   On the 2nd day of January 2015 a company bought a new printer for R150 000.

- The value of the printer decreases by 20% annually on the reducing-balance method.
- When the book value of the printer is R49 152, the company will replace the printer.

6.1.1   Calculate the book value of the printer on the 2nd day of January 2017.   (3)

6.1.2   At the beginning of which year will the company have to replace the printer? Show ALL calculations.   (4)

6.1.3   The cost of a similar printer will be R280 000 at the beginning of 2020. The company will use the R49 152 that it will receive from the sale of the old printer to cover some of the costs of replacing the printer. The company set up a sinking fund to cover the balance. The fund pays interest at 8,5% per annum, compounded quarterly. The first deposit was made on 2 April 2015 and every three months thereafter until 2 January 2020. Calculate the amount that should be deposited every three months to have enough money to replace the printer on 2 January 2020.   (4)

6.2   Lerato wishes to apply for a home loan. The bank charges interest at 11% per annum, compounded monthly. She can afford a monthly instalment of R9 000 and wants to repay the loan over a period of 15 years. She will make the first monthly repayment one month after the loan is granted. Calculate, to the nearest thousand rand, the maximum amount that Lerato can borrow from the bank.   (5)

[16]

QUESTION 7

7.1   Determine $f'(x)$ from first principles if $f(x) = x^2 - 5$.   (5)

7.2   Determine the derivative of: $g(x) = 5x^2 - \dfrac{2x}{x^3}$ (3)

7.3   Given: $h(x) = ax^2$, $x > 0$.
        Determine the value of a if it is given that $h^{-1}(8) = h'(4)$.   (6)

[14]

QUESTION 8

8.1   Determine $f'(x)$ from first principles if $f(x) = 3x^2$ (5)

8.2   John determines $g'(a)$, the derivative of a certain function g at $x = a$, and arrives

        at the answer: $\lim\limits_{h \to 0} \dfrac{\sqrt{4+h} - 2}{h}$

        Write down the equation of g and the value of a.   (2)

8.3   Determine $\dfrac{dy}{dx}$ if $y = \sqrt{x^3} - \dfrac{5}{x^3}$ (4)

8.4 $g(x) = -8x + 20$ is a tangent to $f(x) = x^3 + ax^2 + bx + 18$ at $x = 1$. Calculate the
        values of a and b.   (5)
                                                                                              [16]

QUESTION 9

For a certain function f, the first derivative is given as $f'(x) = 3x^2 + 8x - 3$

9.1   Calculate the x-coordinates of the stationary points of f.   (3)

9.2   For which values of x is f concave down?   (3)

9.3   Determine the values of x for which f is strictly increasing.   (2)

9.4   If it is further given that $f(x) = ax^3 + bx^2 + cx + d$ and $f(0) = -18$, determine the
        equation of f.   (5)
                                                                                              [13]

QUESTION 10

The number of molecules of a certain drug in the bloodstream t hours after it has been taken
is represented by the equation $M(t) = -t^3 + 3t^2 + 72t$, $0 < t < 10$.

10.1   Determine the number of molecules of the drug in the bloodstream 3 hours after the
        drug was taken.   (2)

10.2   Determine the rate at which the number of molecules of the drug in the bloodstream is
        changing at exactly 2 hours after the drug was taken.   (3)

10.3   How many hours after taking the drug will the rate at which the number of molecules
        of the drug in the bloodstream is changing, be a maximum?   (3)
                                                                                              [8]

QUESTION 6

6.1   How long would the price of an asset take to reduce by a third of its original value if it depreciates on a reducing balance at a rate of 4,7% p.a.?   (4)

6.2   Lebogo bought a tractor for Rx on 1 April 2016.

- She will trade in this tractor when she replaces it with a similar one in 5 years' time on 1 April 2021.
- The tractor depreciates by 20% p.a. according to the reducing-balance method.
- The price of a similar tractor increases by 18% annually.
- Lebogo calculated that if she deposited R8 000 per month into a sinking fund, which paid interest at 10% p.a. compounded monthly, she would have enough money to cover the replacement cost of the tractor. She made the first deposit in this fund on 30 April 2016 and will continue to do so at the end of every month until 31 March 2021.

6.2.1   Determine, in terms of x, what the book value of the current tractor will be on 1 April 2021 (that is, 5 years after it was bought). Give your answer correct to FIVE decimal places.   (2)

6.2.2   Determine, in terms of x, what the price of a similar new tractor will be on 1 April 2021. Give your answer correct to FIVE decimal places.   (2)

6.2.3   Calculate the amount accumulated in the sinking fund on 1 April 2021.   (4)

6.2.4   Calculate the value of x, the price of the current tractor. Round off your answer to the nearest thousand.   (4)
[16]

QUESTION 7

7.1   Determine $f'(x)$ from first principles if $f(x) = 3x^2 - 5$ (5)

7.2   Determine $\dfrac{dy}{dx}$ if:

7.2.1 $y = 2x^5 + \dfrac{4}{x^3}$ (3)

7.2.2 $y = \left(\sqrt{x} - x^2\right)^2$ (4)
[12]

QUESTION 6

Given: $f(x) = \dfrac{1}{4}x^2$, $x \leq 0$

6.1 Determine the equation of f^{-1} in the form $f^{-1}(x) = \ldots$ (3)

6.2 On the same system of axes, sketch the graphs of f and f^{-1}. Indicate clearly the intercepts with the axes, as well as another point on the graph of each of f and f^{-1}. (3)

6.3 Is f^{-1} a function? Give a reason for your answer. (2)

[8]

QUESTION 7

7.1 Diane invests a lump sum of R5 000 in a savings account for exactly 2 years. The investment earns interest at 10% p.a., compounded quarterly.

7.1.1 What is the quarterly interest rate for Diane's investment? (1)

7.1.2 Calculate the amount in Diane's savings account at the end of the 2 years. (3)

7.2 Motloi inherits R800 000. He invests all of his inheritance in a fund which earns interest at a rate of 14% p.a., compounded monthly. At the end of each month he withdraws R10 000 from the fund. His first withdrawal is exactly one month after his initial investment.

7.2.1 How many withdrawals of R10 000 will Motloi be able to make from this fund? (5)

7.2.2 Exactly four years after his initial investment Motloi decides to withdraw all the remaining money in his account and to use it as a deposit towards a house.

(a) What is the value of Motloi's deposit, to the nearest rand? (4)

(b) Motloi's deposit is exactly 30% of the purchase price of the house. What is the purchase price of the house, to the nearest rand? (1)

[14]

QUESTION 8

8.1   If $f(x) = x^2 - 3x$, determine $f'(x)$ from first principles. (5)

8.2   Determine:

        8.2.1 $\dfrac{dy}{dx}$ if $y = \left(x^2 - \dfrac{1}{x^2} \right)^2$ (3)

        8.2.2 $D_x \left(\dfrac{x^3 - 1}{x - 1} \right)$

                                                  (3)
                                                  [11]

QUESTION 9

Given: $h(x) = -x^3 + ax^2 + bx$ and $g(x) = -12x$. P and Q(2 ; 10) are the turning points of h. The graph of h passes through the origin.

9.1   Show that $a = \dfrac{3}{2}$ and $b = 6$. (5)

9.2   Calculate the average gradient of h between P and Q, if it is given that $x = -1$ at P. (4)

9.3   Show that the concavity of h changes at $x = \dfrac{1}{2}$. (3)

9.4   Explain the significance of the change in QUESTION 9.3 with respect to h. (1)

9.5   Determine the value of x, given $x < 0$, at which the tangent to h is parallel to g. (4)

                                                    [17]
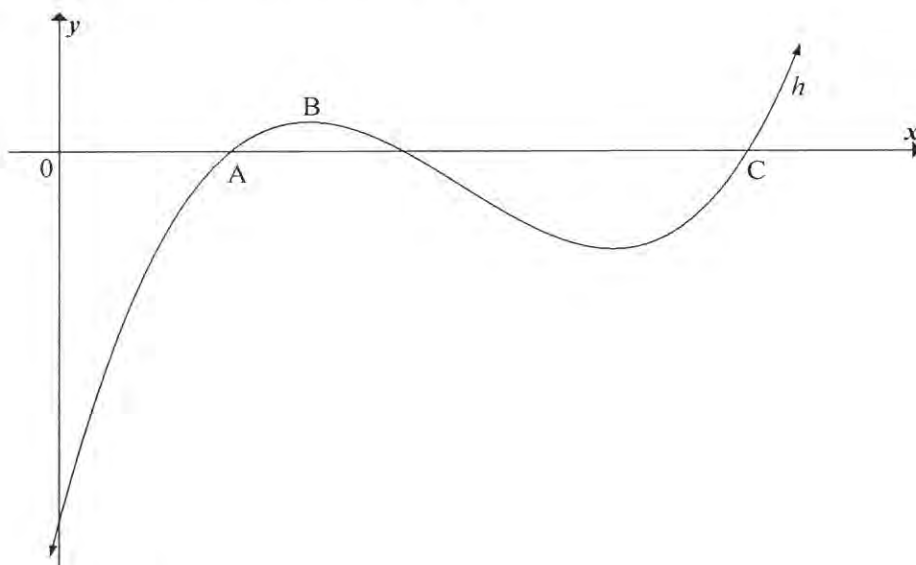
QUESTION 8

8.1　　Determine the derivative of $f(x) = 2x^2 + 4$ from first principles.　　(4)

8.2　　Differentiate:

8.2.1　　$f(x) = -3x^2 + 5\sqrt{x}$　　(3)

8.2.2　　$p(x) = \left(\dfrac{1}{x^3} + 4x\right)^2$　　(4)

8.3　　The sketch below shows the graph of $h(x) = x^3 - 7x^2 + 14x - 8$. The x-coordinate of point A is 1. C is another x-intercept of h.

8.3.1　　Determine $h'(x)$.　　(1)

8.3.2　　Determine the x-coordinate of the turning point B.　　(3)

8.3.3　　Calculate the coordinates of C.　　(4)

8.3.4　　The graph of h is concave down for $x < k$. Calculate the value of k.　　(3)

[22]

QUESTION 7

7.1   Exactly five years ago Mpume bought a new car for R145 000. The current book value of this car is R72 500. If the car depreciates by a fixed annual rate according to the reducing-balance method, calculate the rate of depreciation.   (3)

7.2   Samuel took out a home loan for R500 000 at an interest rate of 12% per annum, compounded monthly. He plans to repay this loan over 20 years and his first payment is made one month after the loan is granted.

    7.2.1   Calculate the value of Samuel's monthly instalment.   (4)

    7.2.2   Melissa took out a loan for the same amount and at the same interest rate as Samuel. Melissa decided to pay R6 000 at the end of every month. Calculate how many months it took for Melissa to settle the loan.   (4)

    7.2.3   Who pays more interest, Samuel or Melissa? Justify your answer.   (2)
    [13]

QUESTION 8

8.1   Determine $f'(x)$ from first principles if $f(x) = x^3$.   (5)

8.2   Determine the derivative of: $f(x) = 2x^2 + \dfrac{1}{2}x^4 - 3$ (2)

8.3   If $y = \left(x^6 - 1\right)^2$, prove that $\dfrac{dy}{dx} = 12x^5\sqrt{y}$, if $x > 1$.   (3)

8.4   Given: $f(x) = 2x^3 - 2x^2 + 4x - 1$. Determine the interval on which f is concave up.   (4)
    [14]

QUESTION 9

Susan buys a car for R350 000. She secures a loan at an interest rate of 7% p.a., compounded monthly. The monthly instalment is R6 300. She pays the first instalment one month after the loan was secured.

9.1   Calculate the effective annual interest rate on the loan. Leave your answer correct to TWO decimal places.   (3)

9.2   How many months will it take to repay the loan?   (5)

9.3   Calculate the value of the final instalment.   (5)

9.4   The value of the car depreciates at i % p.a. After 3 years its value is R252 000. Calculate i.   (3)

[16]

QUESTION 10

10.1   Given: $f(x) = -\dfrac{2}{x}$

           10.1.1   Determine $f'(x)$ from first principles.   (5)

           10.1.2   For which value(s) of x will $f'(x) > 0$? Justify your answer.   (2)

10.2   Evaluate $\dfrac{dy}{dx}$ if $y = \dfrac{1}{4}x^2 - 2x$.   (2)

10.3   Given: $y = 4\left(\sqrt[3]{x^2}\right)$ and $x = w^{-3}$

      Determine $\dfrac{dy}{dw}$.   (4)

10.4   Given: $f(x) = ax^3 + bx^2 + cx + d$

      Draw a possible sketch of $y = f'(x)$ if a, b and c are all NEGATIVE real numbers.   (4)

[17]

QUESTION 6

The graph of g is defined by the equation $g(x) = \sqrt{ax}$. The point $(8 ; 4)$ lies on g.

6.1   Calculate the value of a.   (2)

6.2   If $g(x) > 0$, for what values of x will g be defined?   (1)

6.3   Determine the range of g.   (1)

6.4   Write down the equation of g^{-1}, the inverse of g, in the form $y = ...$ (2)

6.5   If $h(x) = x - 4$ is drawn, determine ALGEBRAICALLY the point(s) of intersection
        of h and g.   (4)

6.6   Hence, or otherwise, determine the values of x for which $g(x) > h(x)$.   (2)
                                                                                                      [12]

QUESTION 7

Siphokazi bought a house. She paid a deposit of R102 000, which is equivalent to 12% of the selling price of the house. She obtained a loan from the bank to pay the balance of the selling price. The bank charges her interest of 9% per annum, compounded monthly.

7.1   Determine the selling price of the house.   (1)

7.2   The period of the loan is 20 years and she starts repaying the loan one month after it
        was granted. Calculate her monthly instalment.   (4)

7.3   How much interest will she pay over the period of 20 years? Round your answer
        correct to the nearest rand.   (2)

7.4   Calculate the balance of her loan immediately after her 85th instalment.   (3)

7.5   She experienced financial difficulties after the 85th instalment and did not pay any
        instalments for 4 months (that is months 86 to 89). Calculate how much Siphokazi
        owes on her bond at the end of the 89th month.   (2)

7.6   She decides to increase her payments to R8 500 per month from the end of the 90th
        month. How many months will it take to repay her bond after the new payment of   (4)
        R8 500 per month?   **[16]**

QUESTION 8

8.1   Determine $f'(x)$ from first principles if $f(x) = 3x^2 - 2$.   (5)

8.2   Determine $\dfrac{dy}{dx}$ if $y = 2x^{-4} - \dfrac{x}{5}$.   (2)
                                                                                                       [7]

QUESTION 7

7.1　　Mpho invests R12 500 for exactly k years. She earns interest at a rate of 9% per annum, compounded quarterly. At the end of k years, her investment is worth R30 440.

　　　　7.1.1　　Calculate the effective annual interest rate of Mpho's investment.　　(2)

　　　　7.1.2　　Determine the value of k.　　(5)

7.2　　Darrel is planning to buy his first home. The bank will allow him to use a maximum of 30% of his monthly salary to repay the bond.

　　　　7.2.1　　Calculate the maximum amount that the bank will allow Darrel to spend each month on his bond repayments, if Darrel earns R18 480 per month.　　(1)

　　　　7.2.2　　Suppose, at the end of each month, Darrel repays the maximum amount allowed by the bank. How much money does Darrel borrow if he takes 25 years to repay the loan at a rate of 8% p.a., compounded monthly? (The first repayment is made one month after the loan is granted.)　　(4)

　　[12]

QUESTION 8

8.1　　Given: $f(x) = 3x^2 - 4$

　　　　8.1.1　　Determine $f'(x)$ from first principles.　　(5)

　　　　8.1.2　　A(x ; 23), where $x > 0$, and B($- 2$; y) are points on the graph of f. Calculate the numerical value of the average gradient of f between A and B.　　(5)

8.2　　Differentiate $y = \dfrac{x+5}{\sqrt{x}}$ with respect to x.　　(3)

8.3　　Determine the gradient of the tangent of the graph of $f(x) = -3x^3 - 4x + 5$ at $x = -1$.　　(4)

　　[17]

Mathematics/P1

8

NSC

DBE/Feb.–Mar. 2013

QUESTION 7

7.1 Raeesa invests R4 million into an account earning interest of 6% per annum, compounded annually. How much will her investment be worth at the end of 3 years? (3)

7.2 Joanne invests R4 million into an account earning interest of 6% per annum, compounded monthly.

    7.2.1 She withdraws an allowance of R30 000 per month. The first withdrawal is exactly one month after she has deposited the R4 million. How many such withdrawals will Joanne be able to make? (6)

    7.2.2 If Joanne withdraws R20 000 per month, how many withdrawals will she be able to make? (3)

[12]

QUESTION 8

Jeffrey invests R700 per month into an account earning interest at a rate of 8% per annum, compounded monthly. His friend also invests R700 per month and earns interest compounded semi-annually (that is every six months) at r% per annum. Jeffrey and his friend's investments are worth the same at the end of 12 months. Calculate r. **[3]**

QUESTION 9

9.1 Use the definition of the derivative (first principles) to determine $f'(x)$ if $f(x) = 2x^3$ (5)

9.2 Determine $\dfrac{dy}{dx}$ if $y = \dfrac{2\sqrt{x}+1}{x^2}$ (4)

9.3 Calculate the values of a and b if $f(x) = ax^2 + bx + 5$ has a tangent at $x = -1$ which is defined by the equation $y = -7x + 3$ (6)

[15]

QUESTION 7

7.1   A business buys a machine that costs R120 000. The value of the machine depreciates at 9% per annum according to the diminishing-balance method.

         7.1.1   Determine the scrap value of the machine at the end of 5 years.   (3)

         7.1.2   After five years the machine needs to be replaced. During this time, inflation remained constant at 7% per annum. Determine the cost of the new machine at the end of 5 years.   (3)

         7.1.3   The business estimates that it will need R90 000 by the end of five years. A sinking fund for R90 000, into which equal monthly instalments must be paid, is set up. Interest on this fund is 8,5% per annum, compounded monthly. The first payment will be made immediately and the last payment will be made at the end of the 5-year period.

                  Calculate the value of the monthly payment into the sinking fund.   (5)

7.2   Lorraine receives an amount of R900 000 upon her retirement. She invests this amount immediately at an interest rate of 10,5% per annum, compounded monthly.

     She needs an amount of R18 000 per month to maintain her current lifestyle. She plans to withdraw the first amount at the end of the first month.

     For how many months will she be able to live from her investment?   (6)

                                                   [17]

QUESTION 8

8.1   Determine $f'(x)$ from first principles if $f(x) = 2x^2 - 5$.   (5)

8.2   Evaluate $\dfrac{dy}{dx}$ if $y = x^{-4} + 2x^3 - \dfrac{x}{5}$.   (3)

8.3   Given: $g(x) = \dfrac{x^2 + x - 2}{x - 1}$

         8.3.1   Calculate $g'(x)$ for $x \neq 1$.   (2)

         8.3.2   Explain why it is not possible to determine $g'(1)$.   (1)

                                                   [11]

QUESTION 8

8.1   Determine $f'(x)$ from first principles if $f(x) = 9 - x^2$.   (5)

8.2   Evaluate:

      8.2.1 $D_x[1 + 6\sqrt{x}]$ (2)

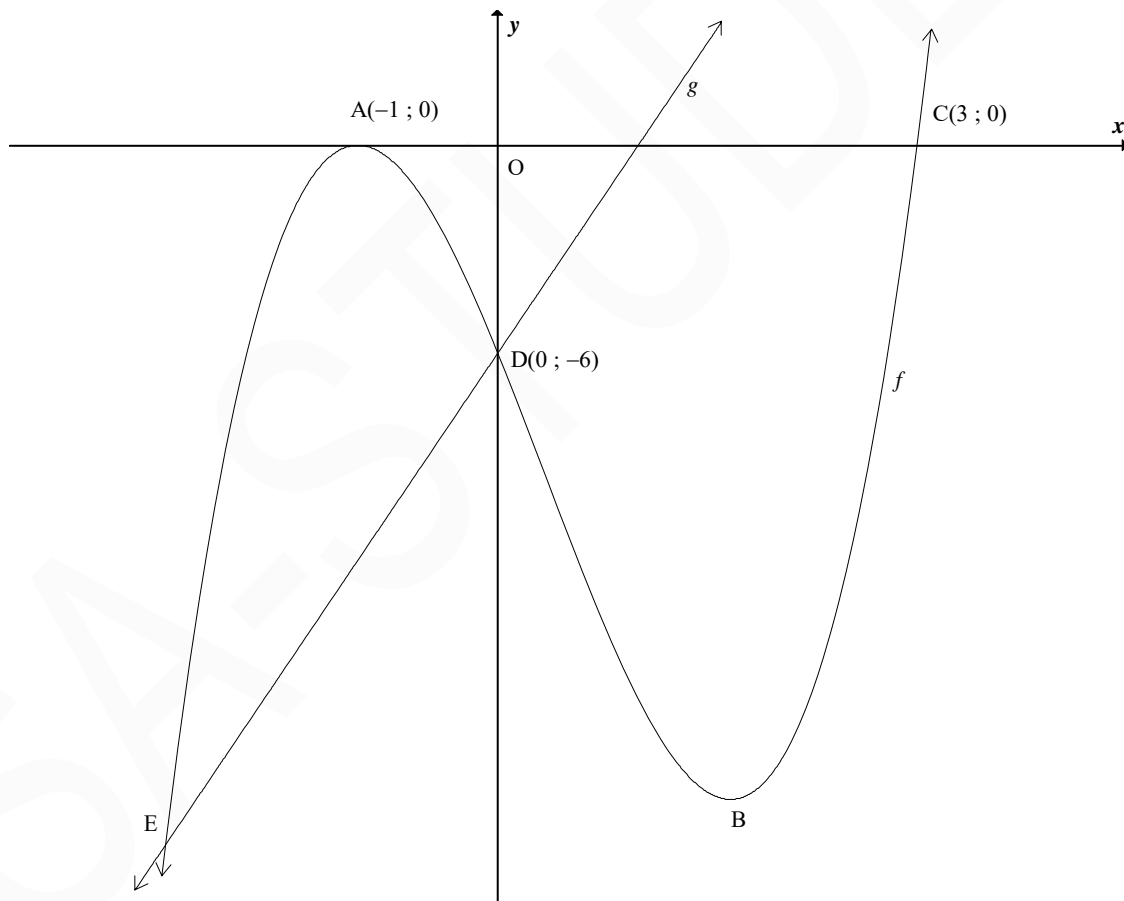   8.2.2 $\dfrac{dy}{dx}$ if $y = \dfrac{8 - 3x^6}{8x^5}$ (4)

[11]

QUESTION 9

The graphs of $f(x) = ax^3 + bx^2 + cx + d$ and $g(x) = 6x - 6$ are sketched below.
A($-1 ; 0$) and C($3 ; 0$) are the x-intercepts of f.
The graph of f has turning points at A and B.
D($0 ; -6$) is the y-intercept of f.
E and D are points of intersection of the graphs of f and g.

9.1   Show that $a = 2$; $b = -2$; $c = -10$ and $d = -6$.   (5)

9.2   Calculate the coordinates of the turning point B.   (5)

9.3 $h(x)$ is the vertical distance between $f(x)$ and $g(x)$, that is $h(x) = f(x) - g(x)$.
      Calculate x such that $h(x)$ is a maximum, where $x < 0$.   (5)

[15]

QUESTION 7

7.1 How many years will it take for an article to depreciate to half its value according to the reducing-balance method at 7% per annum? (4)

7.2 Two friends each receive an amount of R6 000 to invest for a period of 5 years. They invest the money as follows:

- Radesh: 8,5% per annum simple interest. At the end of the 5 years, Radesh will receive a bonus of exactly 5% of the principal amount.
- Thandi: 8% per annum compounded quarterly.

Who will have the bigger investment after 5 years? Justify your answer with appropriate calculations. (6)

7.3 Nicky opened a savings account with a single deposit of R1 000 on 1 April 2011. She then makes 18 monthly deposits of R700 at the end of every month. Her first payment is made on 30 April 2011 and her last payment on 30 September 2012. The account earns interest at 15% per annum compounded monthly.

Determine the amount that should be in her savings account immediately after her last deposit is made (that is on 30 September 2012). (6)

[16]

QUESTION 8

8.1 Determine $f'(x)$ from first principles if $f(x) = -4x^2$. (5)

8.2 Evaluate:

8.2.1 $\dfrac{dy}{dx}$ if $y = \dfrac{3}{2x} - \dfrac{x^2}{2}$ (3)

8.2.2 $f'(1)$ if $f(x) = (7x + 1)^2$ (4)

[12]

QUESTION 7

Consider the function $f(x) = \left(\dfrac{1}{3}\right)^x$.

7.1 Is f an increasing or decreasing function? Give a reason for your answer. (2)

7.2 Determine $f^{-1}(x)$ in the form $y = \ldots$ (2)

7.3 Write down the equation of the asymptote of $f(x) - 5$. (1)

7.4 Describe the transformation from f to g if $g(x) = \log_3 x$. (2)

[7]

QUESTION 8

8.1 R1 430,77 was invested in a fund paying $i\%$ p.a. compounded monthly. After 18 months the fund had a value of R1 711,41. Calculate i. (4)

8.2 A father decided to buy a house for his family for R800 000. He agreed to pay monthly instalments of R10 000 on a loan which incurred interest at a rate of 14% p.a. compounded monthly. The first payment was made at the end of the first month.

8.2.1 Show that the loan would be paid off in 234 months. (4)

8.2.2 Suppose the father encountered unexpected expenses and was unable to pay any instalments at the end of the 120th, 121st, 122nd and 123rd months. At the end of the 124th month he increased his payment so as to still pay off the loan in 234 months by 111 equal monthly payments.
Calculate the value of this new instalment. (7)

[15]

QUESTION 9

9.1 Use the definition to differentiate $f(x) = 1 - 3x^2$. (Use first principles.) (4)

9.2 Calculate $D_x\left[4 - \dfrac{4}{x^3} - \dfrac{1}{x^4}\right]$. (3)

9.3 Determine $\dfrac{dy}{dx}$ if $y = \left(1 + \sqrt{x}\right)^2$. (3)

[10]

QUESTION 9

9.1   A photocopier valued at R24 000 depreciates at a rate of 18% p.a. on the reducing-balance method. After how many years will its value be R15 000?   (4)

9.2   A car that costs R130 000 is advertised in the following way: 'No deposit necessary and first payment due three months after date of purchase.' The interest rate quoted is 18% p.a. compounded monthly.

         9.2.1   Calculate the amount owing two months after the purchase date, which is one month before the first monthly payment is due.   (3)

         9.2.2   Herschel bought this car on 1 March 2009 and made his first payment on 1 June 2009. Thereafter he made another 53 equal payments on the first day of each month.

                 (a) Calculate his monthly repayments.   (3)

                 (b) Calculate the total of all Herschel's repayments.   (1)

         9.2.3   Hashim also bought a car for R130 000. He also took out a loan for R130 000, at an interest rate of 18% p.a. compounded monthly. He also made 54 equal payments. However, he started payments one month after the purchase of the car. Calculate the total of all Hashim's repayments.   (4)

         9.2.4   Calculate the difference between Herschel's and Hashim's total repayments.   (1)

                                                                [16]

QUESTION 10

10.1   Differentiate $f(x)$ from first principles if $f(x) = -2x^2 + 3$.   (5)

10.2   Evaluate: $\dfrac{dy}{dx}$ if $y = x^2 - \dfrac{1}{2x^3}$ (2)

                                                                      [7]

QUESTION 7

7.1 R1 570 is invested at 12% p.a. compound interest. After how many years will the investment be worth R23 000? (4)

7.2 A farmer has just bought a new tractor for R800 000. He has decided to replace the tractor in 5 years' time, when its trade-in value will be R200 000. The replacement cost of the tractor is expected to increase by 8% per annum.

7.2.1 The farmer wants to replace his present tractor with a new one in 5 years' time. The farmer wants to pay cash for the new tractor, after trading in his present tractor for R200 000. How much will he need to pay? (3)

7.2.2 • One month after purchasing his present tractor, the farmer deposited x rands into an account that pays interest at a rate of 12% p.a., compounded monthly.
• He continued to deposit the same amount at the end of each month for a total of 60 months.
• At the end of 60 months he has exactly the amount that is needed to purchase a new tractor, after he trades in his present tractor.

Calculate the value of x. (6)

7.2.3 Suppose that 12 months after the purchase of the present tractor and every 12 months thereafter, he withdraws R5 000 from his account, to pay for maintenance of the tractor. If he makes 5 such withdrawals, what will the new monthly deposit be? (4)

[17]

QUESTION 8

8.1 Determine $f'(x)$ from first principles if $f(x) = -3x^2$. (5)

8.2 Determine, using the rules of differentiation:

$$\frac{dy}{dx} \text{ if } y = \frac{\sqrt{x}}{2} - \frac{1}{6x^3}$$

Show ALL calculations. (3)

[8]