

SA-STUDENT

To pass high school please visit us at:
<https://sa-student.com/>



QUESTION 8

Given: $f(x) = x^3 + 4x^2 - 7x - 10$

- 8.1 Write down the y -intercept of f . (1)
- 8.2 Show that 2 is a root of the equation $f(x) = 0$. (2)
- 8.3 Hence, factorise $f(x)$ completely. (3)
- 8.4 If it is further given that the coordinates of the turning points are approximately at $(0,7 ; -12,6)$ and $(-3,4 ; 20,8)$, draw a sketch graph of f and label all intercepts and turning points. (3)
- 8.5 Use your graph to determine the values of x for which:
- 8.5.1 $f'(x) < 0$ (2)
- 8.5.2 The gradient of a tangent to f will be a minimum (2)
- 8.5.3 $f'(x) \cdot f''(x) \leq 0$ (3)
- [16]

QUESTION 9

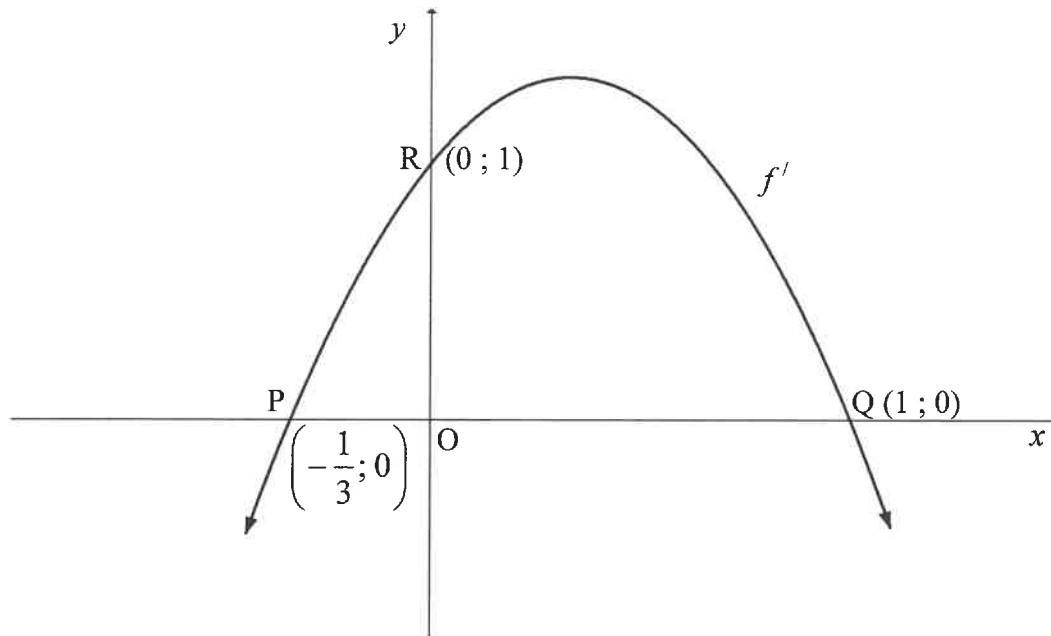
A wire, 12 metres long, is cut into two pieces. One part is bent to form an equilateral triangle and the other a square. A side of the triangle has a length of $2x$ metres.

- 9.1 Write down the length of a side of the square in terms of x . (2)
- 9.2 If this square is now used as the base of a rectangular prism with a height of $4x$ metres, determine the maximum volume of the rectangular prism. (7)
- [9]

QUESTION 8

The graph of $y = f'(x) = mx^2 + nx + k$ is drawn below.

The graph passes the points $P\left(-\frac{1}{3}; 0\right)$, $Q(1; 0)$ and $R(0; 1)$.



- 8.1 Determine the values of m , n and k . (6)
- 8.2 If it is further given that $f(x) = -x^3 + x^2 + x + 2$:
- 8.2.1 Determine the coordinates of the turning points of f . (3)
- 8.2.2 Draw the graph of f . Indicate on your graph the coordinates of the turning points and the intercepts with the axes. (5)
- 8.3 Points E and W are two variable points on f' and are on the same horizontal line.
- h is a tangent to f' at E .
 - g is a tangent to f' at W .
 - h and g intersect at $D(a; b)$.
- 8.3.1 Write down the value of a . (1)
- 8.3.2 Determine the value(s) of b for which h and g will no longer be tangents to f' . (2)

[17]

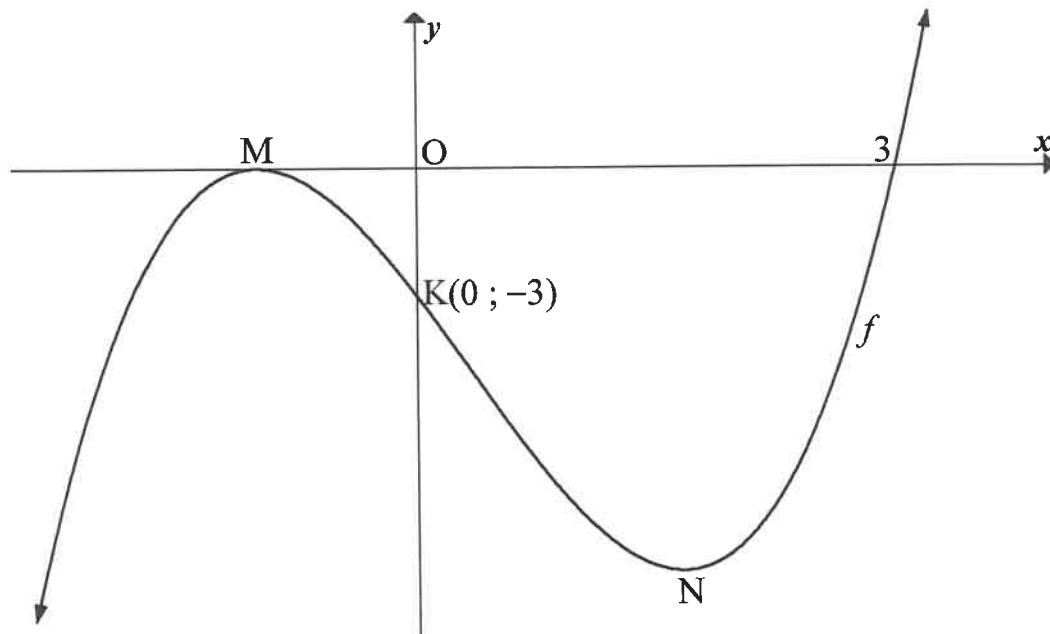
QUESTION 9

Sketched below is the graph of $f(x) = x^3 + ax^2 + bx + c$.

The x -intercepts of f are at $(3; 0)$ and M , where M lies on the negative x -axis.

$K(0; -3)$ is the y -intercept of f .

M and N are the turning points of f .



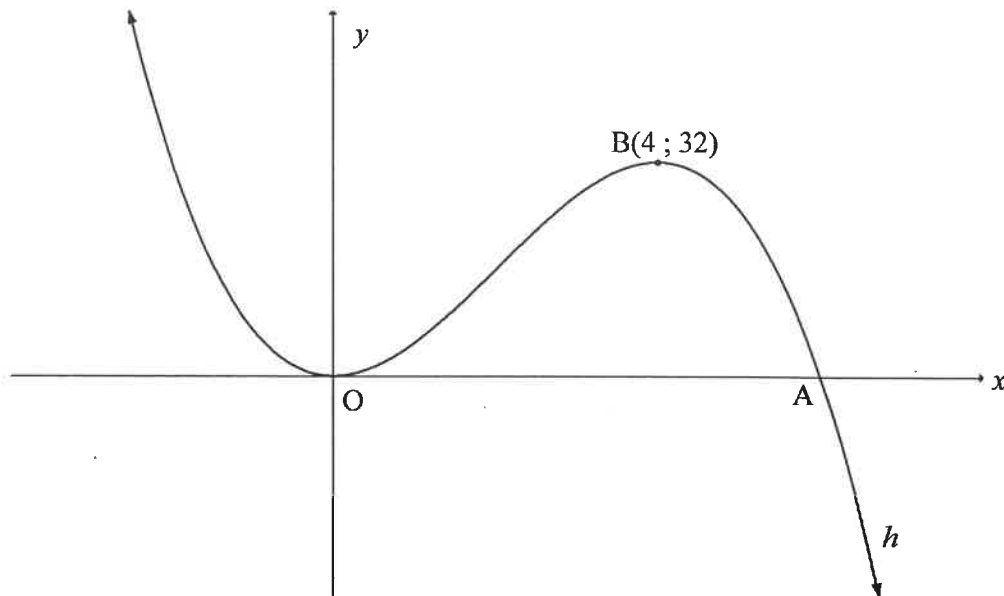
- 9.1 Show that the equation of f is given by $f(x) = x^3 - x^2 - 5x - 3$. (5)
- 9.2 Calculate the coordinates of N . (5)
- 9.3 For which values of x will:
- 9.3.1 $f(x) < 0$ (2)
- 9.3.2 f be increasing (2)
- 9.3.3 f be concave up (3)
- 9.4 Determine the maximum vertical distance between the graphs of f and f' in the interval $-1 < x < 0$. (6)
- [23]**

QUESTION 10

The graph of $h(x) = ax^3 + bx^2$ is drawn.

The graph has turning points at the origin, $O(0 ; 0)$ and $B(4 ; 32)$.

A is an x -intercept of h .



- 10.1 Show that $a = -1$ and $b = 6$. (5)
- 10.2 Calculate the coordinates of A. (3)
- 10.3 Write down the values of x for which h is:
- 10.3.1 Increasing (2)
- 10.3.2 Concave down (2)
- 10.4 For which values of k will $-(x-1)^3 + 6(x-1)^2 - k = 0$ have one negative and two distinct positive roots? (3)
- [15]

QUESTION 8

8.1 Determine $f'(x)$ from first principles if it is given that $f(x) = 3x^2$. (5)

8.2 Determine:

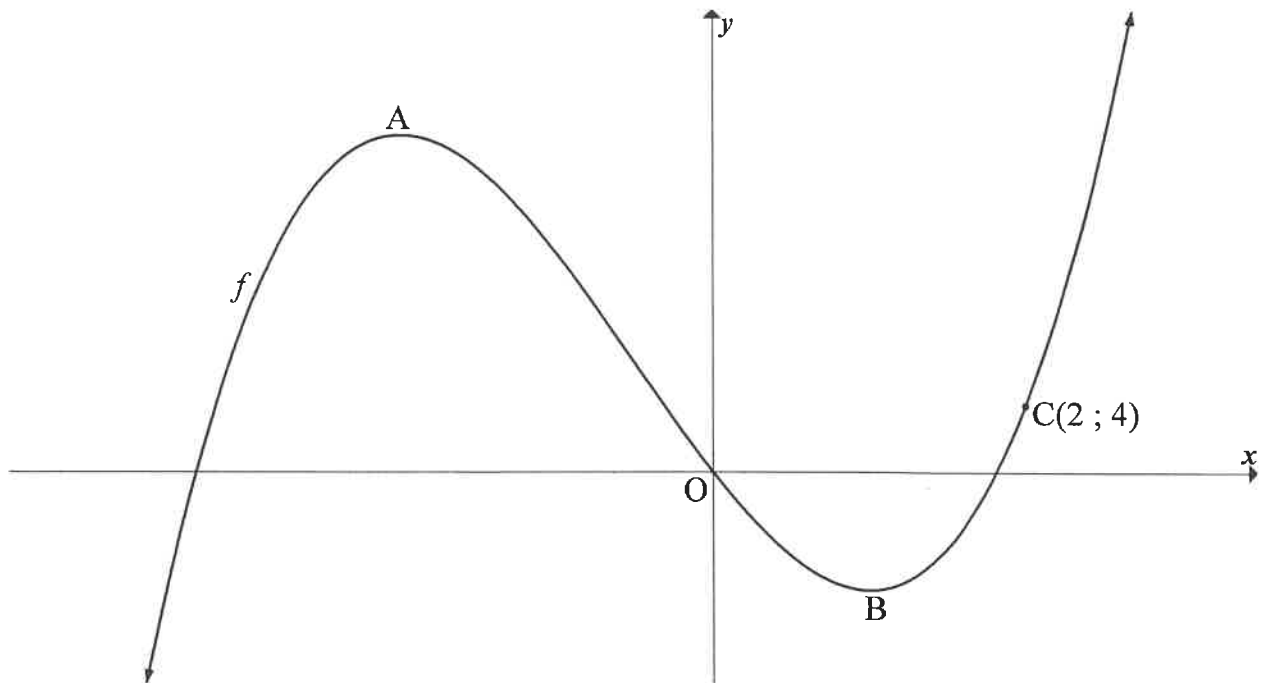
8.2.1 $f'(x)$ if $f(x) = x^2 - 3 + \frac{9}{x^2}$ (3)

8.2.2 $g'(x)$ if $g(x) = (\sqrt{x} + 3)(\sqrt{x} - 1)$ (4)
[12]

QUESTION 9

The graph of $f(x) = 2x^3 + 3x^2 - 12x$ is sketched below.

A and B are the turning points of f . $C(2 ; 4)$ is a point on f .



9.1 Determine the coordinates of A and B. (5)

9.2 For which values of x will f be concave up? (3)

9.3 Determine the equation of the tangent to f at $C(2 ; 4)$. (3)
[11]

QUESTION 10

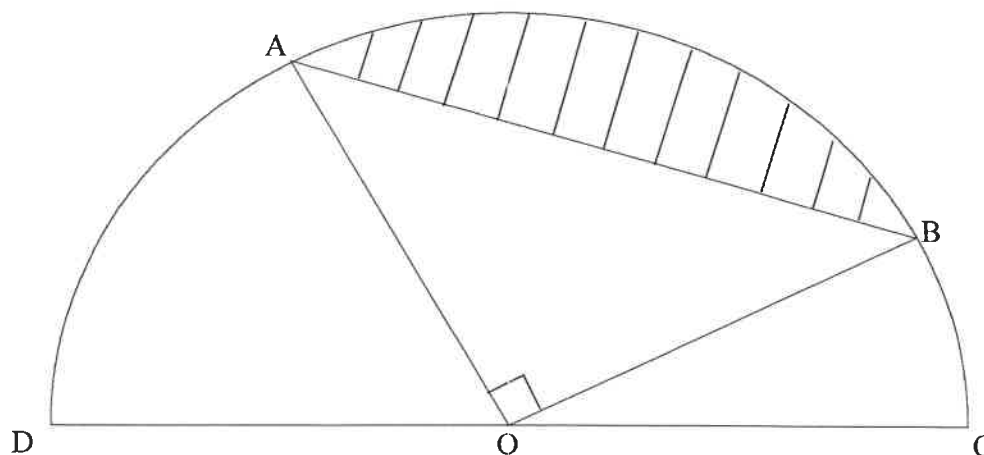
10.1 The graph of $f(x) = ax^3 + bx^2 + cx + d$ has two turning points.

The following information about f is also given:

- $f(2) = 0$
- The x -axis is a tangent to the graph of f at $x = -1$
- $f'(1) = 0$
- $f'\left(\frac{1}{2}\right) > 0$

Without calculating the equation of f , use this information to draw a sketch graph of f , only indicating the x -coordinates of the x -intercepts and turning points. (4)

10.2 O is the centre of a semicircle passing through A, B, C and D. The radius of the semicircle is $(x - x^2)$ units for $0 < x < 1$. $\triangle AOB$ is right-angled at O.



10.2.1 Show that the area of the shaded part is given by:

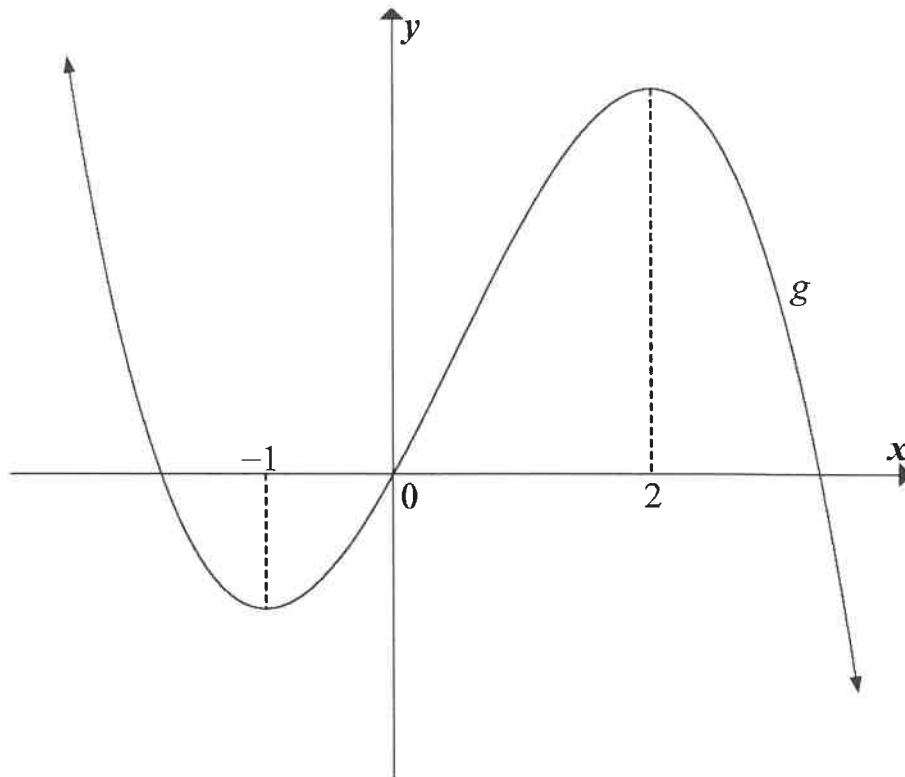
$$\text{Area} = \left(\frac{\pi - 2}{4}\right)(x^4 - 2x^3 + x^2) \quad (5)$$

10.2.2 Determine the value of x for which the shaded area will be a maximum. (4)

[13]

QUESTION 8

The graph of $g(x) = ax^3 + bx^2 + cx$, a cubic function having a y -intercept of 0, is drawn below. The x -coordinates of the turning points of g are -1 and 2 .



- 8.1 For which values of x will g increase? (2)
- 8.2 Write down the x -coordinate of the point of inflection of g . (2)
- 8.3 For which values of x will g be concave down? (2)
- 8.4 If $g'(x) = -6x^2 + 6x + 12$, determine the equation of g . (4)
- 8.5 Determine the equation of the tangent to g that has the maximum gradient. Write your answer in the form $y = mx + c$. (5)
- [15]

QUESTION 8

After flying a short distance, an insect came to rest on a wall. Thereafter the insect started crawling on the wall. The path that the insect crawled can be described by $h(t) = (t - 6)(-2t^2 + 3t - 6)$, where h is the height (in cm) above the floor and t is the time (in minutes) since the insect started crawling.

- 8.1 At what height above the floor did the insect start to crawl? (1)
- 8.2 How many times did the insect reach the floor? (4)
- 8.3 Determine the maximum height that the insect reached above the floor. (4)
- [8]**

QUESTION 9

Given: $f(x) = 3x^3$

- 9.1 Solve $f(x) = f'(x)$ (3)
- 9.2 The graphs f , f' and f'' all pass through the point $(0; 0)$.
- 9.2.1 For which of the graphs will $(0; 0)$ be a stationary point? (1)
- 9.2.2 Explain the difference, if any, in the stationary points referred to in QUESTION 9.2.1. (2)
- 9.3 Determine the vertical distance between the graphs of f' and f'' at $x = 1$. (3)
- 9.4 For which value(s) of x is $f(x) - f'(x) < 0$? (4)
- [13]**

QUESTION 10

The school library is open from Monday to Thursday. Anna and Ben both studied in the school library one day this week. If the chance of studying any day in the week is equally likely, calculate the probability that Anna and Ben studied on:

- 10.1 The same day (2)
- 10.2 Consecutive days (3)
- [5]**

QUESTION 8

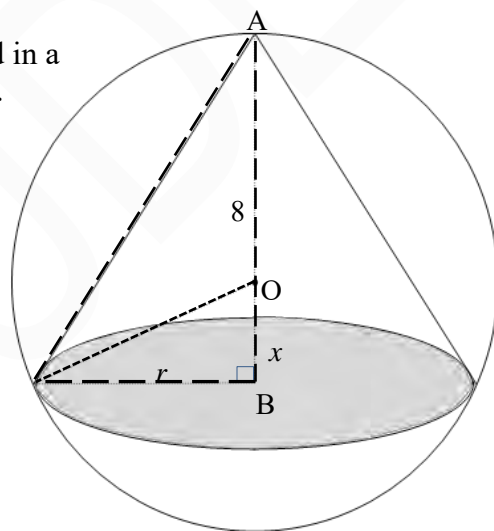
A cubic function $h(x) = -2x^3 + bx^2 + cx + d$ cuts the x -axis at $(-3; 0)$; $\left(-\frac{3}{2}; 0\right)$ and $(1; 0)$.

- 8.1 Show that $h(x) = -2x^3 - 7x^2 + 9$. (3)
- 8.2 Calculate the x -coordinates of the turning points of h . (3)
- 8.3 Determine the value(s) of x for which h will be decreasing. (2)
- 8.4 For which value(s) of x will there be a tangent to the curve of h that is parallel to the line $y - 4x = 7$. (4)
- [12]

QUESTION 9

A cone with radius r cm and height AB is inscribed in a sphere with centre O and a radius of 8 cm. $OB = x$.

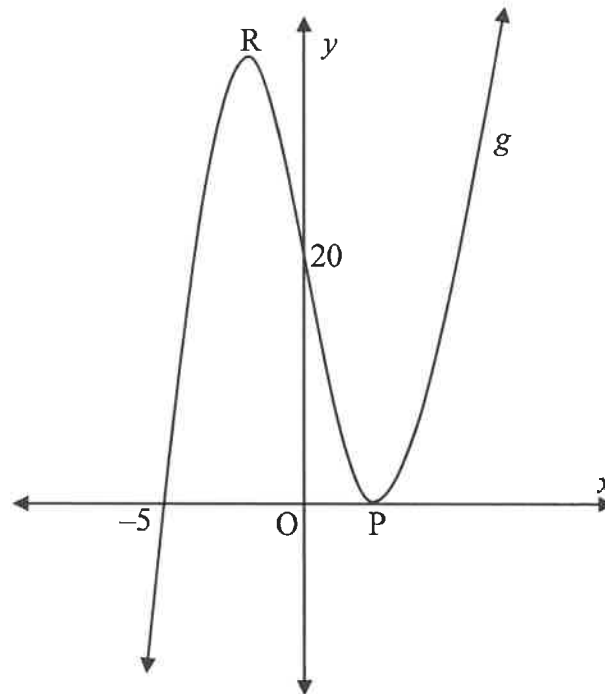
$\text{Volume of sphere} = \frac{4}{3}\pi r^3$ $\text{Volume of cone} = \frac{1}{3}\pi r^2 h$



- 9.1 Calculate the volume of the sphere. (1)
- 9.2 Show that $r^2 = 64 - x^2$. (1)
- 9.3 Determine the ratio between the largest volume of this cone and the volume of the sphere. (7)
- [9]

QUESTION 9

- 9.1 The graph of $g(x) = x^3 + bx^2 + cx + d$ is sketched below.
The graph of g intersects the x -axis at $(-5 ; 0)$ and at P , and the y -axis at $(0 ; 20)$.
 P and R are turning points of g .

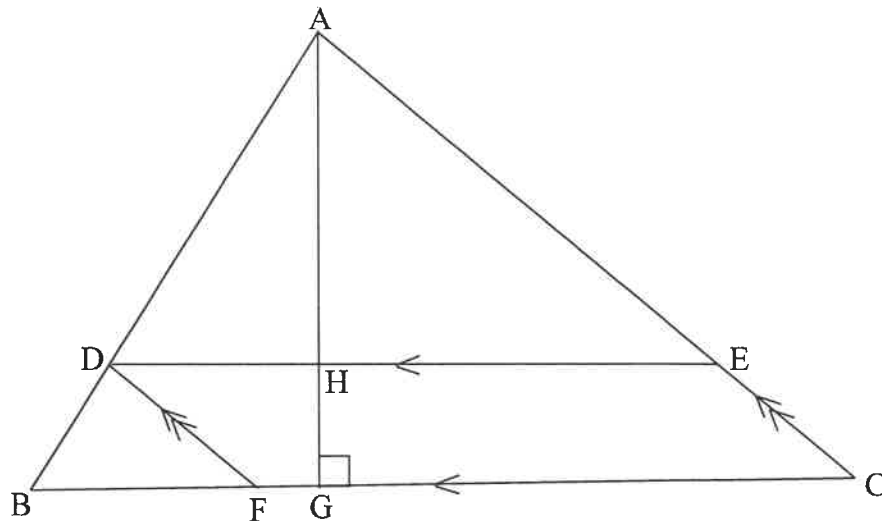


- 9.1.1 Show that $b = 1$, $c = -16$ and $d = 20$. (4)
- 9.1.2 Calculate the coordinates of P and R . (5)
- 9.1.3 Is the graph concave up or concave down at $(0 ; 20)$? Show ALL your calculations. (3)
- 9.2 If g is a cubic function with:
- $g(3) = g'(3) = 0$
 - $g(0) = 27$
 - $g''(x) > 0$ when $x < 3$ and $g''(x) < 0$ when $x > 3$,
- draw a sketch graph of g indicating ALL relevant points. (3)
- [15]

QUESTION 10

In $\triangle ABC$:

- D is a point on AB, E is a point on AC and F is a point on BC such that DECF is a parallelogram.
- $BF : FC = 2 : 3$.
- The perpendicular height AG is drawn intersecting DE at H.
- $AG = t$ units.
- $BC = (5 - t)$ units.



10.1 Write down $AH : HG$. (1)

10.2 Calculate t if the area of the parallelogram is a maximum.
(NOTE: Area of a parallelogram = base \times \perp height) (5)
[6]

QUESTION 11

Given the digits: 3 ; 4 ; 5 ; 6 ; 7 ; 8 and 9

11.1 Calculate how many unique 5-digit codes can be formed using the digits above, if:

11.1.1 The digits may be repeated (2)

11.1.2 The digits may not be repeated (2)

11.2 How many unique 3-digit codes can be formed using the above digits, if:

- Digits may be repeated
 - The code is greater than 400 but less than 600
 - The code is divisible by 5
- (3)
[7]

QUESTION 8

8.1 Determine $f'(x)$ from first principles if $f(x) = 4x^2$. (5)

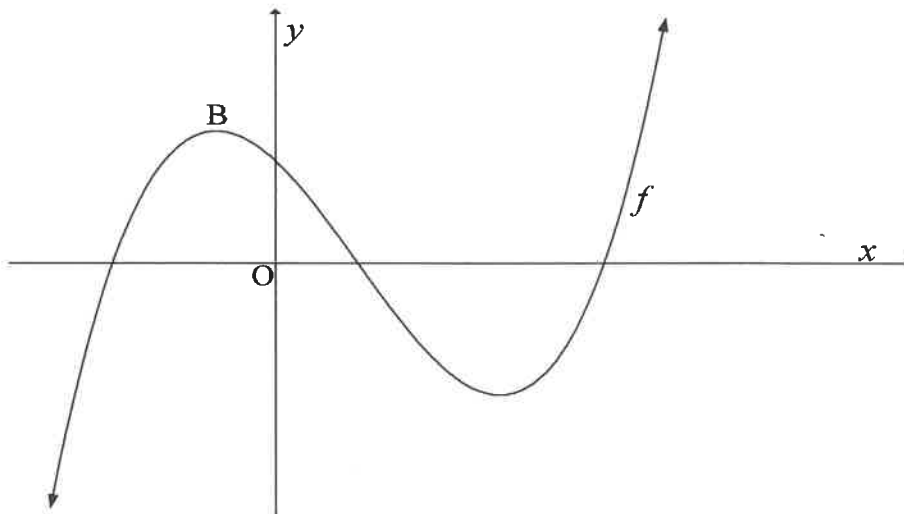
8.2 Determine:

8.2.1 $D_x \left[\frac{x^2 - 2x - 3}{x + 1} \right]$ (3)

8.2.2 $f''(x)$ if $f(x) = \sqrt{x}$ (3)
[11]

QUESTION 9

The sketch below represents the curve of $f(x) = x^3 + bx^2 + cx + d$. The solutions of the equation $f(x) = 0$ are -2 ; 1 and 4 .



9.1 Calculate the values of b , c and d . (4)

9.2 Calculate the x -coordinate of B , the maximum turning point of f . (4)

9.3 Determine an equation for the tangent to the graph of f at $x = -1$. (4)

9.4 In the ANSWER BOOK, sketch the graph of $f''(x)$. Clearly indicate the x - and y -intercepts on your sketch. (3)

9.5 For which value(s) of x is $f(x)$ concave upwards? (2)
[17]

QUESTION 10

Given: $f(x) = -3x^3 + x$.

Calculate the value of q for which $f(x) + q$ will have a maximum value of $\frac{8}{9}$. [6]

QUESTION 11

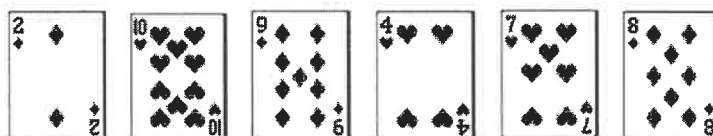
11.1 Veli and Bongi are learners at the same school. Some days they arrive late at school. The probability that neither Veli nor Bongi will arrive late on any day is 0,7.

11.1.1 Calculate the probability that at least one of the two learners will arrive late on a randomly selected day. (1)

11.1.2 The probability that Veli arrives late for school on a randomly selected day is 0,25, while the probability that both of them arrive late for school on that day is 0,15. Calculate the probability that Bongi will arrive late for school on that day. (3)

11.1.3 The principal suspects that the latecoming of the two learners is linked. The principal asks you to determine whether the events of Veli arriving late for school and Bongi arriving late for school are statistically independent or not. What will be your response to him? Show ALL calculations. (3)

11.2 The cards below are placed from left to right in a row.



11.2.1 In how many different ways can these 6 cards be randomly arranged in a row? (2)

11.2.2 In how many different ways can these cards be arranged in a row if the diamonds and hearts are placed in alternating positions? (3)

11.2.3 If these cards are randomly arranged in a row, calculate the probability that ALL the hearts will be next to one another. (3)
[15]

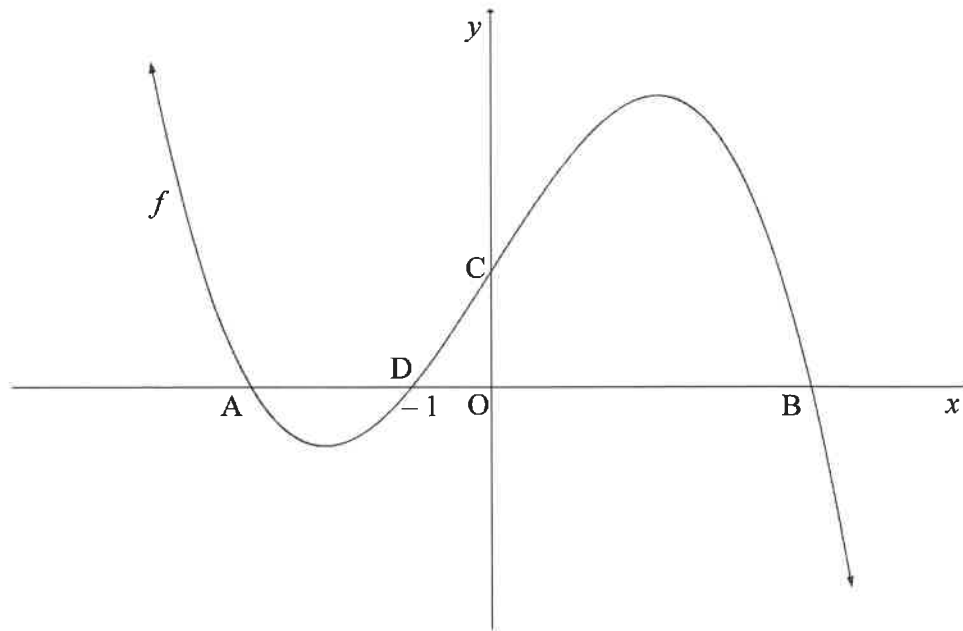
TOTAL: 150

QUESTION 8

The graph of $f(x) = -x^3 + 13x + 12$ is sketched below.

A, B and D(-1 ; 0) are the x -intercepts of f .

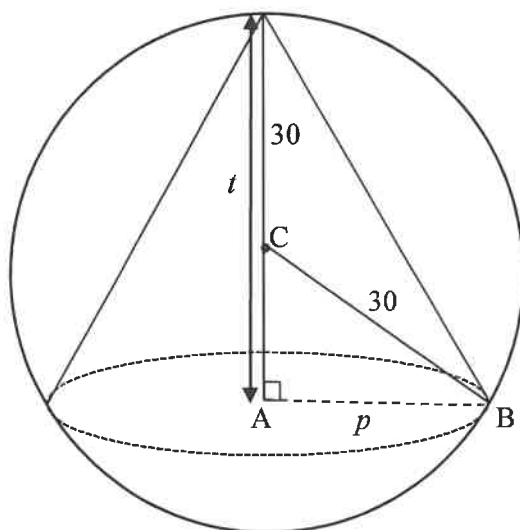
C is the y -intercept of f .



- 8.1 Write down the coordinates of C. (1)
- 8.2 Calculate the coordinates of A and B. (5)
- 8.3 Determine the point of inflection of g if it is given that $g(x) = -f(x)$. (4)
- 8.4 Calculate the value(s) of x for which the tangent to f is parallel to the line $y = -14x + c$. (4)
- [14]

QUESTION 9

A right circular cone with radius p and height t is machined (cut out) from a solid sphere (with centre C) with a radius of 30 cm, as shown in the sketch.



$$\text{Sphere: } V = \frac{4}{3}\pi r^3$$

$$\text{Cone: } V = \frac{1}{3}\pi r^2 h$$

- 9.1 From the given information, express the following:
- 9.1.1 AC in terms of t . (1)
- 9.1.2 p^2 , in its simplest form, in terms of t . (3)
- 9.2 Show that the volume of the cone can be written as $V(t) = 20\pi t^2 - \frac{1}{3}\pi t^3$. (1)
- 9.3 Calculate the value of t for which the volume of the cone will be a maximum. (3)
- 9.4 What percentage of the sphere was used to obtain this cone having maximum volume? (4)

[12]

QUESTION 8

Given: $f(x) = x(x-3)^2$ with $f'(1) = f'(3) = 0$ and $f(1) = 4$

8.1 Show that f has a point of inflection at $x = 2$. (5)

8.2 Sketch the graph of f , clearly indicating the intercepts with the axes and the turning points. (4)

8.3 For which values of x will $y = -f(x)$ be concave down? (2)

8.4 Use your graph to answer the following questions:

8.4.1 Determine the coordinates of the local maximum of h if $h(x) = f(x-2) + 3$. (2)

8.4.2 Claire claims that $f'(2) = 1$.

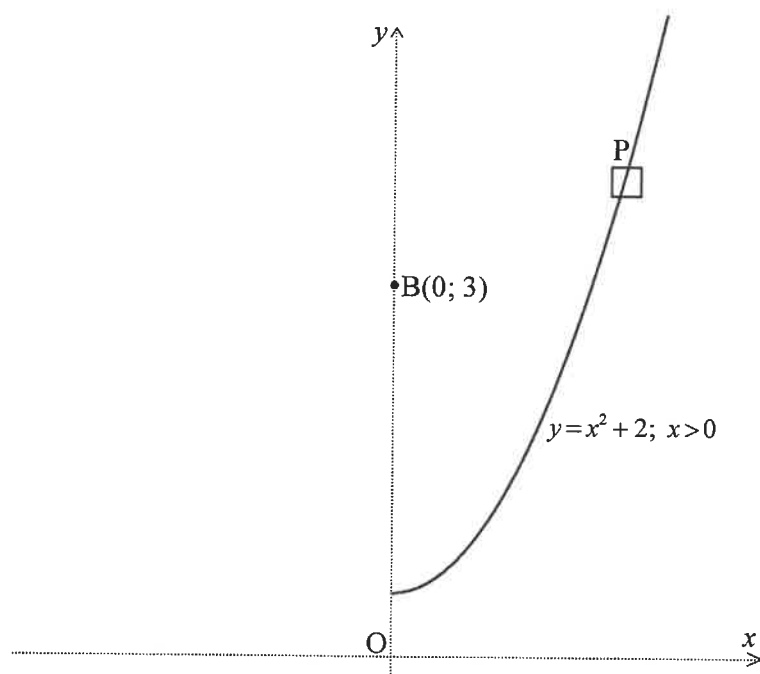
Do you agree with Claire? Justify your answer.

(2)
[15]

QUESTION 9

An aerial view of a stretch of road is shown in the diagram below. The road can be described by the function $y = x^2 + 2$, $x \geq 0$ if the coordinate axes (dotted lines) are chosen as shown in the diagram.

Benny sits at a vantage point $B(0; 3)$ and observes a car, P, travelling along the road.



Calculate the distance between Benny and the car, when the car is closest to Benny.

[7]

QUESTION 7

- 7.1 A company bought a new machine for R500 000. After 3 years, the machine has a book value of R331 527. Calculate the yearly rate of depreciation if the machine was depreciated according to the reducing-balance method. (3)
- 7.2 Musa takes a personal loan from a bank to buy a motorcycle that costs R46 000. The bank charges interest at 24% per annum, compounded monthly.
- How many months will it take Musa to repay the loan, if the monthly instalment is R1 900? (4)
- 7.3 Neil set up an investment fund. Exactly 3 months later and every 3 months thereafter he deposited R3 500 into the fund. The fund pays interest at 7,5% p.a., compounded quarterly. He continued to make quarterly deposits into the fund for $6\frac{1}{2}$ years from the time that he originally set up the fund.
- Neil made no further deposits into the fund, but left the money in the same fund at the same rate of interest. Calculate how much he will have in the fund 10 years after he originally set it up. (6)
- [13]**

QUESTION 8

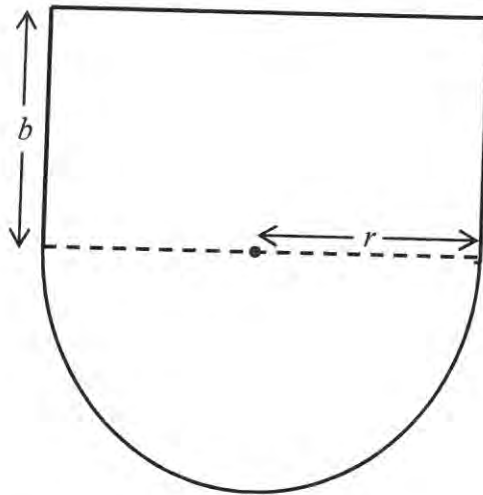
- 8.1 Given $f(x) = 3 - 2x^2$. Determine $f'(x)$, using first principles. (5)
- 8.2 Determine $\frac{dy}{dx}$ if $y = \frac{12x^2 + 2x + 1}{6x}$. (4)
- 8.3 The function $f(x) = x^3 + bx^2 + cx - 4$ has a point of inflection at (2 ; 4). Calculate the values of b and c . (7)
- [16]**

QUESTION 9

Given: $f(x) = x^3 - x^2 - x + 1$

- 9.1 Write down the coordinates of the y -intercept of f . (1)
- 9.2 Calculate the coordinates of the x -intercepts of f . (5)
- 9.3 Calculate the coordinates of the turning points of f . (6)
- 9.4 Sketch the graph of f in your ANSWER BOOK. Clearly indicate all intercepts with the axes and the turning points. (3)
- 9.5 Write down the values of x for which $f'(x) < 0$. (2)
- [17]**

QUESTION 10



The figure above shows the design of a theatre stage which is in the shape of a semicircle attached to a rectangle. The semicircle has a radius r and the rectangle has a breadth b . The perimeter of the stage is 60 m.

- 10.1 Determine an expression for b in terms of r . (2)
- 10.2 For which value of r will the area of the stage be a maximum? (6)
- [8]

QUESTION 8

Given: $f(x) = 2x^3 - 5x^2 + 4x$

8.1 Calculate the coordinates of the turning points of the graph of f . (5)

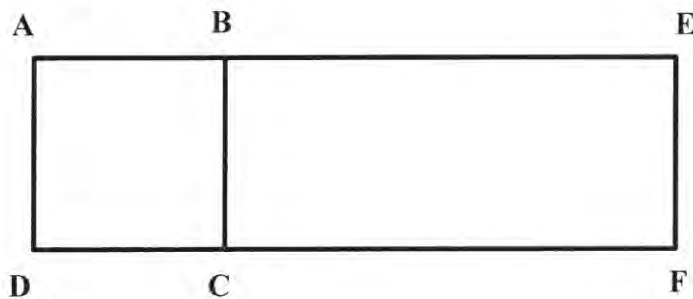
8.2 Prove that the equation $2x^3 - 5x^2 + 4x = 0$ has only one real root. (3)

8.3 Sketch the graph of f , clearly indicating the intercepts with the axes and the turning points. (3)

8.4 For which values of x will the graph of f be concave up? (3)
[14]

QUESTION 9

A piece of wire 6 metres long is cut into two pieces. One piece, x metres long, is bent to form a square ABCD. The other piece is bent into a U-shape so that it forms a rectangle BEFC when placed next to the square, as shown in the diagram below.



Calculate the value of x for which the sum of the areas enclosed by the wire will be a maximum. [7]

QUESTION 8

- 8.1 Determine $f'(x)$ from first principles if $f(x) = 3x^2$ (5)
- 8.2 John determines $g'(a)$, the derivative of a certain function g at $x = a$, and arrives at the answer: $\lim_{h \rightarrow 0} \frac{\sqrt{4+h} - 2}{h}$
Write down the equation of g and the value of a . (2)
- 8.3 Determine $\frac{dy}{dx}$ if $y = \sqrt{x^3} - \frac{5}{x^3}$ (4)
- 8.4 $g(x) = -8x + 20$ is a tangent to $f(x) = x^3 + ax^2 + bx + 18$ at $x = 1$. Calculate the values of a and b . (5)
[16]

QUESTION 9

For a certain function f , the first derivative is given as $f'(x) = 3x^2 + 8x - 3$

- 9.1 Calculate the x -coordinates of the stationary points of f . (3)
- 9.2 For which values of x is f concave down? (3)
- 9.3 Determine the values of x for which f is strictly increasing. (2)
- 9.4 If it is further given that $f(x) = ax^3 + bx^2 + cx + d$ and $f(0) = -18$, determine the equation of f . (5)
[13]

QUESTION 10

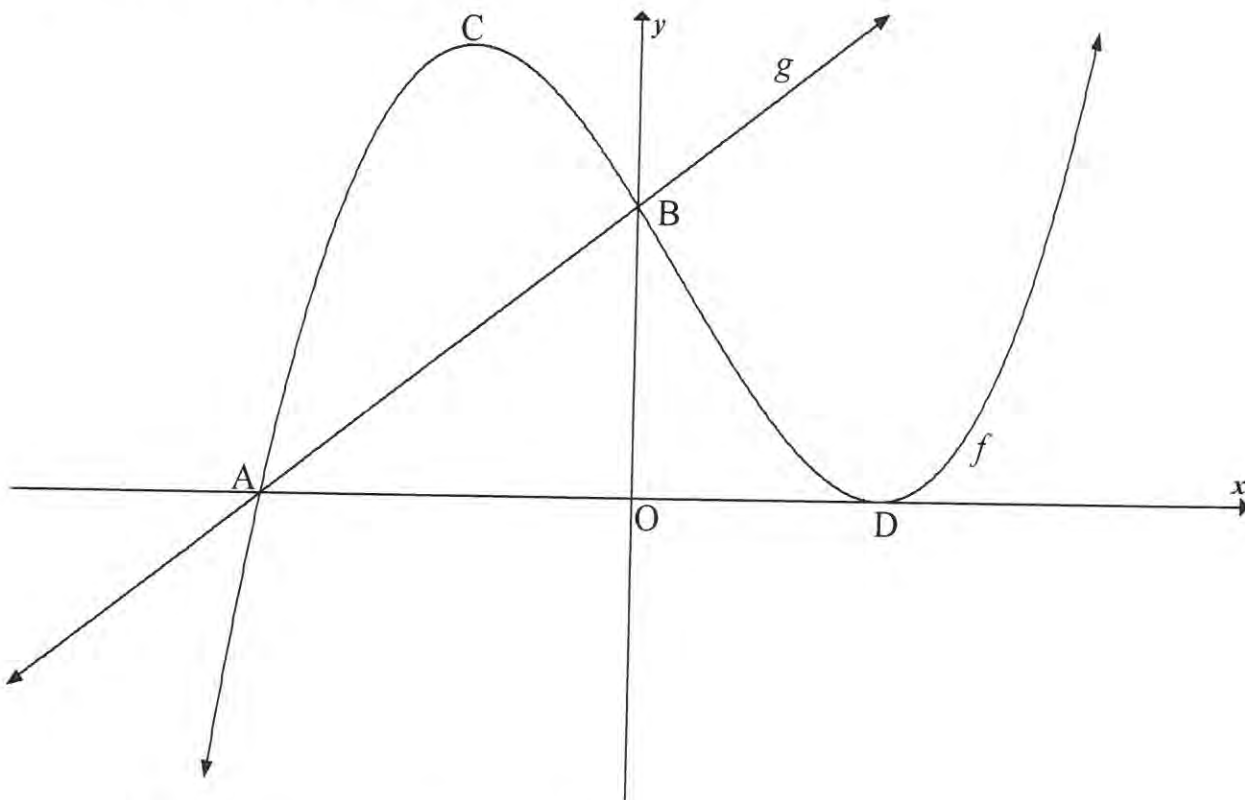
The number of molecules of a certain drug in the bloodstream t hours after it has been taken is represented by the equation $M(t) = -t^3 + 3t^2 + 72t$, $0 < t < 10$.

- 10.1 Determine the number of molecules of the drug in the bloodstream 3 hours after the drug was taken. (2)
- 10.2 Determine the rate at which the number of molecules of the drug in the bloodstream is changing at exactly 2 hours after the drug was taken. (3)
- 10.3 How many hours after taking the drug will the rate at which the number of molecules of the drug in the bloodstream is changing, be a maximum? (3)
[8]

QUESTION 8

Sketched below are the graphs of $f(x) = (x-2)^2(x-k)$ and $g(x) = mx + 12$

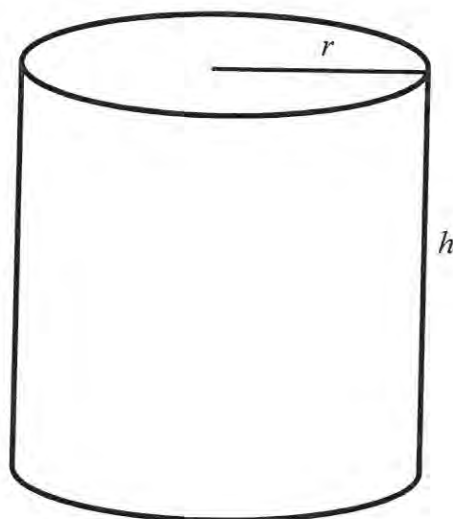
- A and D are the x -intercepts of f .
- B is the common y -intercept of f and g .
- C and D are turning points of f .
- The straight line g passes through A.



- 8.1 Write down the y -coordinate of B. (1)
- 8.2 Calculate the x -coordinate of A. (3)
- 8.3 If $k = -3$, calculate the coordinates of C. (6)
- 8.4 For which values of x will f be concave down? (3)
- [13]**

QUESTION 9

A 340 ml can with height h cm and radius r cm is shown below.



$$1 \text{ ml} = 1 \text{ cm}^3$$

- 9.1 Determine the height of the can in terms of the radius r . (3)
- 9.2 Calculate the length of the radius of the can, in cm, if the surface area is to be a minimum. (6)
- [9]

QUESTION 10

- 10.1 A tournament organiser conducted a survey among 150 members at a local sports club to find out whether they play tennis or not. The results are shown in the table below.

	PLAYING TENNIS	NOT PLAYING TENNIS
Male	50	30
Female	20	50

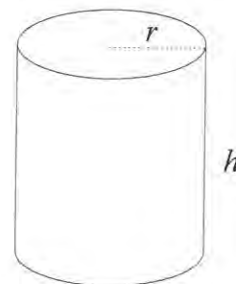
- 10.1.1 What is the probability that a member selected at random is:
- (a) Female (2)
- (b) Female and plays tennis (1)
- 10.1.2 Is playing tennis independent of gender? Motivate your answer with the necessary calculations. (3)

QUESTION 8

- 8.1 Determine $f'(x)$ from first principles if $f(x) = -x^2 + 4$. (5)
- 8.2 Determine the derivative of:
- 8.2.1 $y = 3x^2 + 10x$ (2)
- 8.2.2 $f(x) = \left(x - \frac{3}{x}\right)^2$ (3)
- 8.3 Given: $f(x) = 2x^3 - 23x^2 + 80x - 84$
- 8.3.1 Prove that $(x - 2)$ is a factor of f . (2)
- 8.3.2 Hence, or otherwise, factorise $f(x)$ fully. (2)
- 8.3.3 Determine the x -coordinates of the turning points of f . (4)
- 8.3.4 Sketch the graph of f , clearly labelling ALL turning points and intercepts with the axes. (3)
- 8.3.5 Determine the coordinates of the y -intercept of the tangent to f that has a slope of 40 and touches f at a point where the x -coordinate is an integer. (6)
- [27]**

QUESTION 9

A soft drink can has a volume of 340 cm^3 , a height of $h \text{ cm}$ and a radius of $r \text{ cm}$.



- 9.1 Express h in terms of r . (2)
- 9.2 Show that the surface area of the can is given by $A(r) = 2\pi r^2 + 680r^{-1}$. (2)
- 9.3 Determine the radius of the can that will ensure that the surface area is a minimum. (4)
- [8]**

QUESTION 8

8.1 If $f(x) = x^2 - 3x$, determine $f'(x)$ from first principles. (5)

8.2 Determine:

8.2.1 $\frac{dy}{dx}$ if $y = \left(x^2 - \frac{1}{x^2}\right)^2$ (3)

8.2.2 $D_x \left(\frac{x^3 - 1}{x - 1} \right)$ (3)
[11]

QUESTION 9

Given: $h(x) = -x^3 + ax^2 + bx$ and $g(x) = -12x$. P and Q(2 ; 10) are the turning points of h . The graph of h passes through the origin.

9.1 Show that $a = \frac{3}{2}$ and $b = 6$. (5)

9.2 Calculate the average gradient of h between P and Q, if it is given that $x = -1$ at P. (4)

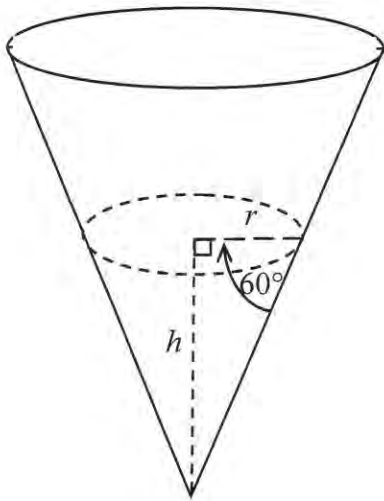
9.3 Show that the concavity of h changes at $x = \frac{1}{2}$. (3)

9.4 Explain the significance of the change in QUESTION 9.3 with respect to h . (1)

9.5 Determine the value of x , given $x < 0$, at which the tangent to h is parallel to g . (4)
[17]

QUESTION 10

A rain gauge is in the shape of a cone. Water flows into the gauge. The height of the water is h cm when the radius is r cm. The angle between the cone edge and the radius is 60° , as shown in the diagram below.



Formulae for volume:

$$V = \pi r^2 h \qquad V = \frac{1}{3} \pi r^2 h$$

$$V = lbh \qquad V = \frac{4}{3} \pi r^3$$

- 10.1 Determine r in terms of h . Leave your answer in surd form. (2)
- 10.2 Determine the derivative of the volume of water with respect to h when h is equal to 9 cm. (5)
- [7]

QUESTION 8

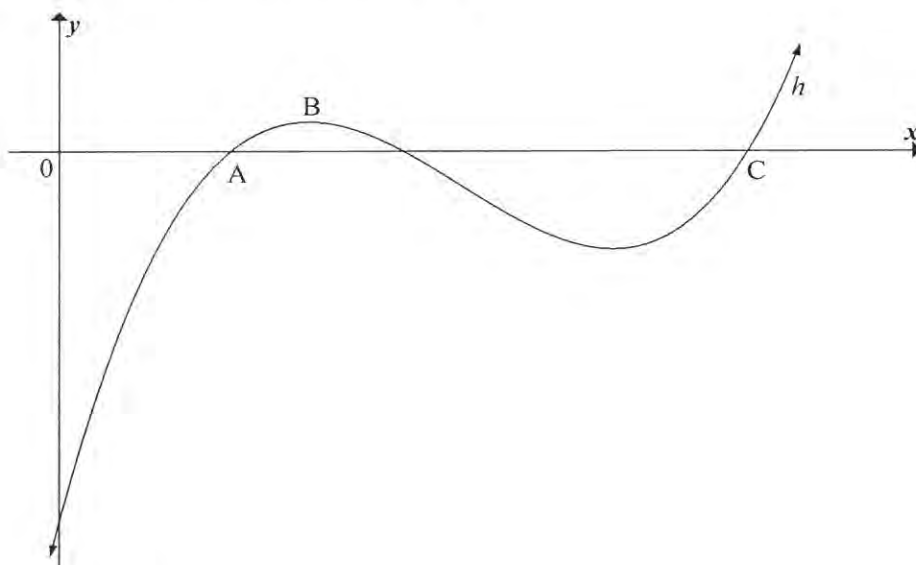
8.1 Determine the derivative of $f(x) = 2x^2 + 4$ from first principles. (4)

8.2 Differentiate:

8.2.1 $f(x) = -3x^2 + 5\sqrt{x}$ (3)

8.2.2 $p(x) = \left(\frac{1}{x^3} + 4x\right)^2$ (4)

8.3 The sketch below shows the graph of $h(x) = x^3 - 7x^2 + 14x - 8$. The x -coordinate of point A is 1. C is another x -intercept of h .



8.3.1 Determine $h'(x)$. (1)

8.3.2 Determine the x -coordinate of the turning point B. (3)

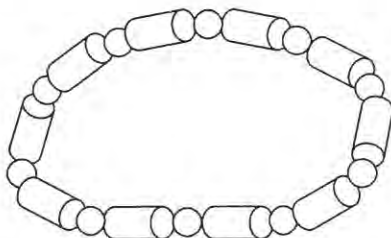
8.3.3 Calculate the coordinates of C. (4)

8.3.4 The graph of h is concave down for $x < k$. Calculate the value of k . (3)

[22]

QUESTION 9

A necklace is made by using 10 wooden spheres and 10 wooden cylinders. The radii, r , of the spheres and the cylinders are exactly the same. The height of each cylinder is h . The wooden spheres and cylinders are to be painted. (Ignore the holes in the spheres and cylinders.)



$$V = \pi r^2 h \quad S = 2\pi r^2 + 2\pi r h$$

$$V = \frac{4}{3} \pi r^3 \quad S = 4\pi r^2$$

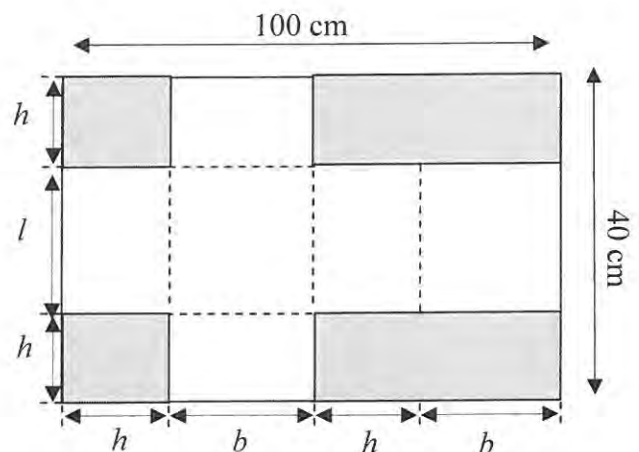
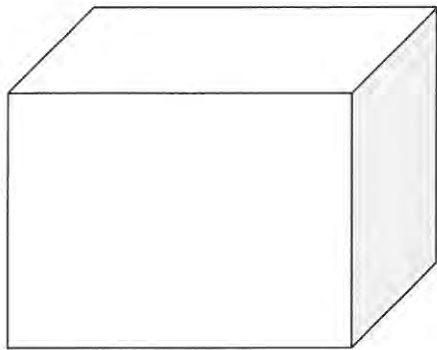
- 9.1 If the volume of a cylinder is 6 cm^3 , write h in terms of r . (1)
- 9.2 Show that the total surface area (S) of all the painted surfaces of the necklace is equal to $S = 60\pi r^2 + \frac{120}{r}$ (4)
- 9.3 Determine the value of r so that the least amount of paint will be used. (4)

[9]

QUESTION 9

Given: $f(x) = (x + 2)(x^2 - 6x + 9)$
 $= x^3 - 4x^2 - 3x + 18$

- 9.1 Calculate the coordinates of the turning points of the graph of f . (6)
- 9.2 Sketch the graph of f , clearly indicating the intercepts with the axes and the turning points. (4)
- 9.3 For which value(s) of x will $x \cdot f'(x) < 0$? (3)
- [13]**

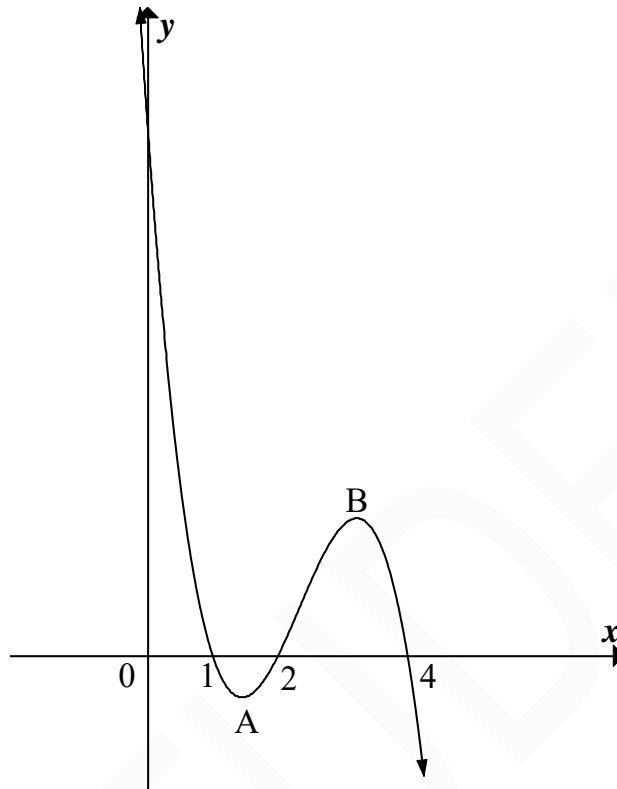
QUESTION 10

A box is made from a rectangular piece of cardboard, 100 cm by 40 cm, by cutting out the shaded areas and folding along the dotted lines as shown in the diagram above.

- 10.1 Express the length l in terms of the height h . (1)
- 10.2 Hence prove that the volume of the box is given by $V = h(50 - h)(40 - 2h)$ (3)
- 10.3 For which value of h will the volume of the box be a maximum? (5)
- [9]**

QUESTION 11

The graph of $f(x) = -x^3 + ax^2 + bx + c$ is sketched below. The x -intercepts are indicated.



- 11.1 Calculate the values of a , b and c . (4)
- 11.2 Calculate the x -coordinates of A and B, the turning points of f . (5)
- 11.3 For which values of x will $f'(x) < 0$? (3)
- [12]**

QUESTION 12

A small business currently sells 40 watches per year. Each of the watches is sold at R144. For each yearly price increase of R4 per watch, there is a drop in sales of one watch per year.

- 12.1 How many watches are sold x years from now? (1)
- 12.2 Determine the annual income from the sale of watches in terms of x . (3)
- 12.3 In what year and at what price should the watches be sold in order for the business to obtain a maximum income from the sale of watches? (4)
- [8]**

QUESTION 13

A sweet factory produces two types of toffees, Taffy and Chewy, and stores them in containers.

- The quantities of butter and sugar (in kilograms) used in each container of toffies are as follows:
 - Taffy toffees contain 40 kg of butter and 64 kg of sugar for every container of toffees.
 - Chewy toffees contain 50 kg of butter and 40 kg of sugar for every container of toffees.
- Each week, the factory has a maximum of 2 000 kg of butter and 2 560 kg of sugar available.
- The factory must produce at least 15 containers of Taffy toffees per week.

Let x and y be the number of containers of Taffy and Chewy toffees produced each week, respectively.

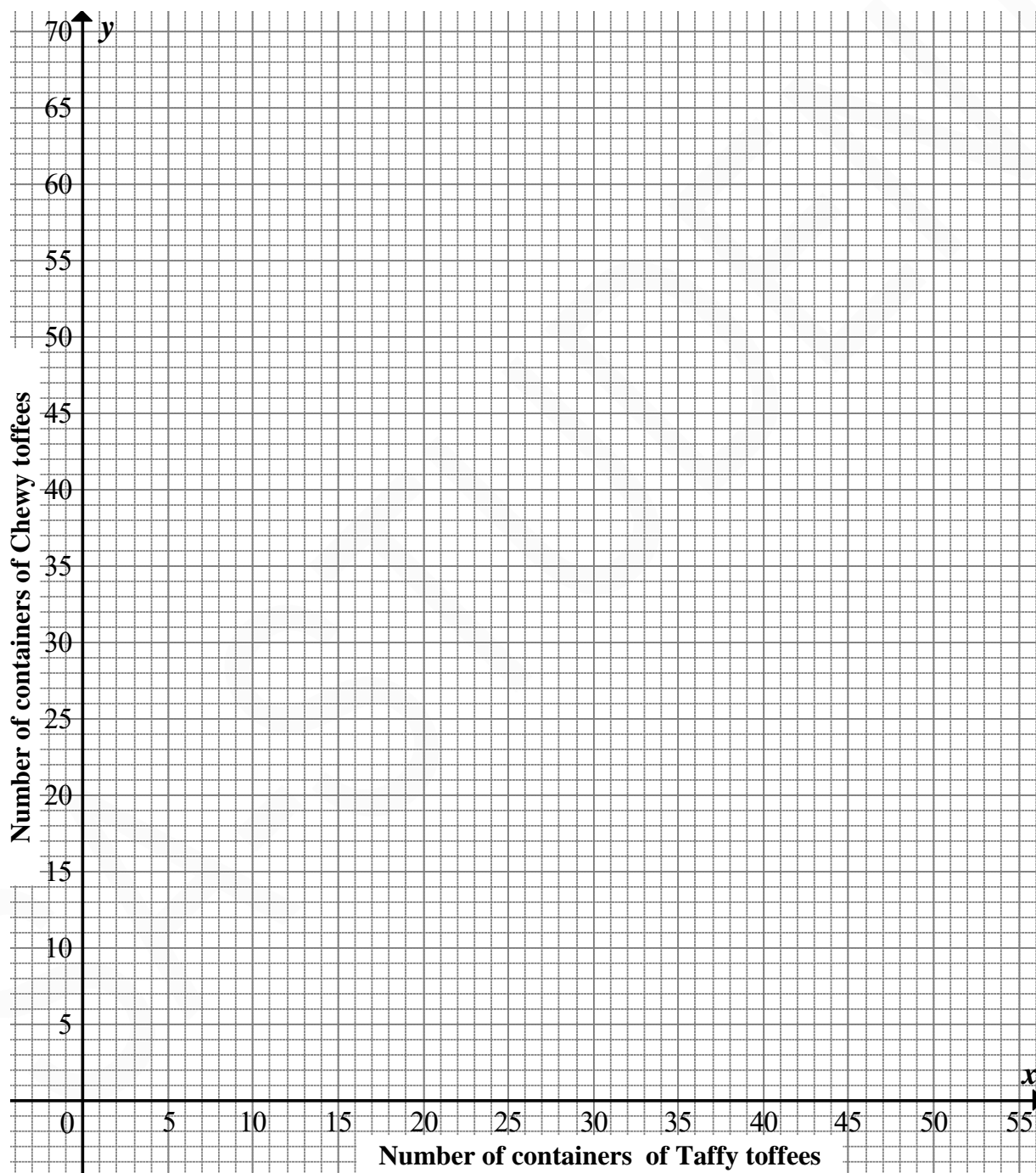
- | | | |
|------|--|-------------|
| 13.1 | Write down all the constraints which describe the production process above. | (5) |
| 13.2 | Sketch the system of constraints (inequalities) on the graph paper on DIAGRAM SHEET 1, clearly indicating the feasible region. | (4) |
| 13.3 | Write down the maximum number of containers of Taffy toffees that can be produced under these conditions. | (2) |
| 13.4 | If the profit earned per week by the factory is R1 400 per container of Taffy toffees and R1 000 per container of Chewy toffees, what amount of each type of toffee needs to be produced in order to make a maximum profit per week? | (3) |
| | | [14] |
| | TOTAL: | 150 |

CENTRE NUMBER:

--	--	--	--	--	--	--	--

EXAMINATION NUMBER:

--	--	--	--	--	--	--	--	--	--	--	--	--

DIAGRAM SHEET 1**QUESTION 13.2**

QUESTION 9

Given: $f(x) = x^3 - 4x^2 - 11x + 30$.

- 9.1 Use the fact that $f(2) = 0$ to write down a factor of $f(x)$. (1)
- 9.2 Calculate the coordinates of the x -intercepts of f . (4)
- 9.3 Calculate the coordinates of the stationary points of f . (5)
- 9.4 Sketch the curve of f in your ANSWER BOOK. Show all intercepts with the axes and turning points clearly. (3)
- 9.5 For which value(s) of x will $f'(x) < 0$? (2)
- [15]**

QUESTION 10

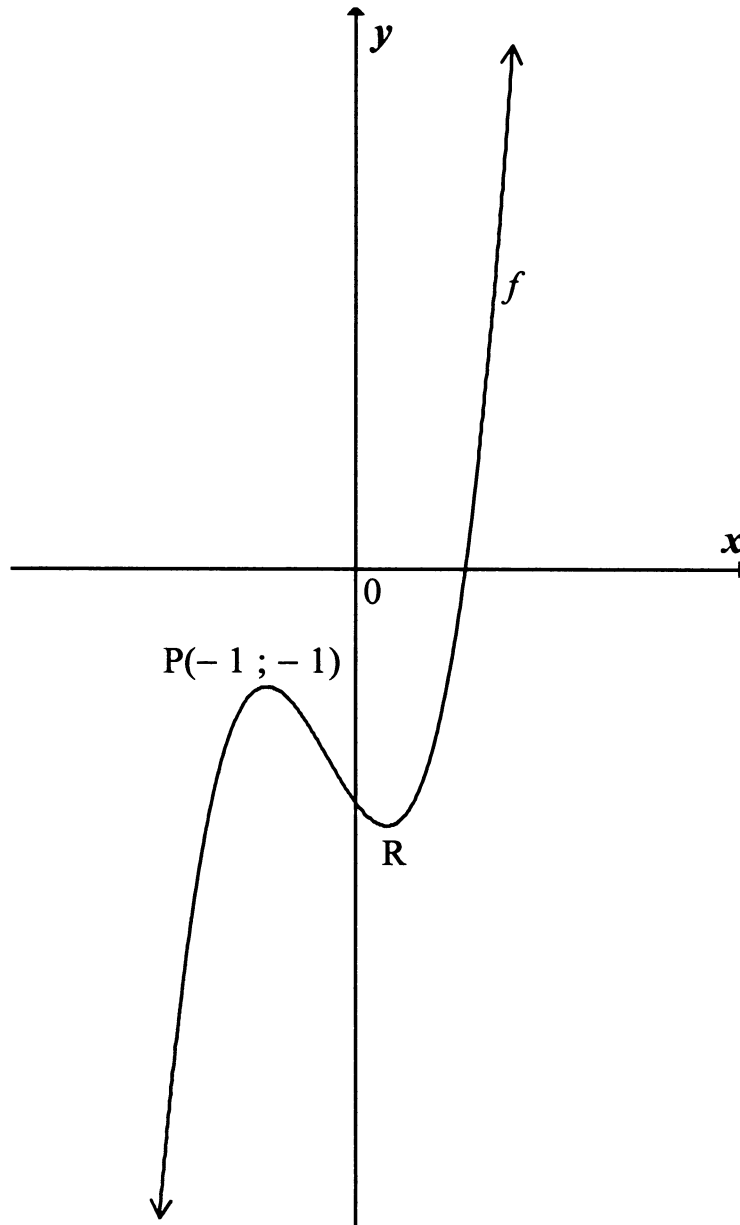
Two cyclists start to cycle at the same time. One starts at point B and is heading due north to point A, whilst the other starts at point D and is heading due west to point B. The cyclist starting from B cycles at 30 km/h while the cyclist starting from D cycles at 40 km/h. The distance between B and D is 100 km. After time t (measured in hours), they reach points F and C respectively.



- 10.1 Determine the distance between F and C in terms of t . (4)
- 10.2 After how long will the two cyclists be closest to each other? (4)
- 10.3 What will the distance between the cyclists be at the time determined in QUESTION 10.2? (2)
- [10]**

QUESTION 9

The function defined by $f(x) = x^3 + ax^2 + bx - 2$ is sketched below.
 $P(-1; -1)$ and R are the turning points of f .



- 9.1 Show that $a = 1$ and $b = -1$. (6)
- 9.2 Hence, or otherwise, determine the x -coordinate of R . (3)
- 9.3 Write down the coordinates of a turning point of h if h is defined by $h(x) = 2f(x) - 4$. (2)
- [11]

QUESTION 10

An industrial process requires water to flow through its system as part of the cooling cycle. Water flows continuously through the system for a certain period of time.

The relationship between the time (t) from when the water starts flowing and the rate (r) at which the water is flowing through the system is given by the equation:

$$r = -0,2t^2 + 10t$$

where t is measured in seconds.

- 10.1 After how long will the water be flowing at the maximum rate? (3)
- 10.2 After how many seconds does the water stop flowing? (3)
- [6]**

QUESTION 11

A company manufactures both short-sleeved shirts and long-sleeved shirts. The constraints below govern the production of the shirts per day.

- No more than 80 short-sleeved shirts can be produced per day.
- A minimum of 50 long-sleeved shirts must be produced per day.
- At most 5 long-sleeved shirts must be manufactured for every short-sleeved shirt.
- Each short-sleeved shirt has 5 buttons and each long-sleeved shirt has 4 buttons.
- At most 800 buttons are available for production per day.

Let the number of short-sleeved shirts be x and the number of long-sleeved shirts be y .

- 11.1 Write down the constraints which govern this system. (4)
- 11.2 Sketch the system of constraints (inequalities) on the graph paper on DIAGRAM SHEET 1. Clearly indicate the feasible region. (5)
- 11.3 A profit of R30 is made on each short-sleeved shirt and a profit of R20 is made on each long-sleeved shirt.
- 11.3.1 Write down the profit function. (1)
- 11.3.2 Determine the number of short-sleeved shirts and long-sleeved shirts that must be manufactured per day to provide the company with maximum profit. (2)
- 11.4 If the objective profit function is given by $P = ax + by$, determine $\frac{a}{b}$ if P is maximised at each value of y between 100 and 160. (2)
- [14]**

TOTAL: 150



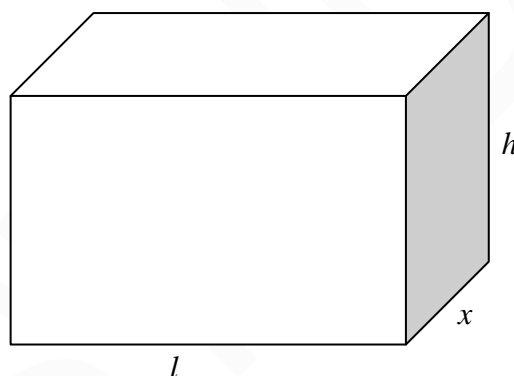
QUESTION 10

Given: $f(x) = -x^3 - x^2 + x + 10$

- 10.1 Write down the coordinates of the y-intercept of f . (1)
- 10.2 Show that (2 ; 0) is the only x-intercept of f . (4)
- 10.3 Calculate the coordinates of the turning points of f . (6)
- 10.4 Sketch the graph of f in your ANSWER BOOK. Show all intercepts with the axes and all turning points. (3)
- [14]**

QUESTION 11

A rectangular box is constructed in such a way that the length (l) of the base is three times as long as its width. The material used to construct the top and the bottom of the box costs R100 per square metre. The material used to construct the sides of the box costs R50 per square metre. The box must have a volume of 9 m^3 . Let the width of the box be x metres.



- 11.1 Determine an expression for the height (h) of the box in terms of x . (3)
- 11.2 Show that the cost to construct the box can be expressed as $C = \frac{1200}{x} + 600x^2$. (3)
- 11.3 Calculate the width of the box (that is the value of x) if the cost is to be a minimum. (4)
- [10]**

QUESTION 12

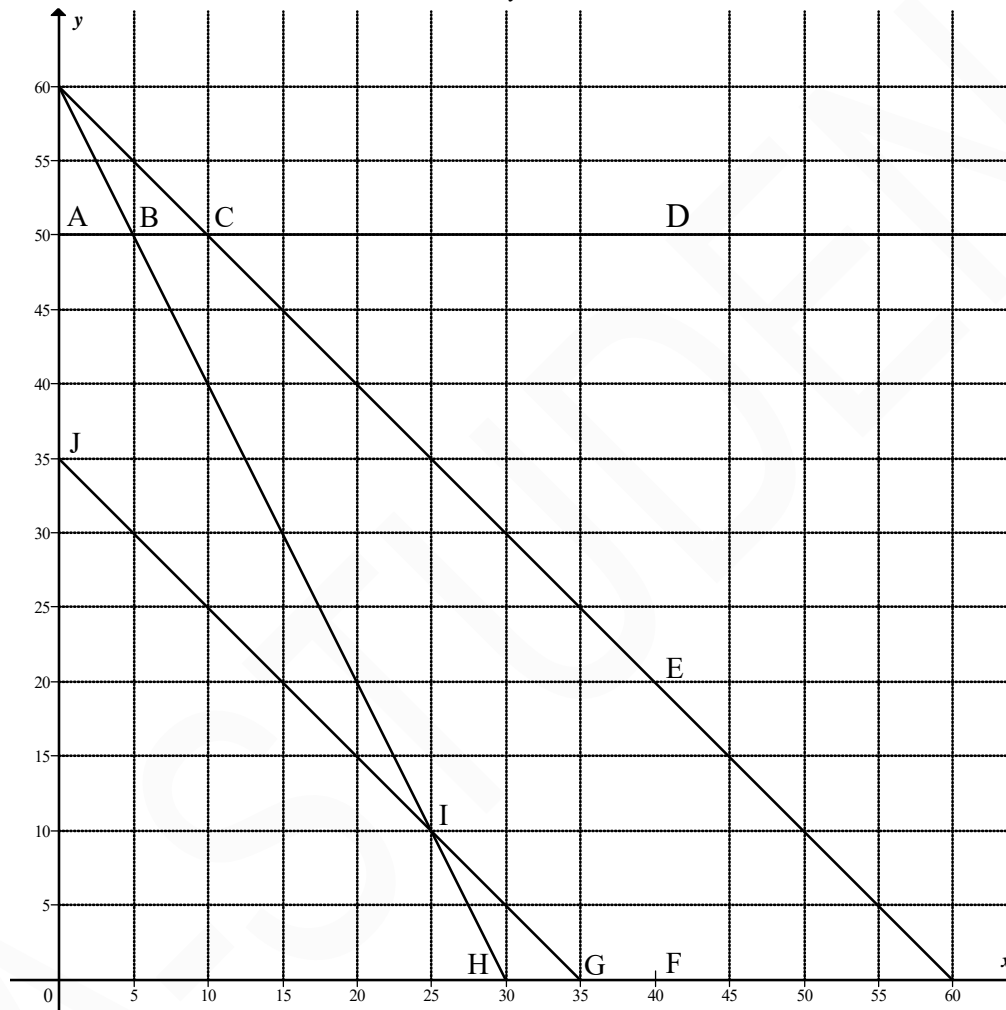
A system of constraints is given below. Their boundary lines are represented graphically in the sketch below. The diagram is reproduced on DIAGRAM SHEET 1. The constraints are:

$$0 \leq y \leq 50$$

$$0 \leq x \leq 40$$

$$2x + y \leq 60$$

$$35 \leq x + y \leq 60$$



- 12.1 Shade the feasible region on DIAGRAM SHEET 1. (2)
- 12.2 Indicate which constraints have no influence on the feasible region. (3)
- 12.3 What is the maximum value of x allowed by these constraints? (1)
- 12.4 If $P = 4x + y$ for $(x ; y)$ in the feasible region, determine the maximum value of P . (4)
- 12.5 If the objective function $C = kx + y$ is minimised at J only, determine ALL possible values of k . (2)

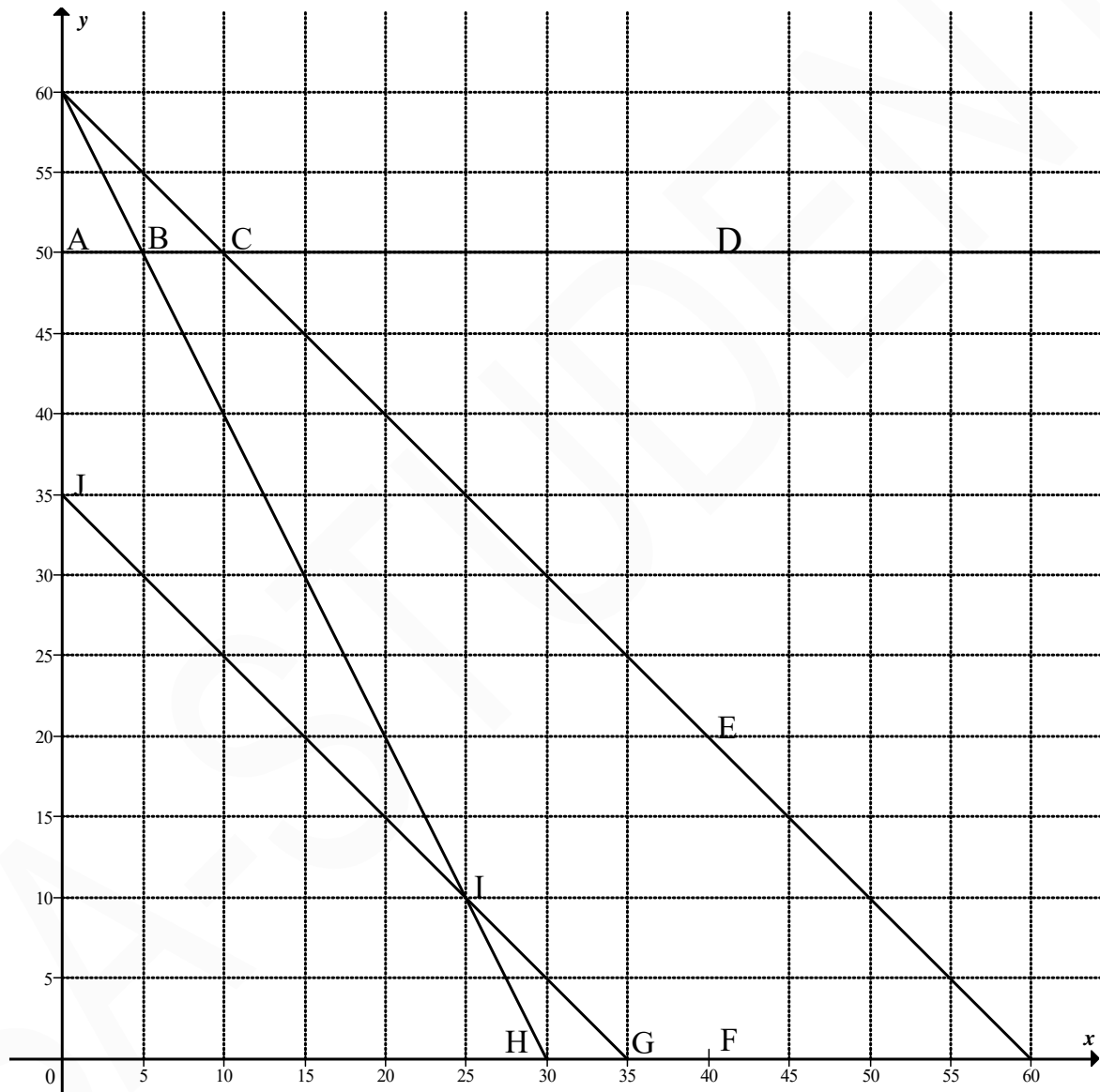
[12]**TOTAL: 150**

CENTRE NUMBER:

--	--	--	--	--	--	--	--

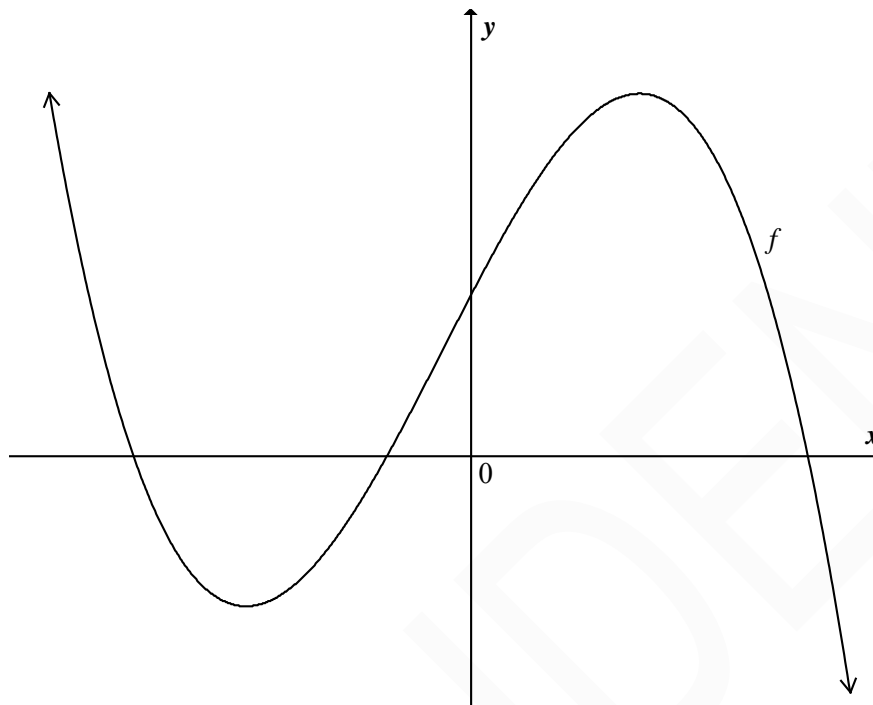
EXAMINATION NUMBER:

--	--	--	--	--	--	--	--	--	--	--	--	--

DIAGRAM SHEET 1**QUESTION 12.1**

QUESTION 9

9.1 The graph of the function $f(x) = -x^3 - x^2 + 16x + 16$ is sketched below.



9.1.1 Calculate the x -coordinates of the turning points of f . (4)

9.1.2 Calculate the x -coordinate of the point at which $f'(x)$ is a maximum. (3)

9.2 Consider the graph of $g(x) = -2x^2 - 9x + 5$.

9.2.1 Determine the equation of the tangent to the graph of g at $x = -1$. (4)

9.2.2 For which values of q will the line $y = -5x + q$ not intersect the parabola? (3)

9.3 Given: $h(x) = 4x^3 + 5x$

Explain if it is possible to draw a tangent to the graph of h that has a negative gradient. Show ALL your calculations. (3)

[17]

QUESTION 10

A particle moves along a straight line. The distance, s , (in metres) of the particle from a fixed point on the line at time t seconds ($t \geq 0$) is given by $s(t) = 2t^2 - 18t + 45$.

10.1 Calculate the particle's initial velocity. (Velocity is the rate of change of distance.) (3)

10.2 Determine the rate at which the velocity of the particle is changing at t seconds. (1)

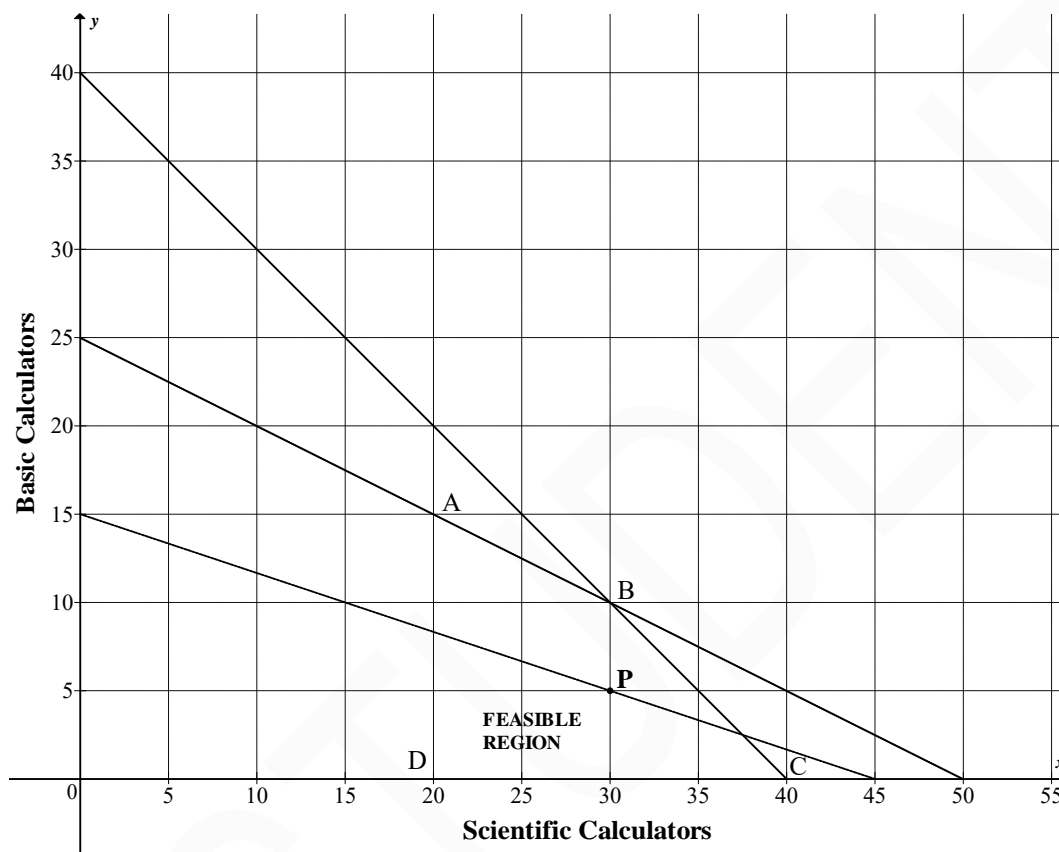
10.3 After how many seconds will the particle be closest to the fixed point? (2)

[6]

QUESTION 11

A calculator company manufactures two kinds of calculators: scientific and basic. The company is able to sell all the calculators that it produces. A system of constraints has been developed for the production of the calculators. The feasible region is shaded below.

Let x and y respectively be the number of scientific and basic calculators produced each day.



- 11.1 Is it possible for the company to manufacture 15 scientific calculators and 5 basic calculators in one day according to their system of constraints? Motivate your answer. (1)
- 11.2 Write down all the algebraic inequalities which describe the constraints related to the manufacturing of the calculators. (6)
- 11.3 The profit Q (in hundreds of rands) is given by $Q = x + 3y$. The dotted line on the graph is a search line associated with the profit function.
- 11.3.1 Identify the point in the region where the profit is a maximum. Use only A, B, C or D. (1)
- 11.3.2 Write down the coordinates of a point on the dotted line (if the point exists) at which the profit is greater than the profit at P. (2)
- 11.3.3 Given that the profit, when given by $Q = ax + by$ ($a > 0$; $b > 0$), is a maximum at B, determine the maximum value of $\frac{a}{b}$. (4)

[14]**TOTAL: 150**

QUESTION 8

8.1 Determine $f'(x)$ from first principles if $f(x) = 9 - x^2$. (5)

8.2 Evaluate:

8.2.1 $D_x[1 + 6\sqrt{x}]$ (2)

8.2.2 $\frac{dy}{dx}$ if $y = \frac{8 - 3x^6}{8x^5}$ (4)
[11]

QUESTION 9

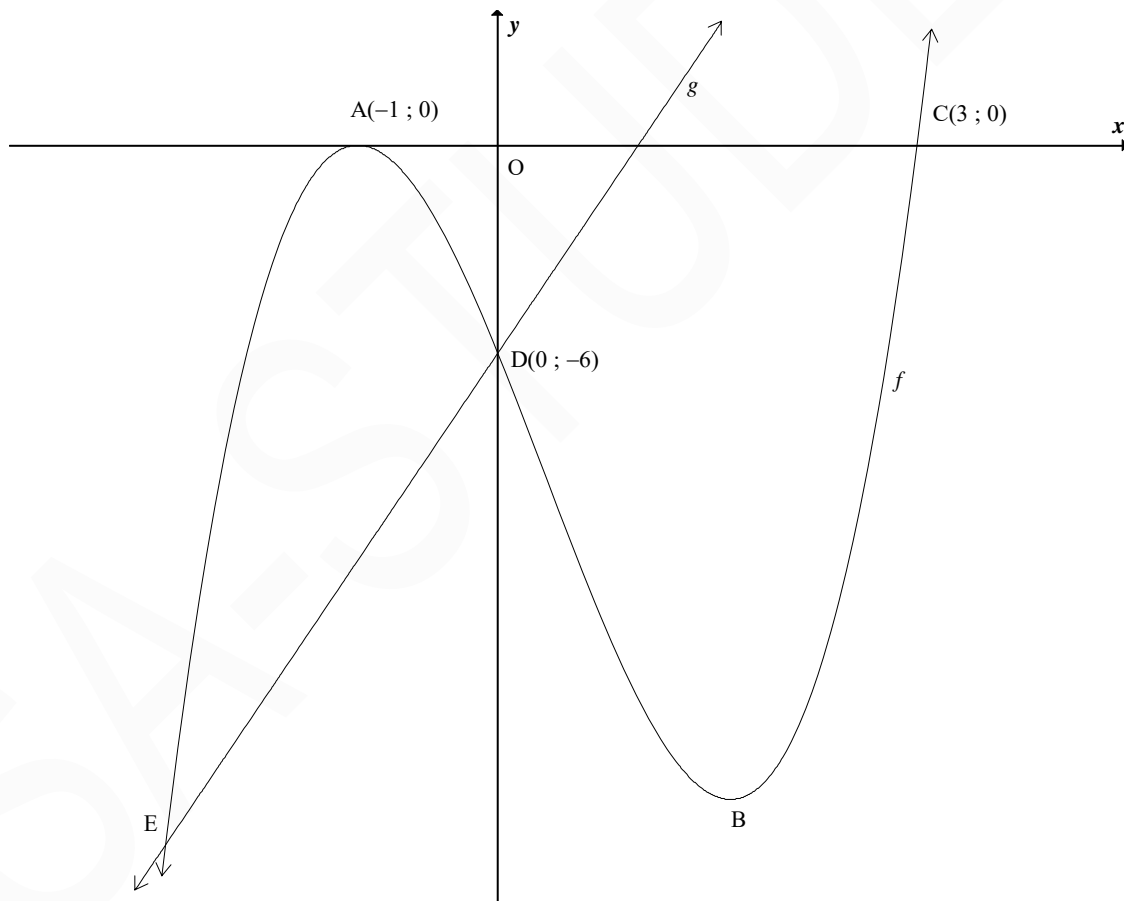
The graphs of $f(x) = ax^3 + bx^2 + cx + d$ and $g(x) = 6x - 6$ are sketched below.

$A(-1; 0)$ and $C(3; 0)$ are the x -intercepts of f .

The graph of f has turning points at A and B.

$D(0; -6)$ is the y -intercept of f .

E and D are points of intersection of the graphs of f and g .



9.1 Show that $a = 2$; $b = -2$; $c = -10$ and $d = -6$. (5)

9.2 Calculate the coordinates of the turning point B. (5)

9.3 $h(x)$ is the vertical distance between $f(x)$ and $g(x)$, that is $h(x) = f(x) - g(x)$.
Calculate x such that $h(x)$ is a maximum, where $x < 0$. (5)
[15]

QUESTION 10

The tangent to the curve of $g(x) = 2x^3 + px^2 + qx - 7$ at $x = 1$ has the equation $y = 5x - 8$.

10.1 Show that $(1 ; -3)$ is the point of contact of the tangent to the graph. (1)

10.2 Hence or otherwise, calculate the values of p and q . (6)
[7]

QUESTION 11

A cubic function f has the following properties:

- $f\left(\frac{1}{2}\right) = f(3) = f(-1) = 0$
- $f'(2) = f'\left(-\frac{1}{3}\right) = 0$
- f decreases for $x \in \left[-\frac{1}{3}; 2\right]$ only

Draw a possible sketch graph of f , clearly indicating the x -coordinates of the turning points and ALL the x -intercepts. [4]

QUESTION 12

A furniture factory produces small tables and large tables. The tables undergo sanding and/or painting processes.

- The factory can produce at most 100 tables in total per week.
- At most 50 hours are available for painting the tables per week and at most 180 hours are available for sanding the tables per week.
- A small table requires 1 hour for painting and 1 hour for sanding.
- A large table requires NO painting and 2 hours for sanding.

Let the number of small tables produced per week be x , and let the number of large tables produced per week be y .

- 12.1 Write down the constraints, in terms of x and y , to represent the above information. (5)
- 12.2 Represent the constraints graphically on the attached DIAGRAM SHEET. Clearly indicate the feasible region. (4)
- 12.3 What is the maximum number of large tables that can be produced in a week? (1)
- 12.4 The profit on a small table is R300 and the profit on a large table is R400. Write down an expression for the total profit made per week. (1)
- 12.5 Determine the number of each type of table the factory needs to produce in a week in order to ensure a maximum total profit. Indicate this point using the letter A. (2)
- 12.6 The profit on a small table tends to fluctuate to q rands per table. The profit on a large table is constant at R400. Determine the values of q for which the total profit will be a maximum at point A. (2)
- [15]**

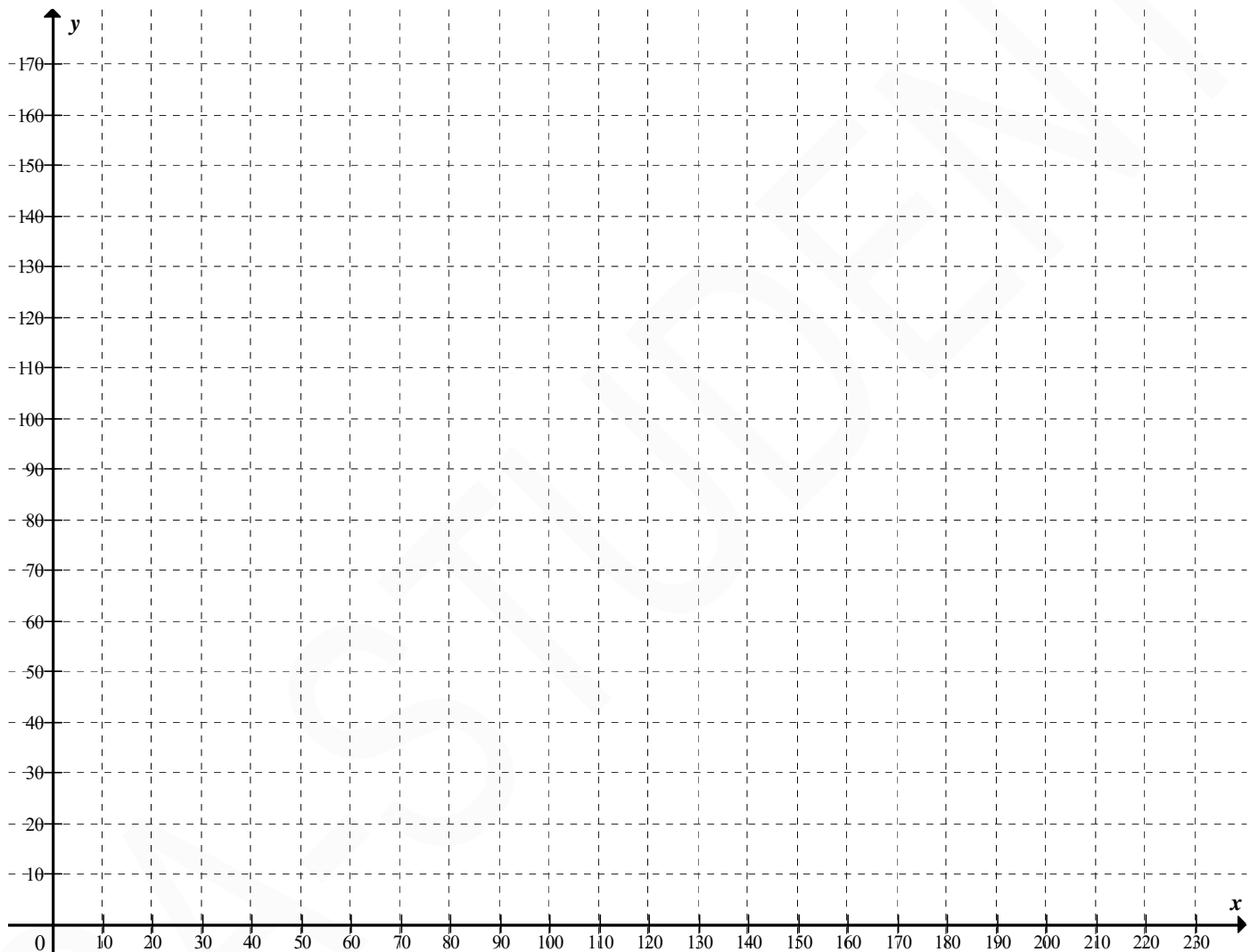
TOTAL: 150

CENTRE NUMBER:

--	--	--	--	--	--	--	--

EXAMINATION NUMBER:

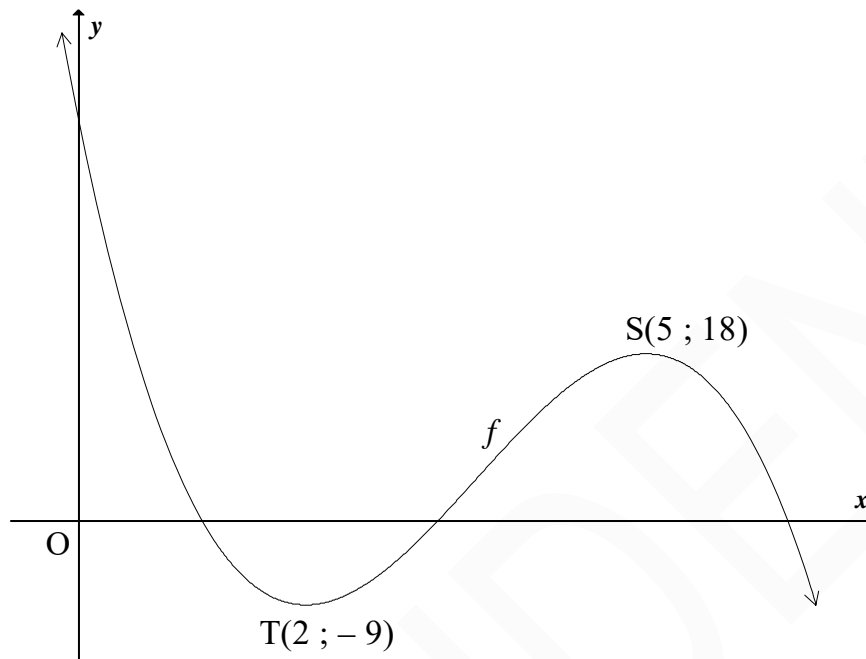
--	--	--	--	--	--	--	--	--	--	--	--	--

DIAGRAM SHEET 1**QUESTION 12.2**

QUESTION 9

The function $f(x) = -2x^3 + ax^2 + bx + c$ is sketched below.

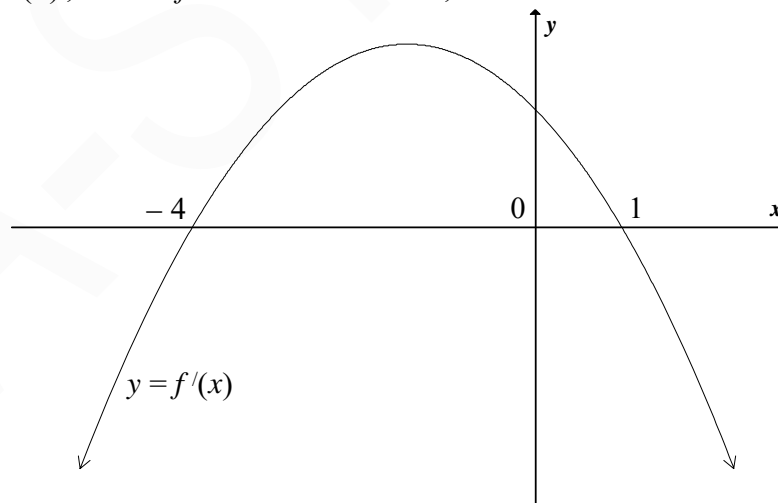
The turning points of the graph of f are $T(2 ; -9)$ and $S(5 ; 18)$.



- 9.1 Show that $a = 21$, $b = -60$ and $c = 43$. (7)
- 9.2 Determine an equation of the tangent to the graph of f at $x = 1$. (5)
- 9.3 Determine the x -value at which the graph of f has a point of inflection. (2)
- [14]**

QUESTION 10

The graph of $y = f'(x)$, where f is a cubic function, is sketched below.



Use the graph to answer the following questions:

- 10.1 For which values of x is the graph of $y = f'(x)$ decreasing? (1)
- 10.2 At which value of x does the graph of f have a local minimum? Give reasons for your answer. (3)
- [4]**

QUESTION 11

Water is flowing into a tank at a rate of 5 litres per minute. At the same time water flows out of the tank at a rate of k litres per minute. The volume (in litres) of water in the tank at time t (in minutes) is given by the formula $V(t) = 100 - 4t$.

- 11.1 What is the initial volume of the water in the tank? (1)
- 11.2 Write down TWO different expressions for the rate of change of the volume of water in the tank. (3)
- 11.3 Determine the value of k (that is, the rate at which water flows out of the tank). (2)
- [6]**

QUESTION 12

A school is planning a trip for 500 learners. The company that will be providing the transport has two types of buses, red buses and blue buses, available.

- Each red bus has 50 seats and each blue bus has 25 seats.
- The company has at most 15 bus drivers available.
- There are at most 8 blue buses available.

Let the number of red buses hired by the school be x and the number of blue buses hired by the school be y .

- 12.1 Write down ALL the constraints, in terms of x and y , to represent the above information. (6)
- 12.2 Represent the constraints graphically on the attached DIAGRAM SHEET. Clearly indicate the feasible region. (4)
- 12.3 The cost of hiring a red bus is R600 for the day and the cost of hiring a blue bus is R300 for the day. Write down the total transport cost. (1)
- 12.4 12.4.1 Determine ALL possible values of x and y so that the cost will be a minimum. (3)
- 12.4.2 Calculate the minimum cost of hiring the buses. (2)
- 12.5 If exactly 12 bus drivers are to be used, determine the number of each type of bus which the school will now need to still ensure minimum cost. (1)
- [17]**

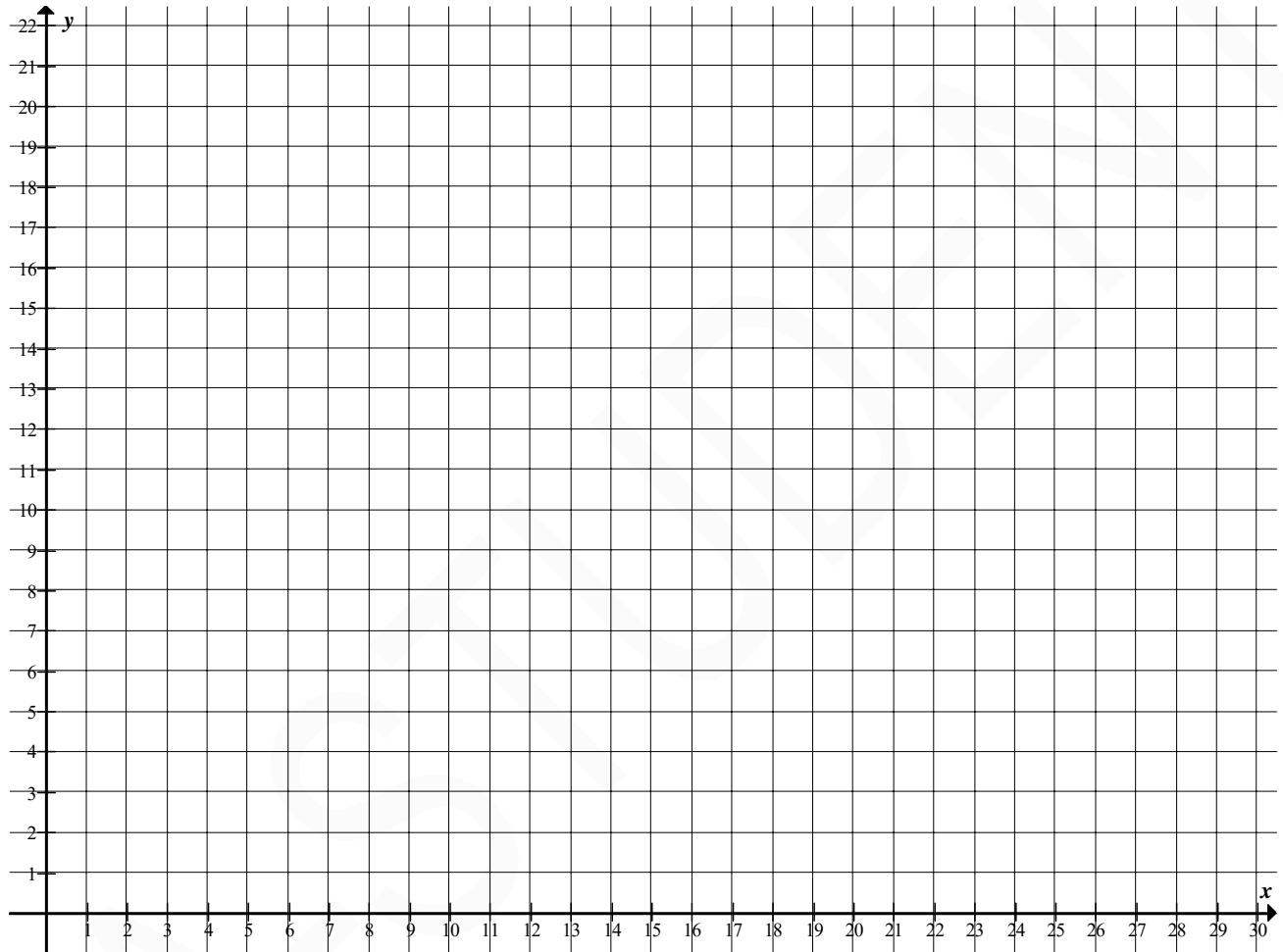
TOTAL: 150

CENTRE NUMBER:

--	--	--	--	--	--	--	--

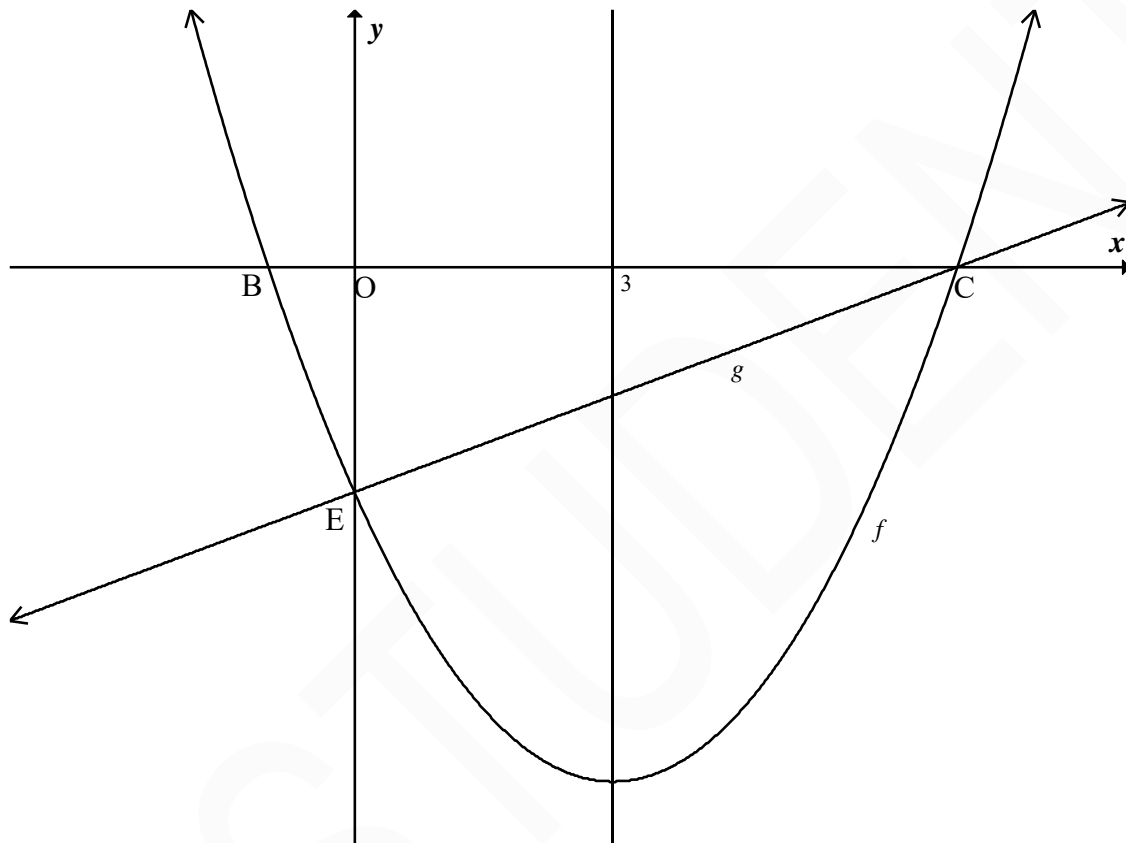
EXAMINATION NUMBER:

--	--	--	--	--	--	--	--	--	--	--	--	--

DIAGRAM SHEET 1**QUESTION 12.2**

QUESTION 6

A parabola f intersects the x -axis at B and C and the y -axis at E. The axis of symmetry of the parabola has equation $x = 3$. The line through E and C has equation $g(x) = \frac{x}{2} - \frac{7}{2}$.



- 6.1 Show that the coordinates of C are (7 ; 0). (1)
- 6.2 Calculate the x -coordinate of B. (1)
- 6.3 Determine the equation of f in the form $y = a(x - p)^2 + q$. (6)
- 6.4 Write down the equation of the graph of h , the reflection of f in the x -axis. (1)
- 6.5 Write down the maximum value of $t(x)$ if $t(x) = 1 - f(x)$. (2)
- 6.6 Solve for x if $f(x^2 - 2) = 0$. (4)
- [15]**

QUESTION 7

Consider the function $f(x) = \left(\frac{1}{3}\right)^x$.

- 7.1 Is f an increasing or decreasing function? Give a reason for your answer. (2)
- 7.2 Determine $f^{-1}(x)$ in the form $y = \dots$ (2)
- 7.3 Write down the equation of the asymptote of $f(x) - 5$. (1)
- 7.4 Describe the transformation from f to g if $g(x) = \log_3 x$. (2)
- [7]**

QUESTION 8

- 8.1 R1 430,77 was invested in a fund paying $i\%$ p.a. compounded monthly. After 18 months the fund had a value of R1 711,41. Calculate i . (4)
- 8.2 A father decided to buy a house for his family for R800 000. He agreed to pay monthly instalments of R10 000 on a loan which incurred interest at a rate of 14% p.a. compounded monthly. The first payment was made at the end of the first month.
- 8.2.1 Show that the loan would be paid off in 234 months. (4)
- 8.2.2 Suppose the father encountered unexpected expenses and was unable to pay any instalments at the end of the 120th, 121st, 122nd and 123rd months. At the end of the 124th month he increased his payment so as to still pay off the loan in 234 months by 111 equal monthly payments. Calculate the value of this new instalment. (7)
- [15]**

QUESTION 9

- 9.1 Use the definition to differentiate $f(x) = 1 - 3x^2$. (Use first principles.) (4)
- 9.2 Calculate $D_x \left[4 - \frac{4}{x^3} - \frac{1}{x^4} \right]$. (3)
- 9.3 Determine $\frac{dy}{dx}$ if $y = (1 + \sqrt{x})^2$. (3)
- [10]**

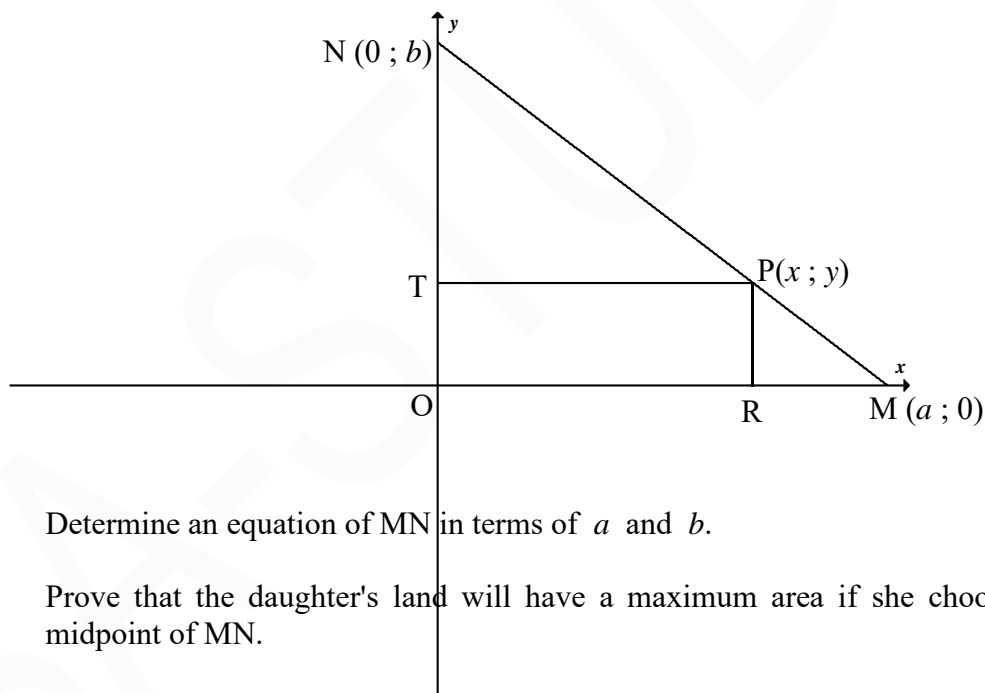
QUESTION 10

Given: $g(x) = (x - 6)(x - 3)(x + 2)$

- 10.1 Calculate the y-intercept of g . (1)
- 10.2 Write down the x-intercepts of g . (2)
- 10.3 Determine the turning points of g . (6)
- 10.4 Sketch the graph of g on DIAGRAM SHEET 2. (4)
- 10.5 For which values of x is $g(x) \cdot g'(x) < 0$? (3)
- [16]**

QUESTION 11

A farmer has a piece of land in the shape of a right-angled triangle OMN, as shown in the figure below. He allocates a rectangular piece of land PTOR to his daughter, giving her the freedom to choose P anywhere along the boundary MN. Let $OM = a$, $ON = b$ and $P(x; y)$ be any point on MN.



- 11.1 Determine an equation of MN in terms of a and b . (2)
- 11.2 Prove that the daughter's land will have a maximum area if she chooses P at the midpoint of MN. (6)
- [8]**

QUESTION 12

While preparing for the 2010 Soccer World Cup, a group of investors decided to build a guesthouse with single and double bedrooms to hire out to visitors. They came up with the following constraints for the guesthouse:

- There must be at least one single bedroom.
- They intend to build at least 10 bedrooms altogether, but not more than 15.
- Furthermore, the number of double bedrooms must be at least twice the number of single bedrooms.
- There should not be more than 12 double bedrooms.

Let the number of single bedrooms be x and the number of double bedrooms be y .

- 12.1 Write down the constraints as a system of inequalities. (6)
- 12.2 Represent the system of constraints on the graph paper provided on DIAGRAM SHEET 3. Indicate the feasible region by means of shading. (7)
- 12.3 According to these constraints, could the guesthouse have 5 single bedrooms and 8 double bedrooms? Motivate your answer. (2)
- 12.4 The rental for a single bedroom is R600 per night and R900 per night for a double bedroom. How many rooms of each type of bedroom should the contractors build so that the guesthouse produces the largest income per night? Use a search line to determine your answer and assume that all bedrooms in the guesthouse are fully occupied. (3)

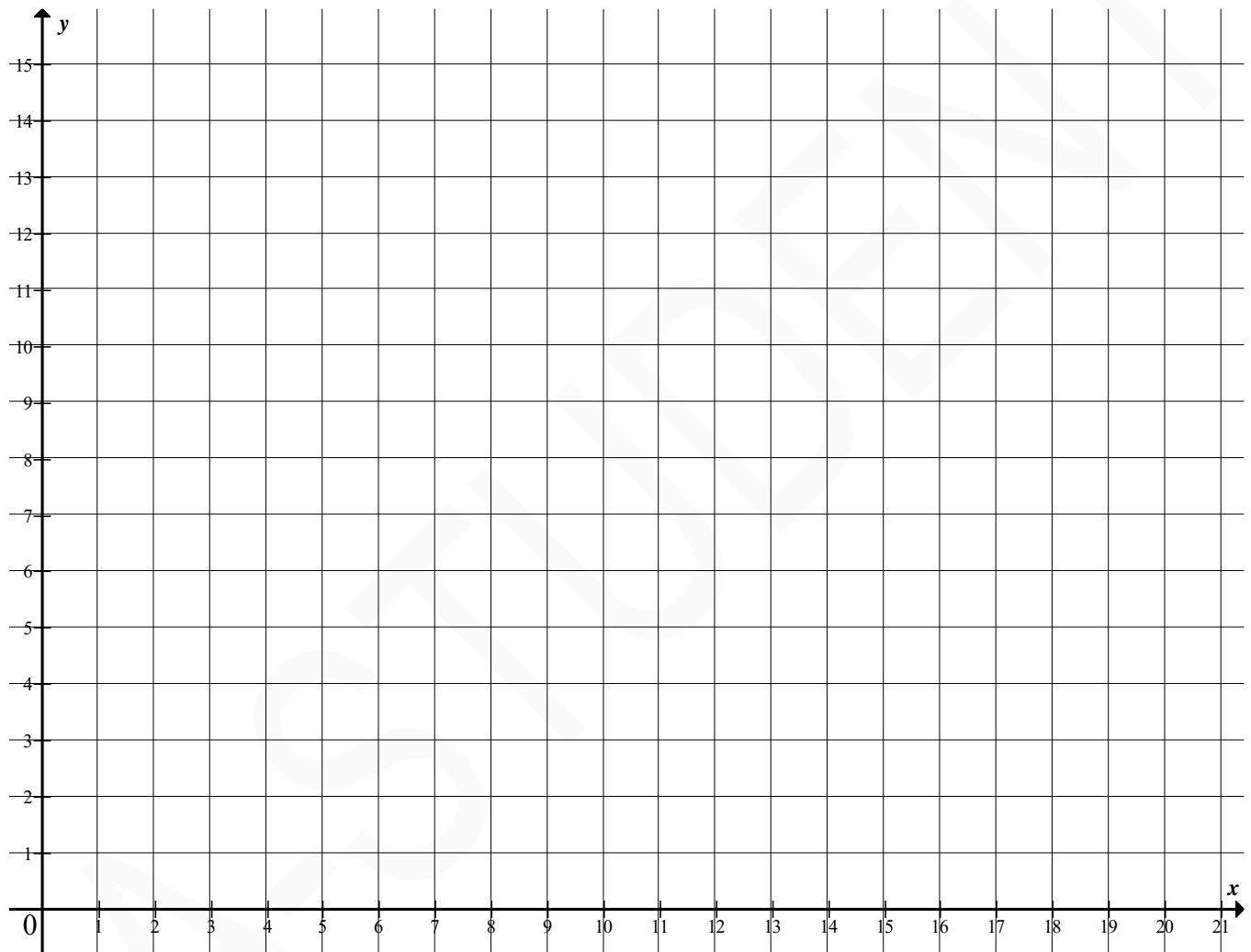
[18]**TOTAL: 150**

CENTRE NUMBER:

--	--	--	--	--	--	--	--

EXAMINATION NUMBER:

--	--	--	--	--	--	--	--	--	--	--	--	--

DIAGRAM SHEET 3**QUESTION 12.2**

QUESTION 11

Given: $f(x) = -x^3 + x^2 + 8x - 12$

- 11.1 Calculate the x -intercepts of the graph of f . (5)
- 11.2 Calculate the coordinates of the turning points of the graph of f . (5)
- 11.3 Sketch the graph of f , showing clearly all the intercepts with the axes and turning points. (3)
- 11.4 Write down the x -coordinate of the point of inflection of f . (2)
- 11.5 Write down the coordinates of the turning points of $h(x) = f(x) - 3$. (2)
- [17]**

QUESTION 12

A tourist travels in a car over a mountainous pass during his trip. The height above sea level of the car, after t minutes, is given as $s(t) = 5t^3 - 65t^2 + 200t + 100$ metres. The journey lasts 8 minutes.

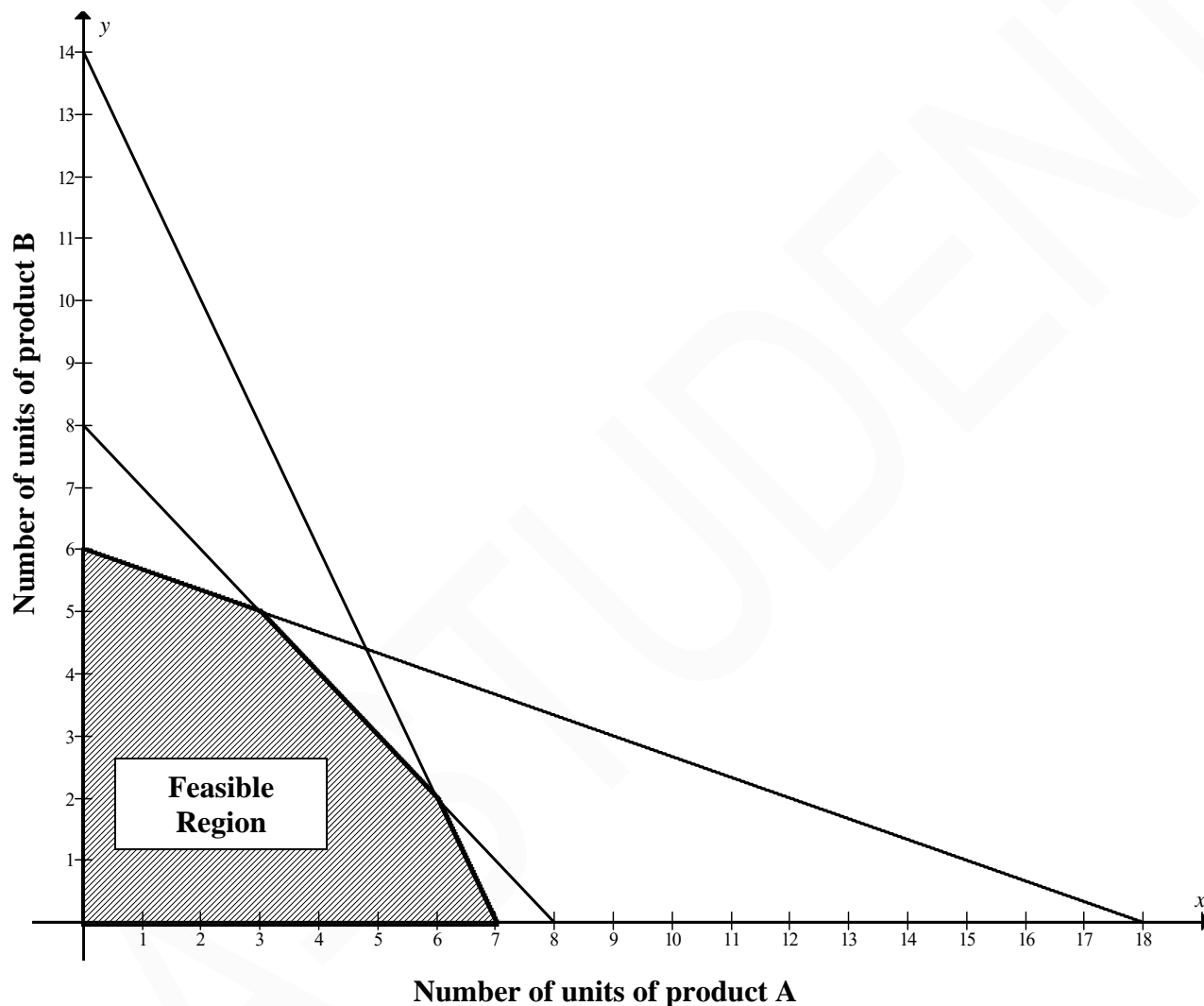
- 12.1 How high is the car above sea level when it starts its journey on the mountainous pass? (2)
- 12.2 Calculate the car's rate of change of height above sea level with respect to time, 4 minutes after starting the journey on the mountainous pass. (3)
- 12.3 Interpret your answer to QUESTION 12.2. (2)
- 12.4 How many minutes after the journey has started will the rate of change of height with respect to time be a minimum? (3)
- [10]**

QUESTION 13

A steel manufacturer makes two kinds of products, product A and B, having parts that must be cut, assembled and finished. The manufacturer is aware that it can sell as many products as it can produce.

Let x and y be the number of units of product A and product B that are manufactured every day respectively.

The constraints that govern the manufacture of the products are represented below and the feasible region is shaded.



- 13.1 Write down the constraints in terms of x and y that represent the above information. (7)
- 13.2 If product A yields a profit of R30 per item and product B yields R40 per item, write down the equation indicating the daily profit in terms of x and y . (2)
- 13.3 Determine the number of units of product A and product B that the manufacturer needs to produce in order to maximise his daily profit. A diagram is provided on DIAGRAM SHEET 1. (2)
- 13.4 The manufacturer would like the maximum profit to be at (6 ; 2) for the profit equation $P = mx + c$. Determine the values of m which will satisfy this condition (2)

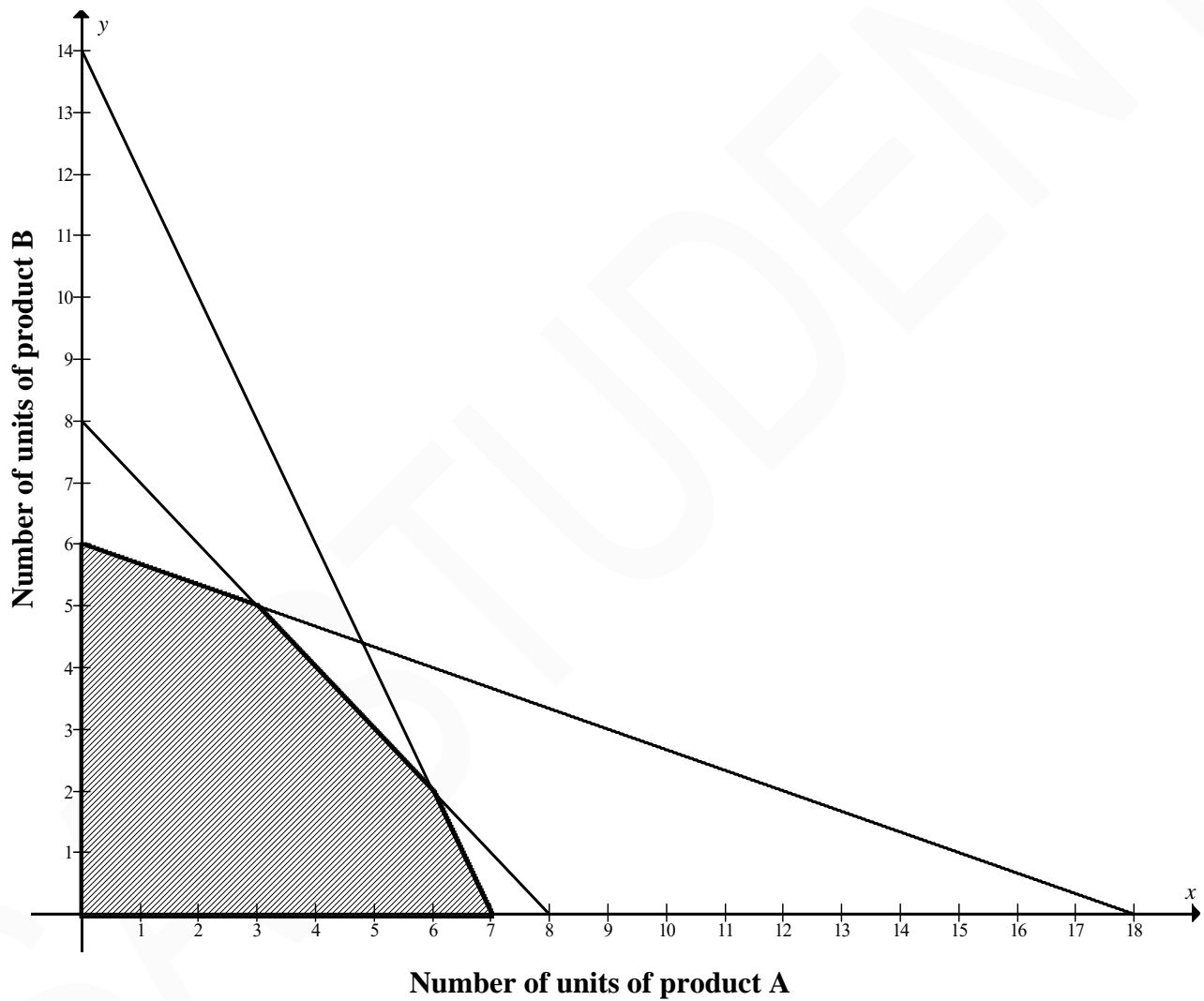
[13]**TOTAL: 150**

CENTRE NUMBER:

--	--	--	--	--	--	--	--

EXAMINATION NUMBER:

--	--	--	--	--	--	--	--	--	--	--	--	--

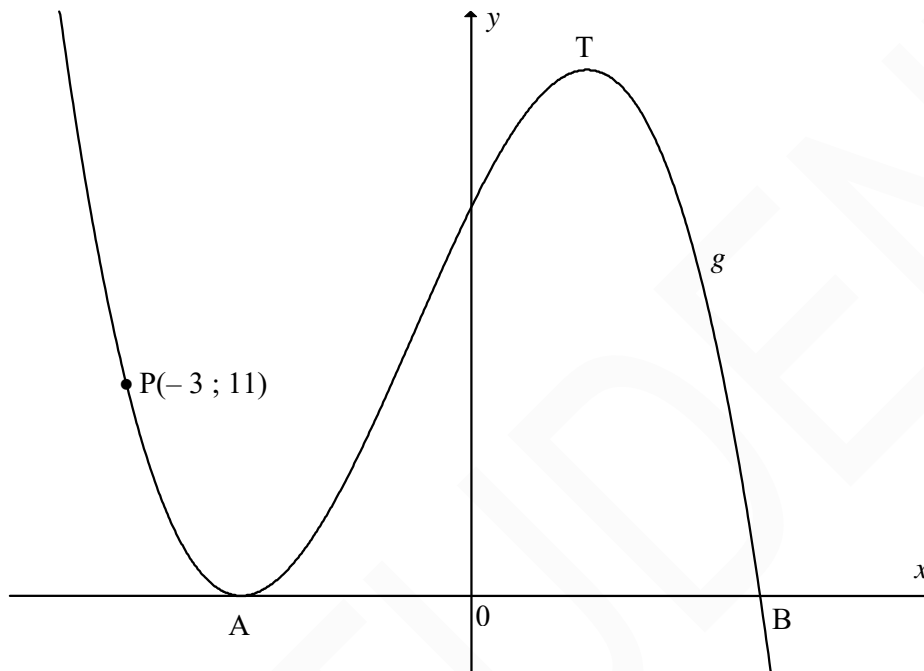
DIAGRAM SHEET 1**QUESTION 13.3**

QUESTION 9

Sketched below is the graph of $g(x) = -2x^3 - 3x^2 + 12x + 20 = -(2x - 5)(x + 2)^2$

A and T are turning points of g . A and B are the x -intercepts of g .

P(-3 ; 11) is a point on the graph.



9.1 Determine the length of AB. (2)

9.2 Determine the x -coordinate of T. (4)

9.3 Determine the equation of the tangent to g at P(-3 ; 11), in the form $y = \dots$ (5)

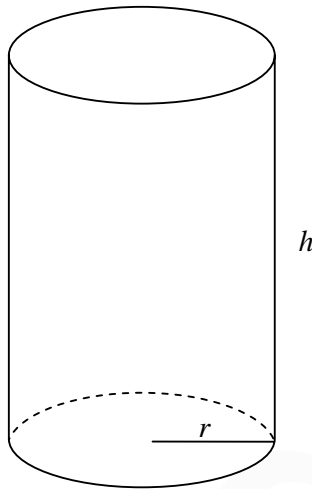
9.4 Determine the value(s) of k for which $-2x^3 - 3x^2 + 12x + 20 = k$ has three distinct roots. (3)

9.5 Determine the x -coordinate of the point of inflection. (4)

[18]

QUESTION 10

A drinking glass, in the shape of a cylinder, must hold 200 mℓ of liquid when full.



- 10.1 Show that the height of the glass, h , can be expressed as $h = \frac{200}{\pi r^2}$. (2)
- 10.2 Show that the total surface area of the glass can be expressed as $S(r) = \pi r^2 + \frac{400}{r}$. (2)
- 10.3 Hence determine the value of r for which the total surface area of the glass is a minimum. (5)
- [9]

QUESTION 11

Amina owns a small factory that manufactures two types of cellular phones, namely Acuna and Matata cellular phones.

- Each Acuna cellular phone requires 10 manhours to manufacture and each Matata cellular phone requires 8 manhours to manufacture.
- Each Acuna cellular phone requires 3 manhours in the testing department and each Matata cellular phone requires 4 manhours in the testing department.
- The manufacturing department has a maximum of 800 manhours available per week.
- The testing department has a maximum of 360 manhours available per week.
- The factory needs to manufacture at least 60 of the Matata models each week.

Let x represent the number of Acuna cellular phones manufactured in one week.

Let y represent the number of Matata cellular phones manufactured in one week.

- 11.1 Write down the constraints, in terms of x and y , that represent the above-mentioned information. (3)
- 11.2 Use the attached graph paper (DIAGRAM SHEET 2) to represent the constraints graphically. (5)
- 11.3 Clearly indicate the feasible region by shading it. (1)
- 11.4 If the profit on one Acuna cellular phone is R200 and the profit on one Matata cellular phone is R250, write down an expression that will represent the profit, P , on the cellular phones. (1)
- 11.5 Using a search line and your graph, determine the number of Acuna and Matata cellular phones that will give a maximum profit, assuming they are all sold out. Draw a search line on your graph. (3)
- 11.6 If the profit function for the factory was $P = 180x + 240y$, would there be any difference in the optimal solution? Give a reason for your answer. (3)

[16]**TOTAL: 150**

EXAMINATION NUMBER:

DIAGRAM SHEET 2

QUESTIONS 11.2 AND 11.3

