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Basic Education
REPUBLIC OF SOUTH AFRICA

NATIONAL
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NASIONALE
SENIOR SERTIFIKAAT

GRADE 12/*GRAAD 12*

MATHEMATICS P2/*WISKUNDE V2*

NOVEMBER 2023

MARKING GUIDELINES/*NASIENRIGLYNE*

MARKS/PUNTE: 150

These marking guidelines consist of 23 pages./
Hierdie nasienriglyne bestaan uit 23 bladsye.

NOTE:

- If a candidate answers a question TWICE, only mark the FIRST attempt.
- If a candidate has crossed out an attempt of a question and not redone the question, mark the crossed out version.
- Consistent accuracy applies in ALL aspects of the Marking Guidelines. Stop marking at the second calculation error.
- Assuming answers/values in order to solve a problem is NOT acceptable.

NOTA:

- *As 'n kandidaat 'n vraag TWEE KEER beantwoord, merk slegs die EERSTE poging.*
- *As 'n kandidaat 'n antwoord van 'n vraag doodtrek en nie oordoen nie, merk die doodgetrekte poging.*
- *Volgehoue akkuraatheid word in ALLE aspekte van die Nasienriglyne toegepas. Hou op nasien by die tweede berekeningsfout.*
- *Aanvaar van antwoorde/waardes om 'n probleem op te los, word NIE toegelaat nie.*

GEOMETRY	
S	**A mark for a correct statement** **(A statement mark is independent of a reason)**
	'n Punt vir 'n korrekte bewering **('n Punt vir 'n bewering is onafhanklik van die rede)**
R	**A mark for the correct reason** **(A reason mark may only be awarded if the statement is correct)**
	'n Punt vir 'n korrekte rede **('n Punt word slegs vir die rede toegeken as die bewering korrek is)**
S/R	**Award a mark if statement AND reason are both correct**
	Ken 'n punt toe as die bewering EN rede beide korrek is

QUESTION/*VRAAG* 1

1.1	$a = -23{,}846\ldots$ $b = 0{,}227\ldots$ $\hat{y} = -23{,}85 + 0{,}23x$	✓ $a = -23{,}846\ldots$ ✓ $b = 0{,}227\ldots$ ✓ equation (3)
1.2	$\hat{y} = -23{,}85 + 0{,}23(550)$ $y = 102{,}65$ **OR** $y = 101{,}02$	✓ substitution of 550 ✓ answer (2) ✓✓ $y = 101{,}02$ (calculator) (2)
1.3	$r = 0{,}98$	✓ $r = 0{,}98$ (1)
1.4	Very strong positive correlation	✓ strong positive (1)

50	100	130	150	180	190	200	200

1.5.1	$\bar{x} = \frac{1200}{8}$ $\bar{x} = 150$ **OR** $\bar{x} = 150$	✓ 1200 ✓ answer (2) ✓✓ $\bar{x} = 150$ (2)
1.5.2	$\sigma = 50{,}50$	✓ $\sigma = 50{,}50$ (1)
1.5.3	$\bar{x} - \sigma$ $= 150 - 50{,}50$ $= 99{,}50$ $\therefore$ 1 stop	✓ calculation of $\bar{x} - \sigma$ ✓ answer (2)
		[12]

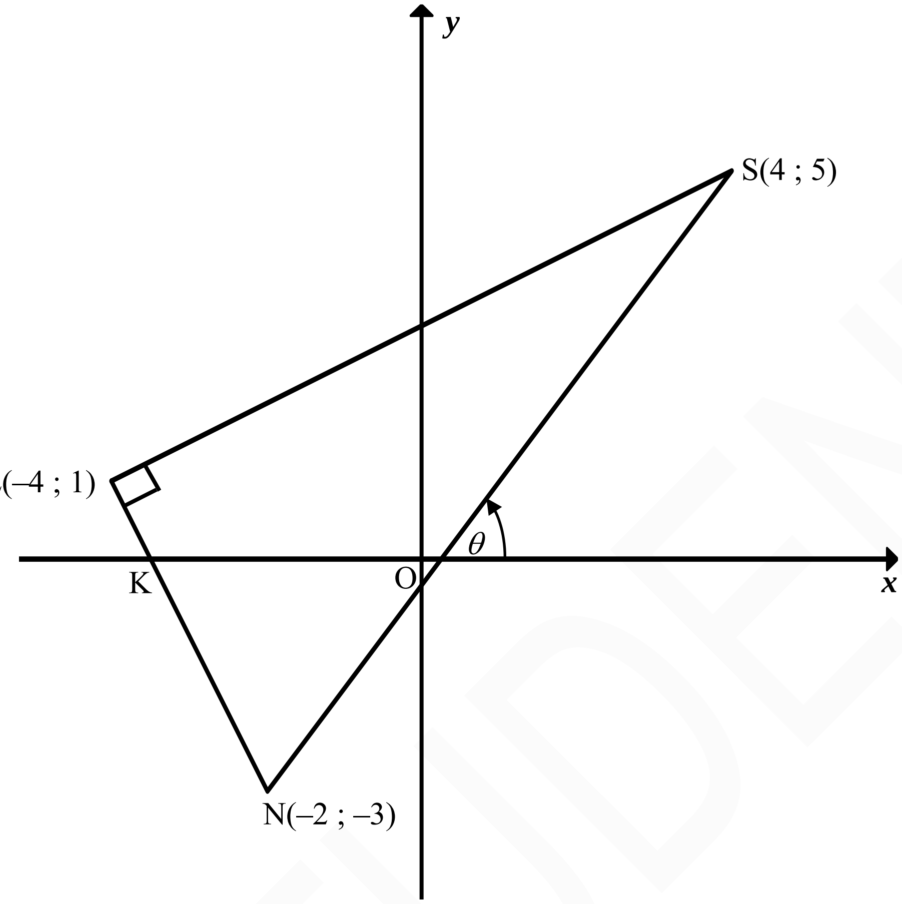
QUESTION/*VRAAG* 2

2.1	**Table:** see below	✓ 5; 20 ✓ 40 (2)
2.2	40 staff members	✓ answer (1)
2.3	33 staff members	✓ answer (1)
2.4	$\bar{x} = \dfrac{\left(1\times\left(5+\frac{k}{2}\right)\right)+(3\times15)+\left(5\times\left(13+\frac{k}{2}\right)\right)+(7\times5)+(9\times2)}{40+k}=4$ $5+\frac{k}{2}+45+65+\frac{5k}{2}+35+18=160+4k$ $3k+168=160+4k$ $k=8$	✓ answer from Q2.2 + k ✓ $\left(1\times\left(5+\frac{k}{2}\right)\right)$ ✓ $\left(5\times\left(13+\frac{k}{2}\right)\right)$ ✓ answer (4)
	OR $\bar{x} = \dfrac{(1\times5)+(15\times3)+(13\times5)+(5\times7)+(2\times9)}{40}$ $=4{,}2$ $\bar{x}_{\text{old}}-\bar{x}_{\text{current}}=4{,}2-4$ $=0{,}2$ $\therefore 0{,}2\times40$ $=8$ teachers	✓ 4,2 ✓ $\bar{x}_{\text{old}}-4$ ✓ difference ✓ answer (4)
		[8]

Table for 2.1:

Number of glasses of water per day	Number of staff members	Cumulative frequency
$0\le x<2$	5	5
$2\le x<4$	15	20
$4\le x<6$	13	33
$6\le x<8$	5	38
$8\le x<10$	2	40

QUESTION/*VRAAG* 3

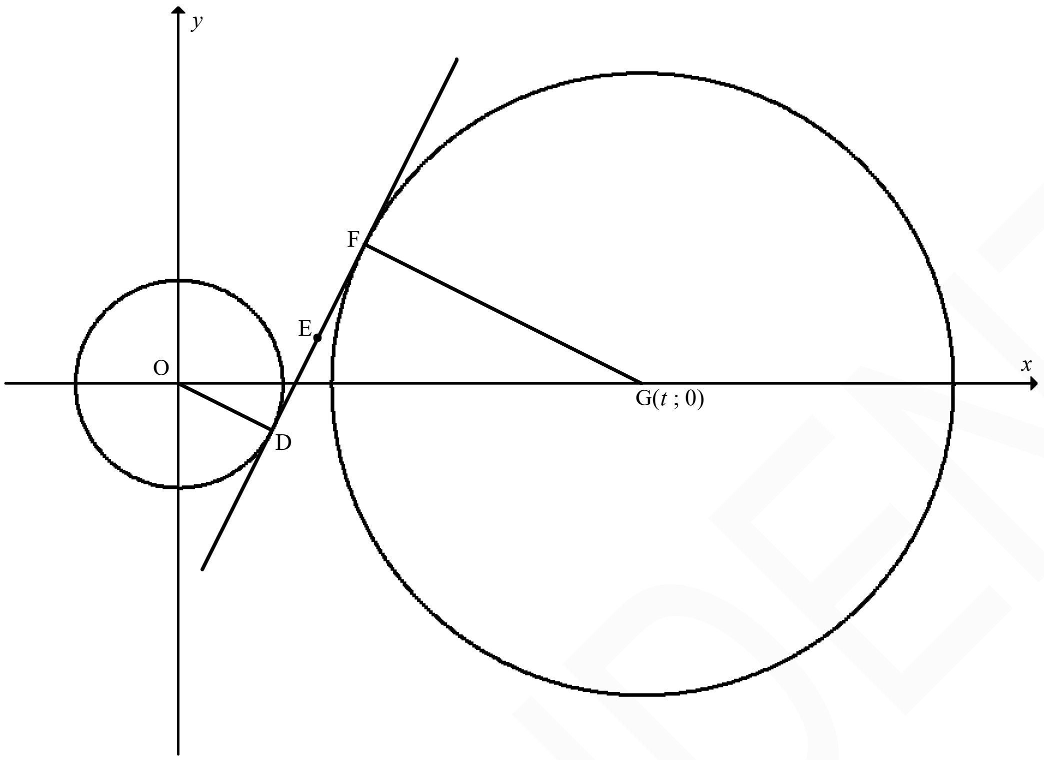


3.1	$SL = \sqrt{(x_S - x_L)^2 + (y_S - y_L)^2}$ $SL = \sqrt{(4-(-4))^2 + (5-1)^2}$ $SL = \sqrt{80} = 4\sqrt{5} = 8{,}94$ units	✓ substitution of S and L into correct formula ✓ answer (2)
3.2	$m_{SN} = \frac{5-(-3)}{4-(-2)}$ $m_{SN} = \frac{4}{3}$	✓ substitution of S and N into correct formula ✓ answer (2)
3.3	$m = \tan\theta = \frac{4}{3}$ $\theta = 53{,}13°$	✓ $\tan\theta = m_{SN}$ ✓ answer (2)
3.4	$m_{LN} = \frac{1-(-3)}{-4-(-2)}$ $m_{LN} = -2$ $L\hat{K}O = 116{,}565...°$ $L\hat{N}S = 116{,}565...° - 53{,}13°$ $L\hat{N}S = 63{,}44°$	✓ $m_{LN} = -2$ ✓ size of $L\hat{K}O$ ✓ answer (3)

	OR	
	SN = 10 units $\sin L\hat{N}S = \frac{4\sqrt{5}}{10}$ $L\hat{N}S = 63{,}44°$	✓ SN = 10 units ✓ correct trig ratio ✓ answer (3)
	OR	
	$LN = 2\sqrt{5}$ units $\tan L\hat{N}S = \frac{4\sqrt{5}}{2\sqrt{5}}$ $L\hat{N}S = 63{,}44°$	✓ $LN = 2\sqrt{5}$ units ✓ correct trig ratio ✓ answer (3)
	OR	
	SN = 10 units $LN = 2\sqrt{5}$ units $\cos L\hat{N}S = \frac{2\sqrt{5}}{10}$ $L\hat{N}S = 63{,}44°$	✓ SN = 10 units and $LN = 2\sqrt{5}$ units ✓ correct trig ratio ✓ answer (3)
3.5	$m = \frac{4}{3}$ $1 = \frac{4}{3}(-4) + c$ $c = \frac{19}{3}$ $y = \frac{4}{3}x + \frac{19}{3}$ **OR** $y - 1 = \frac{4}{3}(x - (-4))$ $y - 1 = \frac{4}{3}x + \frac{16}{3}$ $y = \frac{4}{3}x + \frac{19}{3}$	✓ m_{SN} ✓ substitution of m_{SN} & L ✓ equation (3)
3.6	$SL = 4\sqrt{5}$ $LN = \sqrt{(-4-(-2))^2 + (1-(-3))^2}$ $LN = \sqrt{20} = 2\sqrt{5}$ Area $\Delta LSN = \frac{1}{2}(4\sqrt{5})(2\sqrt{5})$ $= 20$ units^2	✓ $LN = \sqrt{20} = 2\sqrt{5}$ ✓ substitution into formula ✓ answer (3)
	OR	

	SN = 10 units $LN = \sqrt{(-4-(-2))^2 + (1-(-3))^2}$ $LN = \sqrt{20} = 2\sqrt{5}$ Area $\Delta LSN = \frac{1}{2}(10)(2\sqrt{5})\sin 63,44°$ $= 20$ units^2	✓ $LN = \sqrt{20} = 2\sqrt{5}$ ✓ substitution into formula ✓ answer (3)
3.7	$\hat{L} = 90°$ SN is a diameter of circle S, L, N [chord subtends 90° **OR** converse ∠ in semi-circle] Centre of circle = $P\left(\frac{4+(-2)}{2};\frac{5+(-3)}{2}\right)$ $= P(1\,;1)$	✓ SN is a diameter of circle S, L, N ✓ x-value ✓ y-value (3)
	OR Let the coordinates of P be $(a\,;b)$. Then, PL = PN: $(-4-a)^2+(1-b)^2=(-2-a)^2+(-3-b)^2$ $a-2b=-1$ …..equation 1 If PS = PN, then: $4a+2b=6$ ………….. equation 2 Solving simultaneously yields: $a=1$ and $b=1$ and $P(1\,;1)$	✓ 2 correct linear equations ✓ x-value ✓ y-value (3)
	OR If PL = PN, then: $a-2b=-1$ …..equation 1 If PS = PL, then: $2a+b=3$ .....equation 2 Solving simultaneously yields: $a=1$ and $b=1$ and $P(1\,;1)$	✓ 2 correct linear equations ✓ x-value ✓ y-value (3)
3.8	$L\hat{P}N = \theta = 53,13°$ [alt ∠s; LP ‖ x–axis] $\therefore L\hat{P}S = 126,87°$	✓ $L\hat{P}N$ ✓ answer (2)
	OR $L\hat{N}S = 63,44°$ $\therefore L\hat{P}S = 126,88°$ [∠ at centre = 2 ×∠ at circumference]	✓ $L\hat{N}S$ ✓ answer (2)
	OR $L\hat{S}N = 26,56°$ [sum of ∠s in Δ] $S\hat{L}P = 26,56°$ [∠s opp equal radii] $\therefore L\hat{P}S = 126,88°$ [sum of ∠s in Δ]	✓ $L\hat{S}N$ ✓ answer (2)
	OR $(4\sqrt{5})^2 = 5^2+5^2-2(5)(5)\cos L\hat{P}S$ $\cos L\hat{P}S = -\frac{3}{5}$ $\therefore L\hat{P}S = 126,87°$	✓ correct substitution into cosine formula ✓ answer (2)
		[20]

QUESTION/*VRAAG* 4

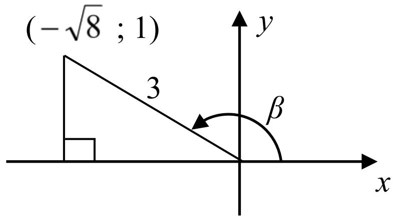


4.1	D(p ; −2) $x^2 + y^2 = 20$ $p^2 + (-2)^2 = 20$ $p^2 = 16$ $p = \pm 4$ $p = 4$	✓ substitution of point D$(p\ ;-2)$ ✓ $p^2 = 16$ (2)
4.2	$\frac{4 + x_F}{2} = 6$ $\quad$ $\frac{-2 + y_F}{2} = 2$ $x_F = 8$ $\quad$ $y_F = 6$ $F(8;6)$ **OR** $x_E - x_D = 6 - 4$ $\quad$ $y_E - y_D = 2 - (-2)$ $= 2$ $\quad$ $= 4$ $x_F = 6 + 2 = 8$ $\quad$ $y_F = 2 + 4 = 6$ $F(8;6)$	✓ method ✓ x-value ✓ y-value (3) ✓ method ✓ x-value ✓ y-value (3)

4.3	$m_{DE} = \frac{-2-2}{4-6}$ $m_{DE} = 2$ $-2 = 2(4) + c$ **OR** $y - (-2) = 2(x-4)$ $c = -10$ $\quad$ $y + 2 = 2x - 8$ $y = 2x - 10$ $\quad$ $y = 2x - 10$	✓ correct substitution ✓ gradient of DE, DF or EF ✓ substitution of point D(4 ; –2) or E(6 ; 2) or F(8 ; 6) ✓ answer (4)
	OR $m_{OD} = -\frac{2}{4} = -\frac{1}{2}$ $\therefore m_{DE} = 2$ [tan ⊥ radius] $-2 = 2(4) + c$ **OR** $y - (-2) = 2(x-4)$ $c = -10$ $\quad$ $y + 2 = 2x - 8$ $y = 2x - 10$ $\quad$ $y = 2x - 10$	✓ correct gradient of OD ✓ gradient of DE ✓ substitution of point D(4 ; –2) or E(6 ; 2) or F(8 ; 6) ✓ answer (4)
4.4	$m_{DE} = 2$ $\therefore m_{GF} = -\frac{1}{2}$ [tan ⊥ radius] $\frac{0-6}{t-8} = -\frac{1}{2}$ $-(t-8) = 2(-6)$ $t = 20$	✓ correct gradient of GF ✓ substitution of F ✓ answer (3)
	OR $y = 2x - 10$ $0 = 2x - 10$ $x = 5$ A(5 ; 0) In ΔAFG: FA ⊥ FG $FA^2 = (6-0)^2 + (8-5)^2 = 45$ $FG^2 = (t-8)^2 + (0-6)^2$ $= t^2 - 16t + 100$ $GA^2 = (t-5)^2$ $= t^2 - 10t + 25$ $\therefore GA^2 = GF^2 + FA^2$ $t^2 - 10t + 25 = t^2 - 16t + 100 + 45$ $6t = 120$ $t = 20$	✓ x-intercept of DF ✓ substitution into Pythagoras ✓ answer (3)

4.5	$F(8;6)$ G(20 ; 0) $(8-20)^2+(6-0)^2=r^2$ $r^2=180$ $(x-20)^2+y^2=180$ $x^2+y^2-40x+220=0$	✓ substitution of F and G ✓ value of r^2 ✓ equation of circle ✓ answer (4)
4.6	Smaller circle $r=2\sqrt{5}$ Larger circle $r=6\sqrt{5}$ G(20 ; 0) $k=20-(6\sqrt{5}-2\sqrt{5})$ or $k=20+(6\sqrt{5}-2\sqrt{5})$ $=20-4\sqrt{5}$ $=20+4\sqrt{5}$ = 11,06 units = 28,94 units **OR** Smaller circle $r=2\sqrt{5}$ $k=2\left(2\sqrt{5}\right)+20-8\sqrt{5}$ or $k=2\left(6\sqrt{5}\right)+20-8\sqrt{5}$ $=20-4\sqrt{5}$ $=20+4\sqrt{5}$ = 11,06 units = 28,94 units **OR** $x^2+y^2-40x+220=0$ $y=0$ $\therefore x^2-40x+220=0$ $\therefore x=20+6\sqrt{5}$ or $x=20-6\sqrt{5}$ $\therefore k=20+6\sqrt{5}-\sqrt{20}$ or $k=20-6\sqrt{5}+\sqrt{20}$ $\therefore k=20+4\sqrt{5}$ $\therefore k=20-4\sqrt{5}$ = 11,06 units = 28,94 units	✓ $r=2\sqrt{5}$ ✓ method ✓ answer ✓ answer (4) ✓ $r=2\sqrt{5}$ ✓ method ✓ answer ✓ answer (4) ✓ x-intercepts ✓ method ✓ answer ✓ answer (4)
		[20]

QUESTION/VRAAG 5

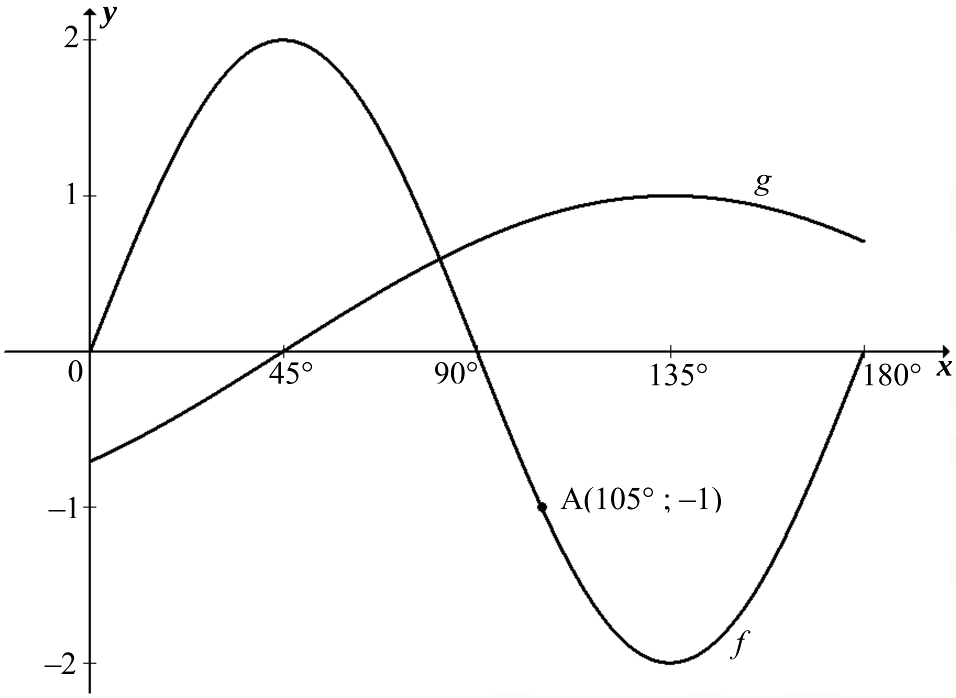
5.1.1	$\sin\beta = \frac{1}{3}$ $\quad\beta \in (90°;270°)$  $x = -\sqrt{8} = -2\sqrt{2}$ $\cos\beta$ $= \frac{-2\sqrt{2}}{3}$	✓ $x^2 + y^2 = r^2$ ✓ $x = -2\sqrt{2}$ ✓ answer (3)
	OR	
	$\sin\beta = \frac{1}{3}$ $\quad\beta \in (90°;270°)$ $\cos^2\beta = 1 - \sin^2\beta$ $\cos^2\beta = 1 - \left(\frac{1}{3}\right)^2$ $\cos^2\beta = \frac{8}{9}$ $\cos\beta = \frac{-\sqrt{8}}{3}$ $= \frac{-2\sqrt{2}}{3}$	✓ square identity ✓ $\cos^2\beta$ ✓ answer (3)
5.1.2	$\sin 2\beta$ $= 2\sin\beta\cos\beta$ $= 2\left(\frac{1}{3}\right)\left(\frac{-\sqrt{8}}{3}\right)$ $= \frac{-2\sqrt{8}}{9}$ **OR** $2\left(\frac{-2\sqrt{2}}{9}\right)$ $= \frac{-4\sqrt{2}}{9}$	✓ double angle ✓ substitution ✓ answer (3)
5.1.3	$\cos(450° - \beta)$ $= \cos(90° - \beta)$ $= \sin\beta$ $= \frac{1}{3}$ **OR**	✓ $\cos(90° - \beta)$ ✓ co-ratio ✓ answer (3)

	$\cos(450° - \beta)$ $= \cos 450° \cos\beta + \sin 450° \sin\beta$ $= \cos 90° \cos\beta + \sin 90° \sin\beta$ $= \sin\beta$ $= \frac{1}{3}$	✓ expansion ✓ reduction ✓ answer (3)
5.2.1	LHS $= \frac{\cos^4 x + \sin^2 x.\cos^2 x}{1+\sin x}$ $= \frac{\cos^2 x(\cos^2 x + \sin^2 x)}{1+\sin x}$ $= \frac{1-\sin^2 x}{1+\sin x}$ $= \frac{(1-\sin x)(1+\sin x)}{1+\sin x}$ $= 1-\sin x$ = RHS **OR** LHS $= \frac{\cos^4 x + \sin^2 x.\cos^2 x}{1+\sin x}$ $= \frac{\cos^4 x + (1-\cos^2 x)\cos^2 x}{1+\sin x}$ $= \frac{\cos^4 x + \cos^2 x - \cos^4 x}{1+\sin x}$ $= \frac{1-\sin^2 x}{1+\sin x}$ $= \frac{(1-\sin x)(1+\sin x)}{1+\sin x}$ $= 1-\sin x$ = RHS **OR** RHS $= 1-\sin x$ $= (1-\sin x) \times \frac{1+\sin x}{1+\sin x}$ $= \frac{1-\sin^2 x}{1+\sin x}$ $= \frac{\cos^2 x}{1+\sin x}$ $= \frac{\cos^2 x(\sin^2 x + \cos^2 x)}{1+\sin x}$ $= \frac{\cos^4 x + \cos^2 x.\sin^2 x}{1+\sin x}$ = LHS	✓ factors ✓ $\sin^2 x + \cos^2 x = 1$ ✓ $\cos^2 x = 1-\sin^2 x$ ✓ factors (4) ✓ $\sin^2 x = 1-\cos^2 x$ ✓ expansion ✓ $\cos^2 x = 1-\sin^2 x$ ✓ factors (4) ✓ $\times \frac{1+\sin x}{1+\sin x}$ ✓ product ✓ $1-\sin^2 x = \cos^2 x$ ✓ $1 = \cos^2 x + \sin^2 x$ (4)

5.2.2	$\sin x+1=0$ $\sin x=-1$ ref. $\angle = 90°$ $x = 270°$	✓ $\sin x+1=0$ ✓ $x = 270°$ (2)
5.2.3	$y=\dfrac{\cos^4 x+\sin^2 x.\cos^2 x}{1+\sin x}$ $y=1-\sin x$ $\therefore$ Minimum = 0	✓✓ Minimum = 0 (2)
5.3.1	sin (A – B) $=\cos\left[90°-(A-B)\right]$ $=\cos\left[(90°-A)-(-B)\right]$ $=\cos(90°-A)\cos(-B)+\sin(90°-A)\sin(-B)$ $=\sin A\cos B+\cos A(-\sin B)$ $=\sin A\cos B-\cos A\sin B$ **OR** sin (A – B) $=\cos\left[90°-(A-B)\right]$ $=\cos\left[(90°+B)-A\right]$ $=\cos(90°+B)\cos A+\sin(90°+B)\sin A$ $=-\sin B\cos A+\cos B\sin A$ $=\sin A\cos B-\cos A\sin B$	✓ co-ratio ✓ compound angle ✓ reduction (3) ✓ co-ratio ✓ compound angle ✓ reduction (3)
5.3.2	$\sin 48°\cos x-\cos 48°\sin x=\cos 2x$ $\sin(48°-x)=\cos 2x$ $\sin(48°-x)=\sin(90°-2x)$ $48°-x=90°-2x+k.360°$ or $48°-x=180°-(90°-2x)+k.360°$ $x=42°+k.360°$ $-3x=42°+k.360°$ $x=-14°-k.120°$; $k\in Z$ **OR** $\sin 48°\cos x-\cos 48°\sin x=\cos 2x$ $\sin(48°-x)=\cos 2x$ $\cos(90°-48°+x)=\cos 2x$ $\cos(42°+x)=\cos 2x$ $42°+x=2x+k.360°$ or $42°+x=360°-2x+k.360°$ $-x=-42°+k.360°$ $3x=318°+k.360°$ $x=42°-k.360°$ $x=106°+k.120°$; $k\in Z$	✓ compound angle ✓ co-ratio ✓ both equations ✓ general solution ✓ general solution; $k\in Z$ (5) ✓ compound angle ✓ co-ratio ✓ both equations ✓ general solution ✓ general solution; $k\in Z$ (5)

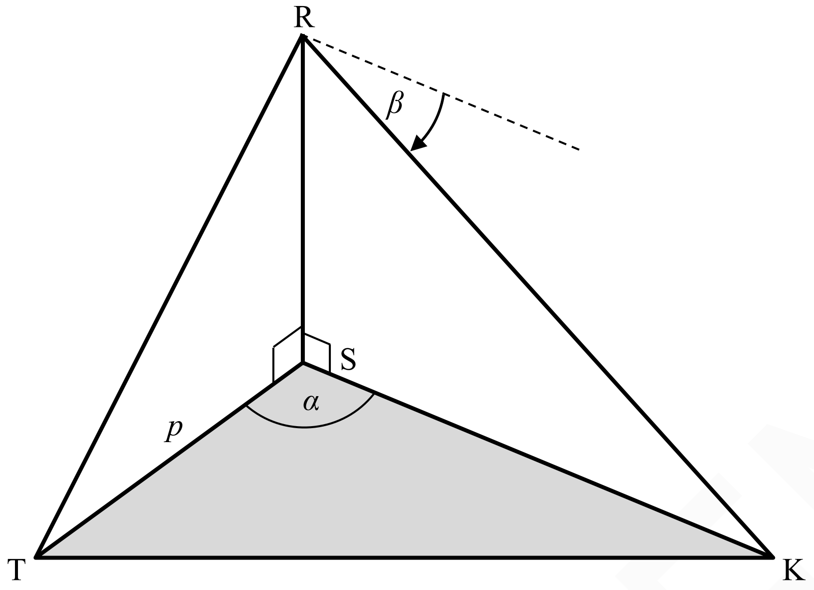
5.4	$\frac{\sin 3x+\sin x}{\cos 2x+1}$	
	$=\frac{\sin(2x+x)+\sin(2x-x)}{\cos 2x+1}$	✓ $3x=(2x+x)$
	$=\frac{\sin 2x\cos x+\cos 2x\sin x+\sin 2x\cos x-\cos 2x\sin x}{2\cos^2 x-1+1}$	✓ expansion ✓ double angle of cos2x
	$=\frac{2\sin 2x\cos x}{2\cos^2 x}$	✓ simplification
	$=\frac{2(2\sin x\cos x)\cos x}{2\cos^2 x}$	✓ $\sin 2x=2\sin x\cos x$
	$=\frac{4\sin x\cos^2 x}{2\cos^2 x}$	
	$=2\sin x$	✓ answer (6)
	OR	
	$\frac{\sin 3x+\sin x}{\cos 2x+1}$	
	$=\frac{\sin(2x+x)+\sin x}{2\cos^2 x-1+1}$	✓ $3x=(2x+x)$ ✓ double angle of cos2x
	$=\frac{\sin 2x\cos x+\cos 2x\sin x+\sin x}{2\cos^2 x}$	✓ expansion
	$=\frac{2\sin x\cos x\cos x+\cos 2x\sin x+\sin x}{2\cos^2 x}$	✓ $\sin 2x=2\sin x\cos x$
	$=\frac{\sin x\left(2\cos^2 x+\cos 2x+1\right)}{2\cos^2 x}$	✓ common factor
	$=\frac{\sin x\left(2\cos^2 x+2\cos^2 x-1+1\right)}{2\cos^2 x}$	
	$=2\sin x$	✓ answer (6)
		[31]

QUESTION/*VRAAG* 6



6.1	Period = 180°	✓ 180° (1)
6.2	$y \in \left[-\frac{\sqrt{2}}{2}\ ;\ 1\right]$ **OR** $y \in [-0{,}71;\ 1]$ **OR** $-\frac{\sqrt{2}}{2} \leq y \leq 1$	✓ $-\frac{\sqrt{2}}{2}$ ✓ $y \in \left[-\frac{\sqrt{2}}{2}\ ;\ 1\right]$ (2)
6.3.1	$x \in (45°\ ;\ 90°)$ **OR** $45° < x < 90°$	✓✓ $x \in (45°\ ;\ 90°)$ (2)
6.3.2	$f(x)+1 \leq 0$ $f(x) \leq -1$ $x \in [105°\ ;\ 165°]$ **OR** $105° \leq x \leq 165°$	✓✓ $x \in [105°\ ;\ 165°]$ (2)
6.4	$p(x) = -2\sin 2x$ $-2\sin 2x = -1$ **OR** $2\sin 2x = 1$ $k = 15°$ or $k = 75°$	✓ reading off $f(x) = 1$ or $-f(x) = -1$ ✓ 15° ✓ 75° (3)
6.5	$g(x) = -\cos(x+45°)$ $h(x) = -\cos(x+90°)$ $h(x) = \sin x$	✓ $-\cos(x+90°)$ ✓ answer (2)
		[12]

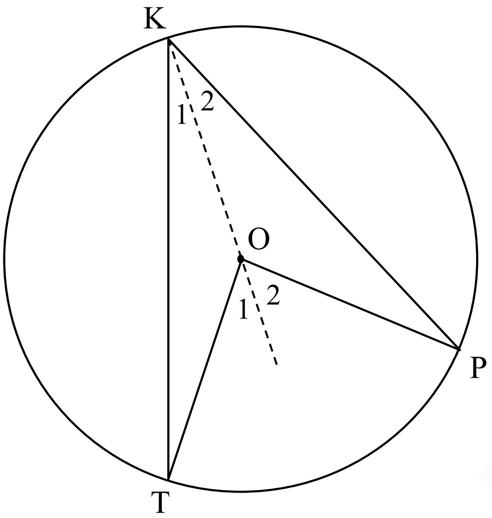
QUESTION/*VRAAG* 7



7.1	Area $\Delta STK = \frac{1}{2}p(SK)\sin\alpha$ $q = \frac{1}{2}p(SK)\sin\alpha$ $SK = \frac{q}{\frac{1}{2}p\sin\alpha}$ $= \frac{2q}{p\sin\alpha}$	✓ substitution into the correct formula ✓ answer (2)
7.2	$R\hat{K}S = \beta$ $\frac{RS}{SK} = \tan\beta$ $RS = \frac{2q\tan\beta}{p\sin\alpha}$ **OR** $\frac{RS}{\sin\beta} = \frac{SK}{\sin(90° - \beta)}$ $RS\cos\beta = SK\sin\beta$ $RS = SK\tan\beta$ $RS = \frac{2q\tan\beta}{p\sin\alpha}$	✓ $R\hat{K}S = \beta$ ✓ correct trig ratio (2) ✓ $R\hat{K}S = \beta$ ✓ $\tan\beta = \frac{\sin\beta}{\cos\beta}$ (2)
7.3	$70 = \frac{2(2500)\tan 42°}{80\sin\alpha}$ $\sin\alpha = \frac{25}{28}\tan 42°$ **OR** $\sin\alpha = 0{,}80...$ $\alpha = 53{,}51°$	✓ correct substitution of values into RS ✓ value of $\sin\alpha$ ✓ answer (3)
		[7]

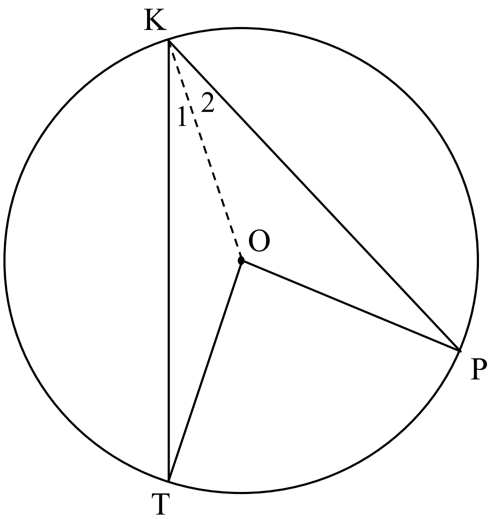
QUESTION/*VRAAG* 8

8.1



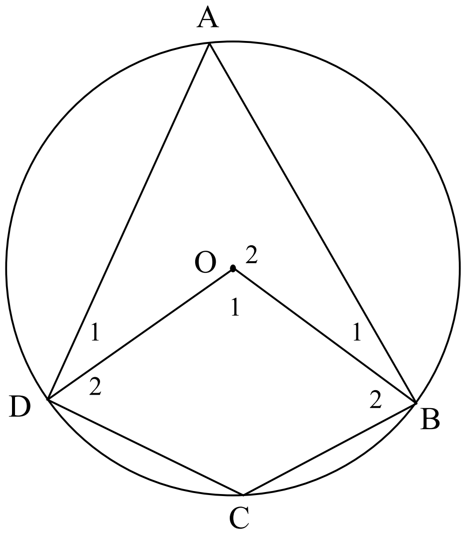
8.1	Construction: Draw KO produced $\hat{O}_1 = \hat{K}_1 + \hat{T}$ [ext ∠ of Δ] But $\hat{K}_1 = \hat{T}$ [∠s opp equal sides] $\therefore \hat{O}_1 = 2\hat{K}_1$ $\hat{O}_2 = \hat{K}_2 + P$ [ext ∠ of Δ] But $\hat{K}_2 = P$ [∠s opp equal sides] $\therefore \hat{O}_2 = 2\hat{K}_2$ $\therefore \hat{O}_1 + \hat{O}_2 = 2\hat{K}_1 + 2\hat{K}_2$ $= 2\left(\hat{K}_1 + \hat{K}_2\right)$ $\therefore T\hat{O}P = 2T\hat{K}P$ **OR**	✓ construction ✓ S / R ✓ S ✓ S ✓ S (5)

8.1



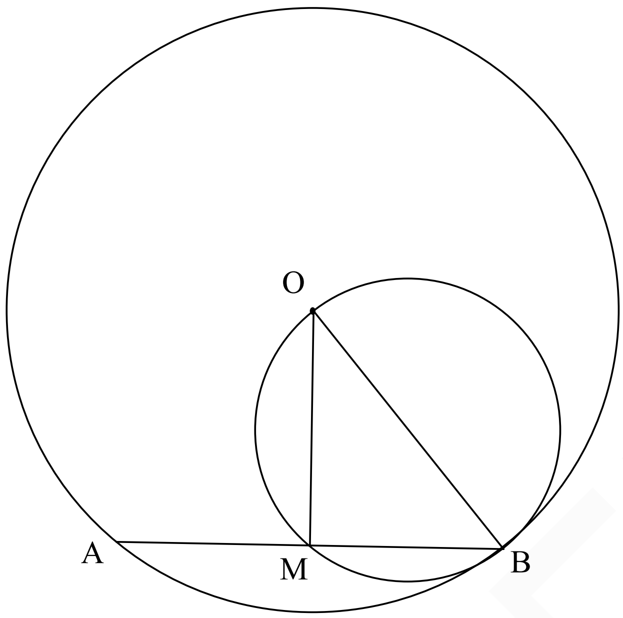
8.1	Construction: Draw KO $\hat{T} = \hat{K}_1$ [$\angle$s opp. equal sides] $\therefore K\hat{O}T = 180° - 2\hat{K}_1$ [sum of $\angle$s of ΔKOT] $\hat{P} = \hat{K}_2$ [$\angle$s opp. equal sides] $\therefore K\hat{O}P = 180° - 2\hat{K}_2$ [sum of $\angle$s of ΔKOP] $T\hat{O}P = 360° - \left(K\hat{O}T + K\hat{O}P\right)$ [$\angle$s around a point] $= 360° - \left(180° - 2\hat{K}_1 + 180° - 2\hat{K}_2\right)$ $= 2\hat{K}_1 + 2\hat{K}_2$ $= 2\left(\hat{K}_1 + \hat{K}_2\right)$ $\therefore T\hat{O}P = 2T\hat{K}P$	✓ construction ✓ S / R ✓ S ✓ S ✓ S (5)

8.2



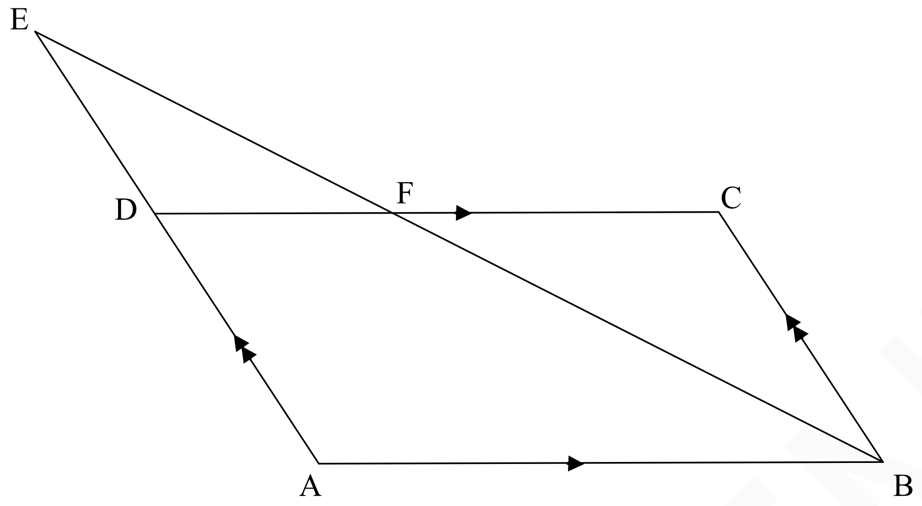
8.2	$\hat{O}_1 = 4x + 100°$ [given] $\therefore \hat{A} = 2x + 50°$ [∠ at centre = 2 ×∠ at circumference] $x + 34° + 2x + 50° = 180°$ [opp ∠s of cyclic quad] $3x = 96°$ $x = 32°$ **OR** $\hat{O}_2 = 2x + 68°$ [∠ at centre = 2 ×∠ at circumference] $4x + 100° + 2x + 68° = 360°$ [∠s round a pt] $6x = 192°$ $x = 32°$ **OR** $\hat{O}_2 = -4x + 260°$ [∠s round a pt] $2\hat{C} = -4x + 260°$ [∠ at centre = 2 ×∠ at circumference] $\hat{C} = -2x + 130°$ $x + 34° = -2x + 130°$ $3x = 96°$ $x = 32°$	 ✓ S ✓ R ✓ S ✓ R ✓ answer (5) ✓ S ✓ R ✓ S ✓ R ✓ answer (5) ✓ S ✓ R ✓ S ✓ R ✓ answer (5)

8.3



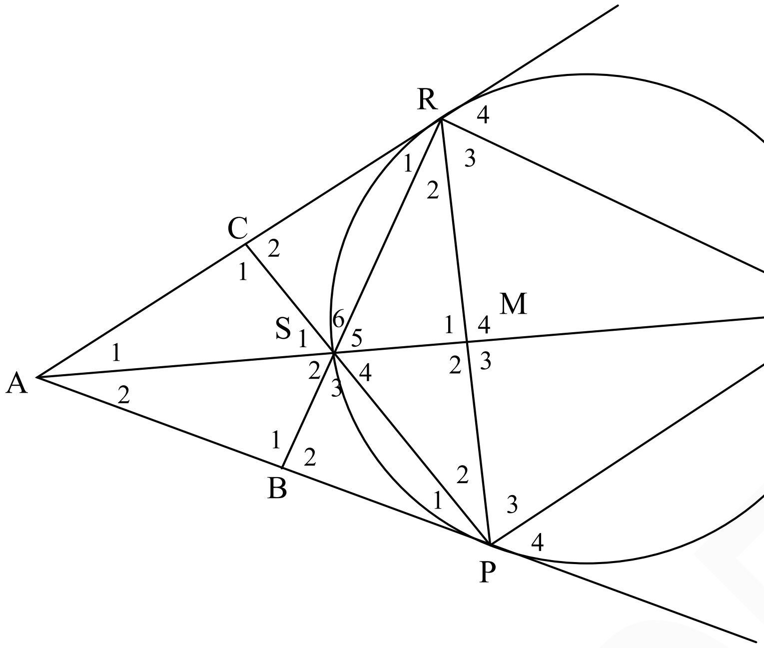
8.3.1	$O\hat{M}B = 90°$ [$\angle$ in semi circle]	✓ S ✓ R (2)
8.3.2	$AB = \sqrt{300} = 10\sqrt{3}$ $\therefore MB = 5\sqrt{3}$ [line from centre $\perp$ to chord] $OB^2 = OM^2 + MB^2$ [Pythagoras] $OB^2 = 5^2 + \left(5\sqrt{3}\right)^2$ $OB = 10$ units	 ✓ S ✓ R ✓ S ✓ answer (4)
		[16]

QUESTION/*VRAAG* 9



9.1	$\frac{FB}{EB}=\frac{DA}{EA}$ [prop theorem; DC ∥ AB] **OR** [line ∥ one side of Δ] $FB=\frac{4p\times 21}{7p}$ FB = 12 units	✓ S ✓ R ✓ answer (3)
9.2	In ΔEDF and ΔEAB: $\hat{E}$ is common $E\hat{D}F=\hat{A}$ [corresp ∠s; EA ∥ CB] $E\hat{F}D=E\hat{B}A$ [corresp ∠s; DC ∥ AB] ΔEDF ||| ΔEAB [∠;∠;∠]	✓ S ✓ S/R ✓ S **OR** R (3)
9.3	$\frac{DF}{AB}=\frac{ED}{EA}$ [||| Δs] $DF=\frac{3p\times 14}{7p}$ DF = 6 units FC = 8 units [DC = AB = 14 units; opp sides of $\|\|^m$] **OR** ΔEDF ||| ΔBCF [∠;∠;∠] $\frac{ED}{BC}=\frac{DF}{CF}$ [||| Δs] $\frac{3}{4}=\frac{14-FC}{FC}$ [BC = AD; opp sides of $\|\|^m$] $3FC=56-4FC$ $FC=8$	✓ S ✓ DF = 6 ✓ FC = 14 – DF (3) ✓ ΔEDF ||| ΔBCF ✓ $\frac{3}{4}=\frac{14-FC}{FC}$ ✓ answer (3)
		[9]

QUESTION/*VRAAG* 10



10.1	$\hat{S}_3 = P\hat{Q}R$ [ext $\angle$ of cyclic quad] $\hat{R}_3 = P\hat{Q}R$ [$\angle$s opp equal sides] $\therefore \hat{S}_3 = \hat{R}_3$ But $\hat{S}_4 = \hat{R}_3$ [$\angle$s in the same seg] $\therefore \hat{S}_3 = \hat{S}_4$	✓ S ✓ R ✓ S / R ✓ S ✓ R (5)
10.2	$\hat{R}_1 + \hat{R}_2 = P\hat{Q}R$ [tan chord theorem] $\hat{S}_4 = P\hat{Q}R$ [proved in 10.1] $\therefore \hat{S}_4 = \hat{R}_1 + \hat{R}_2$ SMRC is a cyclic quad [converse ext $\angle$ of cyclic quad]	✓ S ✓ R ✓ S ✓ R (4)
10.3	$\hat{S}_3 = \hat{R}_2 + \hat{P}_2$ [ext $\angle$ of Δ] $\hat{S}_4 = \hat{P}_1 + \hat{A}_2$ [ext $\angle$ of Δ] $\therefore \hat{R}_2 + \hat{P}_2 = \hat{A}_2 + \hat{P}_1$ But $\hat{P}_1 = \hat{R}_2$ [tan chord theorem] $\therefore \hat{P}_2 = \hat{A}_2$ RP is a tangent to the circle [converse tan chord theorem] **OR** [$\angle$ between line and chord] **OR** [converse alt seg theorem] **OR**	✓ S ✓ R ✓ S ✓ S ✓ R ✓ R (6)

	In ΔMSP and ΔMPA	
	$\hat{M}_2$ is common	✓ S
	AR = AP [tans from same point]	✓ S / R
	$\hat{R}_1 + \hat{R}_2 = \hat{P}_1 + \hat{P}_2$ [$\angle$s opp equal sides]	✓ S
	$\hat{S}_4 = \hat{R}_1 + \hat{R}_2$ [proved in 10.2]	
	$\therefore \hat{S}_4 = \hat{P}_1 + \hat{P}_2$	✓ S
	$\therefore \hat{P}_2 = \hat{A}_2$ [sum of $\angle$s in Δ]	✓ S
	RP is a tangent to the circle [converse tan chord theorem]	✓ R (6)
		[15]

TOTAL/TOTAAL: 150

basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

SENIOR CERTIFICATE EXAMINATIONS/
SENIORSERTIFIKAAT-EKSAMEN
NATIONAL SENIOR CERTIFICATE EXAMINATIONS/
NASIONALE SENIORSERTIFIKAAT-EKSAMEN

MATHEMATICS P2/*WISKUNDE V2*

MARKING GUIDELINES/*NASIENRIGLYNE*

MAY/JUNE/*MEI/JUNIE* 2023

MARKS: 150
PUNTE: 150

These marking guidelines consist of 21 pages./
Hierdie nasienriglyne bestaan uit 21 bladsye.

NOTE:

- If a candidate answers a question TWICE, mark only the FIRST attempt.
- If a candidate has crossed out an attempt at an answer and not redone the question, mark the crossed-out version.
- Consistent accuracy applies in ALL aspects of the marking guidelines. Stop marking at the second calculation error.
- Assuming answers/values in order to solve a problem is NOT acceptable.

LET WEL:

- *As 'n kandidaat 'n vraag TWEE KEER beantwoord, sien slegs die EERSTE poging na.*
- *As 'n kandidaat 'n antwoord van 'n vraag doodtrek en nie oordoen nie, merk die doodgetrekte poging.*
- *Volgehoue akkuraatheid word in ALLE aspekte van die nasienriglyne toegepas. Hou op nasien by die tweede berekeningsfout.*
- *Aanvaar van antwoorde/waardes om 'n probleem op te los, word NIE toegelaat nie.*

GEOMETRY • *MEETKUNDE*	
S	**A mark for a correct statement** ***(A statement mark is independent of a reason)***
	'n Punt vir 'n korrekte bewering ***('n Punt vir 'n bewering is onafhanklik van die rede)***
R	**A mark for the correct reason** ***(A reason mark may only be awarded if the statement is correct)***
	'n Punt vir 'n korrekte rede ***('n Punt word slegs vir die rede toegeken as die bewering korrek is)***
S/R	**Award a mark if statement AND reason are both correct**
	Ken 'n punt toe as die bewering EN rede beide korrek is

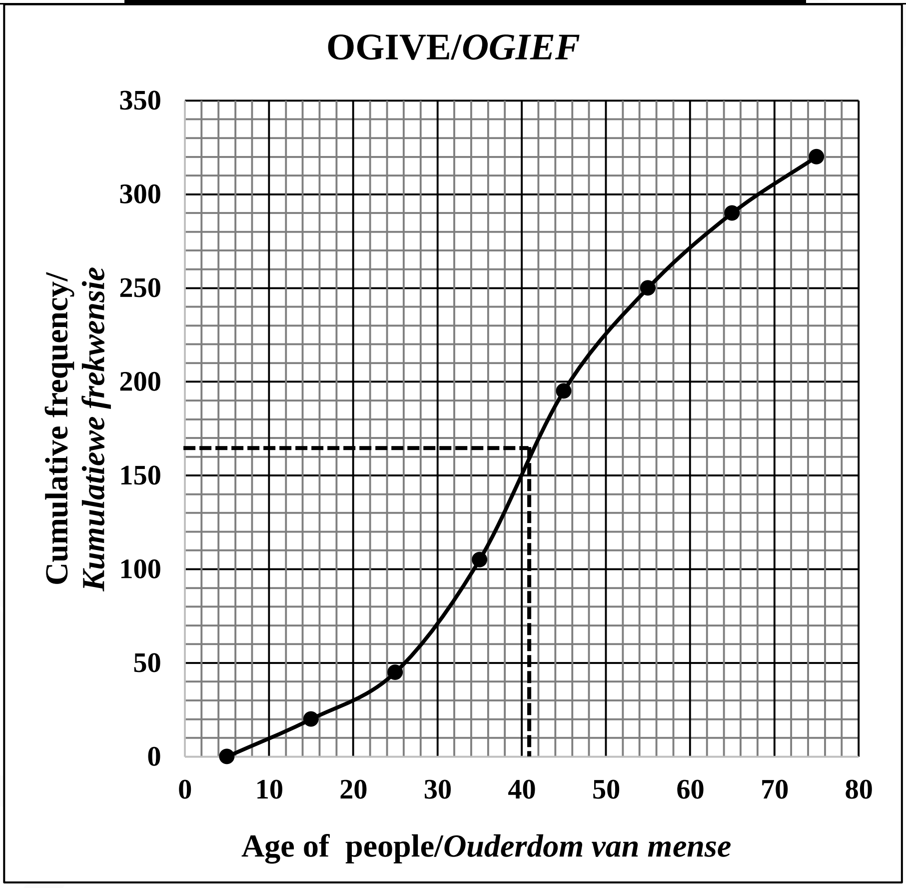
QUESTION/*VRAAG* 1

1.1.1	$a = 1730{,}22$ $b = 13{,}96$ $\hat{y} = 1730{,}22 + 13{,}96x$	✓ $a = 1730{,}22$ ✓ $b = 13{,}96$ ✓ equation (3)
1.1.2	$\hat{y} = 1730{,}22 + 13{,}96x$ $\hat{y} = 1730{,}22 + 13{,}96(28\,500)$ $\hat{y} = \text{R}399\ 590{,}22$ **OR/*OF*** $\hat{y} =$ R399 599,64 (calc)	✓ substitution ✓ answer (2) ✓✓ answer (2)
1.1.3	$r = 0{,}98002 \ldots$ $r = 0{,}98$	✓ answer (1)
1.1.4	There is a very strong positive correlation between the amount spent on advertising and sales. / *Daar is 'n baie sterk positiewe korrelasie tussen die bedrag spandeer op advertensie en die verkope.*	✓ strong positive (1)
1.2.1	$\bar{x} = \dfrac{1\,552\,195}{9}$ $\bar{x} = 172\,466{,}11$	✓ $\bar{x} = \dfrac{1\,552\,195}{9}$ ✓ answer (2)
1.2.2	$\sigma = 56\,950{,}09$	✓ answer (1)
1.2.3	$\bar{x} + \sigma$ $= 172\,466{,}11 + 56\,950{,}09$ $= 229\,416{,}20$ 2 years/*jaar*	✓ $\bar{x} + \sigma$ ✓ answer (2)
		[12]

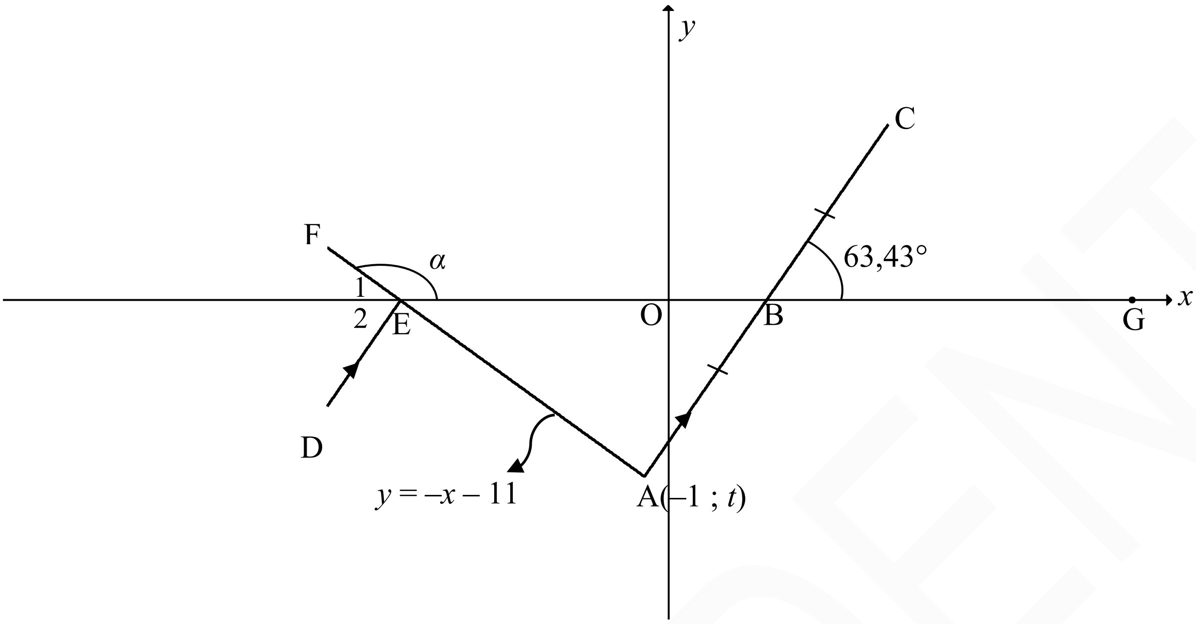
QUESTION/*VRAAG* 2

2.1	$35 < x \leq 45$	✓ answer (1)
2.2	320 people/*mense*	✓ answer (1)
2.3	see table and ogive below	✓ cumulative frequency ✓ grounding ✓ plotting at upper limit ✓ shape (4)
2.4	Median = 41	✓✓ answer (2)
		[8]

AGE	NUMBER OF PEOPLE	CUMULATIVE FREQUENCY
$5 < x \leq 15$	20	20
$15 < x \leq 25$	25	45
$25 < x \leq 35$	60	105
$35 < x \leq 45$	90	195
$45 < x \leq 55$	55	250
$55 < x \leq 65$	40	290
$65 < x \leq 75$	30	320



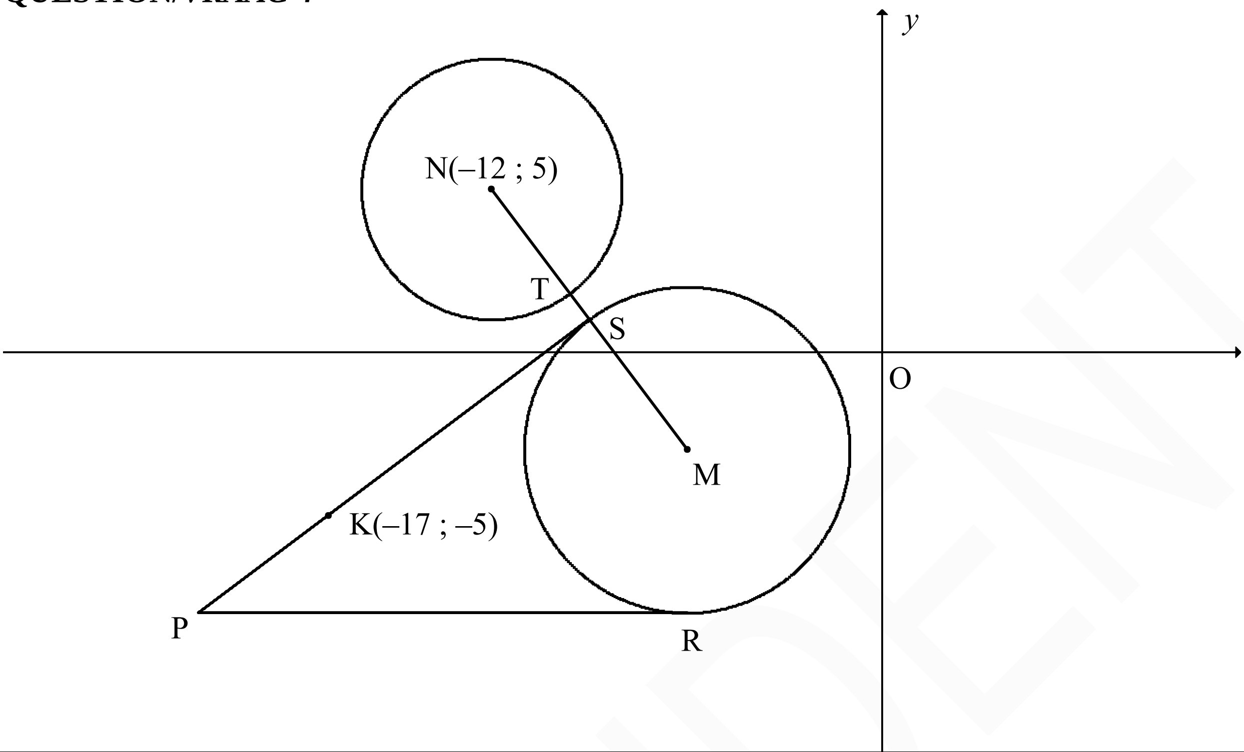
QUESTION/*VRAAG* 3



3.1.1	$y=-x-11$ A(–1 ; t) $t=-(-1)-11$ $t=-10$	✓ substitution ✓ value of t (2)
3.1.2	$\tan\alpha=-1$ $ref.\angle=45°$ $\therefore\alpha=135°$	✓ $\tan\alpha=-1$ ✓ 135° (2)
3.1.3	$\tan 63{,}43°=m_{AC}$ $m_{AC}=2$	✓ $\tan 63{,}43°=m_{AC}$ ✓ answer (2)
3.2	$m_{AC}=2$ A(–1 ; –10) $y=2x+k$ $-10=2(-1)+k$ $k=-8$ $y=2x-8$ **OR/*OF*** $y-y_1=2(x-x_1)$ $y-(-10)=2(x-(-1))$ $y=2x-8$	✓ substitution of m and A ✓ equation (2)

3.3.1	$y = 2x - 8$ $0 = 2x - 8$ $x_B = 4$ $\frac{x_C + (-1)}{2} = 4$ $\quad$ $\frac{y_C + (-10)}{2} = 0$ $x_C = 9$ $\quad$ $y_C = 10$ **OR/*OF*** by translation / *met translasie* $A \to B\ (x; y) \to (x+5; y+10)$ $B \to C\ (4;0) \to (4+5;\ 0+10) = (9\ ;\ 10)$	✓ $x_B = 4$ ✓ $x_C = 9$ ✓ $y_C = 10$ (3) ✓ $(x+5; y+10)$ ✓ $x_C = 9$ ✓ $y_C = 10$ (3)
3.3.2	$A\hat{B}E = 63,43°$ [vert. opp ∠'s =] $\hat{E}_2 = 63,43°$ [corres. ∠'s, DE ‖ AB] $\hat{E}_1 = 45°$ [∠s on a str line] $F\hat{E}D = 108,43°$ **OR/*OF*** $E\hat{A}B = 135° - 63,43°$ $E\hat{A}B = 71,57°$ $D\hat{E}A = E\hat{A}B = 71,57°$ $F\hat{E}D = 108,43°$ **OR/*OF*** $A\hat{B}E = 63,43°$ [vert. opp ∠'s] $D\hat{E}O = 116,57°$ [co-int. ∠'s, DE ‖ AB] $F\hat{E}D = 360° - (116,57° + 135°)$ $= 108,43°$	✓ $A\hat{B}E = 63,43°$ ✓ $\hat{E}_1 = 45°$ ✓ $F\hat{E}D = 108,43°$ (3) ✓ $E\hat{A}B = 71,57°$ ✓ $D\hat{E}A = E\hat{A}B = 71,57°$ ✓ $F\hat{E}D = 108,43°$ (3) ✓ $A\hat{B}E = 63,43°$ ✓ $D\hat{E}O = 116,57°$ ✓ $F\hat{E}D = 108,43°$ (3)
3.4	$y = 0$ $x_E = -11$ $\frac{x_G + (-11)}{2} = 4$ $x_G = 19$ $(x-19)^2 + y^2 = 15^2$ $(x-19)^2 + y^2 = 225$	✓ $x_E = -11$ ✓ $x_G = 19$ ✓ $(x-19)^2 + y^2$ ✓ 225 (4)
		[18]

QUESTION/*VRAAG* 4

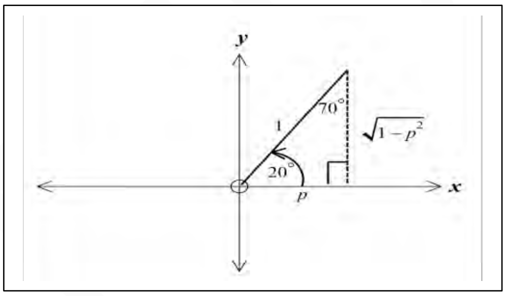


4.1	$M(-6;-3)$	✓ –6 ✓ –3 (2)
4.2.1	$x^2+y^2+24x-10y+153=0$ $(x+12)^2+(y-5)^2=-153+144+25$ $(x+12)^2+(y-5)^2=16$ $r^2=16$ $r=4$ units	✓ $r^2=-153+144+25$ ✓ length of radius (2)
4.2.2	$NM=\sqrt{(-12-(-6))^2+(5-(-3))^2}$ NM = 10 units SM = 5 units $\therefore$TS = 10 – 5 – 4 = 1 unit	✓ substitution into distance formula ✓ NM = 10 units ✓ SM = 5 units ✓ answer (4)
4.3.1	R(–6 ; –8) $y=-8$	✓ $y_R=-8$ ✓ answer (2)

4.3.2	$m_{NM} = \dfrac{5-(-3)}{-12-(-6)}$ $m_{NM} = -\dfrac{4}{3}$ $m_{tangent} = \dfrac{3}{4}$ $-5 = \dfrac{3}{4}(-17)+c$ **OR/*OF*** $y - y_1 = \dfrac{3}{4}(x - x_1)$ $c = \dfrac{31}{4}$ $y-(-5) = \dfrac{3}{4}(x-(-17))$ $y = \dfrac{3}{4}x + \dfrac{31}{4}$ $y = \dfrac{3}{4}x + \dfrac{31}{4}$	✓ substitution ✓ $m_{NM} = -\dfrac{4}{3}$ ✓ $m_{tangent} = \dfrac{3}{4}$ ✓ substitution of m and N ✓ equation (5)
	OR/OF NS = SM = 5 $S\left(\dfrac{-12-6}{2}\ ;\ \dfrac{5-3}{2}\right)$ S (– 9 ; 1) $m_{SK} = \dfrac{1-(-5)}{-9+17}$ $= \dfrac{6}{8} = \dfrac{3}{4}$ $y+5 = \dfrac{3}{4}(x+17)$ $y = \dfrac{3}{4}x + \dfrac{31}{4}$ *or* $y = \dfrac{3}{4}x + 7\dfrac{3}{4}$	✓ S midpoint ✓ coordinates of S ✓ $m_{tangent} = \dfrac{3}{4}$ ✓ substitution of m and K(–17 ; –5) or S ✓ equation (5)
4.4.1	$-8 = \dfrac{3}{4}x + \dfrac{31}{4}$ $-32 = 3x + 31$ $3x = -63$ $x = -21$ P(–21 ; –8) R(–6 ; –8) PR = PS = 15 units [tangents from same point] MS = MR = 5 units Perimeter PSMR = 15 + 15 + 5 + 5 = 40 units	✓ $-8 = \dfrac{3}{4}x + \dfrac{31}{4}$ ✓ $x = -21$ ✓ PR = PS = 15 units ✓ MS = MR = 5 units ✓ answer (5)

4.4.2	$\dfrac{\text{area of } \Delta\text{NPS}}{\text{area of quadrilateral PSMR}}$ $\dfrac{\frac{1}{2}\text{NS.SP}}{\frac{1}{2}\text{SP.MS}+\frac{1}{2}\text{MR.PR}}$ $=\dfrac{\frac{1}{2}(5)(15)}{2\left(\frac{1}{2}\right)(5)(15)}$ $=\dfrac{1}{2}$ **OR** $\Delta\text{NPS} \equiv \Delta\text{SPM} \equiv \Delta\text{MPR}$ $\dfrac{\text{area of } \Delta\text{NPS}}{\text{area of quadrilateral PSMR}}$ $=\dfrac{1}{2}$	✓ substitution ✓ answer (2) ✓ congruent ✓ answer (2)
		[22]

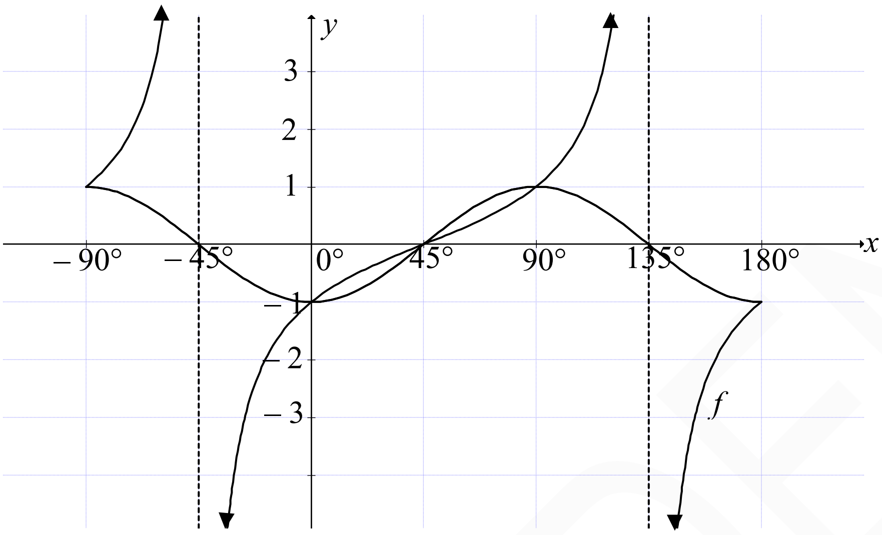
QUESTION/*VRAAG* 5

5.1	$\frac{1-\sin(-\theta)\cos(90°+\theta)}{\cos(\theta-360°)}$ $=\frac{1-(-\sin\theta)(-\sin\theta)}{\cos\theta}$ $=\frac{1-\sin^2\theta}{\cos\theta}$ $=\frac{\cos^2\theta}{\cos\theta}$ $=\cos\theta$	✓ $-\sin\theta$ ✓ $-\sin\theta$ ✓ $\cos\theta$ ✓ $\cos^2\theta$ ✓ answer (5)
5.2.1	$\cos 200°$ $=-\cos 20°$ $=-p$	✓ reduction ✓ answer (2)
5.2.2	$\sin(-70°)$ $=-\sin 70°$ $=-\cos 20°$ $=-p$ **OR/*OF*** $\sin(-70°)$ $=-\sin 70°$ $=-p$  	✓ reduction ✓ answer (2) ✓ reduction ✓ answer (2)
5.2.3	$\sin 10°$ $\cos(2(10°))=1-2\sin^2 10°$ $2\sin^2 10°=1-\cos 20°$ $\sin 10°=\sqrt{\frac{1-\cos 20°}{2}}$ $\sin 10°=\sqrt{\frac{1-p}{2}}$ **OR/*OF*** $\sin 10°$ $\sin(30°-20°)$ $=\sin 30°\cos 20°-\cos 30°\sin 20°$ $=\frac{1}{2}p-\frac{\sqrt{3}}{2}\sqrt{1-p^2}=\frac{p-\sqrt{3}\sqrt{1-p^2}}{2}$ **OR/*OF***	✓ double angle ✓ $\sin 10°$ as subject ✓ answer (3) ✓ using special angle ✓ expanding ✓ answer (3)

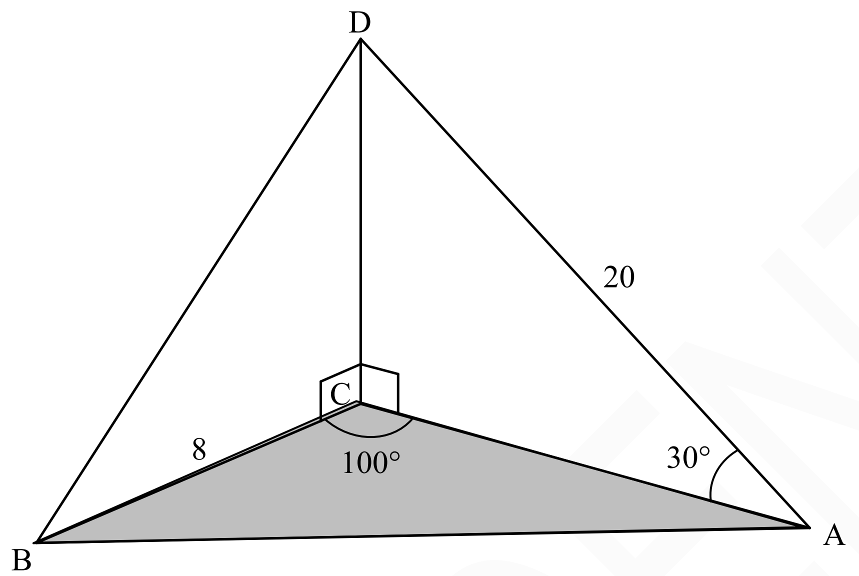
	$\sin 10°$ $\sin(70° - 60°)$ $= \sin 70° \cos 60° - \cos 70° \sin 60°$ $= p.\frac{1}{2} - \sqrt{1-p^2} \times \frac{\sqrt{3}}{2} = \frac{p - \sqrt{3}\sqrt{1-p^2}}{2}$ **OR/*OF*** $\sin 10°$ $= \cos 80°$ $\cos(60° + 20°)$ $= \cos 60° \cos 20° - \sin 60° \sin 20°$ $= \frac{1}{2}p - \frac{\sqrt{3}}{2}.\sqrt{1-p^2}$	✓ using special angle ✓ expanding ✓ answer (3) ✓ using special angle ✓ expanding ✓ answer (3)
5.3	$\cos(A + 55°)\cos(A + 10°) + \sin(A + 55°)\sin(A + 10°)$ $= \cos[A + 55° - (A + 10°)]$ $= \cos 45°$ $= \frac{1}{\sqrt{2}}$ or $\frac{\sqrt{2}}{2}$	✓✓ compound identity ✓ answer (3)
5.4.1	$\text{LHS} = \frac{\cos 2x + \sin 2x - \cos^2 x}{\sin x - 2\cos x}$ $\quad$ $\text{RHS} = -\sin x$ $= \frac{\cos^2 x - \sin^2 x + 2\sin x \cos x - \cos^2 x}{\sin x - 2\cos x}$ $= \frac{-\sin^2 x + 2\sin x \cos x}{\sin x - 2\cos x}$ $= \frac{-\sin x(\sin x - 2\cos x)}{\sin x - 2\cos x}$ $= -\sin x$ $\therefore \text{LHS} = \text{RHS}$	✓ $\cos^2 x - \sin^2 x$ ✓ $2\sin x \cos x$ ✓ common factor of $-\sin x$ (3)
5.4.2	$\frac{\cos 2x + \sin 2x - \cos^2 x}{-3\sin^2 x + 6\sin x \cos x}$ $= \frac{\cos 2x + \sin 2x - \cos^2 x}{-3\sin x(\sin x - 2\cos x)}$ $= \frac{\cos 2x + \sin 2x - \cos^2 x}{(\sin x - 2\cos x)} \times \frac{1}{-3\sin x}$ $= (-\sin x) \times \frac{1}{-3\sin x}$ $= \frac{1}{3}$	✓ common factor of $-3\sin x$ ✓ substitution ✓ answer (3)

5.5.1	$3\tan 4x = -2\cos 4x$ $3\left(\dfrac{\sin 4x}{\cos 4x}\right) = -2\cos 4x$ $3\sin 4x + 2\cos^2 4x = 0$ $3\sin 4x + 2\left(1-\sin^2 4x\right) = 0$ $-2\sin^2 4x + 3\sin 4x + 2 = 0$ $2\sin^2 4x - 3\sin 4x - 2 = 0$ $(2\sin 4x + 1)(\sin 4x - 2) = 0$ $\sin 4x = -\dfrac{1}{2}$ or $\sin 4x \neq 2$	✓ identity ✓ $1-\sin^2 4x$ ✓ standard form ✓ factors (4)
5.5.2	$\sin 4x = -\dfrac{1}{2}$ *ref.* $\angle = 30°$ $4x = 210° + k.360°$ or $4x = 330° + k.360°$ $x = 52{,}5° + k.90°$; $k \in Z$ $x = 82{,}5° + k.90°$; $k \in Z$	✓ 210° ; 330° ✓ 52,5° ; 82,5° ✓ $k.90°$; $k \in Z$ (3)
		[28]

QUESTION/VRAAG 6

6.1	Period = 180°	✓ answer (1)
6.2		✓ x-intercepts ✓ turning points ✓ end points (3)
6.3	$y \in [-1;1]$ **OR/*OF*** $-1 \leq y \leq 1$	✓ answer (1)
6.4	$g(x) = -\cos 2x$ $g(x+45°) = -\cos 2(x+45°)$ $= -\cos(2x+90°)$ $= \sin 2x$	✓ $-\cos 2(x+45°)$ ✓ answer (2)
6.5.1	$x \in (-90°;-45°)$ **OR/*OF*** $-90° < x < -45°$	✓✓ $x \in (-90°;-45°)$ (2)
6.5.2	$2\cos 2x - 1 > 0$ $\cos 2x > \frac{1}{2}$ $-\cos 2x < -\frac{1}{2}$ $x \in (-30°;30°)$ **OR/*OF*** $-30° < x < 30°$	✓ $\cos 2x > \frac{1}{2}$ ✓ $-\cos 2x < -\frac{1}{2}$ ✓ $x = \pm 30°$ ✓ interval (4)
		[13]

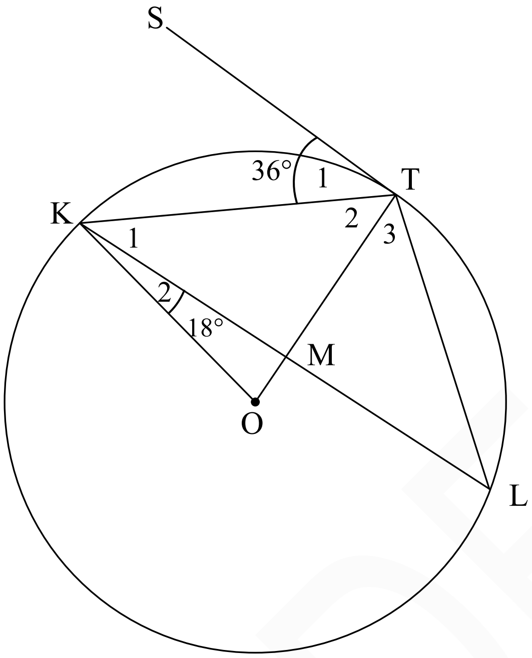
QUESTION/*VRAAG* 7



7.1.1	$\frac{AC}{20} = \cos 30°$ $AC = 20\cos 30°$ $AC = 10\sqrt{3} = 17{,}32$ units **OR/*OF*** $\frac{AC}{\sin 60°} = \frac{20}{\sin 90°}$ $\therefore AC = 20\sin 60 = 17{,}32$	✓ trig ratio ✓ answer (2) ✓ trig ratio ✓ answer (2)
7.1.2	$AB^2 = AC^2 + BC^2 - 2AC.BC\cos A\hat{C}B$ $AB^2 = (10\sqrt{3})^2 + 8^2 - 2(10\sqrt{3})(8)\cos 100°$ $AB = 20{,}30$ units	✓ cosine formula ✓ substitution into cosine formula ✓ answer (3)
7.2	$\frac{\sin A\hat{D}B}{AB} = \frac{\sin A\hat{B}D}{AD}$ $\frac{\sin A\hat{D}B}{20{,}3} = \frac{\sin 73{,}4°}{20}$ $\sin A\hat{D}B = \frac{20{,}3\sin 73{,}4°}{20}$ $A\hat{D}B = 76{,}58°$	✓ sine formula in ΔABD ✓ substitution into sine formula ✓ answer (3)
		[8]

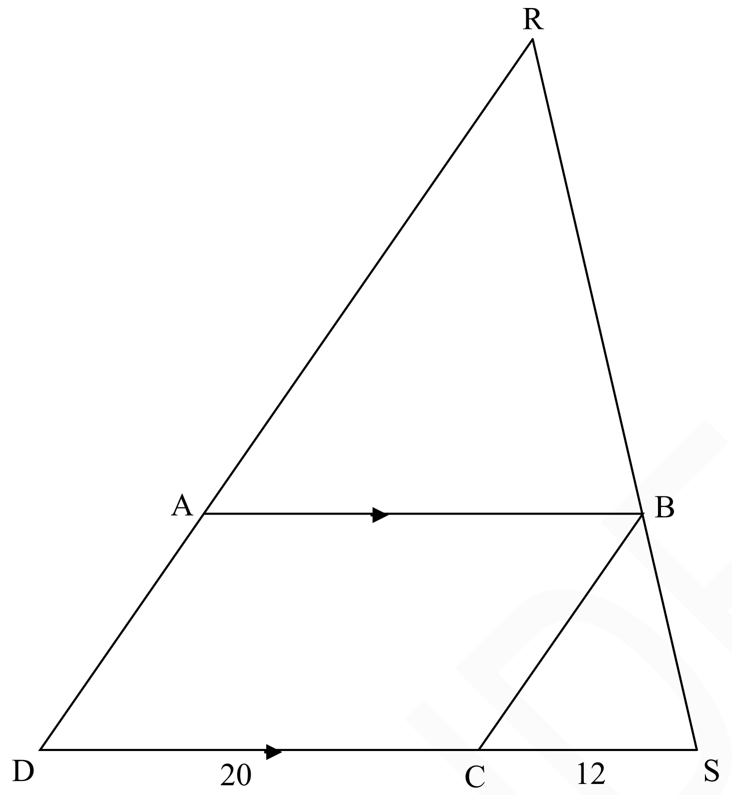
QUESTION/*VRAAG* 8

8.1



8.1.1(a)	$\hat{T}_2 = 54°$ [tan ⊥ rad]	✓ S ✓R (2)
8.1.1(b)	$\hat{L} = 36°$ [tan - chord theorem]	✓ S ✓R (2)
8.1.1(c)	$K\hat{O}T = 72°$ [∠ at centre = 2×∠ at circumference] **OR/*OF*** $O\hat{K}T = \hat{T}_2 = 54°$ [∠s opposite = radii] $K\hat{O}T = 180° - (54° + 54°)$ [sum of int ∠'s of Δ] $= 72°$	✓ S ✓R (2) ✓ S/R ✓ S (2)
8.1.2	$K\hat{M}O = 180° - (18° + 72°)$ $= 90°$ [sum of int ∠'s of Δ] $\therefore$ KM = ML [line from centre ⊥ to chord] **OR/*OF*** $O\hat{K}T = 54°$ [∠s opposite = radii] $\hat{K}_1 = 54° - 18° = 36°$ $T\hat{M}K = 90°$ [sum of int ∠'s of Δ] $\therefore$ KM = ML [line from centre ⊥ to chord]	✓ S ✓ S ✓ R (3) ✓ S ✓ S ✓ R (3)

8.2

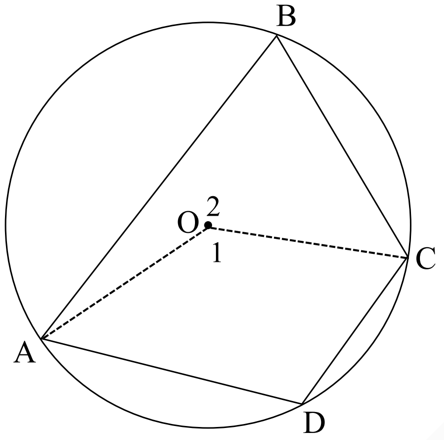


8.2.1	$\frac{DC}{CS}=\frac{20}{12}=\frac{5}{3}$ $\therefore \frac{DC}{CS}=\frac{RB}{BS}$ $\therefore$ BC || DR [converse line || one side of Δ OR sides in the $\therefore$ BC || AD same proportion]	✓ S ✓ S ✓ R (3)
8.2.2	$\frac{AR}{AD}=\frac{RB}{BS}$ [line || one side of Δ] **OR**[Prop Theorem AB || DS] $\frac{AR}{AD}=\frac{5}{3}$ $\frac{48-AD}{AD}=\frac{5}{3}$ $\therefore$ 5AD = 144 − 3AD AD = 18 AB = 20 [opp sides of parm] $\therefore$ AD : AB = 18 : 20 = 9 : 10	✓ $\frac{AR}{AD}=\frac{5}{3}$ ✓ AD = 18 ✓ ratio (3)

	OR/OF $\frac{AR}{RD}=\frac{5}{8}$.............prop thm $AB \parallel DS$ $\frac{AR}{48}=\frac{5}{8}$ $\therefore AR = 30$ and $AD = 18$ $\therefore \frac{AR}{RD}=\frac{AB}{DS}$........$\|\|\|$ $\Delta's$ $\therefore AB = 20$ $\therefore AB : AD = 18 : 20 = 9 : 10$	✓ $\frac{AR}{RD}=\frac{5}{8}$ ✓ AD = 18 ✓ ratio (3)
		[15]

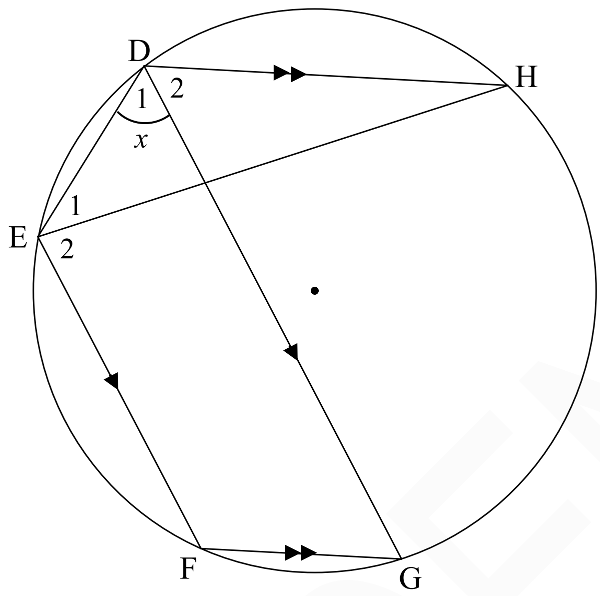
QUESTION/*VRAAG 9*

9.1



9.1	Constr: Draw radii OA and OC. Proof: $\hat{O}_1 = 2\hat{B}$ [$\angle$ at centre = $2\times\angle$ at circumference] $\hat{O}_2 = 2\hat{D}$ [$\angle$ at centre = $2\times\angle$ at circumference] $\hat{O}_1 + \hat{O}_2 = 360°$ [revolution] $2\hat{B} + 2\hat{D} = 360°$ [revolution] $\therefore \hat{B} + \hat{D} = 180°$	✓ Construction ✓ S ✓ R ✓ S/R ✓ S (5)

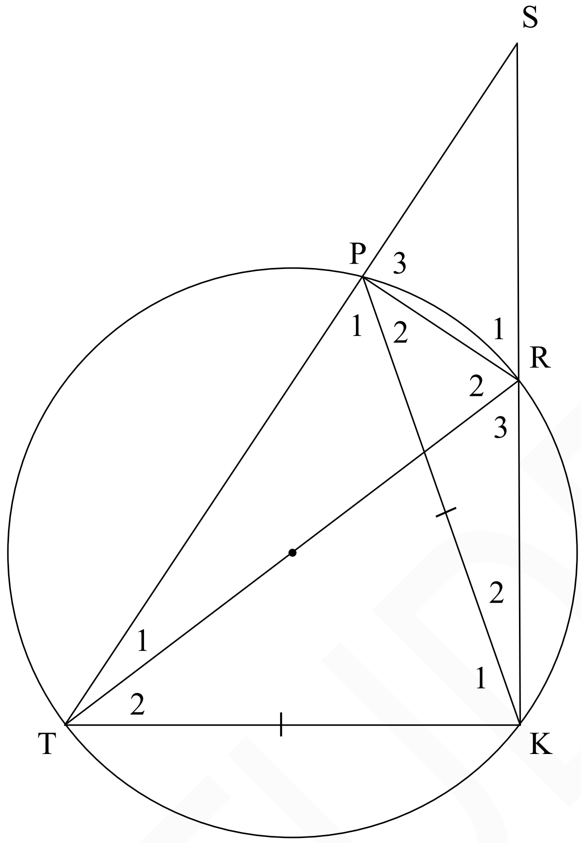
9.2



9.2	$\hat{EFG} = 180° - \hat{D}_1$ [opp $\angle$'s of cyclic quad] $\therefore \hat{EFG} = 180° - x$ $\hat{EFG} = 180° - \hat{G}$ [co-int $\angle$'s; EF || DG] $\hat{G} = x$ But $\hat{G} = \hat{D}_2$ [alt $\angle$'s; DH || FG] $\therefore \hat{D}_1 = \hat{D}_2 = x$	✓S ✓ R ✓S / R ✓ S / R (4)
		[9]

QUESTION/VRAAG 10

10.1



10.1.1	$T\hat{P}R = 90°$ $S\hat{P}R = 90°$ $\therefore$ SR is a diameter **OR** $T\hat{K}R = 90°$ $S\hat{P}R = 90°$ $\therefore$ SR is a diameter	[$\angle$ in semi-circle] [$\angle$'s on a straight line] [converse $\angle$ in semi-circle] [$\angle$ in semi-circle] [ext $\angle$ of cyclic quad] [converse $\angle$ in semi-circle] **OR** [chord subtends a right angle]	✓S ✓R ✓S ✓R (4) ✓S ✓R ✓S ✓R (4)

10.1.2	$\hat{R}_1 = P\hat{T}K$ [ext $\angle$ of cyclic quad] $\hat{P}_1 = P\hat{T}K = \hat{R}_1$ [$\angle$s opp equal sides] $\hat{S} + \hat{R}_1 = \hat{P}_1 + P_2$ [ext $\angle$ of Δ] $\therefore \hat{S} = \hat{P}_2$ [$\hat{R}_1 = \hat{P}_1$]	✓S ✓R ✓S /R ✓S ✓R (5)
10.1.3	In ΔSPK and ΔPRK $\hat{S} = \hat{P}_2$ [proved] $\hat{K}_2 = \hat{K}_2$ [common] $\Delta SPK \mid\mid\mid \Delta PRK$ [$\angle, \angle, \angle$] **OR/*OF*** In ΔSPK and ΔPRK $\hat{S} = \hat{P}_2$ [proved] $\hat{K}_2 = \hat{K}_2$ [common] $S\hat{P}K = P\hat{R}K$ [sum of $\angle$s in Δ] $\Delta SPK \mid\mid\mid \Delta PRK$	✓S ✓S ✓S/R (3) ✓S ✓S ✓S/R (3)
10.2	$\frac{PK}{RK} = \frac{SK}{PK}$ [$\Delta SPK \mid\mid\mid \Delta PRK$] $PK^2 = SK.RK$ $ST^2 = SK^2 + TK^2$ [Pythagoras] $TK = PK$ [Given] $ST^2 = SK^2 + PK^2$ $ST^2 = SK^2 + SK.RK$ $ST^2 = (2RK)^2 + 2RK.RK$ $ST^2 = 6RK^2$ $ST = \sqrt{6}RK$	✓S ✓S ✓ $PK^2 = SK.RK$ ✓$SK = 2RK$ ✓ $ST^2 = 6RK^2$ (5)
		[17]

TOTAL/TOTAAL: 150

basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

NATIONAL
SENIOR CERTIFICATE/
NASIONALE
SENIOR SERTIFIKAAT

GRADE 12/*GRAAD 12*

MATHEMATICS P2/*WISKUNDE V2*

NOVEMBER 2022

MARKING GUIDELINES/*NASIENRIGLYNE*

MARKS/*PUNTE*: 150

These marking guidelines consist of 24 pages.
Hierdie nasienriglyne bestaan uit 24 bladsye.

NOTE:

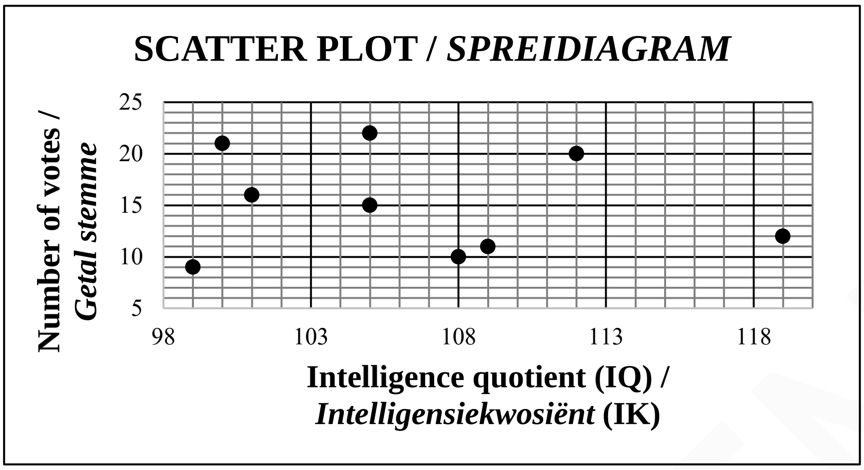
- **If a candidate answers a question TWICE, only mark the FIRST attempt.**
- **If a candidate has crossed out an attempt of a question and not redone the question, mark the crossed out version.**
- **Consistent accuracy applies in ALL aspects of the marking memorandum. Stop marking at the second calculation error.**
- **Assuming answers/values in order to solve a problem is NOT acceptable.**

NOTA:

- ***As 'n kandidaat 'n vraag TWEE KEER beantwoord, merk slegs die EERSTE poging.***
- ***As 'n kandidaat 'n antwoord van 'n vraag doodtrek en nie oordoen nie, merk die doodgetrekte poging.***
- ***Volgehoue akkuraatheid word in ALLE aspekte van die memorandum toegepas. Hou op nasien by die tweede berekeningsfout.***
- ***Aanvaar van antwoorde/waardes om 'n probleem op te los, word NIE toegelaat nie.***

GEOMETRY/*MEETKUNDE*	
S	**A mark for a correct statement** **(A statement mark is independent of a reason)**
	'n Punt vir 'n korrekte bewering ***('n Punt vir 'n bewering is onafhanklik van die rede)***
R	**A mark for the correct reason** **(A reason mark may only be awarded if the statement is correct)**
	'n Punt vir 'n korrekte rede ***('n Punt word slegs vir die rede toegeken as die bewering korrek is)***
S/R	**Award a mark if statement AND reason are both correct**
	Ken 'n punt toe as die bewering EN rede beide korrek is

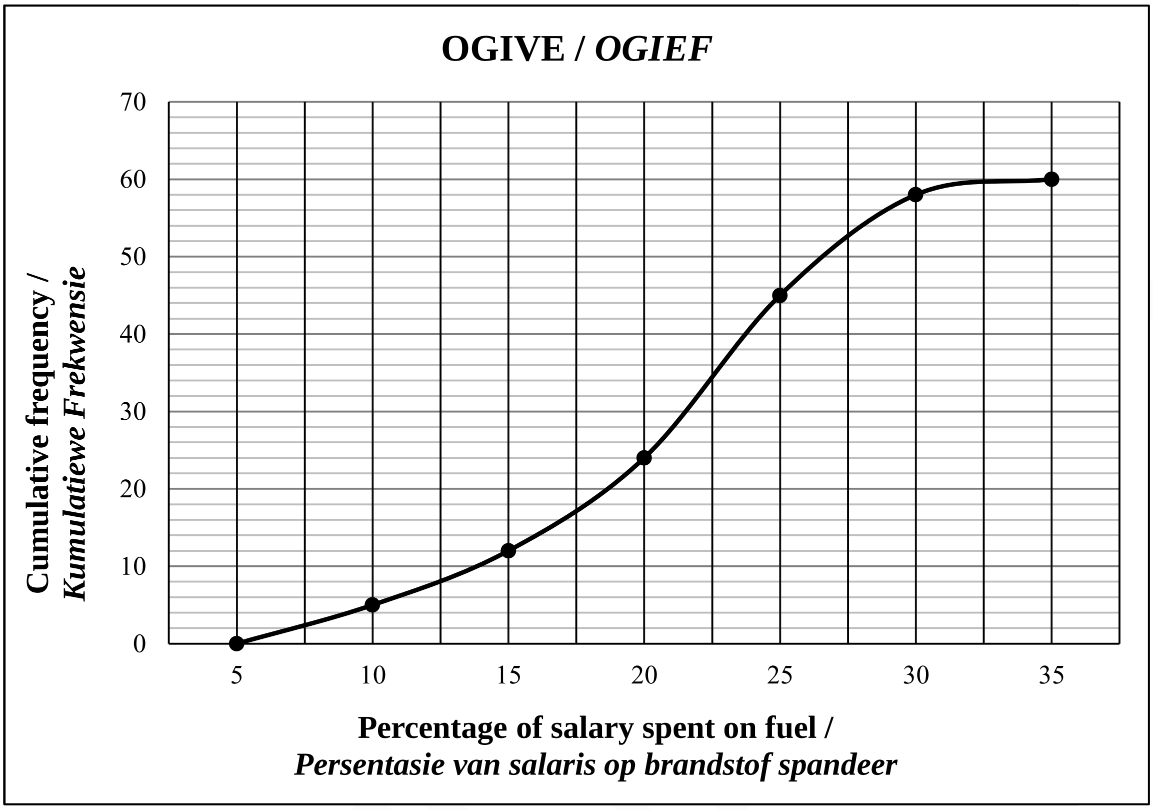
QUESTION/*VRAAG* 1



Popularity score (x) ***Gewildheidspunt* (x)**	32	89	35	82	50	59	81	40	79	65
Number of votes (y) ***Getal stemme* (y)**	9	22	10	21	11	15	20	12	19	16

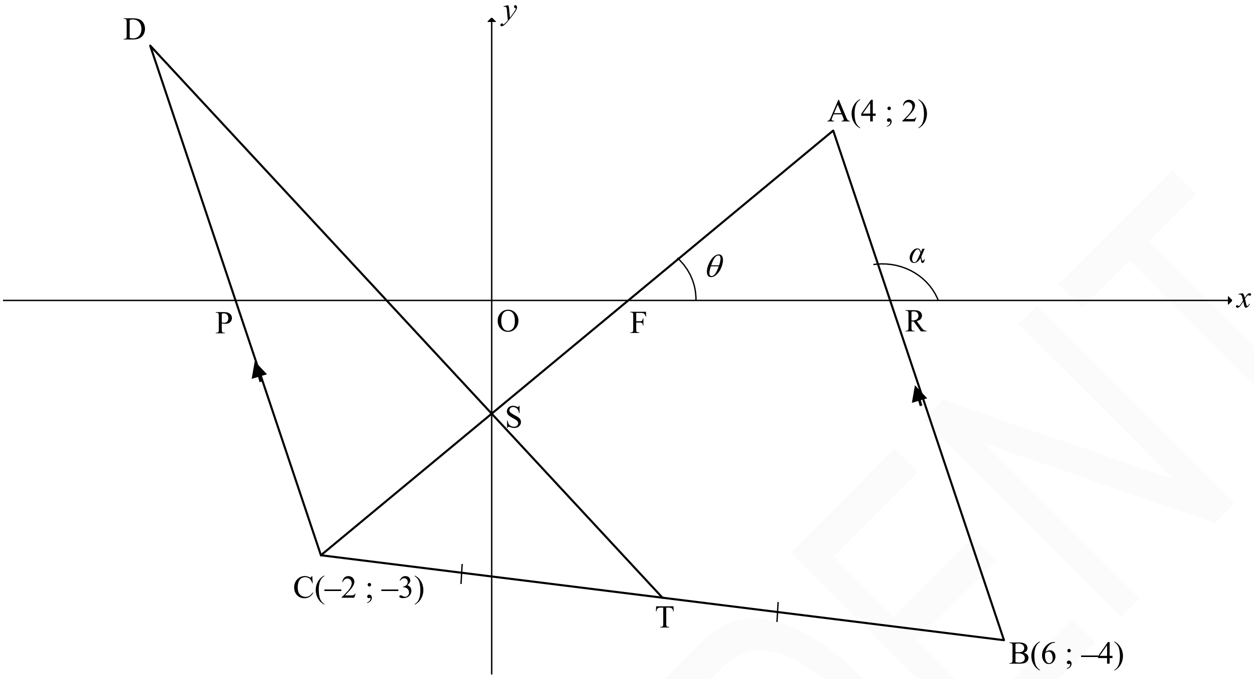
1.1.1	$\bar{y} = \frac{155}{10}$ $= 15,5$ ANSWER ONLY: Full marks	✓155 ✓answer (2)
1.1.2	SD $= 4,59$	✓ answer (1)
1.2	$\bar{y} - \text{SD}$ $= 15,5 - 4,59$ $= 10,91$ $\therefore\ 10 - 2 = 8$ learners	✓ value of $\bar{y} - \text{SD}$ ✓ answer (2)
1.3	$a = 1,7709\ldots$ $b = 0,2243\ldots$ $\hat{y} = 1,77 + 0,22x$	✓ a ✓ b ✓ equation (3)
1.4	$\hat{y} = 1,77 + 0,22(72)$ $= 17,61$ ≈ 18 votes **OR/*OF*** $\hat{y} = 17,92 \approx 18$ votes	✓ substitution ✓ answer (2) ✓✓ answer (2)
1.5.1	Points are all scattered therefore low correlation and unrealistic prediction./*Punte is versprei daarom 'n lae korrelasie en onrealistiese voorspelling.*	✓ R (1)
1.5.2	$r = 0,98$/correlation very strong/*korrelasie baie sterk* $\therefore$ a reliable prediction/*'n betroubare voorspelling*	✓ S (1)
		[12]

QUESTION/*VRAAG* 2



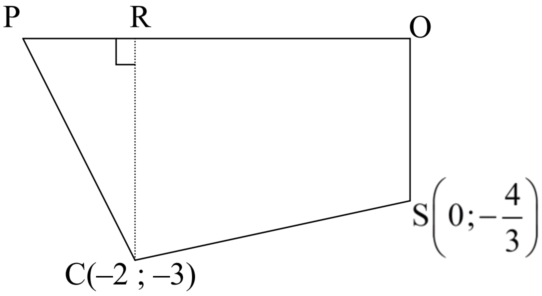
2.1	60 employees	✓ answer (A) (1)
2.2	$20 < x \leq 25$	✓ answer (1)
2.3	$60 - 34$ $= 26$ employees ANSWER ONLY: Full marks	✓ 34 ✓ answer (2)
2.4	Salary $= \frac{100}{7} \times 2400$ Salary = R34 285,71 ANSWER ONLY: Full marks	✓ method ✓ answer (2)
2.5	∴ Ogive/Cumulative frequency graph will shift to the right/will become steeper. ∴ *Ogief/Kumulatiewe frekwensie grafiek sal na regs skuif/sal steiler wees.*	✓✓ answer (2)
		[8]

QUESTION/*VRAAG* 3



3.1.1	$m_{AB}=\dfrac{2-(-4)}{4-6}$ **OR** $m_{AB}=\dfrac{-4-2}{6-4}$ $m_{AB}=-3$ ANSWER ONLY: Full marks	✓ substitution ✓ answer (2)
3.1.2	$\tan\alpha=m_{AB}=-3$ $\alpha=108{,}43°$ ANSWER ONLY: Full marks	✓ $\tan\alpha=m_{AB}=-3$ ✓ answer (2)
3.1.3	$T\left(\dfrac{x_1+x_2}{2};\dfrac{y_1+y_2}{2}\right)$ $T\left(\dfrac{-2+6}{2};\dfrac{-3-4}{2}\right)$ $T\left(2\,;\dfrac{-7}{2}\right)$	✓ $x_T=2$ ✓ $y_T=\dfrac{-7}{2}$ (2)
3.1.4	$5(0)-6y=8$ $y=-\dfrac{4}{3}$ $S\left(0;\dfrac{-4}{3}\right)$	✓ $x_S=0$ ✓ $y_S=\dfrac{-4}{3}$ (2)
3.2	$m_{CD}=m_{AB}=-3$ $-3=-3(-2)+c$ OR $y-(-3)=-3(x-(-2))$ $c=-9$ $\quad$ $y=-3x-9$ $y=-3x-9$	✓ gradient ✓ substitution of C(–2 ; –3) ✓ equation (3)

3.3.1	$5x-6y=8$ $y=\frac{5}{6}x-\frac{8}{6}$ $\tan\theta=m_{AC}=\frac{5}{6}$ $\theta=39{,}81°$ $\hat{A}=108{,}43°-39{,}81°$ $=68{,}62°$ $D\hat{C}A=68{,}62°$ [alt ∠s ; DC‖AB]	✓ $\tan\theta=m_{AC}=\frac{5}{6}$ ✓ $\theta=39{,}81°$ ✓ $\hat{A}=68{,}62°$ ✓ answer (4)
3.3.2	$P(-3;0)$ and F(1,6 ; 0) Area POSC = Area ΔFPC – Area ΔOFS $=\frac{1}{2}(4{,}6)(3)-\frac{1}{2}(1{,}6)\left(\frac{4}{3}\right)$ = 6,9 – 1,07 = 5,83 units^2 **OR/*OF*** $P(-3;0)$ $FC=\sqrt{\left(-2-\frac{8}{5}\right)^2+(-3-0)^2}=\frac{3\sqrt{61}}{5}$ Area $\Delta PFC=\frac{1}{2}(PF)(FC)\sin O\hat{F}S$ $=\frac{1}{2}\left(\frac{23}{5}\right)\left(\frac{3\sqrt{61}}{5}\right)\sin 39{,}81°$ $=6{,}90$ Area $\Delta OFS=\frac{1}{2}\left(\frac{8}{5}\right)\left(\frac{4}{3}\right)$ = 1,07 Area POSC = 6,90 – 1,07 = 5,83 units^2 **OR/*OF***	✓ $P(-3;0)$ ✓ method ✓ $\frac{1}{2}(4{,}6)(3)$ ✓ $\frac{1}{2}(1{,}6)\left(\frac{4}{3}\right)$ ✓ answer (5) ✓ $P(-3;0)$ ✓ $\frac{1}{2}\left(\frac{23}{5}\right)\left(\frac{3\sqrt{61}}{5}\right)\sin 39{,}81°$ ✓ $\frac{1}{2}\left(\frac{8}{5}\right)\left(\frac{4}{3}\right)$ ✓ method ✓ answer (5)



$P(-3;0)$ ✓ $P(-3;0)$

Area of POSC = Area of OSCR + Area of ΔPRC ✓ method

$$=\frac{1}{2}\left(\frac{4}{3}+3\right)\times 2+\frac{1}{2}(1\times 3)$$

✓ $\frac{1}{2}\left(\frac{4}{3}+3\right)\times 2$ ✓ $\frac{1}{2}(1\times 3)$

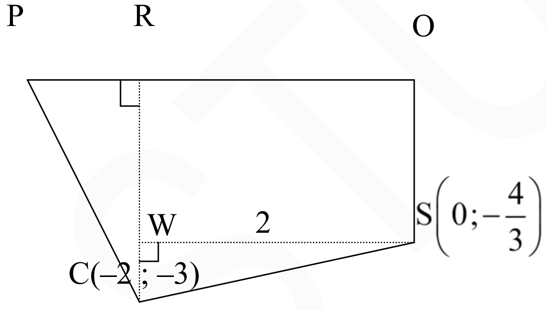
$$=\frac{35}{6}$$

$$=5{,}83\,\text{units}^2$$

✓ answer

(5)

OR/
OF



$P(-3;0)$ ✓ $P(-3;0)$

Area POSC = Area ROSW + Area ΔPRC + Area ΔWSC ✓ method

$$=\left(\frac{4}{3}\right)(2)+\frac{1}{2}(1)(3)+\frac{1}{2}(2)\left(\frac{5}{3}\right)$$

✓ $\frac{1}{2}(1)(3)$

✓ $\left(\frac{4}{3}\right)(2)+\frac{1}{2}(2)\left(\frac{5}{3}\right)$

$$=\frac{35}{6}$$

$$=5{,}83\text{units}^2$$

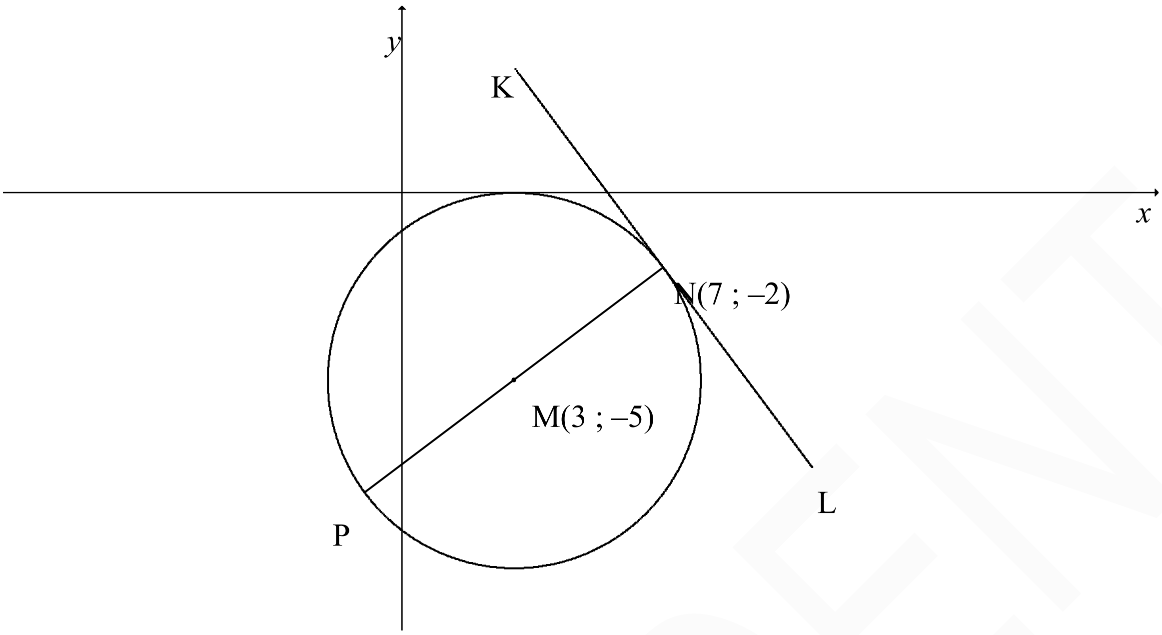
✓ answer

(5)

OR/*OF*

	$P(-3;0)$ Area of $\Delta PSC = \frac{1}{2}(PC)(CS)\sin D\hat{C}A$ $= \frac{1}{2}\left(\sqrt{10}\right)\left(\frac{\sqrt{61}}{3}\right)\sin 68,62°$ $= 3,833..$ Area of $\Delta POS = \frac{1}{2}(PO)(OS)$ $= \frac{1}{2}(3)\left(\frac{4}{3}\right)$ $= 2$ Area POSC = 3,833… + 2 $= 5,83\text{units}^2$	✓ $P(-3;0)$ ✓ $\frac{1}{2}\left(\sqrt{10}\right)\left(\frac{\sqrt{61}}{3}\right)\sin 68,62°$ ✓ $\frac{1}{2}(3)\left(\frac{4}{3}\right)$ ✓ method ✓ answer (5)
		[20]

QUESTION/*VRAAG* 4



4.1	$P(x;y);\ N(7;-2)\ ;\ M(3;-5)$ $\frac{x+7}{2}=3 \qquad \frac{y-2}{2}=-5$ $x=-1 \qquad y=-8$ $P(-1;-8)$	✓ $x_P=-1$ ✓ $y_P=-8$ (2)
4.2.1	$r^2=(7-3)^2+(-2-(-5))^2$ **OR/*OF*** $r^2=(-1-3)^2+(-8-(-5))^2$ $r^2=25$ $(x-3)^2+(y+5)^2=25$	✓ substitution into distance formula ✓ $(x-3)^2+(y+5)^2$ ✓ $r^2=25$ (3)
4.2.2	$m_{\text{radius}}=\frac{-5-(-2)}{3-7}=\frac{3}{4}$ $m_{\text{tangent}}=-\frac{4}{3}$ [radius ⊥ tangent/*raaklyn* ⊥ *radius*] $-2=-\frac{4}{3}(7)+c$ **OR** $y-(-2)=-\frac{4}{3}(x-7)$ $c=\frac{22}{3}$ $\qquad y=-\frac{4}{3}x+\frac{22}{3}$ $y=-\frac{4}{3}x+\frac{22}{3}$	✓ substitution ✓ $m_{\text{radius}}=\frac{-3}{-4}=\frac{3}{4}$ ✓ $m_{\text{tangent}}=-\frac{4}{3}$ ✓ substitution of m and N(7 ; –2) ✓ equation (5)
4.3	$-8=-\frac{4}{3}(-1)+c$ $\therefore\ c=-\frac{28}{3}$ $-\frac{28}{3}<k<\frac{22}{3}$	✓ subst m and P ✓ value of c ✓✓ answer (4)

4.4.1	$AB^2 = AM^2 - MB^2$ $AB^2 = \left[(t-3)^2 + (t+5)^2\right] - 5^2$ $= t^2 - 6t + 9 + t^2 + 10t + 25 - 25$ $AB = \sqrt{2t^2 + 4t + 9}$	✓ substitution into Pythagoras ✓ simplification (A) (2)
4.4.2	$t = \frac{-4}{2(2)}$ $= -1$ Minimum at $t = -1$ $AB = \sqrt{2(-1)^2 + 4(-1) + 9}$ $AB = \sqrt{7}$	✓ substitution into correct formula ✓ $t = -1$ ✓ substitution ✓ answer (4)
	OR/*OF* $4t + 4 = 0$ $t = -1$ Minimum at $t = -1$ $AB = \sqrt{2(-1)^2 + 4(-1) + 9}$ $AB = \sqrt{7}$	✓ derivative = 0 ✓ $t = -1$ ✓ substitution ✓ answer (4)
	OR/*OF* Length of $AB = \sqrt{2t^2 + 4t + 9}$ $= \sqrt{2\left(t^2 + 2t + \frac{9}{2}\right)}$ $= \sqrt{2\left[(t+1)^2 + \frac{7}{2}\right]}$ $= \sqrt{2(t+1)^2 + 7}$ Minimum at $t = -1$ $AB = \sqrt{2(-1)^2 + 4(-1) + 9}$ $AB = \sqrt{7}$	✓ completing of the square ✓ $t = -1$ ✓substitution ✓answer (4)
		[20]

QUESTION/*VRAAG* 5

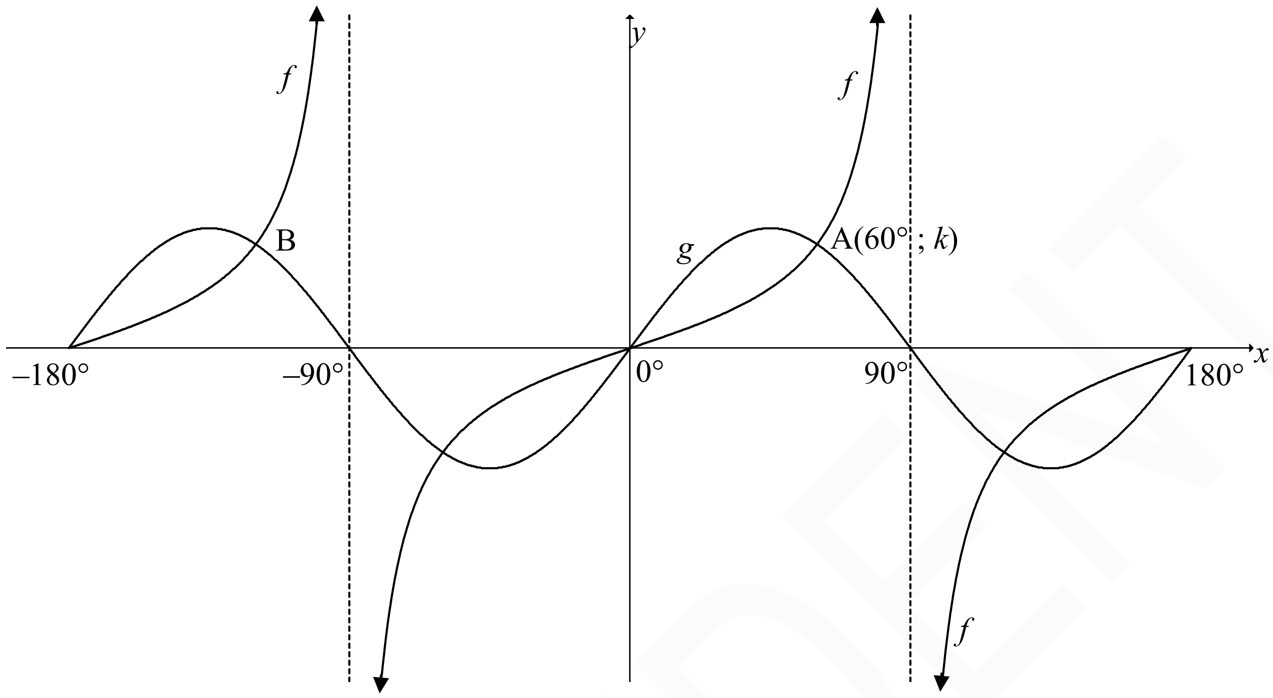
5.1.1	$\sin(360°+x)$ $=\sin x$	✓ + ✓ $\sin x$ (2)
5.1.2	$x-\text{coordinate}=\sqrt{\left(\sqrt{13}\right)^2-(-3)^2}$ $=-2$ $\tan x=\frac{-3}{-2}$ $=\frac{3}{2}$ **OR/*OF*** $x-\text{coordinate}=\sqrt{\left(\sqrt{13}\right)^2-(3)^2}$ $=2$ $\tan x=\frac{3}{2}$	✓✓ substitution ✓ method (3) ✓✓ substitution ✓ method (3)
5.1.3	$\cos(180°+x)$ $=-\cos x$	✓ – ✓ $\cos x$ (2)
5.2	$\frac{\cos(90°+\theta)}{\sin(\theta-180°)+3\sin(-\theta)}$ $=\frac{-\sin\theta}{\sin(-(180°-\theta))-3\sin\theta}$ $=\frac{-\sin\theta}{-\sin\theta-3\sin\theta}$ $=\frac{-\sin\theta}{-4\sin\theta}$ $=\frac{1}{4}$	✓ $-\sin\theta$ ✓ $-3\sin\theta$ ✓ $-\sin\theta$ ✓ simplification ✓ answer (5)

5.3	$(\cos x + 2\sin x)(3\sin 2x - 1) = 0$ $\cos x + 2\sin x = 0$ or $3\sin 2x - 1 = 0$ $\tan x = -\frac{1}{2}$ $\sin 2x = \frac{1}{3}$ ref $\angle$=26,565...° ref $\angle$=19,471...° $x = 153{,}43° + k.180°$; $k \in Z$ $x = 9{,}74° + k.180°$; $k \in Z$ **OR/*OF*** or $x = 153{,}43° + k.360°$; $k \in Z$ $x = 80{,}26° + k.180°$; $k \in Z$ or $x = 333{,}43° + k.360°$; $k \in Z$	✓ both equations ✓ $\tan x = -\frac{1}{2}$ ✓ $\sin 2x = \frac{1}{3}$ ✓ $x = 153{,}43°$ **OR** $x = 153{,}43°$ &333,43° ✓ $x = 9{,}74°$ & 80,26° ✓ $+ k.180°$; $k \in Z$ (6)
5.4.1	LHS $=\cos(x+y).\cos(x-y)$ $=[\cos x.\cos y - \sin x.\sin y][\cos x.\cos y + \sin x.\sin y]$ $= \cos^2 x.\cos^2 y - \sin^2 x.\sin^2 y$ $=(1-\sin^2 x)(1-\sin^2 y) - \sin^2 x.\sin^2 y$ $=1+\sin^2 x.\sin^2 y - \sin^2 x - \sin^2 y - \sin^2 x.\sin^2 y$ $=1-\sin^2 x - \sin^2 y$ = RHS	✓ expansion ✓ simplification ✓ square identity ✓ product (4)
5.4.2	$1-\sin^2 45° - \sin^2 15°$ $= \cos(45°+15°).\cos(45°-15°)$ $= \cos 60°.\cos 30°$ $= \left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$ $= \frac{\sqrt{3}}{4}$ **OR/*OF***	✓ identifying x and y ✓ substitution ✓ answer (3)

	$1-\sin^2 45°-\sin^2 15°$ $=\sin^2 15°+\cos^2 15°-\sin^2 45°-\sin^2 15°$ $=\cos^2 15°-\left(\frac{\sqrt{2}}{2}\right)^2$ $=\cos^2 15°-\frac{1}{2}$ $=\frac{2\cos^2 15°-1}{2}$ $=\frac{\cos 30°}{2}$ $=\frac{\sqrt{3}}{2}\times\frac{1}{2}$ $=\frac{\sqrt{3}}{4}$	✓ identity ✓ substitution ✓ answer (3)
	OR $1-\sin^2 45°-\sin^2 15°$ $=\cos^2 45°-\sin^2(45°-30°)$ $=\left(\frac{1}{\sqrt{2}}\right)^2-(\sin 45°\cos 30°-\cos 45°\sin 30°)^2$ $=\frac{1}{2}-\left(\frac{1}{\sqrt{2}}\times\frac{\sqrt{3}}{2}-\frac{1}{\sqrt{2}}\times\frac{1}{2}\right)^2$ $=\frac{1}{2}-\left(\frac{\sqrt{3}}{2\sqrt{2}}-\frac{1}{2\sqrt{2}}\right)^2$ $=\frac{1}{2}-\left(\frac{\sqrt{3}}{2\sqrt{2}}-\frac{1}{2\sqrt{2}}\right)^2$ $=\frac{1}{2}-\left(\frac{3}{8}-\frac{\sqrt{3}}{4}+\frac{1}{8}\right)$ $=\frac{\sqrt{3}}{4}$	✓ expansion ✓ substitution ✓ answer (3)

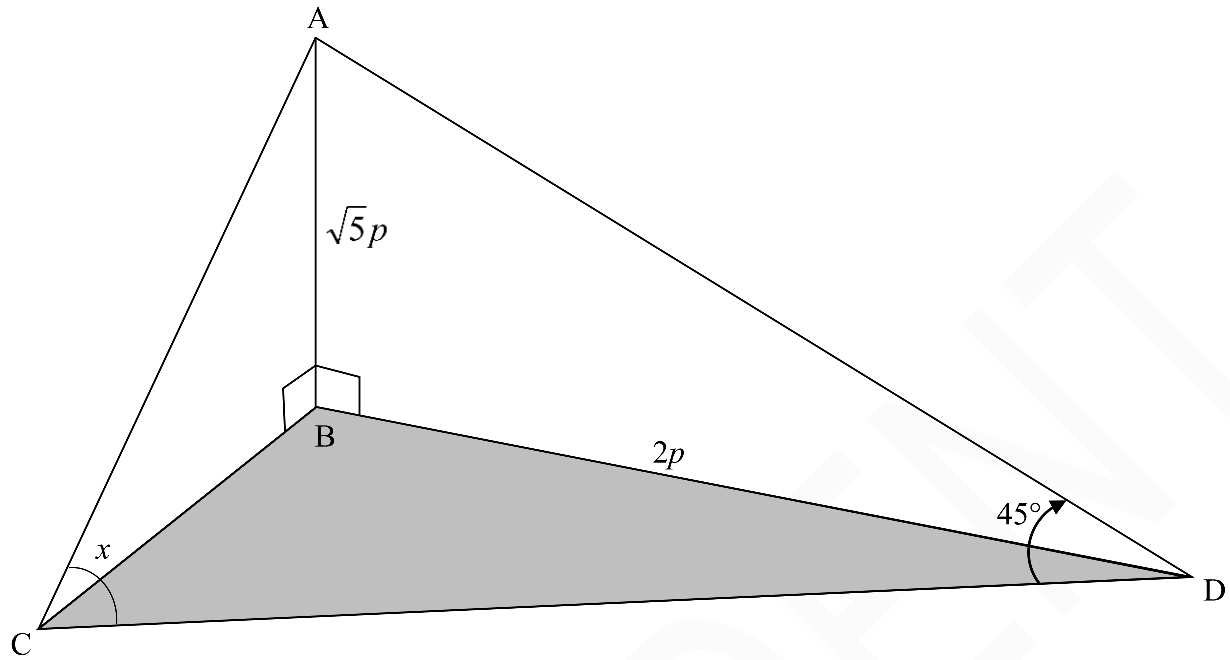
5.5.1	$16\sin x.\cos^3 x - 8\sin x.\cos x$ $= 8\sin x.\cos x(2\cos^2 x - 1)$ $= 4\sin 2x(\cos 2x)$ $= 2\sin 4x$ **OR/*OF*** $16\sin x.\cos^3 x - 8\sin x.\cos x$ $= 16\cos^2 x\left(\frac{1}{2}\sin 2x\right) - 8\left(\frac{1}{2}\sin 2x\right)$ $= 8\left(2\cos^2 x - 1\right)\left(\frac{1}{2}\sin 2x\right)$ $= 4\sin 2x.\cos 2x$ $= 2\sin 4x$	✓ factorisation ✓ 4sin2x ✓ cos2x ✓ double angle (4) ✓ factorisation ✓ $4\sin 2x$ ✓ cos2x ✓ double angle (4)
5.5.2	$16\sin x.\cos^3 x - 8\sin x.\cos x = 2\sin 4x$ Minimum at $x = 67{,}5°$	✓ answer (1)
		[30]

QUESTION/*VRAAG* 6



6.1	180°	✓ answer (1)
6.2.1	$k=\sqrt{3}=1,73$	✓ answer (1)
6.2.2	$B\left(-120°;\sqrt{3}\right)$	✓ $x=-120°$ (1)
6.3	Range of g : $y\in[-2;\ 2]$ Range of $2g(x)$: $y\in[-4;\ 4]$ **OR/*OF*** ANSWER ONLY: Full marks Range of g : $-2\le y\le 2$ Range of $2g(x)$: $-4\le y\ \le 4$	✓ $y\in[-2;\ 2]$ ✓ answer (2) ✓ $-2\le y\le 2$ ✓ answer (2)
6.4	$x\in[-65°;\ -5°]$ **OR/*OF*** $-65°\le x\ \le -5°$	✓✓ $x\in[-65°;\ -5°]$ (2) ✓✓ $-65°\le x\le -5°$ (2)
6.5	$\sin x.\cos x=p$ $4\sin x.\cos x=4p$ $2\sin 2x=4p$ $4p\ =\pm 2$ $\therefore\ p\ =-\frac{1}{2}$ or $\frac{1}{2}$ ANSWER ONLY: Full marks	✓ $2\sin 2x=4p$ ✓ $4p\ =\pm 2$ ✓ answers (3)
		[10]

QUESTION/*VRAAG* 7

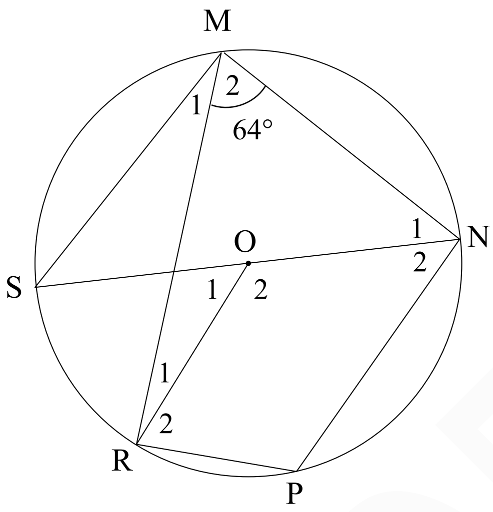


7.1	$AD^2 = AB^2 + BD^2$ $AD^2 = \left(\sqrt{5}p\right)^2 + (2p)^2$ $AD^2 = 9p^2$ $AD = 3p$	✓ substitution in Pythagoras ✓ answer (2)
7.2	$\dfrac{CD}{\sin(135° - x)} = \dfrac{3p}{\sin x}$ $CD = \dfrac{3p\sin(135° - x)}{\sin x}$ $CD = \dfrac{3p(\sin 135° \cos x - \cos 135° \sin x)}{\sin x}$ $CD = \dfrac{3p(\sin 45° \cos x + \cos 45° \sin x)}{\sin x}$ $CD = \dfrac{3p\left(\dfrac{\sqrt{2}}{2}\cos x + \dfrac{\sqrt{2}}{2}\sin x\right)}{\sin x}$ $CD = \dfrac{3p\left(\dfrac{\sqrt{2}}{2}\right)(\cos x + \sin x)}{\sin x}$ $CD = \dfrac{3p(\sin x + \cos x)}{\sqrt{2}\sin x}$	✓ correct use of sine rule ✓ $135° - x$ ✓ compound angle ✓ special values ✓ factorisation (5)

7.3	$\text{Area } \Delta\text{ADC} = \frac{1}{2}(\text{AD})(\text{CD})\sin\text{A}\hat{\text{D}}\text{C}$ $= \frac{1}{2}(3p)\left(\frac{3p(\sin x + \cos x)}{\sqrt{2}\sin x}\right)(\sin 45°)$ $= \frac{1}{2}(30)\left(\frac{30(\sin 110° + \cos 110°)}{\sqrt{2}\sin 110°}\right)\sin 45°$ $= 143{,}11 m^2$	✓ correct use of area rule ✓ substitution in area rule ✓ answer (3)
		[10]

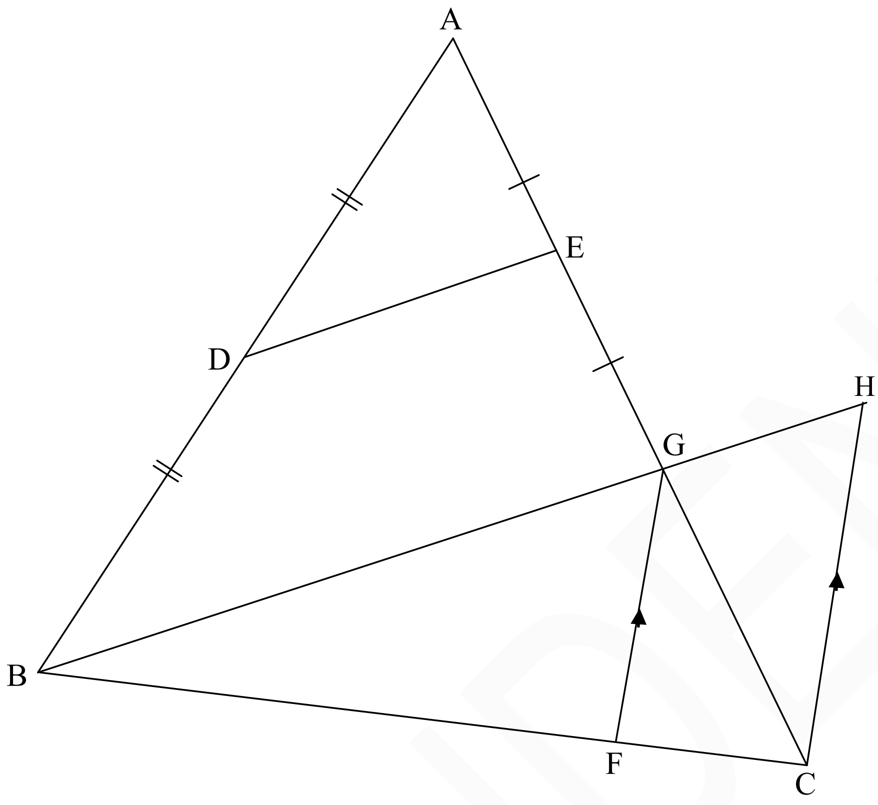
QUESTION/*VRAAG* 8

8.1



8.1.1	$\hat{P} = 116°$ [opp ∠s of cyclic quad/*teenoorst. ∠e van kvh*]	✓ S ✓ R (2)
8.1.2	$\hat{M}_1 + 64° = 90°$ [∠ in semi-circle/*∠ in halwe sirkel*] $\hat{M}_1 = 26°$	✓ R ✓ S (2)
8.1.3	$\hat{O}_1 = 52°$ [∠ at centre = 2 x ∠ at circumference/*midpts. ∠ = 2 x omtreks. ∠*]	✓ S ✓R (2)

8.2

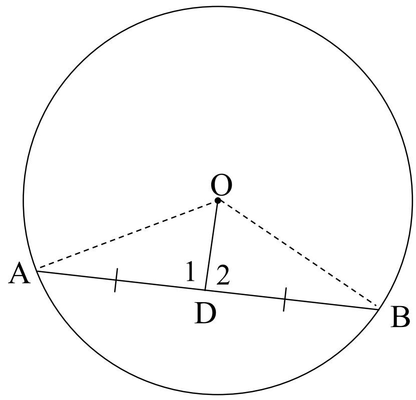


8.2.1	Midpt theorem/*Midpt. Stelling* **OR/*OF*** Converse prop intercept theorem	✓ R (1) ✓ R (1)
8.2.2	BG = 2DE or $6x - 2$ [Midpt theorem/*Midpt. stelling*] BG = $6x - 2$ $\frac{GH}{BG} = \frac{FC}{BF}$ [line || one side of Δ **OR** prop theorem; FG || CH / *lyn || een sy v.* Δ] $\frac{x+1}{6x-2} = \frac{1}{4}$ $4x + 4 = 6x - 2$ $2x = 6$ $x = 3$ **OR/*OF***	✓ S ✓ R ✓ S ✓R ✓ equation into x ✓ answer (6)

	$\frac{BF}{FC} = \frac{BG}{GH}$ [line ‖ one side of Δ **OR** prop theorem; FG ‖ CH / *lyn ‖ een sy v.* Δ] $\frac{AE}{AG} = \frac{DE}{BG}$ [ΔADE ‖‖ ΔABG] $BG = 4x + 4$ $\frac{1}{2} = \frac{3x-1}{4x+4}$ $\therefore 4x + 4 = 6x - 2$ $\therefore x = 3$	✓ S ✓ R ✓ S ✓R ✓ equation into x ✓ answer (6)
		[13]

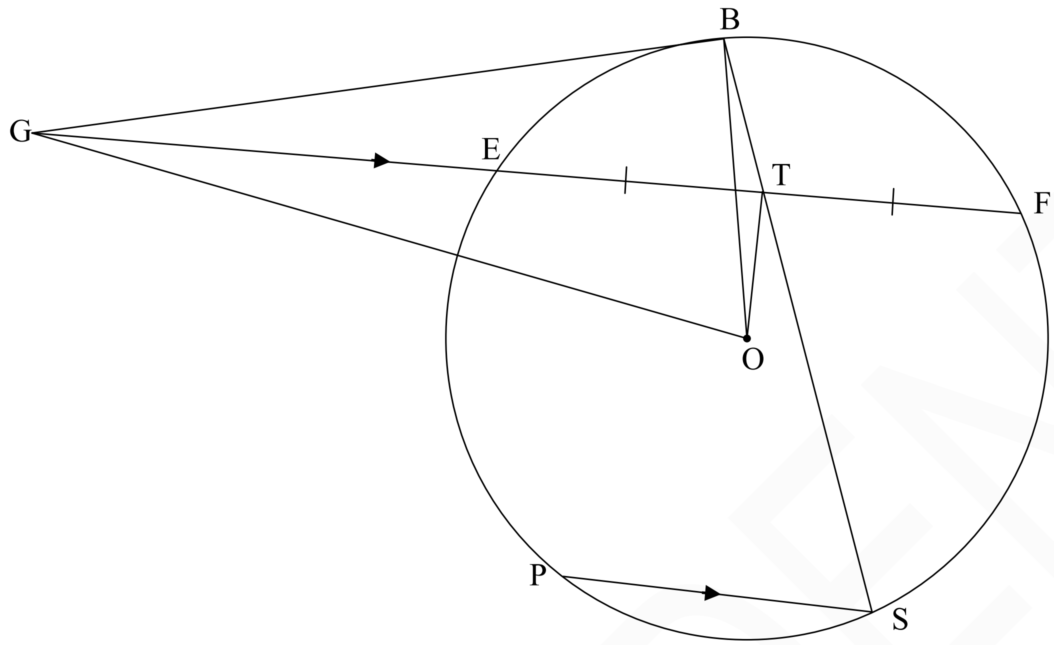
QUESTION/*VRAAG* 9

9.1



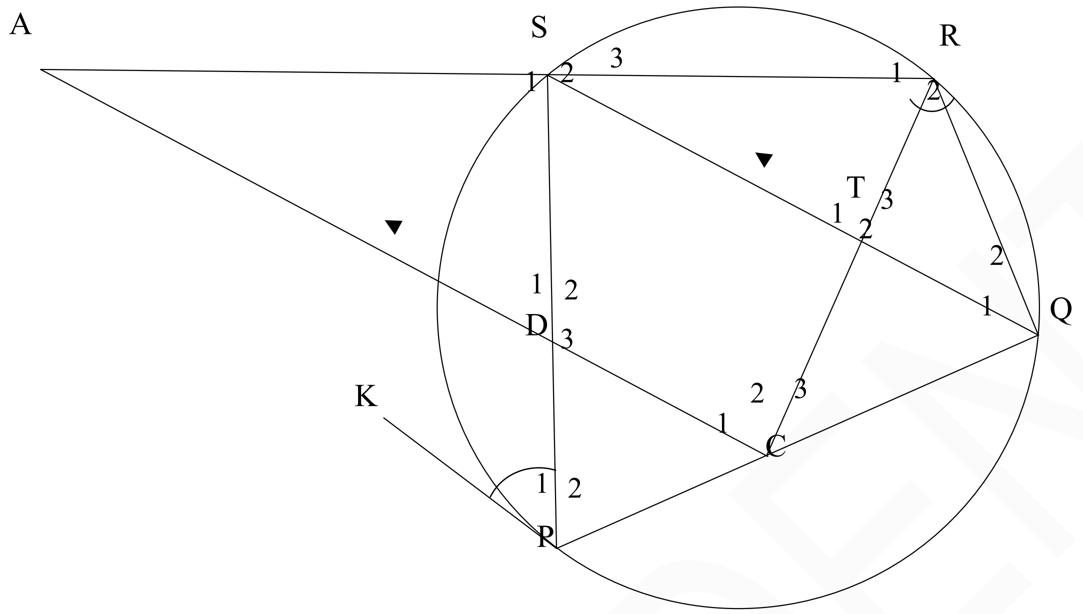
9.1.1	Construction:		
	Draw OA and OB		✓ construction
	In ΔADO and ΔBDO		
	OA = OB	[radii/*radiusse*]	
	OD = OD	[common side/*gemeenskaplike sy*]	✓ first pair of sides
	AD = DB	[given/*gegee*]	✓ other 2 pairs
	$\therefore \Delta ADO \equiv \Delta BDO$	[S;S;S]	✓ R
	ADB is a straight line		
	$\therefore \hat{D}_1 = \hat{D}_2$	$\Delta ADO \equiv \Delta BDO$	✓ R
	$\therefore OD \perp AB$	[$\angle$s on a str line/$\angle$*e op 'n reguitlyn*]	
			(5)
	OR/*OF*		
	Construction:		
	Draw OA and OB		✓ construction
	In ΔADO and ΔBDO		
	AD = DB	[given/*gegee*]	✓ first pair of sides
	$\hat{A} = \hat{B}$	[$\angle$s opp; $\angle$s sides /$\angle$*e teenoor gelyke sye*]	
	OA = OB	[radii/*radiusse*]	✓ other 2 pairs
	$\therefore \Delta ADO \equiv \Delta BDO$	[S;$\angle$;S]	✓ R
	ADB is a straight line		
	$\therefore \hat{D}_1 = \hat{D}_2$	$\Delta ADO \equiv \Delta BDO$	✓ R
	$\therefore OD \perp AB$	[$\angle$s on a str line/$\angle$*e op 'n reguitlyn*]	(5)

9.2



9.2.1	$\hat{OTG} = 90°$ [line from centre to midpt of chord/ *midpt. sirkel; midpt. koord*] $\hat{OBG} = 90°$ [tan ⊥ radius/*raaklyn ⊥ radius*] $\therefore \hat{OTG} = \hat{OBG} = 90°$ $\therefore$ OTBG is a cyclic quadrilateral [line subtends equal ∠s **OR** converse ∠s in the same segment/ *lyn onderspan gelyke ∠e*]	✓S ✓R ✓S ✓R ✓R (5)
9.2.2	$\hat{S} = \hat{BTG}$ [corresp ∠s; GF || PS / *ooreenk.* ∠s; GF || PS] But $\hat{BTG} = \hat{GOB}$ [∠s in the same segment/∠*e in dies. sirkelsegment*] $\hat{GOB} = \hat{S}$	✓S ✓R ✓S ✓R (4)
		[14]

QUESTION/*VRAAG* 10



10.1	$\hat{P}_1 = \hat{Q}_1$ [tan-chord theorem/$\angle$ *tussen raaklyn en koord*] $\hat{S}_1 = \hat{Q}_1 + \hat{Q}_2$ [ext $\angle$ of cyclic quad/*buite* $\angle$ *v. kvh*] $\therefore \hat{S}_1 = \hat{P}_1 + \hat{Q}_2$ $\hat{T}_2 = \hat{R}_2 + \hat{Q}_2$ [ext $\angle$ of Δ/*buite* $\angle$ *v.* Δ] but $\hat{P}_1 = \hat{R}_2$ [given/*gegee*] $\hat{T}_2 = \hat{P}_1 + \hat{Q}_2$ $\therefore \hat{S}_1 = \hat{T}_2 = \hat{P}_1 + \hat{Q}_2$	✓ S ✓ S / R ✓ S ✓ S (4)
10.2	In Δ ASD and ΔACR $\hat{A} = \hat{A}$ [common $\angle$/*gemeenskaplike* $\angle$] $\hat{S}_1 = \hat{T}_2$ [proven/*reeds bewys*] $\hat{T}_2 = \hat{C}_2$ [alt $\angle$s; QS || CA/verw. $\angle$*e*; QS || CA] $\therefore \hat{S}_1 = \hat{C}_2$ $\hat{D}_1 = \hat{R}_1$ [sum of $\angle$s in Δ/$\angle$*e v.* Δ] ΔASD ||| ΔACR $\therefore \frac{AD}{AR} = \frac{AS}{AC}$ [corresponding sides in proportion/ *ooreenstemmende sy in dies. verhouding*] **OR/*OF***	✓ identifying Δ's ✓ S ✓ S / R ✓ S ✓ S (5)

	In Δ ASD and ΔACR $\hat{A} = \hat{A}$ [common $\angle$/*gemeenskaplike* $\angle$] $\hat{S}_1 = \hat{T}_2$ [proven/*gegee*] $\hat{T}_2 = \hat{C}_2$ [alt $\angle$s; QS || CA/verw. $\angle e$; QS || CA] $\therefore \hat{S}_1 = \hat{C}_2$ $\Delta ASD \parallel\!\mid \Delta ACR$ [$\angle;\angle;\angle$] $\therefore \frac{AD}{AR} = \frac{AS}{AC}$ [corresponding sides in proportion/ *ooreenstemmende sy in dies. verhouding*]	✓ identifying Δ's ✓ S ✓ S/R ✓ S ✓ R (5)
10.3	$\frac{AS}{AC} = \frac{SD}{CR}$ [$\Delta ASD \parallel\!\mid \Delta ACR$] $\therefore AS = \frac{AC \times SD}{CR}$ $\frac{AS}{AR} = \frac{CT}{CR}$ [line || one side of Δ OR prop theorem; TS || CA/*lyn* || *een sy v.* Δ] $\therefore AS = \frac{AR \times CT}{CR}$ $\therefore \frac{AC \times SD}{CR} = \frac{AR \times CT}{CR}$ $\therefore AC \times SD = AR \times CT$	✓ S ✓ S ✓ R ✓ equating (4)
		[13]

TOTAL/*TOTAAL*: 150

basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

SENIOR CERTIFICATE EXAMINATIONS/
SENIORSERTIFIKAAT-EKSAMEN
NATIONAL SENIOR CERTIFICATE EXAMINATIONS/
NASIONALE SENIORSERTIFIKAAT-EKSAMEN

MATHEMATICS P2/*WISKUNDE V2*

MARKING GUIDELINES/*NASIENRIGLYNE*

2022

MARKS: 150
PUNTE: 150

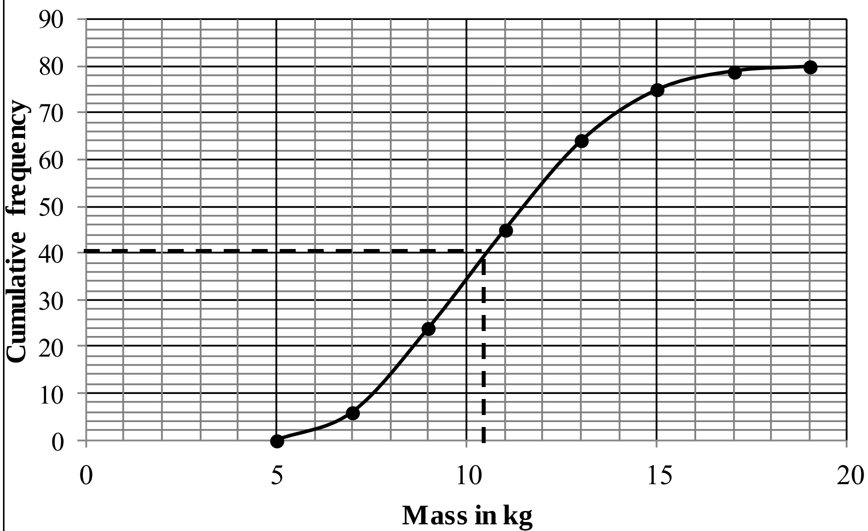
These marking guidelines consist of 20 pages./
Hierdie nasienriglyne bestaan uit 20 bladsye.

NOTE:
- If a candidate answers a question TWICE, mark only the FIRST attempt.
- If a candidate has crossed out an attempt at an answer and not redone the question, mark the crossed-out version.
- Consistent accuracy applies in ALL aspects of the marking guidelines. Stop marking at the second calculation error.
- Assuming answers/values in order to solve a problem is NOT acceptable.

LET WEL:
- *As 'n kandidaat 'n vraag TWEE KEER beantwoord, sien slegs die EERSTE poging na.*
- *As 'n kandidaat 'n antwoord van 'n vraag doodtrek en nie oordoen nie, merk die doodgetrekte poging.*
- *Volgehoue akkuraatheid word in ALLE aspekte van die nasienriglyne toegepas. Hou op nasien by die tweede berekeningsfout.*
- *Aanvaar van antwoorde/waardes om 'n probleem op te los, word NIE toegelaat nie.*

GEOMETRY • *MEETKUNDE*	
S	**A mark for a correct statement** ***(A statement mark is independent of a reason)***
	'n Punt vir 'n korrekte bewering ***('n Punt vir 'n bewering is onafhanklik van die rede)***
R	**A mark for the correct reason** ***(A reason mark may only be awarded if the statement is correct)***
	'n Punt vir 'n korrekte rede ***('n Punt word slegs vir die rede toegeken as die bewering korrek is)***
S/R	**Award a mark if statement AND reason are both correct**
	Ken 'n punt toe as die bewering EN rede beide korrek is

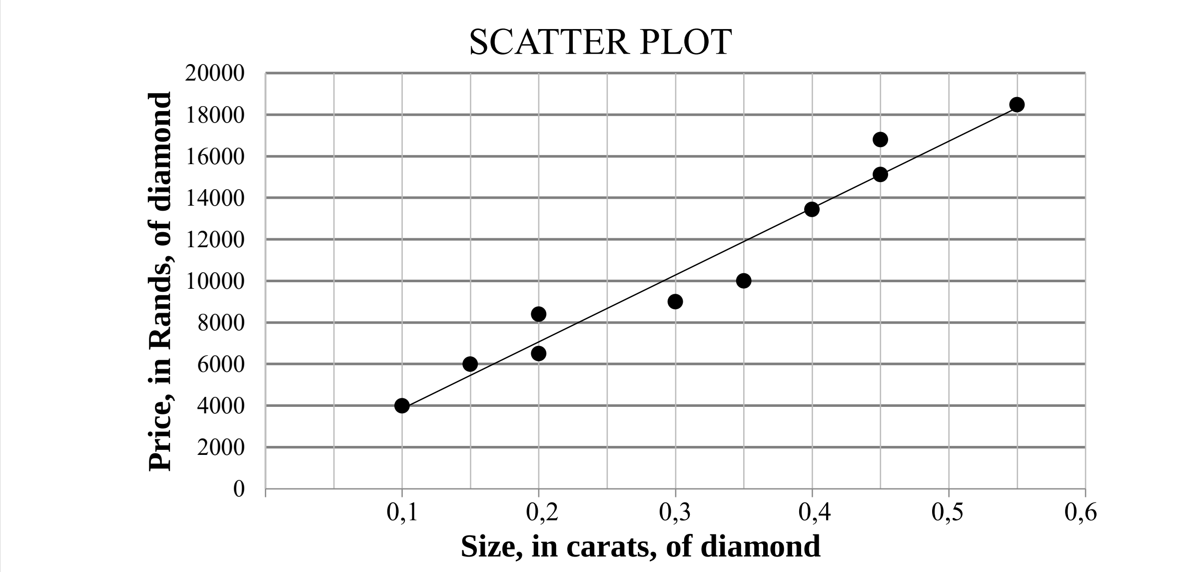
QUESTION/*VRAAG* 1

1.1	Modal class: $9 < m \leq 11$	✓ answer (1)
1.2	<table><tr><th>Mass (in kg)</th><th>Frequency</th><th>Cumulative frequency</th></tr><tr><td>$5 < m \leq 7$</td><td>6</td><td>6</td></tr><tr><td>$7 < m \leq 9$</td><td>18</td><td>24</td></tr><tr><td>$9 < m \leq 11$</td><td>21</td><td>45</td></tr><tr><td>$11 < m \leq 13$</td><td>19</td><td>64</td></tr><tr><td>$13 < m \leq 15$</td><td>11</td><td>75</td></tr><tr><td>$15 < m \leq 17$</td><td>4</td><td>79</td></tr><tr><td>$17 < m \leq 19$</td><td>1</td><td>80</td></tr></table>	✓ adding ✓ 80 (2)
1.3		✓ grounding (5 ; 0) ✓ points ✓ shape (3)
1.4	Median mass: 10,5 kg	✓✓ answer (2)
1.5.1	$\bar{x} = \frac{(6\times6+18\times8+21\times10+19\times12+11\times14+4\times16+1\times18)}{80}$ $= \frac{854}{80}$ $= 10{,}68$ Answer only 2/2	✓ 854 ✓ answer (2)
1.5.2	Learners' bags are heavier than the stipulated international guideline. Estimated mean = 10,68 kg 10% of 80 kg = 8 kg 10,68 kg > 8 kg	✓ answer ✓ 8 kg (2)

	OR/ *OF* Learners' bags are heavier than the stipulated international guideline. Estimated mean $= \frac{10,68}{80} \times 100$ $= 13,35\%$ $13,35\% > 10\%$	✓ answer ✓ 13,35% (2)
		[12]

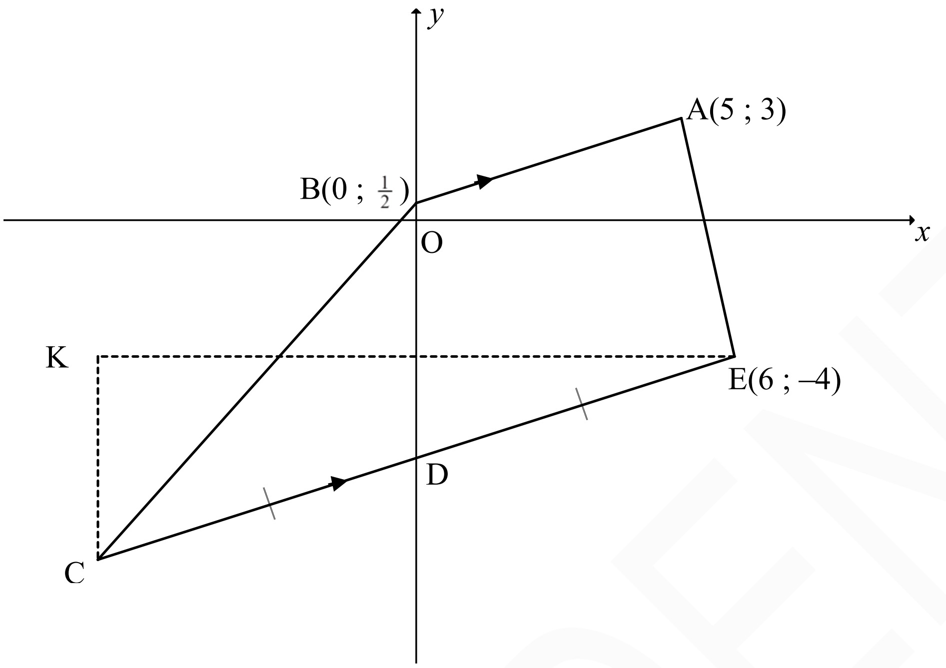
QUESTION/*VRAAG* 2

Size, in carats, of diamond (x)	0,1	0,15	0,2	0,2	0,3	0,35	0,4	0,45	0,45	0,55
Price, in rands, of diamond (y)	4 000	6 000	6 500	8 400	9 000	10 000	13 440	15 120	16 800	18 480



2.1	$a = 634{,}382\ldots$ $b = 32\ 189{,}263\ldots$ $\hat{y} = 634{,}38 + 32189{,}26x$ Answer only 3/3	✓ a ✓ b ✓ equation (3)
2.2	$\hat{y} = 634{,}38 + 32189{,}26(0{,}25)$ $= R8\ 681{,}70$ **OR/*OF*** $\hat{y} = R8\ 681{,}70$ (if using calculator)	✓ substitution ✓ answer (2) ✓ ✓ answer (2)
2.3	Average price increase $= R\dfrac{32189{,}26}{20}$ per 0,05 carat $= R1\ 609{,}46$ per 0,05 carat **OR/*OF*** Average price increase $= 0{,}05 \times 32\ 189{,}26$ $= R1\ 609{,}46$ per 0,05 carat **OR/*OF*** at 0,3: $\hat{y} = R10\ 291{,}16$ $\therefore$ Average price increase $= 10\ 291{,}16 - 8\ 681{,}70$ $= R1\ 609{,}46$ per 0,05 carat Answer only 2/2	✓ divide gradient by 20 ✓ answer (2) ✓ multiply gradient by 0,05 ✓ answer (2) ✓ Estimated price of a 0,3 carat diamond ✓ answer (2)
2.4	The point (0,35 ; 11500) is closer to the least squares regression line.	✓ reason (1)
		[8]

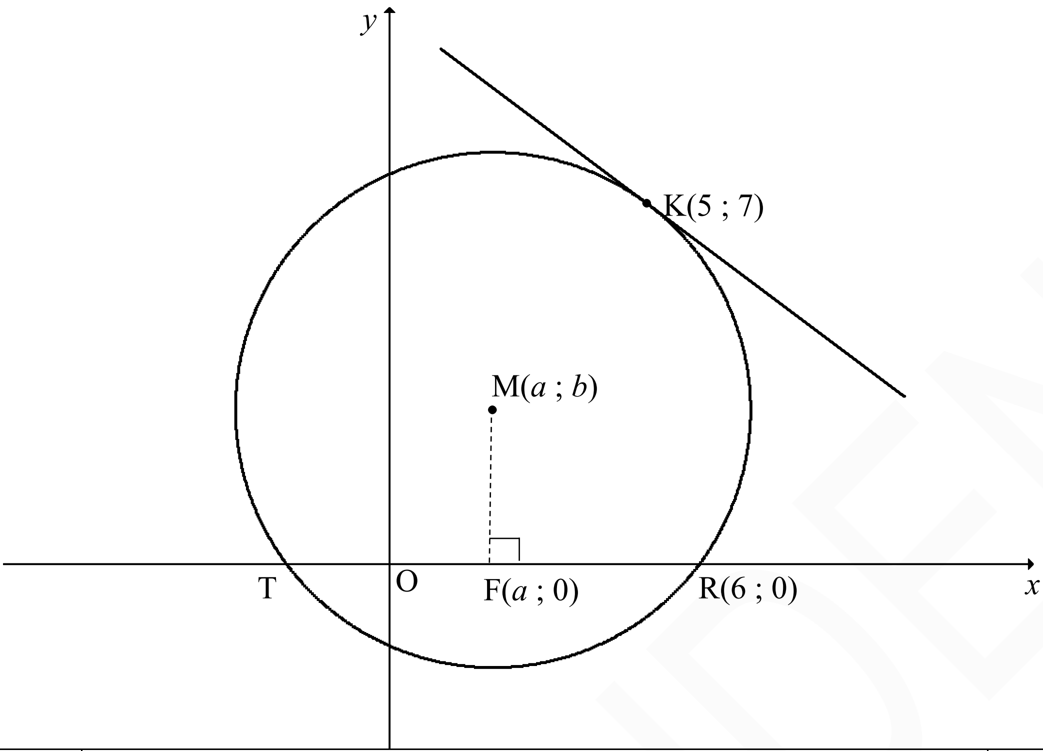
QUESTION/*VRAAG* 3



3.1	$m_{AB} = \dfrac{3-\frac{1}{2}}{5-0}$ $m_{AB} = \dfrac{1}{2}$ Answer only 2/2	✓ substitution ✓ answer (2)
3.2	$m_{CE} = m_{BA} = \dfrac{1}{2}$ $-4 = \dfrac{1}{2}(6) + c$ **OR/*OF*** $y-(-4) = \dfrac{1}{2}(x-6)$ $c = -7$ $y = \dfrac{1}{2}x - 7$	✓ gradient ✓ substitution of E ✓ answer (3)
3.3.1	D(0 ; –7) $\dfrac{x_C + 6}{2} = 0$ $\dfrac{y_C + (-4)}{2} = -7$ $x_C = -6$ $y_C = -10$ C(–6 ; –10) Answer only 3/3	✓ D(0 ; –7) ✓ $x_C = -6$ ✓ $y_C = -10$ (3)
3.3.2	Area Δ BCD $= \dfrac{1}{2}(7{,}5)(6)$ $= 22{,}5$ Area Δ ABD $= \dfrac{1}{2}(7{,}5)(5)$ $= 18{,}75$ Area ABCD $= 22{,}5 + 18{,}75 = 41{,}25$ units^2	✓ subst of correct base and height into the area formula ✓ area ΔBCD = 22,5 ✓ area ΔABD = 18,75 ✓ answer (4)

3.4.1	K(–6 ; –4)	✓ $x_K = -6$ ✓ $y_K = -4$ (2)
3.4.2a	KC = 6 units; KE = 12 units; $CE = \sqrt{(6)^2 + (12)^2}$ [Pythagoras] $CE = \sqrt{180} = 6\sqrt{5} = 13,42$ Perimeter ΔKEC = 6 + 12 + $\sqrt{180}$ = 31,42 units	✓ KC = 6 units ✓ KE = 12 units ✓ CE ✓ answer (4)
3.4.2b	$\tan K\hat{C}E = \frac{KE}{KC} = \frac{12}{6} = 2$ $K\hat{C}E = 63,43°$ **OR/*OF*** $\sin K\hat{C}E = \frac{KE}{CE} = \frac{12}{\sqrt{180}} = \frac{2\sqrt{5}}{5}$ $K\hat{C}E = 63,43°$ **OR/*OF*** $m_{CE} = \frac{1}{2}$ $\tan\theta = \frac{1}{2}$ $\theta = 26,57°$ $K\hat{C}E = 90° - 26,57°$ $K\hat{C}E = 63,43°$ **OR/*OF*** $KE^2 = KC^2 + CE^2 - 2(KC)(CE)\cos K\hat{C}E$ $(12)^2 = (6)^2 + (\sqrt{180})^2 - 2(6)(\sqrt{180})(\cos K\hat{C}E)$ $\cos K\hat{C}E = \frac{\sqrt{5}}{5}$ $K\hat{C}E = 63,43°$	✓ trig ratio ✓ $\tan K\hat{C}E = 2$ ✓ answer (3) ✓ trig ratio ✓ $\sin K\hat{C}E = \frac{12}{\sqrt{180}}$ ✓ answer (3) ✓ $\tan\theta = \frac{1}{2}$ ✓ $\theta = 26,57°$ ✓ answer (3) ✓ substitution into cosine rule ✓ trig ratio ✓ answer (3)
		[21]

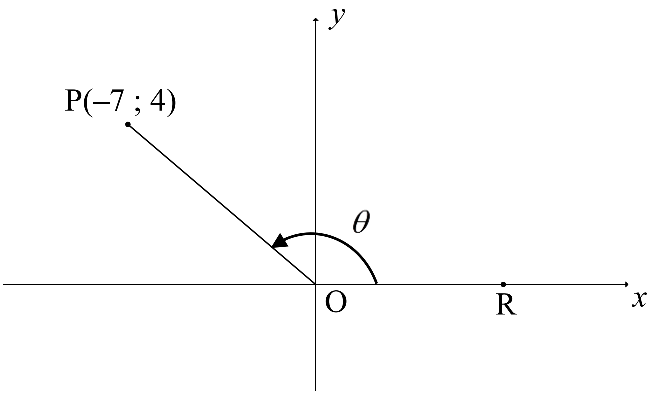
QUESTION/*VRAAG* 4



4.1.1	$y = x + 1$ $b = a + 1$	✓ $b = a + 1$ (1)
4.1.2	$MR^2 = MK^2$ $(a-6)^2 + (b-0)^2 = (a-5)^2 + (b-7)^2$ $(a-6)^2 + (a+1)^2 = (a-5)^2 + (a+1-7)^2$ $a^2 + 2a + 1 = a^2 - 10a + 25$ $12a = 24$ $a = 2$ $b = 3$ $\therefore M(2 ; 3)$	✓ equating radii / solving simultaneously ✓ substitution $b = a + 1$ ✓ $12a = 24$ ✓ $a = 2$ ✓ $b = 3$ (5)
4.2.1	$(6-2)^2 + (0-3)^2 = r^2$ $r = 5$ **OR/*OF*** $(2-5)^2 + (3-7)^2 = r^2$ $r = 5$ Answer only 2/2	✓ substitution R and M ✓ $r = 5$ (2) ✓ substitution K and M ✓ $r = 5$ (2)

4.2.2	T(–2 ; 0) TR = 8 units [line from centre ⊥ to chord] **OR/*OF*** M(2 ; 3) F(a ; 0) FR = 4 units TR = 8 units [line from centre ⊥ to chord] **OR/*OF*** $(x-2)^2+(0-3)^2=25$ $x^2-4x+4+9=25$ $x^2-4x-12=0$ $(x-6)(x+2)=0$ $x=6$ or $x=-2$ TR = 8 units Answer only 2/2	✓ T(–2 ; 0) ✓ answer (2) ✓ 4 units ✓ answer (2) ✓ x values ✓ answer (2)
4.3	$m_{\text{radius}}=\frac{7-3}{5-2}$ $m_{\text{radius}}=\frac{4}{3}$ $m_{\text{tangent}}=-\frac{3}{4}$ $7=-\frac{3}{4}(5)+c$ **OR/*OF*** $y-7=-\frac{3}{4}(x-5)$ $c=\frac{43}{4}$ $y=-\frac{3}{4}x+\frac{43}{4}$ $y=-\frac{3}{4}x+\frac{43}{4}$	✓ substitution ✓ $m_{\text{radius}}=\frac{4}{3}$ ✓ $m_{\text{tangent}}=-\frac{3}{4}$ ✓ substitution ✓ answer (5)
4.4.1	N(2 ; –2)	✓ $x_N=2$ ✓ $y_N=-2$ (2)
4.4.2	$(-2-2)^2+(0+2)^2=r^2$ $r^2=20$ $(x-2)^2+(y+2)^2=20$	✓ substitution ✓ $r^2=20$ ✓ answer (3)
		[20]

QUESTION/*VRAAG* 5



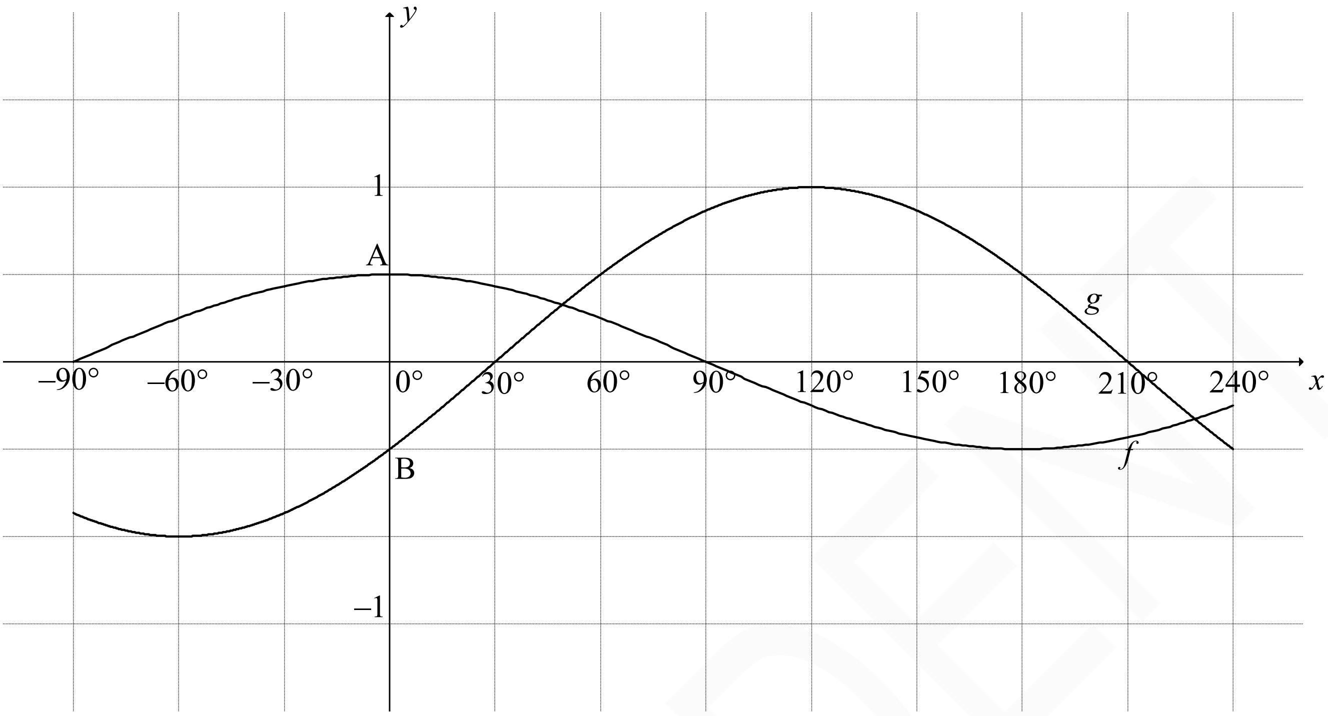
5.1.1	$\text{OP}=\sqrt{(-7)^2+(4)^2}$ $=\sqrt{65}$ Answer only 2/2	✓ substitution ✓ answer (2)
5.1.2(a)	$\tan\theta=\dfrac{4}{-7}$	✓ answer (1)
5.1.2(b)	$\cos(\theta-180°)=-\cos\theta$ $=\dfrac{7}{\sqrt{65}}$	✓ reduction ✓ answer (2)
5.2	$\sin x\cos x+\sin x=3\cos^2 x+3\cos x$ $\sin x\cos x+\sin x-3\cos^2 x-3\cos x=0$ $\sin x(\cos x+1)-3\cos x(\cos x+1)=0$ $(\cos x+1)(\sin x-3\cos x)=0$ $\cos x=-1$ or $\sin x=3\cos x$ $\tan x=3$ $x=180°+k.360°$ or $x=71{,}57°+k.180°$; $k\in Z$ **OR/*OF*** $\sin x\cos x+\sin x=3\cos^2 x+3\cos x$ $\sin x\cos x+\sin x-3\cos^2 x-3\cos x=0$ $\sin x(\cos x+1)-3\cos x(\cos x+1)=0$ $(\cos x+1)(\sin x-3\cos x)=0$ $\cos x=-1$ or $\sin x=3\cos x$ $\tan x=3$ $x=180°+k.360°$ or $x=71{,}57°+k.360°$ or $x=251{,}57°+k.360°$; $k\in Z$	✓ RHS = 0 ✓ grouping ✓ factors ✓ both equations ✓ $x=180°$ ✓ $x=71{,}57°$ ✓ $+k.180°$; $k\in Z$ (7) ✓ RHS = 0 ✓ grouping ✓ factors ✓ both equations ✓ $x=180°$ ✓ $x=71{,}57°$ and $251{,}57°$ ✓ $+k.360°$; $k\in Z$ (7)

5.3.1	$\text{LHS} = \frac{\sin 3x}{1-\cos 3x} \times \frac{1+\cos 3x}{1+\cos 3x}$ $= \frac{(\sin 3x)(1+\cos 3x)}{(1-\cos 3x)(1+\cos 3x)}$ $= \frac{(\sin 3x)(1+\cos 3x)}{1-\cos^2 3x}$ $= \frac{(\sin 3x)(1+\cos 3x)}{\sin^2 3x}$ $= \frac{1+\cos 3x}{\sin 3x}$ $= \text{RHS}$ **OR/*OF*** $\text{LHS} = \frac{\sin 3x}{1-\cos 3x} \times \frac{\sin 3x}{\sin 3x}$ $= \frac{\sin^2 3x}{\sin 3x(1-\cos 3x)}$ $= \frac{1-\cos^2 3x}{\sin 3x(1-\cos 3x)}$ $= \frac{(1-\cos 3x)(1+\cos 3x)}{\sin 3x(1-\cos 3x)}$ $= \frac{1+\cos 3x}{\sin 3x}$ $= \text{RHS}$	✓ multiply by “1” ✓ $1-\cos^2 3x$ ✓ square identity (3) ✓ multiply by “1” ✓ square identity ✓ factors (3)
5.3.2	undefined when $\sin 3x = 0$ and $1-\cos 3x = 0$ $3x = 0°$ or $3x = 180°$ and $3x = 0°$ or $3x = 360°$ $x = 0°$ or $x = 60°$	✓ $\sin 3x = 0$ and $1-\cos 3x = 0$ ✓ $0°$ ✓ $60°$ (3)
		[18]

QUESTION/*VRAAG* 6

6.1	$\frac{\sin 10°}{\cos 440°} + \tan(360° - \theta).\sin 2\theta$ $= \frac{\cos 80°}{\cos 80°} - \tan\theta(2\sin\theta\cos\theta)$ $= 1 - \frac{\sin\theta}{\cos\theta}(2\sin\theta\cos\theta)$ $= 1 - 2\sin^2\theta$ $= \cos 2\theta$	✓ $-\tan\theta$ ✓ cos 80° ✓ co-ratio ✓ double angle ✓ quotient identity ✓ answer (6)
6.2.1	$\sin(60° + 2x) + \sin(60° - 2x) = k\cos 2x$ $(\sin 60°\cos 2x + \cos 60°\sin 2x) + (\sin 60°\cos 2x - \cos 60°\sin 2x) = k\cos 2x$ $2\sin 60°\cos 2x = k\cos 2x$ $2\left(\frac{\sqrt{3}}{2}\right)\cos 2x = k\cos 2x$ $\therefore k = \sqrt{3}$	✓ both expansions correct ✓ special ∠s ✓ answer (3)
6.2.2	$\tan 60°[\sin(60° + 2x) + \sin(60° - 2x)]$ $= \tan 60°[k\cos 2x]$ $= \sqrt{3}(\sqrt{3}\cos 2x)$ $= 3(2\cos^2 x - 1)$ $= 3\left(2\left(\sqrt{t}\right)^2 - 1\right)$ $= 6\left(\sqrt{t}\right)^2 - 3$ $= 6t - 3$	✓ special ∠ ✓ double ∠s ✓ answer i.t.o t (3)
		[12]

QUESTION/*VRAAG* 7



7.1	A$\left(0;\frac{1}{2}\right)$ B$\left(0;-\frac{1}{2}\right)$ $AB=\frac{1}{2}-\left(-\frac{1}{2}\right)$ $=1$ unit Answer only 2/2	✓ y-values ✓ answer (2)
7.2	Range of f : $y\in\left[-\frac{1}{2};\ \frac{1}{2}\right]$ Range of $3f(x)+2$: $y\in\left[\frac{1}{2};\ 3\frac{1}{2}\right]$ **OR/*OF*** $\frac{1}{2}\le y\le 3\frac{1}{2}$	✓ critical values ✓ answer (2)
7.3	$x=90°$	✓✓ $x=90°$ (2)
7.4.1	$x\in(30°;\ 90°)\cup(210°;240°]$ **OR/*OF*** $30°<x<90°$ or $210°<x\le 240°$	✓ $x\in(30°;\ 90°)$ ✓ $(210°;240°]$ (2) ✓ $30°<x<90°$ ✓ $210°<x\le 240°$ (2)
7.4.2	$x\in(-55°;\ 125°)$ **OR/*OF*** $-55°<x<125°$	✓ critical values ✓ answer (2) ✓ critical values ✓ answer (2)
		[10]

QUESTION/*VRAAG* 8

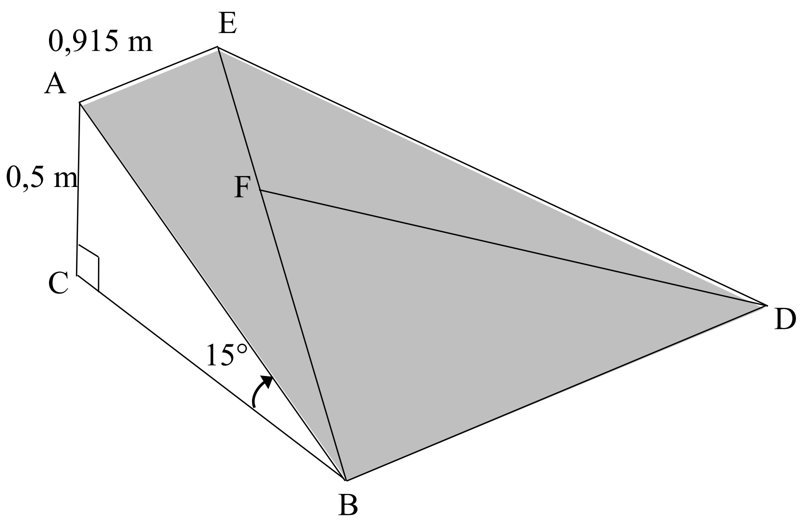


FIGURE I

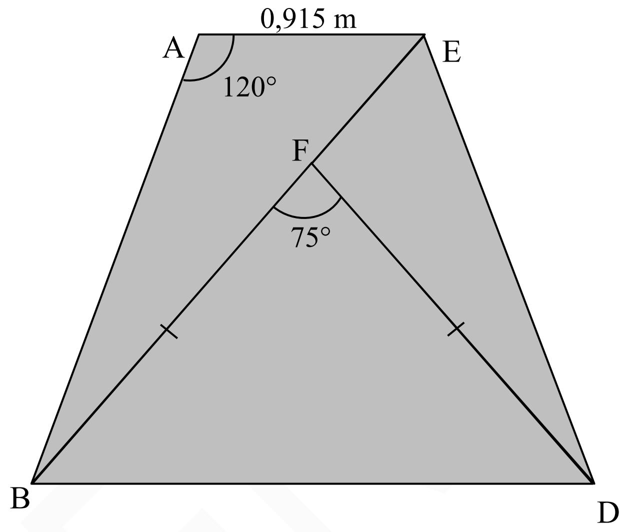
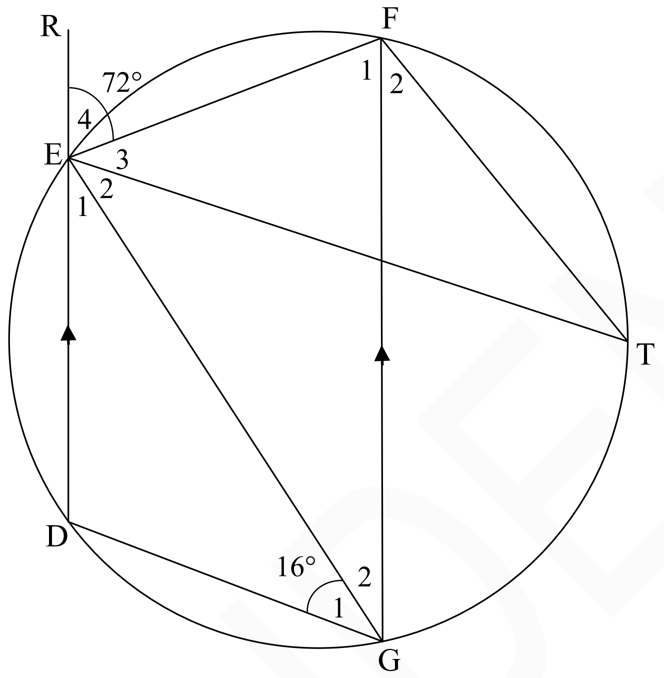


FIGURE II (top view)

8.1	$\frac{0,5}{AB} = \sin 15°$ $AB = \frac{0,5}{\sin 15°}$ $AB = 1,93\ \text{m}$ $\boxed{\text{Answer only 2/2}}$	✓ trig ratio ✓ answer (2)
8.2	$BE^2 = AB^2 + AE^2 - 2(AB)(AE)\cos B\hat{A}E$ $BE^2 = (1,93)^2 + (0,915)^2 - 2(1,93)(0,915)(\cos 120°)$ $BE = 2,52\ \text{m}$	✓ correct use of cosine rule ✓ substitution ✓ answer (3)
8.3	$BF = FD = \frac{5}{7}(2,52) = 1,80\ \text{m}$ $\text{Area } \Delta BFD = \frac{1}{2}(BF)(FD)\sin B\hat{F}D$ $= \frac{1}{2}(1,8)(1,8)(\sin 75°)$ $= 1,56\ \text{m}^2$	✓ BF ✓ correct substitution into the area rule ✓ answer (3)
		[8]

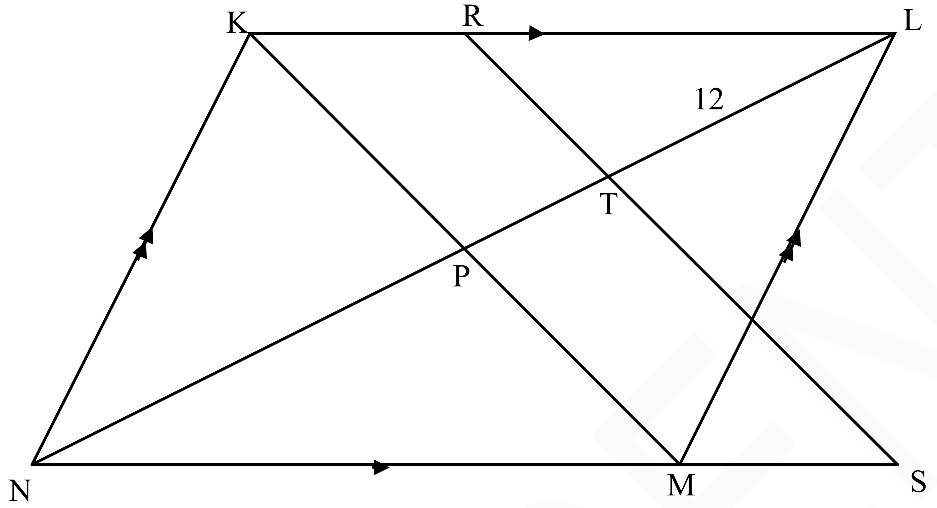
QUESTION/*VRAAG* 9

9.1



9.1.1	$D\hat{G}F = \hat{E}_4 = 72°$ [ext ∠ of cyclic quad/ *buite ∠ v kvh*]	✓ S ✓ R (2)
9.1.2	$\hat{G}_2 = 72° - 16° = 56°$ $\hat{T} = \hat{G}_2 = 56°$ [∠s in the same seg/ *∠e in dies. ⊙ segment*]	✓ S ✓ S / R (2)
9.1.3	$\hat{F}_1 = \hat{E}_4 = 72°$ [alt ∠s; DE ∥ GF / *verw. ∠e*; DE ∥ GF] $\therefore G\hat{E}F = 52°$ [sum of ∠s in Δ / *∠e van Δ*] **OR/*OF*** $\hat{E}_1 = 56°$ [alt ∠s; DE ∥ GF / *verw. ∠e*; DE ∥ GF] $\therefore G\hat{E}F = 52°$ [∠s on a str. line/ *∠e op 'n reguitlyn*]	✓ S / R ✓ S (2) ✓ S / R ✓ S (2)

9.2

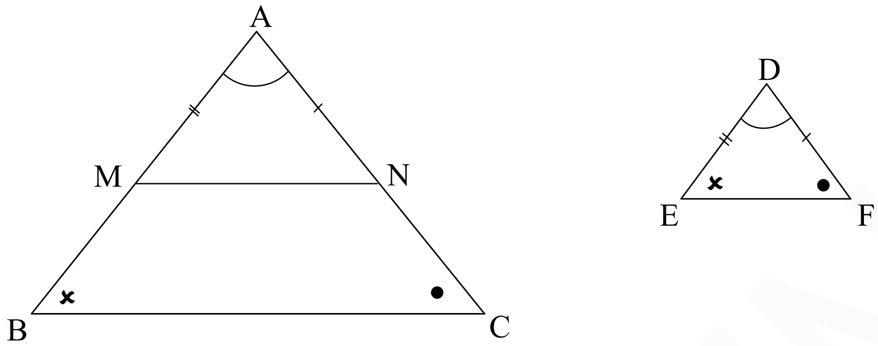


9.2.1	NP = PL = 16 [diag of ||m / *hoeklyne van ||m*] PT = 4 NP : PT = 16 : 4 = 4 : 1	✓ S ✓ R ✓ S ✓ answer (4)
9.2.2	NM : MS = 4 : 1 NP : PT = NM : MS KM || RS [line divides two sides of Δ in prop / *Lyn verdeel 2 sye v Δ eweredig*] **OR/*OF*** [converse prop theorem / *omgekeerde lyn || een sy v Δ*]	✓ S ✓ R (2)
9.2.3	$\frac{RL}{KL} = \frac{TL}{LP}$ [prop theorem; KM || RS **OR** line || one side of Δ / *Lyn || een sy v Δ*] $RL = \frac{12 \times 21}{16}$ $= 15{,}75$	✓ S ✓ R ✓ S ✓ answer (4)

	OR / *OF* NM : MS = 4 : 1 KR = MS = 5,25 [opp side of $\|\|^{m}$ / *teenoorst. sye van* $\|\|^{m}$] KL = NM = 21 RL + 5,25 = 21 RL = 15,75	 ✓ S ✓ R ✓ S ✓ answer (4)
		[16]

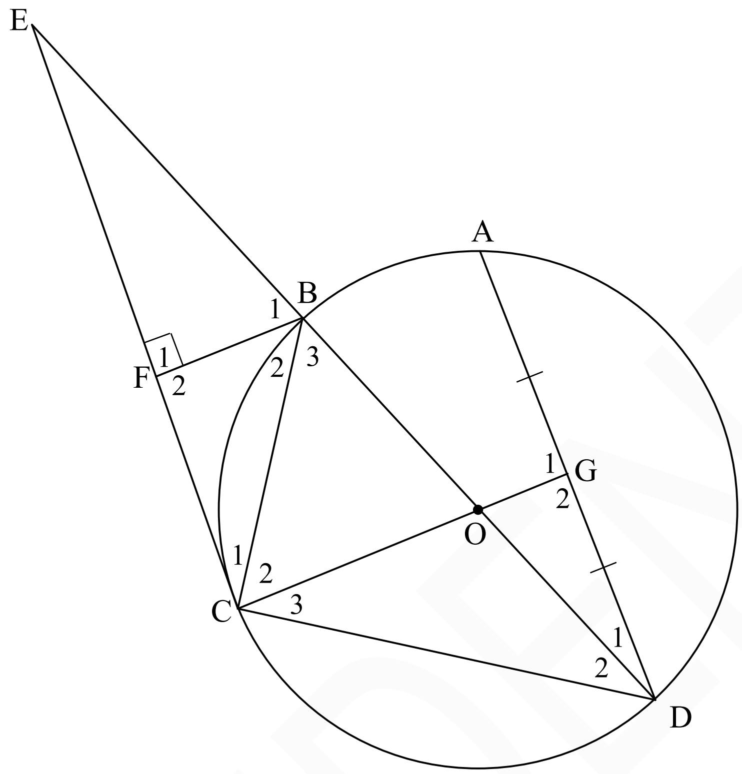
QUESTION/*VRAAG* 10

10.1



10.1	Constr: Let M and N lie on AB and AC respectively such that AM = DE and AN = DF. Draw MN. Proof: In Δ AMN and Δ DEF AM = DE [Constr / *Konstruksie*] AN = DF [Constr / *Konstruksie*] $\hat{A} = \hat{D}$ [Given /*Gegee*] $\therefore\ \Delta AMN \equiv \Delta DEF$ $[s, \angle, s]$ $\therefore A\hat{M}N = \hat{E} = \hat{B}$ MN ‖ BC [corresp ∠'s are equal/ *ooreenk. ∠ e gelyk*] $\frac{AB}{AM} = \frac{AC}{AN}$ [line ‖ one side of Δ **OR/*OF*** prop theorem; MN ‖BC / *Lyn* ‖ *een sy v* Δ] $\therefore \frac{AB}{DE} = \frac{AC}{DF}$ [AM = DE and AN = DF]	✓Constr ✓S ✓R ✓S /R ✓S ✓R (6)

10.2



10.2.1(a)	$\hat{FCO} = 90°$ [tan ⊥ radius / *raaklyn* ⊥ *radius*] $\hat{F}_1 = 90°$ [BF ⊥ EC] $\therefore \hat{FCO} = \hat{F}_1 = 90°$ FB ∥ CG [corresp ∠s = / *ooreenk.* ∠ *gelyk*]	✓ S / R ✓ S ✓ R (3)
10.2.1(b)	In ΔFCB and ΔCDB $\hat{BCD} = 90°$ [∠ in semi-circle / ∠ $\frac{1}{2}$ ⊙] $\hat{F}_2 = 90°$ [BF ⊥ EC] $\therefore \hat{F}_2 = \hat{BCD} = 90°$ $\hat{C}_1 = \hat{D}_2$ [tan chord theorem / ∠ *tussen rkl en koord*] $\hat{B}_2 = \hat{B}_3$ [sum of ∠s in Δ / ∠*e van* Δ] ∴ ΔFCB ∥∣ ΔCDB **OR/*OF*** In ΔFCB and ΔCDB $\hat{BCD} = 90°$ [∠ in semi-circle / ∠ $\frac{1}{2}$ ⊙] $\hat{F}_2 = 90°$ [BF ⊥ EC] $\therefore \hat{F}_2 = \hat{BCD} = 90°$ $\hat{C}_1 = \hat{D}_2$ [tan chord theorem / ∠ *tussen rkl en koord*] ∴ ΔFCB ∥∣ ΔCDB [∠,∠,∠]	✓ S /R ✓ S ✓ S ✓ R ✓ S ✓ S / R ✓ S ✓ S ✓ R ✓ R (5)

10.2.2	$\hat{G}_1 = 90°$ [line from centre to midpt of chord / *midpt.* ⊙; *midpt. koord*]	✓ R (1)
10.2.3	In ΔGCD and ΔCDB $\hat{G}_2 = B\hat{C}D = 90°$ $\hat{C}_3 = \hat{D}_2$ [∠s opp equal sides /∠*e teenoor gelyke sye*] $G\hat{D}C = \hat{B}_3$ [sum of ∠s in Δ /∠*e van* Δ] $\therefore \Delta GCD \parallel\!\mid \Delta CDB$ [∠,∠,∠] $\therefore \frac{CD}{DB} = \frac{CG}{CD}$ [||| Δs] $\therefore CD^2 = CG.DB$	✓ identifying Δs ✓ S ✓ S / R ✓ S **OR** ✓ R ✓ S (5)
10.2.4	$\frac{BC}{DB} = \frac{FB}{BC}$ [ΔFCB ||| ΔCDB] $\therefore BC^2 = DB.FB$ $CD^2 + BC^2 = CG.DB + DB.FB$ $DB^2 = DB(CG + FB)$ $DB = CG + FB$	✓ S ✓ R ✓ S ✓ sum ✓ $DB^2 = CD^2 + BC^2$ (5)
		[25]

TOTAL/*TOTAAL*: 150

basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE/
NASIONALE SENIOR SERTIFIKAAT

GRADE/*GRAAD* 12

MATHEMATICS P2/*WISKUNDE V2*

NOVEMBER 2021

MARKING GUIDELINES/*NASIENRIGLYNE*

MARKS/PUNTE: 150

These marking guidelines consist of 24 pages.
Hierdie nasienriglyne bestaan uit 24 bladsye.

NOTE:

- If a candidate answers a question TWICE, only mark the FIRST attempt.
- If a candidate has crossed out an attempt of a question and not redone the question, mark the crossed out version.
- Consistent accuracy applies in ALL aspects of the marking memorandum. Stop marking at the second calculation error.
- Assuming answers/values in order to solve a problem is NOT acceptable.

NOTA:

- *As 'n kandidaat 'n vraag TWEE KEER beantwoord, sien slegs die EERSTE poging na.*
- *As 'n kandidaat 'n antwoord van 'n vraag doodtrek en nie oordoen nie, sien die doodgetrekte poging na.*
- *Volgehoue akkuraatheid word in ALLE aspekte van die nasienriglyne toegepas. Hou op nasien by die tweede berekeningsfout.*
- *Om antwoorde/waardes te aanvaar om 'n probleem op te los, word NIE toegelaat NIE.*

GEOMETRY • *MEETKUNDE*	
S	**A mark for a correct statement** **(A statement mark is independent of a reason)**
	'n Punt vir 'n korrekte bewering ***('n Punt vir 'n bewering is onafhanklik van die rede)***
R	**A mark for the correct reason** **(A reason mark may only be awarded if the statement is correct)**
	'n Punt vir 'n korrekte rede ***('n Punt word slegs vir die rede toegeken as die bewering korrek is)***
S/R	**Award a mark if statement AND reason are both correct**
	Ken 'n punt toe as die bewering EN rede beide korrek is

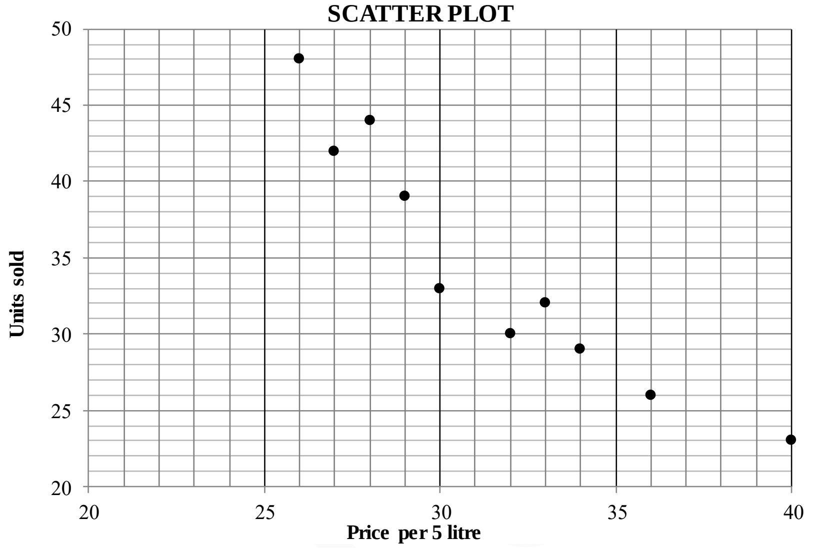
QUESTION/*VRAAG* 1

10	11	13	14	14	15	16	18	18
19	19	20	21	35	35	37	40	41

1.1.1	$\bar{x} = \frac{396}{18}$ $\bar{x} = 22$ Answer only: Full marks *Slegs antw: Volpunte*	✓ 396 ✓ answer (2)
1.1.2	$\sigma = 10{,}1707 \approx 10{,}17$	✓ answer (1)
1.1.3	$\bar{x} + \sigma = 32{,}17$ $\therefore 5$ days	✓ 32,17 ✓ 5 (2)
1.2	$22 \times 18 = 396$ ordered/*bestel* $20 \times 18 = 360$ sold/*verkoop* Total not sold/*Totaal nie verkoop nie*: 36 **OR/*OF*** 22–20 = 2 $2 \times 18 = 36$	✓ $18\bar{x}_1$ and $18\bar{x}_2$ ✓ answer (2) ✓ $\bar{x}_1 - \bar{x}_2$ ✓ answer (2)
1.3.1	Option B/*Opsie B* Any one of the following reasons/*Enige een van die vlg redes*: • Median/*Mediaan* = 18,5 • $Q_1 = 14$ • IQR = 21 • Mean > Median, therefore the data is skewed to the right	✓ B ✓ reason (2)
1.3.2	Data is positively skewed/skewed to the right *Data is positief skeef/skeef na regs*	✓ answer (1)
		[10]

QUESTION/*VRAAG* 2

Price of milk in rands per 5-litre container (x) ***Prys van melk in rand, per 5 liter-houer (x)***	26	32	36	28	40	33	29	34	27	30
Number of 5-litre containers of milk sold (y) ***Aantal 5 liter-houers melk verkoop (y)***	48	30	26	44	23	32	39	29	42	33

<table>
<tr><td>2.1</td><td>


</td><td>1 mark:
3 to 5 points plotted correctly

2 marks:
6 to 9 points plotted correctly

3 marks:
all points plotted correctly

(3)</td></tr>
<tr><td>2.2</td><td>$a = 90{,}478... \approx 90{,}48$
$b = -1{,}773... \approx -1{,}77$
$\hat{y} = 90{,}48 - 1{,}77x$

Answer only: Full marks
Slegs antw: Volpunte</td><td>✓ a
✓ b
✓ equation
(3)</td></tr>
<tr><td>2.3</td><td>$y = 23{,}069\ldots \approx 23{,}07$ units/eenhede (calculator/sakrekenaar)

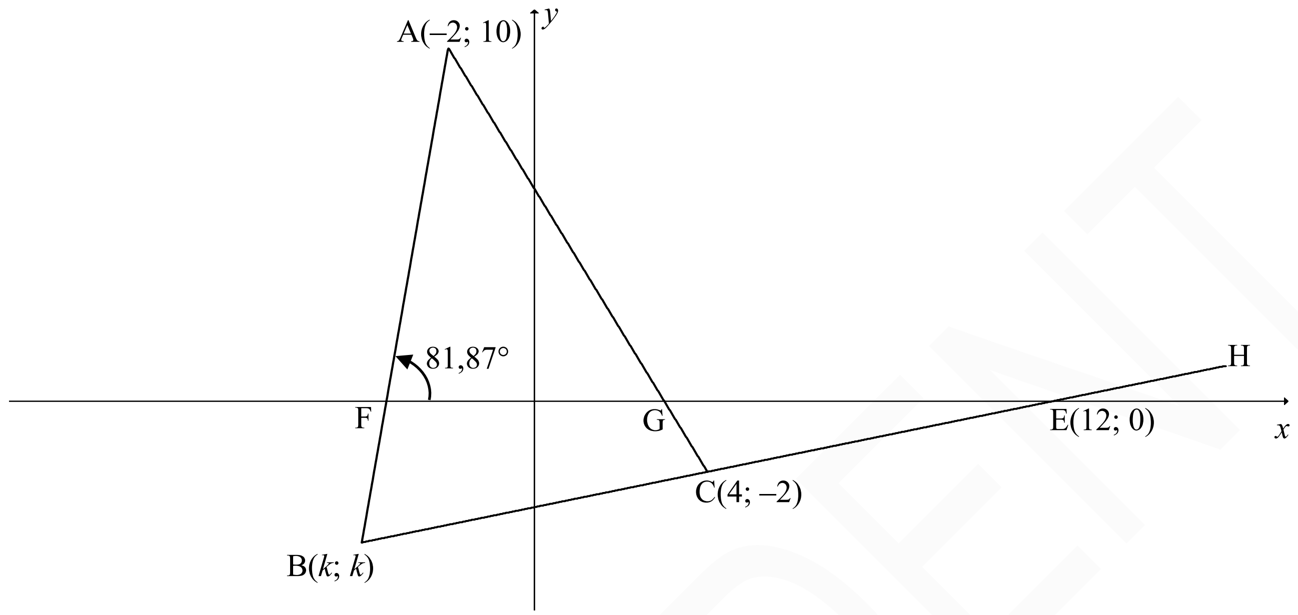
OR/OF

$y = 90{,}48 - 1{,}77(38)$
$y = 23{,}22$ units/eenhede</td><td>✓✓ answer
(2)

✓ substitution
✓ answer
(2)</td></tr>
<tr><td>2.4</td><td>$r = -0{,}94$
The value of r indicates a strong relationship between the cost per 5 litre and the number of units sold $\therefore$ there is a good chance of the prediction being accurate./
Die waarde van r dui 'n sterk vewantskap tussen die koste per 5 liter en die aantal eenhede verkoop aan $\therefore$ daar is 'n goeie kans dat die voorspelling akkuraat is</td><td>✓ value of r OR/OF strong relationship/sterk verwantskap

✓ accurate/akkuraat
(2)</td></tr>
<tr><td></td><td></td><td>[10]</td></tr>
</table>

QUESTION/*VRAAG* 3

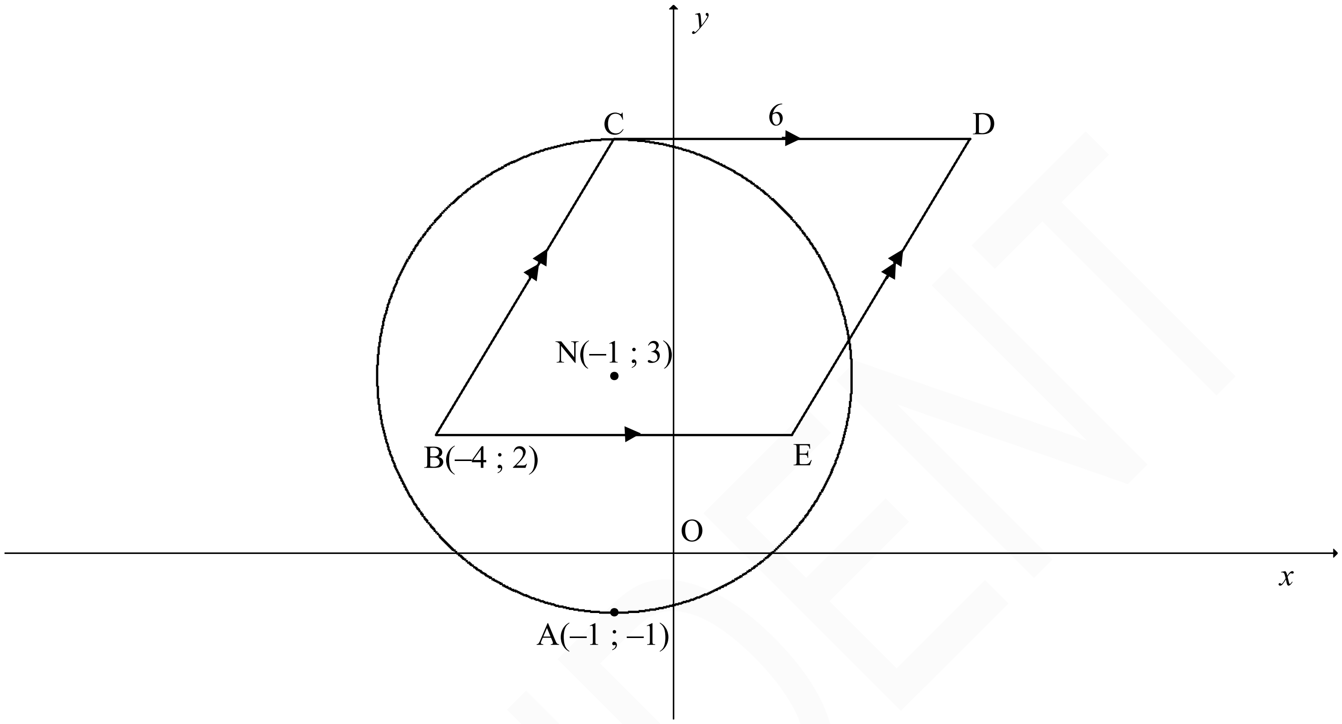


3.1.1	$m_{BE} = m_{CE} = \dfrac{0-(-2)}{12-4}$ **OR/*OF*** $m_{BE} = m_{CE} = \dfrac{-2-0}{4-12}$ $= \dfrac{1}{4}$ $= \dfrac{1}{4}$	✓ substitution C & E ✓ answer (2)
3.1.2	$m_{AB} = \tan 81{,}87°$ $m_{AB} = 7$ Answer only: Full marks *Slegs antw: Volpunte*	✓ substitution ✓ answer (2)
3.2	$y = mx + c$ $y - y_1 = m(x - x_1)$ $0 = \frac{1}{4}(12) + c$ or $y - 0 = \frac{1}{4}(x - 12)$ $c = -3$ $y = \frac{1}{4}x - 3$ $y = \frac{1}{4}x - 3$ **OR/*OF*** $y = mx + c$ $y - y_1 = m(x - x_1)$ $-2 = \frac{1}{4}(4) + c$ or $y - (-2) = \frac{1}{4}(x - 4)$ $c = -3$ $y = \frac{1}{4}x - 3$ $y = \frac{1}{4}x - 3$	✓ substitution of E ✓ answer (2) ✓ substitution of C ✓ answer (2)

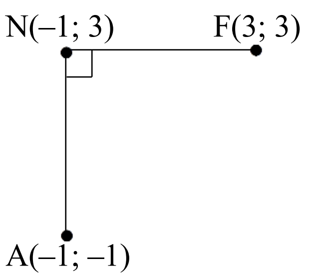
3.3.1	$y=\frac{1}{4}x-3$ $k=\frac{1}{4}k-3$ $\frac{3}{4}k=-3$ $k=-4$ $\therefore B(-4;-4)$	✓ substitution ✓ answer (2)
	OR/*OF* $m_{BE}=\frac{1}{4}$ **OR/*OF*** $m_{BE}=\frac{1}{4}$ $\frac{0-k}{12-k}=\frac{1}{4}$ $\frac{k}{k-12}=\frac{1}{4}$ $-4k=12-k$ $4k=k-12$ $k=-4$ $k=-4$ $\therefore B(-4;-4)$	✓ substitution ✓ answer (2)
	OR/*OF* $m_{AB}=\tan 81{,}87°$ $m_{AB}=7$ $m_{AB}=\frac{10-k}{-2-k}$ $7(-2-k)=10-k$ $-14-7k=10-k$ $-6k=24$ $k=-4$ $\therefore B(-4;-4)$	✓ substitution ✓ answer (2)
	OR/*OF* EB: $y=\frac{1}{4}x-3$ and AB: $y=7x+24$ $\frac{1}{4}x-3=7x+24$ $\frac{27}{4}x=-27$ $x=k=-4$ $\therefore B(-4;-4)$	✓ equating EB & AB ✓ answer (2)

3.3.2	In ΔAFG: $m_{AC} = \frac{10-(-2)}{-2-4} = -2$ $\tan\theta = m_{AC} = -2$ $\theta = 180° - 63{,}43...°$ $\therefore \theta = 116{,}57°$ $\therefore \hat{A} = 116{,}57° - 81{,}87°$ [ext ∠ of Δ] $\therefore \hat{A} = 34{,}70°$	✓ $m_{AC} = -2$ ✓ $\tan\theta = -2$ ✓ $\theta = 116{,}57°$ ✓ answer (4)
	OR/*OF* In ΔABC: $a = BC = 2\sqrt{17}; b = AC = 6\sqrt{5}; c = AB = 10\sqrt{2}$ $a^2 = b^2 + c^2 - 2bc.\cos A$ $\left(2\sqrt{17}\right)^2 = \left(6\sqrt{5}\right)^2 + \left(10\sqrt{2}\right)^2 - 2(6\sqrt{5})(10\sqrt{2}).\cos A$ $\cos A = \frac{\left(6\sqrt{5}\right)^2 + \left(10\sqrt{2}\right)^2 - \left(2\sqrt{17}\right)^2}{2\left(6\sqrt{5}\right)\left(10\sqrt{2}\right)}$ $= 0{,}822...$ $\therefore A = 34{,}7°$	✓ all 3 lengths ✓ substitution into the correct cosine rule ✓ cos A subject ✓ answer (4)
3.3.3	$M\left(\frac{12+(-2)}{2}; \frac{10+(0)}{2}\right)$ Diagonals intersect at the point (5 ; 5)	✓ x-value ✓ y-value (2)
3.4.1	BE = ET $4\sqrt{17} = \sqrt{(12-p)^2 + (0-p)^2}$ $\left(4\sqrt{17}\right)^2 = \left(\sqrt{(12-p)^2 + (0-p)^2}\right)^2$ $272 = 144 - 24p + p^2 + p^2$ $p^2 - 12p - 64 = 0$ $(p-16)(p+4) = 0$ $\therefore p = 16$ or $p = -4$ (n.a.) $\therefore$ T(16; 16)	✓ substitution of E & T ✓ equating ✓ standard form ✓ factors ✓ $p = 16$ (5)
3.4.2a	$(x-12)^2 + y^2 = \left(4\sqrt{17}\right)^2 = 272$	✓ LHS ✓ RHS (2)
3.4.2b	$m_{radius} = \frac{1}{4}$ $m_{tangent} = -4$ $y = -4x + c$ $-4 = -4(-4) + c$ $c = -20$ $y = -4x - 20$ **OR/*OF*** $y - y_1 = -4(x - x_1)$ $y - (-4) = -4(x - (-4))$ $y = -4x - 20$	✓ $m_{tangent}$ ✓ substitution of B ✓ equation (3)
		[24]

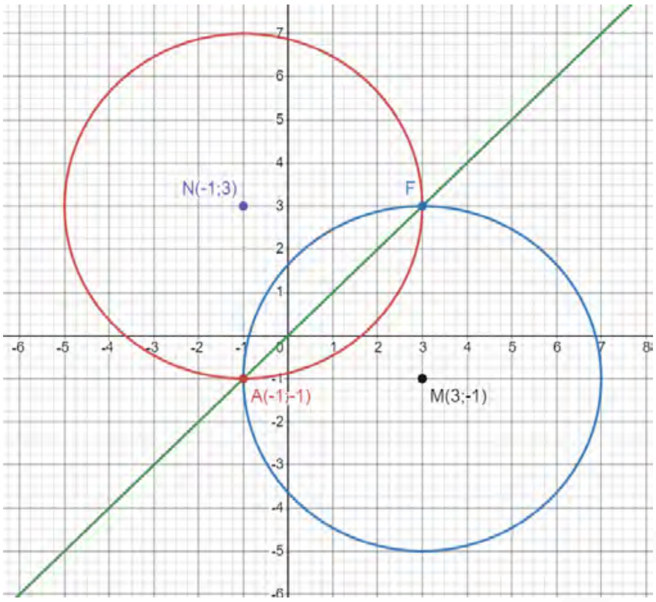
QUESTION/*VRAAG* 4

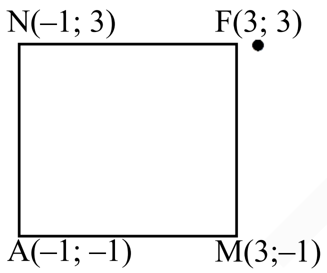


4.1	Radius = 4 units/*eenhede*	✓ answer (1)
4.2.1	$CD \perp CN$ $\therefore C(-1\ ;\ 7)$	✓ x value ✓ y value (2)
4.2.2	CD = 6 units $\therefore D(5\ ;\ 7)$	✓ x value ✓ y value (2)
4.2.3	$\perp h = 5$ units DC = 6 units $\text{Area } \Delta BCD = \frac{1}{2}(6)(5)$ $= 15 \text{ units}^2$ **OR/*OF*** $\perp h = 5$ units DC = 6 units $\text{Area } \Delta BCD = \frac{1}{2}[\text{Area of } \parallel^{m}]$ $= \frac{1}{2}[(5)(6)]$ $= 15 \text{ units}^2$	✓ $\perp h = 5$ units ✓ substitution into Area formula ✓ answer (3) ✓ $\perp h = 5$ units ✓ substitution into Area formula ✓ answer (3)

	OR/*OF* Let angle of inclination of BC = α $\tan\alpha = \frac{5}{3}$ $\alpha = 59{,}036...°$ $\hat{BCD} = 180° - \alpha$ $\hat{BCD} = 180° - 59{,}036...°$ $\hat{BCD} = 120{,}96°$ Area $\Delta BCD = \frac{1}{2}\left(\sqrt{34}\right)(6)\sin 120{,}96°$ $= 15$ units^2	✓ $\hat{BCD} = 120{,}96°$ ✓ substitution into Area rule ✓ answer (3)
4.3.1	M(3 ; –1) [reflection of N(–1 ; 3) about the line $y = x$] $\therefore MN = \sqrt{(3-(-1))^2 + (-1-3)^2}$ $MN = \sqrt{32} = 4\sqrt{2} = 5{,}66$ units	✓ coordinates of M (A) ✓ substitution of M&N ✓ answer (3)
4.3.2	M(3 ; –1) $m_{MN} = \frac{3-(-1)}{-1-3} = -1$ MN: $-1 = -(3) + c$ or $y - 3 = -1(x+1)$ $c = 2$ $y - 3 = -x - 1$ $\therefore y = -x + 2$ $y = -x + 2$ $x = -x + 2$ $2x = 2$ $x = 1$ $\therefore y = 1$ midpoint (1 ; 1) **OR/*OF***  N(–1 ; 3) $y_F = y_N = 3$ Reflected about $y = x$ $\therefore$ F(3 ; 3) midpoint $\left(\frac{-1+3}{2} ; \frac{-1+3}{2}\right) = (1 ; 1)$	✓ equation of MN ✓ equating AF & MN ✓x value ✓ y value (4) ✓✓ coordinates of F ✓x value ✓ y value (4)

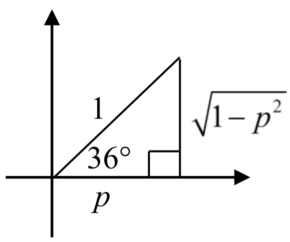
OR/*OF*





	NAMF is a square (NA=NF=AM=MF and NA $\perp$ AM) Midpoint NM $=(1\,;1)$ $=$ Midpoint of AF	✓ NAMF = square ✓x ✓ y of midpt NM ✓ midpt AF (4)
		[15]

QUESTION/*VRAAG* 5

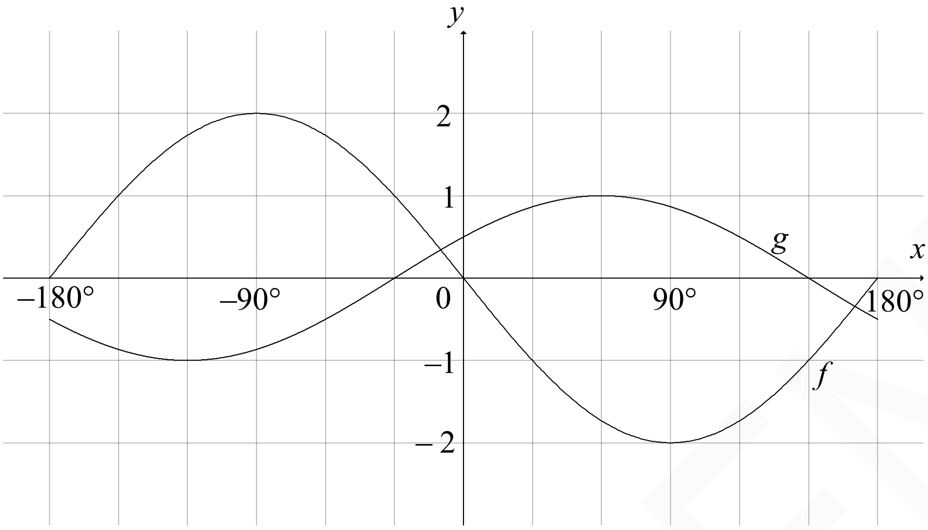
5.1	$\dfrac{\sin 140°.\sin(360°-x)}{\cos 50°.\tan(-x)}$ $=\dfrac{\sin 40°(-\sin x)}{\sin 40°(-\tan x)}$ $=\dfrac{-\sin x}{-\dfrac{\sin x}{\cos x}}$ $=\cos x$	✓ sin40° ✓ $-\sin x$ ✓ co-ratio ✓ $-\tan x$ ✓ $\tan x=\dfrac{\sin x}{\cos x}$ ✓ answer (6)
5.2	$\text{LHS}=\dfrac{-2\sin^2 x+\cos x+1}{1-\cos(540°-x)}$ $\qquad \text{RHS}=2\cos x-1$ $\text{LHS}=\dfrac{-2(1-\cos^2 x)+\cos x+1}{1-(-\cos x)}$ $\text{LHS}=\dfrac{-2+2\cos^2 x+\cos x+1}{1+\cos x}$ $\text{LHS}=\dfrac{2\cos^2 x+\cos x-1}{1+\cos x}$ $\text{LHS}=\dfrac{(2\cos x-1)(\cos x+1)}{1+\cos x}$ $\text{LHS}=2\cos x-1$ $\therefore \text{LHS}=\text{RHS}$	✓ identity i. t. o. $\cos x$ ✓ $\cos(540°-x)=-\cos x$ ✓ standard form ✓ factors (4)
5.3.1	$\sin 36°=\sqrt{1-p^2}$ $\tan 36°=\dfrac{\sqrt{1-p^2}}{p}$  **OR/*OF*** $\cos^2 36°=1-\sin^2 36°$ $\cos 36°=\sqrt{1-(1-p^2)}$ $=p$ $\tan 36°=\dfrac{\sin 36°}{\cos 36°}$ $=\dfrac{\sqrt{1-p^2}}{p}$	✓ method ✓ value of p ✓ answer (3) ✓ method ✓ $\cos 36°=p$ ✓ answer (3)

5.3.2	$\cos 108°$ $= -\cos 72°$ $= -\cos(2\times 36°)$ $= -(2\cos^2 36° - 1)$ $= -2p^2 + 1$	✓ reduction ✓ double angle ✓ expansion ✓ answer i. t. o. p (4)
	OR/*OF* $\cos 108°$ $= -\cos 72°$ $= -\cos(2\times 36°)$ $= -(1 - 2\sin^2 36°)$ $= -1 + 2\left(\sqrt{1-p^2}\right)^2$ $= -1 + 2(1-p^2)$ $= -2p^2 + 1$	✓ reduction ✓ double angle ✓ expansion ✓ answer i. t. o. p (4)
	OR/*OF* $\cos 108°$ $= -\cos 72°$ $= -\cos(2\times 36°)$ $= -(\cos^2 36° - \sin^2 36°)$ $= -\left(p^2 - \left(\sqrt{1-p^2}\right)^2\right)$ $= -\left(p^2 - (1-p^2)\right)$ $= -2p^2 + 1$	✓ reduction ✓ double angle ✓ expansion ✓ answer i. t. o. p (4)
	OR/*OF* $\cos 108°$ $= \cos(2\times 54°)$ $= 2\cos^2 54° - 1$ $= 2(1-p^2) - 1$ $= 1 - 2p^2$	✓ double angle ✓✓ expansion ✓ answer i. t. o. p (4)
	OR/*OF* $\cos 108° = \cos(72° + 36°)$ $= \cos 72° \cos 36° - \sin 72° \sin 36°$ $= (2\cos^2 36° - 1)\cos 36° - (2\sin 36° \cos 36°)\sin 36°$ $= 2\cos^3 36° - \cos 36° - 2\cos 36° \sin^2 36°$ $= 2p^3 - p - 2p\left(\sqrt{1-p^2}\right)^2$ $= 2p^3 - p - 2p + 2p^3$ $= 4p^3 - 3p$	✓ expansion ✓ both double angle identities ✓ value of sin 36° ✓ answer i. t. o. p (4)
		[17]

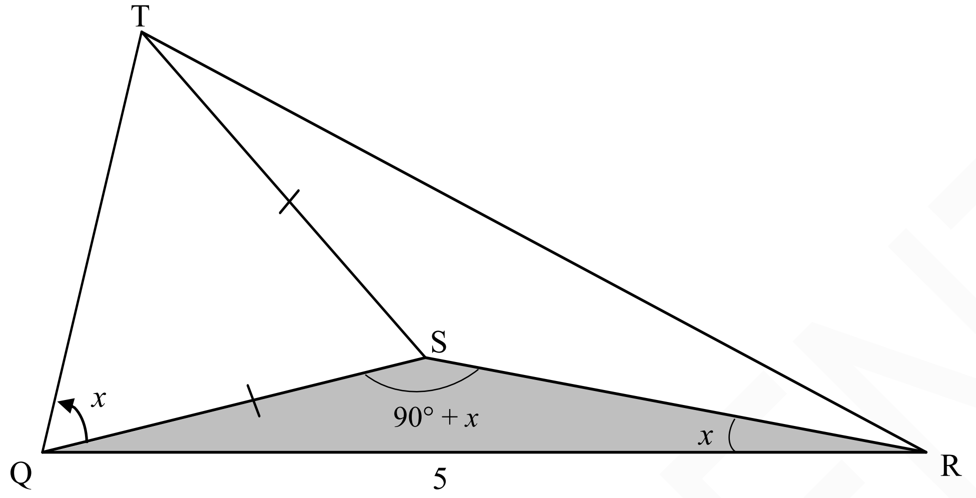
QUESTION/*VRAAG* 6

6.1.1	$\cos(\alpha+\beta)$ $=\cos(\alpha-(-\beta))$ $=\cos\alpha\cos(-\beta)+\sin\alpha\sin(-\beta)$ $=\cos\alpha\cos\beta+\sin\alpha(-\sin\beta)$ $=\cos\alpha\cos\beta-\sin\alpha\sin\beta$	✓ $\cos(\alpha-(-\beta))$ ✓ expansion ✓ reduction (3)
6.1.2	$2\cos 6x\cos 4x-\cos 10x+2\sin^2 x$ $=2\cos 6x\cos 4x-\cos(6x+4x)+2\sin^2 x$ $=2\cos 6x\cos 4x-(\cos 6x\cos 4x-\sin 6x\sin 4x)+2\sin^2 x$ $=\cos 6x\cos 4x+\sin 6x\sin 4x+2\sin^2 x$ $=\cos 2x+2\sin^2 x$ $=1-2\sin^2 x+2\sin^2 x$ $=1$	✓ $\cos 10x=\cos(6x+4x)$ ✓ expansion of $\cos(6x+4x)$ ✓ $\cos 2x$ ✓ $1-2\sin^2 x$ ✓ answer (5)
6.2	$\tan x=2\sin 2x$ $\frac{\sin x}{\cos x}=2(2\sin x\cos x)$ $\sin x=4\sin x\cos^2 x$ $4\sin x\cos^2 x-\sin x=0$ $\sin x(4\cos^2 x-1)=0$ $\sin x=0$ or $\cos^2 x=\frac{1}{4}$ $\cos x=-\frac{1}{2}$ $x=180°+k.360°;\ k\in Z$ or $x=120°+k.360°;\ k\in Z$ $x=240°+k.360°;\ k\in Z$ **OR/*OF*** $\tan x=2\sin 2x$ $\frac{\sin x}{\cos x}=4\sin x\cos x$ $\sin x=4\sin x\cos^2 x$ $4\sin x\cos^2 x-\sin x=0$ $4\sin x(1-\sin^2 x)-\sin x=0$ $3\sin x-4\sin^3 x=0$ $\sin x(3-4\sin^2 x)=0$ $\sin x=0$ or $\sin^2 x=\frac{3}{4}$ $\sin x=\frac{\sqrt{3}}{2}$ or $\sin x=-\frac{\sqrt{3}}{2}$ $x=180^\circ+k.360^\circ,k\in Z$ or $x=120^\circ+k.360^\circ,k\in Z$ or $x=240^\circ+k.360^\circ,k\in Z$	✓ quotient identity ✓ double angle identity ✓ factors ✓ both equations ✓ $x=180°$ ✓ $x=120°$& $240°$ **OR/OF** $x=\pm120°$ ✓ $k.360°;\ k\in Z$ (7) ✓ quotient identity ✓ identity ✓ factors ✓ both equations ✓ $x=180°$ ✓ $x=120°$& $240°$ **OR/OF** $x=\pm120°$ ✓ $k.360°;\ k\in Z$ (7)
		[15]

QUESTION/*VRAAG* 7

7.1		✓ both turning points ✓ both x intercepts (–30° & 150°) ✓ shape (3)
7.2	Period = 120°	✓✓ answer (2)
7.3	$x = -30°$	✓ answer (1)
7.4	Range of/*waardeversameling van g*: $y \in [-1\,;1]$ Range of/*Waardeversameling van* $\frac{1}{2}g$: $y \in \left[-\frac{1}{2}\,;\frac{1}{2}\right]$ Range of/*Waardeversameling van* $\frac{1}{2}g+1$: $y \in \left[\frac{1}{2}\,;\frac{3}{2}\right]$ **OR/*OF*** Range of/*Waardeversameling van* $\frac{1}{2}g+1$: $\frac{1}{2} \le y \le \frac{3}{2}$	✓ critical values ✓ correct notation (2) ✓ critical values ✓ correct notation (2)
		[8]

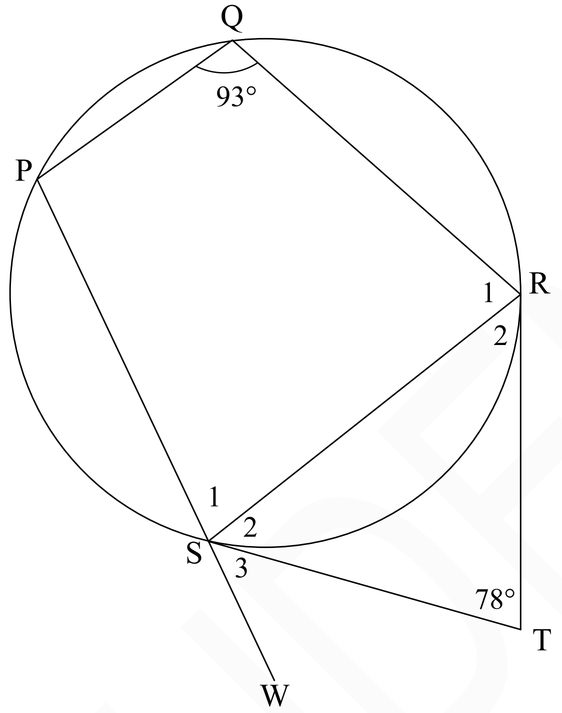
QUESTION/*VRAAG* 8



8.1	In ΔSQR: $\dfrac{QS}{\sin x} = \dfrac{QR}{\sin(90° + x)}$ $\dfrac{QS}{\sin x} = \dfrac{5}{\cos x}$ $QS = \dfrac{5\sin x}{\cos x}$ $QS = 5\tan x$	✓ correct use of sine rule ✓ $\sin(90° + x) = \cos x$ ✓ $QS = \dfrac{5\sin x}{\cos x}$ (3)
8.2	$\dfrac{QT}{\sin(180° - 2x)} = \dfrac{TS}{\sin x}$ $\dfrac{QT}{\sin 2x} = \dfrac{5\tan x}{\sin x}$ $QT = \dfrac{5\tan x \sin 2x}{\sin x}$ $QT = \dfrac{5\left(\dfrac{\sin x}{\cos x}\right)(2\sin x \cos x)}{\sin x}$ $QT = \dfrac{5\sin x\,(2\sin x)}{\sin x}$ $QT = 10\sin x$	✓ correct use of sine rule ✓ TS = QS = 5tan$x$ ✓ $QT = \dfrac{5\tan x \sin 2x}{\sin x}$ ✓ $\tan x = \dfrac{\sin x}{\cos x}$ ✓ $\sin 2x = 2\sin x \cos x$ (5)

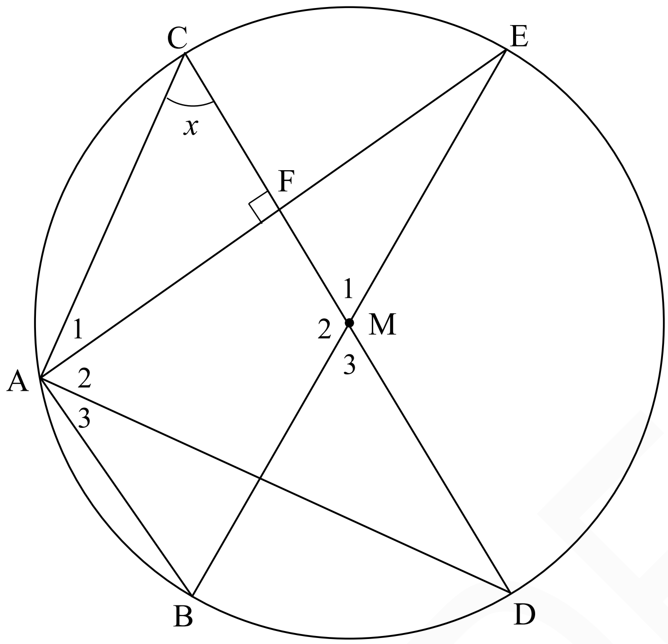
	OR/*OF*	
	$QT^2 = QS^2 + TS^2 - 2QS.TS\cos Q\hat{S}T$ $QT^2 = (5\tan x)^2 + (5\tan x)^2 - 2(5\tan x).(5\tan x)\cos(180° - 2x)$ $QT^2 = 50\tan^2 x - 50\tan^2 x(-\cos 2x)$ $QT^2 = 50\tan^2 x(1+\cos 2x)$ $QT^2 = 50\tan^2 x(1+2\cos^2 x - 1)$ $QT^2 = 50\tan^2 x(2\cos^2 x)$ $QT^2 = 100\dfrac{\sin^2 x}{\cos^2 x}(\cos^2 x)$ $QT^2 = 100\sin^2 x$ $QT = 10\sin x$	✓ correct use of cos rule ✓ TS = QS = 5tan$x$ ✓ $\cos 2x = 2\cos^2 x - 1$ & reduction ✓ $\tan x = \dfrac{\sin x}{\cos x}$ ✓ $QT^2 = 100\sin^2 x$ (5)
	OR/*OF* $TS^2 = QS^2 + TQ^2 - 2QS.TQ.\cos x$ $(5\tan x)^2 = (5\tan x)^2 + TQ^2 - 2(5\tan x).TQ.\cos x$ $0 = TQ^2 - 2(5\tan x).TQ.\cos x$ $0 = TQ[TQ - 10\tan x.\cos x]$ $TQ = 10\tan x.\cos x \quad (TQ \neq 0)$ $= 10\dfrac{\sin x}{\cos x}.\cos x$ $= 10\sin x$	✓ correct use of cos rule ✓ TS = QS = 5tan$x$ ✓ quadratic equation ito TQ ✓ TQ = 10tan x . cos x ✓ $\tan x = \dfrac{\sin x}{\cos x}$ (5)
8.3	$\text{Area of } \Delta TQR = \dfrac{1}{2}.TQ.QR\sin T\hat{Q}R$ $= \dfrac{1}{2}(10\sin 25°)(5)(\sin 70°)$ $= 9{,}93 \text{ unit}^2$	✓ correct substitution into the area rule ✓ answer (2)
		[10]

QUESTION/VRAAG 9



9.1	tangents from same(common) point/*raaklyne vanaf dieselfde punt*	✓ R (1)
9.2.1	$\hat{S}_2 = S\hat{R}T$ [∠s opp equal sides/*∠e teenoor gelyke sye*] $\therefore \hat{S}_2 = 51°$ [sum of ∠s in Δ/*som van ∠e in Δ*]	✓ R ✓ S (2)
9.2.2	$\hat{S}_2 + \hat{S}_3 = 93°$ [ext ∠ of cyclic quad/*buite∠ van koordevh*] $\hat{S}_3 = 42°$ **OR/*OF*** $\hat{S}_1 = 87°$ [opp ∠s of cyclic quad/*teenoorst ∠e v kdvh*] $\hat{S}_3 = 180° - (87° + 51°)$ $\hat{S}_3 = 42°$ [∠s on a str line/*∠e op reguitlyn*]	✓ R ✓ answer (2) ✓ R ✓ answer (2)
		[5]

QUESTION/*VRAAG* 10

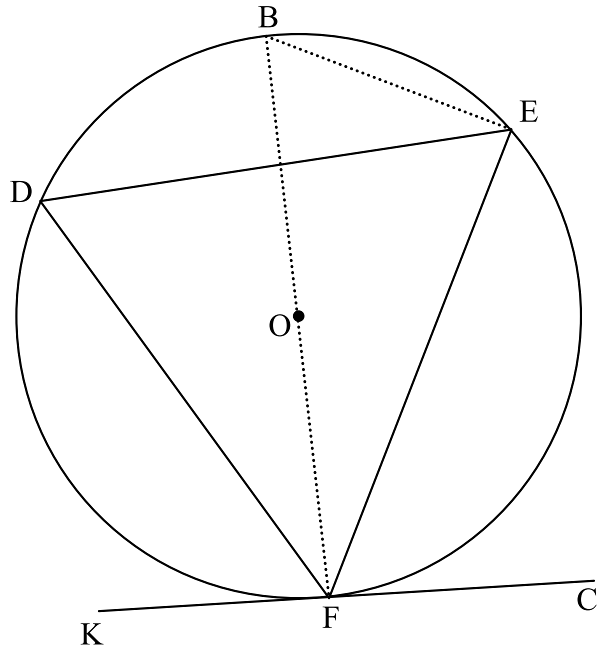


10.1	line from centre ⊥ to chord/*lyn vanaf middelpunt ⊥ op koord*	✓ R (1)
10.2	$\therefore \hat{A}_1 = 90° - x$ [sum of ∠s in Δ/*som van ∠e in Δ*] $\therefore \hat{M}_1 = 180° - 2x$ [∠at centre=2× at circumf/*midpts*∠= 2×omtreks∠]	✓ S ✓ S ✓ R (3)
10.3	$C\hat{A}D = 90°$ [∠ in semi circle/∠ *in halfsirkel*] $\hat{A}_2 = 90° - (90° - x)$ $\hat{A}_2 = x$ $\therefore \hat{A}_2 = \hat{C} = x$ ∴ AD is a tangent [converse tan-chord theorem/*omgek rkl-kd st.*]	✓ S ✓ R ✓ S ✓ R (4)
	OR/*OF* $E\hat{M}D = 2x$ [adj suppl ∠s/*aanligg suppl ∠e*] $\therefore \hat{A}_2 = x$ [∠at centre= 2×∠ at circumf/*midpts*∠= 2 ×omtreks∠] $\therefore \hat{A}_2 = C = x$ ∴ AD is a tangent [converse tan-chord theorem/*omgek rkl-kd st.*]	 ✓ S ✓ S ✓ R ✓ R (4)
	OR/*OF* $\hat{M}_3 = 180° - 2x$ [vert. opp/ *regoorstaande ∠e*] $\therefore \hat{A}_3 = 90° - x$ [∠at centre= 2×∠ at circumf/*midpts*∠= 2 ×omtreks∠] $B\hat{A}E = 90°$ [∠ in semi-circle/∠ *in halfsirkel*] $\therefore \hat{A}_2 = C = x$ ∴ AD is a tangent [converse tan-chord theorem/*omgek rkl-kd st.*] **OR/*OF***	 ✓ S ✓ R ✓ S ✓ R (4)

	CD ∥ AB [midpt. Thm/ *middelpuntst.*] $B\hat{A}E = 90^\circ$ [∠ in semi-circle/∠ *in halfsirkel*] $\therefore \hat{A}_3 = \hat{D} = 90^\circ - x$ [alt.∠s; CD∥AB/*verwiss ∠e*] $\therefore \hat{A}_2 = x =$ C ∴ AD is a tangent [converse tan-chord theorem/*omgek rkl-kd st.*] **OR/*OF*** $C\hat{A}D = 90^\circ$ [∠ in semi circle/∠ *in halfsirkel*] AC = diameter [converse ∠ in semi circle/*omgek ∠ in halfsirkel*] ∴ AD is a tangent [converse radius ⊥ tangent/*omgek radius⊥rkl*]	✓ S ✓ R ✓ S ✓ R (4) ✓ S ✓ R ✓ S ✓ R (4)
10.4	AF = FE and BM = ME [given & radii] ∴ FM = $\frac{1}{2}$AB = 12 units [Midpt Theorem/*middelptstelling*] EM = MB = CM = 18 units [radii] ∴ EB = 36 units [diameter = 2 radius] $\therefore AE^2 = (36)^2 - (24)^2$ [Pythagoras] AE = $12\sqrt{5}$ or 26,83 units **OR/*OF*** AF = FE and BM = ME [given & radii] ∴ FM = $\frac{1}{2}$AB = 12 units [Midpt Theorem/*middelpuntstelling*] EM = MB = CM = 18 units [radii] $\therefore FE^2 = (18)^2 - (12)^2$ [Pythagoras] FE = $6\sqrt{5}$ AE = $12\sqrt{5}$ or 26,83 units	✓ FM = 12 ✓ R ✓ EB = 36 ✓ using Pyth correctly ✓ answer (5) ✓ FM = 12 ✓ R ✓ EM = 18 ✓ using Pyth correctly ✓ answer (5)
		[13]

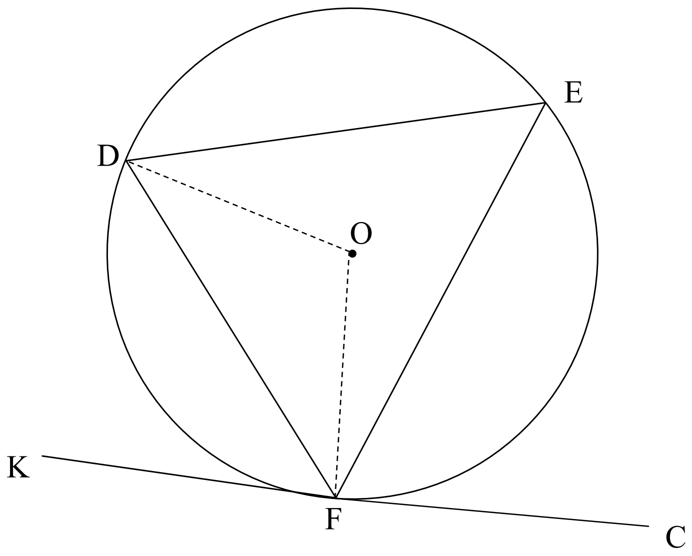
QUESTION/*VRAAG* 11

11.1



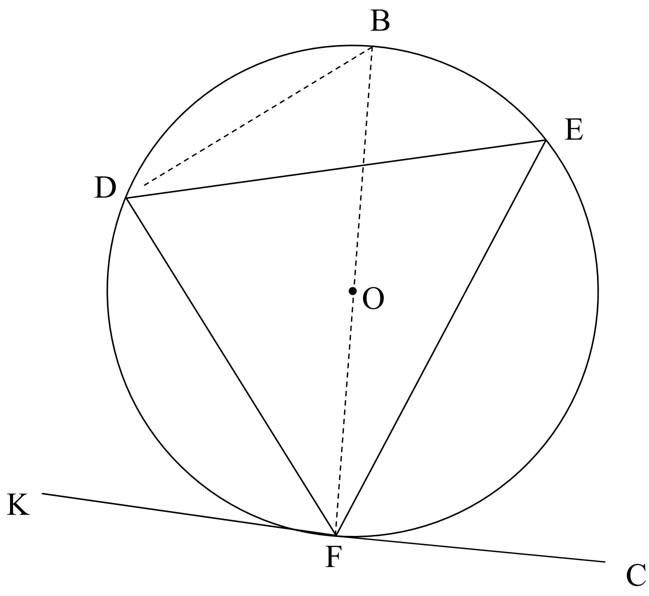
	Construction: Draw diameter BF and draw BE *Konstruksie: Trek middellyn* BF *en verbind* BE	✓Constr
	$B\hat{F}K = 90°$ or $D\hat{F}K = 90° - B\hat{F}D$ [radius ⊥ tangent/*raaklyn*]	✓ S ✓ R
	$B\hat{E}F = 90°$ [∠ in semi-circle/*semi-sirkel*]	✓ S
	$\therefore D\hat{E}F = 90° - B\hat{E}D$	
	$= 90° - B\hat{F}D$ [∠s same segment/*∠e dieselfde segment*]	✓ S/R
	$\therefore D\hat{F}K = D\hat{E}F$	(5)

OR/*OF*



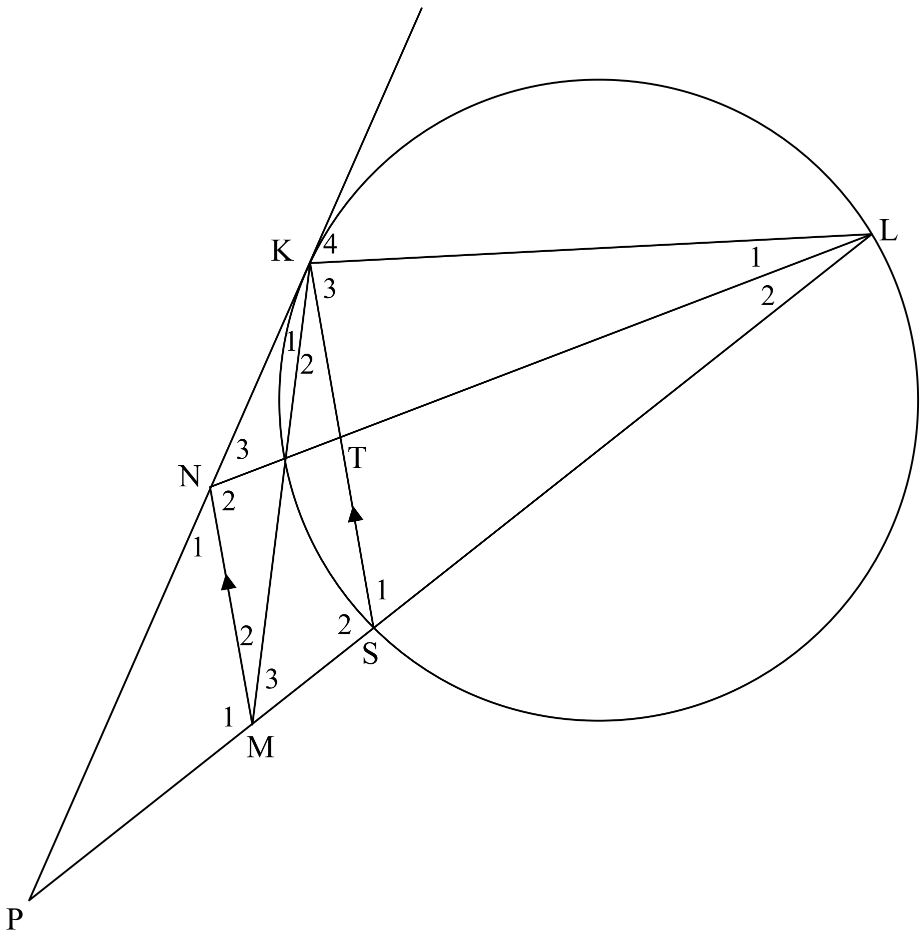
	Construction: Draw radii DO and OF *Konstruksie: Trek radii DO en OF* $O\hat{F}K = 90°$ or $D\hat{F}K = 90° - O\hat{F}D$ radius⊥ tangent/*raaklyn*] $O\hat{D}F = O\hat{F}D$ [∠s opp = sides/*∠e teenoor = sye*] $\therefore D\hat{O}F = 180° - 2O\hat{F}D$ [∠s of Δ/*∠e van Δ*] $D\hat{E}F = 90° - O\hat{F}D$ [∠ at centre = 2×∠ circumf/ *midpts∠ = 2× omtreks∠*] $\therefore D\hat{F}K = D\hat{E}F$	✓ construction ✓ S ✓R ✓ S ✓ S/R (5)

OR/OF



	Construction: Draw diameter BF and join BD. *Konstruksie: Trek middellyn BF en verbind BD.* $B\hat{F}K = 90°$ or $D\hat{F}K = 90° - B\hat{F}D$ [radius⊥ tangent/*raaklyn*] $F\hat{D}B = 90°$ [∠ in half circle/*semi-sirkel*] $\hat{B} = 90° - B\hat{F}D$ $\therefore D\hat{F}K = \hat{B}$ but $\hat{B} = \hat{E}$ [∠s same segment/*∠e dieselfde segment*] $\therefore D\hat{F}K = \hat{E}$	✓ construction ✓ S ✓/R ✓ S ✓ S/R (5)

11.2



11.2.1(a)	$\hat{K}_4 = \hat{S}_1$ [tan chord theorem/*raaklynkoordstelling*] $\hat{M}_2 + \hat{M}_3 = \hat{S}_1$ [corresp ∠s; / *ooreenk ∠s;* MN ∥ KS] $\therefore \hat{K}_4 = \hat{M}_2 + \hat{M}_3 = N\hat{M}L$	✓ S ✓ R ✓ S ✓ R (4)
11.2.1(b)	$\therefore \hat{K}_4 = \hat{M}_2 + \hat{M}_3 = N\hat{M}L$ ∴ KLMN is a cyclic quad [ext ∠ of quad = opp int ∠ / *buite ∠ van vh = teenoorst binne ∠*] **OR/OF** $N_1 = \hat{K}_1 + \hat{K}_2 = N\hat{K}S$ [corresp ∠s; / *ooreenk ∠s;* MN ∥ KS] $N\hat{K}S = K\hat{L}S$ [tan chord theorem / *raaklynkoordstelling*] $\hat{N}_1 = K\hat{L}S$ ∴ KLMN is a cyclic quad [ext ∠ of quad = opp int ∠ / *buite ∠ van vh = teenoorst binne ∠*] **OR/OF**	✓ R (1) ✓ R (1)

	$\text{NKL} = 180^\circ - \text{K}_4$ [adj. suppl.] $\therefore \text{NKL} = 180^\circ - \text{NML}$ [proved] $\therefore$ KLMN is a cyclic quad [opp.∠s supplementary]	✓ R (1)
11.2.2	In $\Delta\text{LKN} \lvert\lvert\lvert \Delta\text{KSM}$: $\hat{N}_3 = \hat{M}_3$ [∠s in the same seg / *∠e in dieselfde sirkel segm*] $\hat{L}_1 = \hat{M}_2$ [∠s in the same seg / *∠e in dieselfde sirkel segm*] $= \hat{K}_2$ [alt ∠s; / *verw∠e;* MN‖KS] $\text{N}\hat{\text{K}}\text{L} = \text{M}\hat{\text{S}}\text{K}$ [∠s of Δ / *∠e van Δ*] $\Delta\text{LKN} \lvert\lvert\lvert \Delta\text{KSM}$ **OR/OF** In $\Delta\text{LKN} \lvert\lvert\lvert \Delta\text{KSM}$: $\hat{N}_3 = \hat{M}_3$ [∠s in the same seg / *∠e in dieselfde sirkel segm*] $\text{N}\hat{\text{K}}\text{L} = \hat{M}_1$ [ext ∠ of cyclic quad/*buite∠ van koordevh*] $= \hat{S}_2$ [corresp ∠s/*ooreenk ∠e*; KS ‖ NM] $\Delta\text{LKN} \lvert\lvert\lvert \Delta\text{KSM}$ [∠,∠,∠] **OR/OF** In $\Delta\text{LKN} \lvert\lvert\lvert \Delta\text{KSM}$: $\hat{N}_3 = \hat{M}_3$ [∠s in the same seg / *∠e in dieselfde sirkel segm*] $\hat{K}_4 + \text{N}\hat{\text{K}}\text{L} = \hat{S}_1 + \hat{S}_2$ [∠s on straight line/*∠e op reguitlyn*] $\therefore \text{N}\hat{\text{K}}\text{L} = \hat{S}_2$ [$\hat{K}_4 = \hat{S}_1$] $\Delta\text{LKN} \lvert\lvert\lvert \Delta\text{KSM}$ [∠,∠,∠]	✓ S ✓ R ✓ S ✓ S/R ✓ S (5) ✓ S ✓ R ✓ S/R ✓ S ✓ R (5) ✓ S ✓ R ✓ S/R ✓ S ✓ R (5)
11.2.3	$\frac{\text{LK}}{\text{KS}} = \frac{\text{KN}}{\text{SM}}$ [$\Delta\text{LKN} \lvert\lvert\lvert \Delta\text{KSM}$] $\therefore \frac{12}{\text{KS}} = \frac{4}{3}$ KS = 9 units	✓ S ✓ R ✓ substitution ✓ answer (4)
11.2.4	4SM = 3KN $\text{SM} = \frac{3(8)}{4}$ SM = 6 $\frac{\text{LT}}{\text{NL}} = \frac{\text{LS}}{\text{ML}}$ [line ‖ one side of Δ / *lyn ‖ een sy v Δ*] $\frac{\text{LT}}{16} = \frac{13}{19}$ $\text{LT} = \frac{208}{19} = 10{,}95$	✓ SM = 6 ✓ S ✓ R ✓ answer (4)
		[23]

TOTAL/*TOTAAL*: 150

basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

SENIOR CERTIFICATE EXAMINATIONS/
SENIORSERTIFIKAAT-EKSAMEN
NATIONAL SENIOR CERTIFICATE EXAMINATIONS/
NASIONALE SENIORSERTIFIKAAT-EKSAMEN

MATHEMATICS P2/
WISKUNDE V2

MARKING GUIDELINES/*NASIENRIGLYNE*

2021

MARKS: 150
PUNTE: 150

These marking guidelines consist of 23 pages.
Hierdie nasienriglyne bestaan uit 23 bladsye.

NOTE:

- If a candidate answers a question TWICE, only mark the FIRST attempt.
- If a candidate has crossed out an attempt of a question and not redone the question, mark the crossed out version.
- Consistent accuracy applies in ALL aspects of the marking memorandum. Stop marking at the second calculation error.
- Assuming answers/values in order to solve a problem is NOT acceptable.

LET WEL:

- *As 'n kandidaat 'n vraag TWEE KEER beantwoord, sien slegs die EERSTE poging na.*
- *As 'n kandidaat 'n antwoord van 'n vraag doodtrek en nie oordoen nie, sien die doodgetrekte poging na.*
- *Volgehoue akkuraatheid word in ALLE aspekte van die memorandum toegepas. Hou op nasien by die tweede berekeningsfout.*
- *Aanvaar van antwoorde/waardes om 'n probleem op te los, word NIE toegelaat nie.*

GEOMETRY	
S	**A mark for a correct statement** **(A statement mark is independent of a reason)**
	'n Punt vir 'n korrekte bewering ***('n Punt vir 'n bewering is onafhanklik van die rede)***
R	**A mark for the correct reason** **(A reason mark may only be awarded if the statement is correct)**
	'n Punt vir 'n korrekte rede ***('n Punt word slegs vir die rede toegeken as die bewering korrek is)***
S/R	**Award a mark if statement AND reason are both correct**
	Ken 'n punt toe as die bewering EN rede beide korrek is

QUESTION/VRAAG 1

1.1

26	13	3	18	12	34	24	58	16	10	15	69	20	17	40

1.1.1(a)	$\bar{x} = \frac{375}{15}$ $\bar{x} = 25\,\text{MB}$ Answer only: Full marks	✓ 375 ✓ answer (2)
1.1.1(b)	$\sigma = 17{,}65$ MB	✓answer (1)
1.1.2	25 + 17,65 = 42,65 $\therefore$ 2 days	✓ 42,65 ✓ 2 (2)
1.1.3	Overall $\bar{x} = \frac{80}{100} \times 25$ $= 20$ MB $\frac{375 + x}{30} = 20$ $x = 600 - 375$ $= 225$ maximum total amount of data that Sam must use for the remainder of the month: **225** MB	✓ Overall $\bar{x} = 20$ ✓ $\frac{375 + x}{30} = 20$ ✓ answer (3)

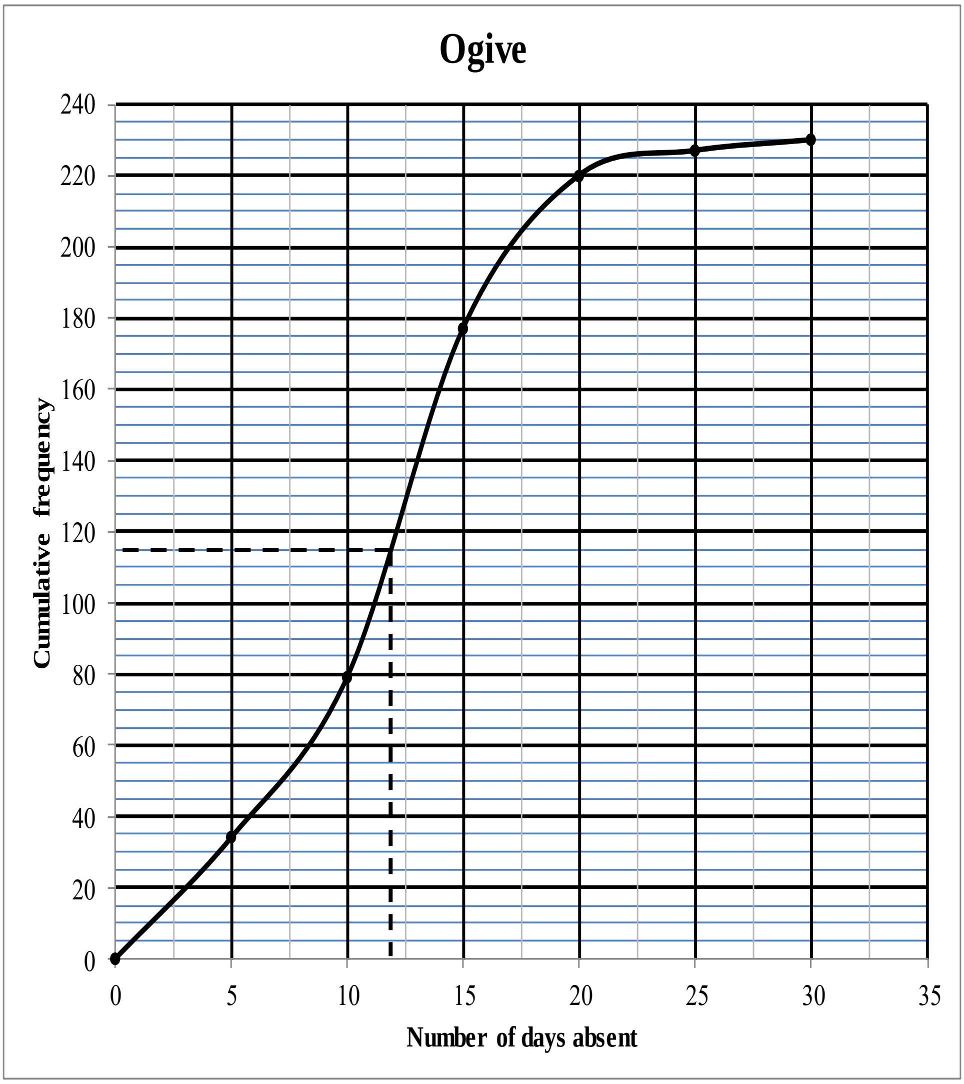
1.2

Wind speed in km/h (*x*)	2	6	15	20	25	17	11	24	13	22
Temperature in °C (*y*)	28	26	22	22	16	20	24	19	26	19

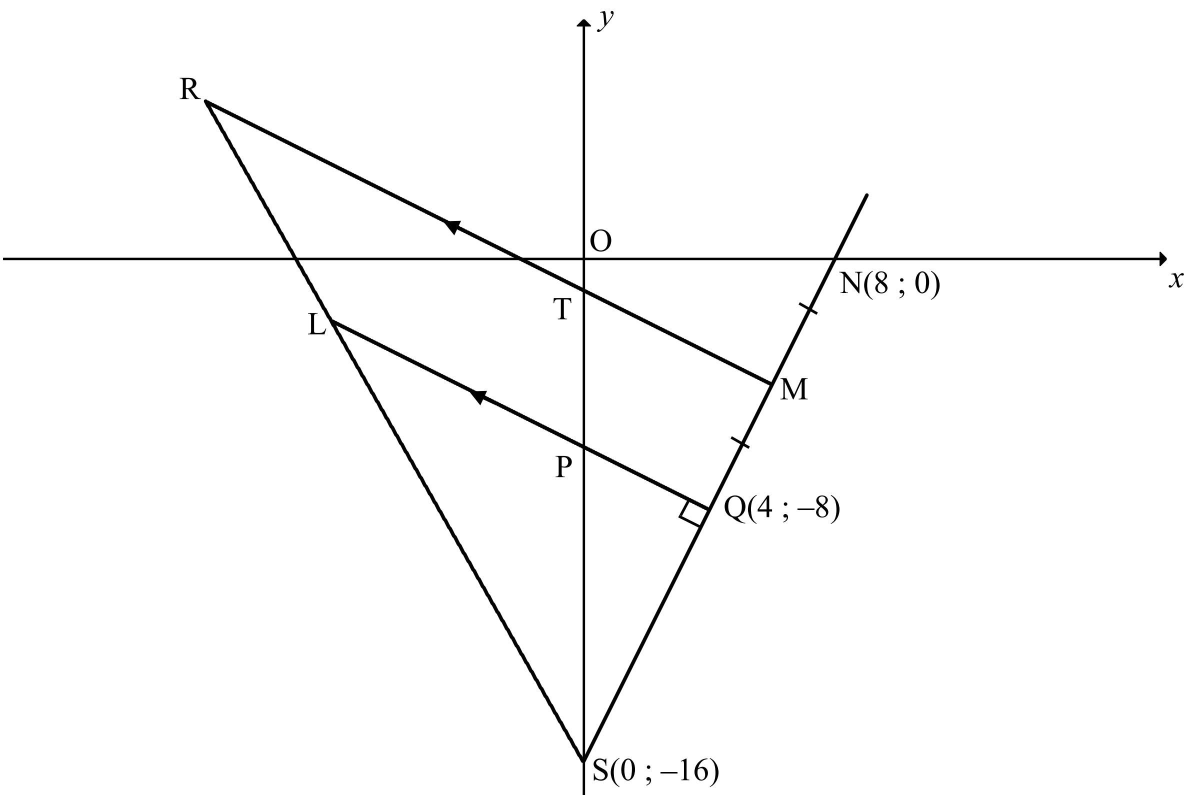
1.2.1	$a = 29{,}35$ $b = -0{,}46$ $\hat{y} = 29{,}35 - 0{,}46x$	✓ a ✓ b ✓ equation (3)
1.2.2	$y = 25{,}20$ °C (calculator) OR $\hat{y} = 29{,}35 - 0{,}46(9)$ $y = 25{,}21$ °C	✓✓ answer (2) ✓ substitution ✓ answer (2)
1.2.3	$b < 0$, indicating that as the wind speed increases the temperature decreases.	✓ interpretation (1)
		[14]

QUESTION/VRAAG 2

Number of days absent	Number of learners	Cumulative frequency
$0 \leq x < 5$	34	34
$5 \leq x < 10$	45	79
$10 \leq x < 15$	98	177
$15 \leq x < 20$	43	220
$20 \leq x < 25$	7	227
$25 \leq x < 30$	3	230

2.1	Modal class: $10 \leq x < 15$	✓ answer (1)
2.2	177 learners	✓ answer (1)
2.3	230 learners	✓ answer (1)
2.4		✓ grounding at (0; 0) ✓ shape ✓ upper limits ✓ All other points correct (4)
2.5	The median is at position 115. $\therefore$value of median is 12 days (accept 11 – 14) Answer only: Full marks	✓ reading off at 115 ✓ answer (2)
		[9]

QUESTION/VRAAG 3

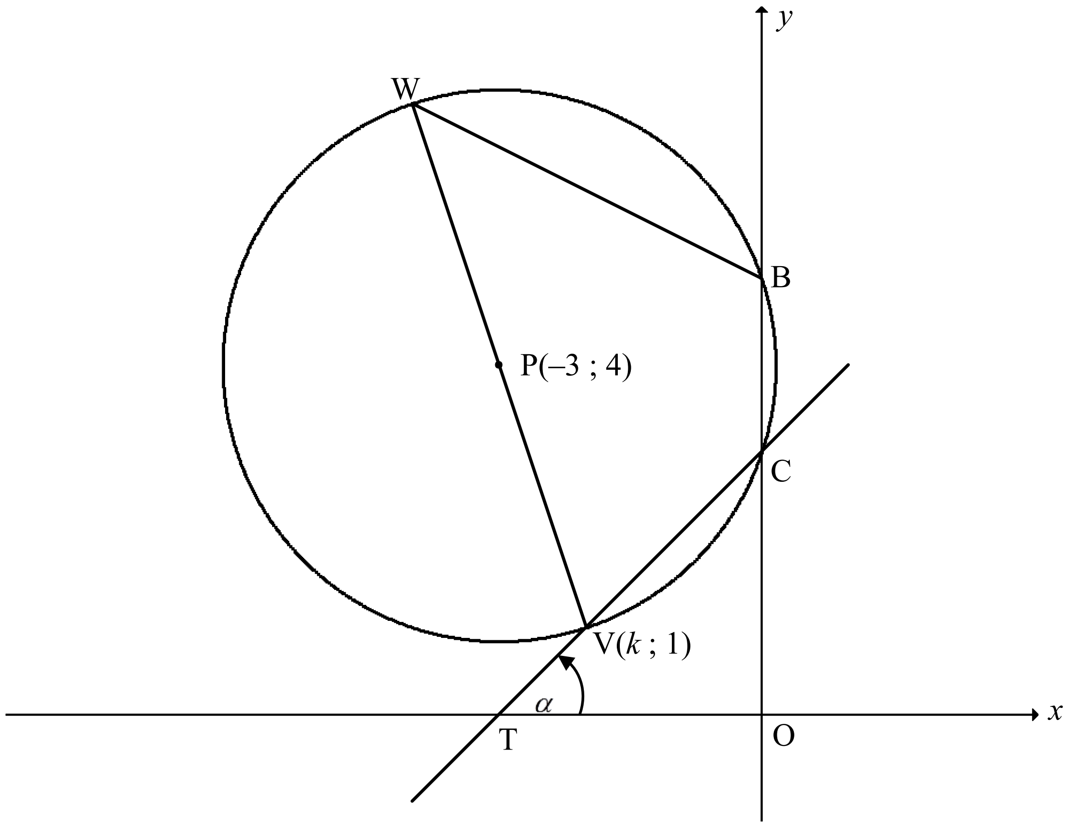


3.1	$M\left(\frac{4+8}{2};\frac{-8+0}{2}\right)$ $M(6;-4)$	✓ x_M ✓ y_M (2)
3.2	$m_{NS}=\frac{0-(-16)}{8-0}$ or $m_{NQ}=\frac{0-(-8)}{8-4}$ or $m_{QS}=\frac{-8-(-16)}{4-0}$ $=2$	✓ subst N and Q or N and Q or Q and S into gradient formula ✓ answer (2)
3.3	$m_{LQ}\times 2=-1$ [LQ $\perp$ NS] $\therefore m_{LQ}=-\frac{1}{2}$ $-8=-\frac{1}{2}(4)+c$ **OR** $y+8=-\frac{1}{2}(x-4)$ $c=-6$ $y+8=-\frac{1}{2}x+2$ $\therefore y=-\frac{1}{2}x-6$	✓ m_{LQ} ✓ substitution of Q ✓ calculation of c or simplification (3)
3.4	OS is the radius of circle passing through S $(x-0)^2+(y-0)^2=(16)^2$ $x^2+y^2=256$ Answer only: Full marks	✓ identifying radius = 16 ✓ Equation of circle (2)

3.5	$m_{RM} = m_{LQ} = -\frac{1}{2}$ [RM ∥ LQ] $-4 = -\frac{1}{2}(6) + c$ **OR** $y + 4 = -\frac{1}{2}(x - 6)$ $c = -1$ $y + 4 = -\frac{1}{2}x + 3$ $\therefore y = -\frac{1}{2}x - 1$ $T(0; -1)$	✓ m_{RM} ✓ substitution of $M(6; -4)$ ✓ coordinates of T (3)
3.6	$T(0; -1)$, $P(0; -6)$ and $S(0; -16)$ $\therefore$ PS = 10 units and TS = 15 units $\frac{LS}{RS} = \frac{PS}{TS} = \frac{2}{3}$ [prop theorem; RM ∥ LP] **OR** [line ∥ one side of Δ/*lyn* ∥ *een sy v Δ*] Answer only: Full marks **OR** M(6 ; –4), Q(4 ; –8) and S(0 ; –16) MS = $\sqrt{180} = 6\sqrt{5}$ and QS = $\sqrt{80} = 4\sqrt{5}$ $\frac{LS}{RS} = \frac{QS}{MS} = \frac{2}{3}$ [prop theorem; RM ∥ LQ] **OR** [line ∥ one side of Δ/*lyn* ∥ *een sy v Δ*] Answer only: Full marks	✓ PS = 10 units ✓ TS = 15 units ✓ answer (3) ✓ MS $= 6\sqrt{5}$ units ✓ QS $= 4\sqrt{5}$ units ✓ answer (3)
3.7	area of PTMQ = area of ΔTSM – area of ΔPSQ $= \frac{1}{2}.ST. \perp h_M - \frac{1}{2}.PS. \perp h_Q$ $= \frac{1}{2}(15)(6) - \frac{1}{2}(10)(4)$ = 45 – 20 = 25 square units **OR** $TM = \sqrt{45} = 3\sqrt{5} = 6{,}71$ $MQ = \sqrt{20} = 2\sqrt{5} = 4{,}47$ $PQ = \sqrt{20} = 2\sqrt{5} = 4{,}47$ area of trapezium PTMQ = $\frac{1}{2}\left(3\sqrt{5} + 2\sqrt{5}\right)\left(2\sqrt{5}\right)$ $= \frac{1}{2}\left(5\sqrt{5}\right)\left(2\sqrt{5}\right)$ = 25 square units	✓ area of ΔTSM – area of ΔPSQ ✓ area ΔTSM = 45 ✓ area ΔPSQ = 20 ✓ answer (4) ✓ TM = $3\sqrt{5}$ MQ = $2\sqrt{5}$ PQ = $2\sqrt{5}$ ✓ area of trapezium = $\frac{1}{2}$ (sum of ∥sides)(height) ✓ substitute into formula ✓ answer (4)

	OR $\text{MQ} = \sqrt{20} = 2\sqrt{5}$ $\text{PQ} = \sqrt{20} = 2\sqrt{5}$ $\text{TP} = 5$ area of PTMQ= area of ΔMTP+ area of ΔPQM $\boxed{\text{area of PTMQ} = \frac{1}{2}\text{TP} \times \perp h_M + \frac{1}{2}\text{MQ} \times \text{PQ}}$ area of $\text{PTMQ} = \frac{1}{2}(5) \times 6 + \frac{1}{2}(2\sqrt{5})(2\sqrt{5})$ area of $\text{PTMQ} = 10 + 15 = 25$	✓ area of ΔMTP + area of ΔPQM ✓ area ΔMTP = 10 ✓ area ΔPQM = 15 ✓ answer (4)
		[19]

QUESTION 4



4.1	$PV = r = \sqrt{10}$ $PV = \sqrt{(k-(-3))^2 + (1-4)^2} = \sqrt{10}$ $(PV)^2 = (k-(-3))^2 + (1-4)^2 = 10$ $k^2 + 6k + 9 + 9 = 10$ **OR** $(k+3)^2 + 9 = 10$ $k^2 + 6k + 8 = 0$ $\quad (k+3)^2 = 1$ $(k+4)(k+2) = 0$ $\quad k+3 = 1$ or $k+3 = -1$ $k = -4$ or $k = -2$ $\therefore k = -2$	✓ $PV = r = \sqrt{10}$ ✓ substitution into distance formula ✓ standard form ✓ factors ✓ answer (5)
4.2	$x^2 + 6x + y^2 - 8y + 15 = 0$ y-intercepts: $(0)^2 + 6(0) + y^2 - 8y + 15 = 0$ $(y-3)(y-5) = 0$ $y_C = 3$ or $y_B = 5$ $\therefore BC = 2$ units	✓ $x = 0$ ✓ factors ✓ both values ✓ answer (4)

4.3.1	$m_{TC} = \frac{3-1}{0-(-2)}$ $= 1$ $\tan\alpha = 1$ $\therefore \alpha = 45°$ **OR** $y = mx + 3$ $1 = m(-2) + 3$ $m_{TC} = 1$ $\tan\alpha = 1$ $\therefore \alpha = 45°$	✓ substitution into gradient formula ✓ $\tan\alpha = 1$ ✓ answer (3) ✓ substitution into equation of a line ✓ $\tan\alpha = 1$ ✓ answer (3)
4.3.2	$B\hat{C}V = 135°$ [ext ∠ of Δ/*buite* ∠ v Δ] $\therefore V\hat{W}B = 45°$ [opp ∠s of cyclic quad/*teenoorst. ∠e v kvh*] Answer only: Full marks **OR** $T\hat{C}O = 45°$ [∠s of Δ/∠*e v* Δ] $\therefore V\hat{W}B = 45°$ [ext ∠s of cyclic quad/*buite ∠ v kvh*] Answer only: Full marks	✓ $B\hat{C}V = 135°$ ✓ answer (2) ✓ $T\hat{C}O = 45°$ ✓ answer (2)
4.4.1	$Q(-3;-2)$	✓ x_Q ✓ y_Q (2)
4.4.2	$(x+3)^2 + (y+2)^2 = 10$	✓ LHS ✓ RHS (2)
4.4.3	$x = -2$ or $x = -4$	✓ $x = -2$ ✓ $x = -4$ (2)
		[20]

QUESTION/VRAAG 5

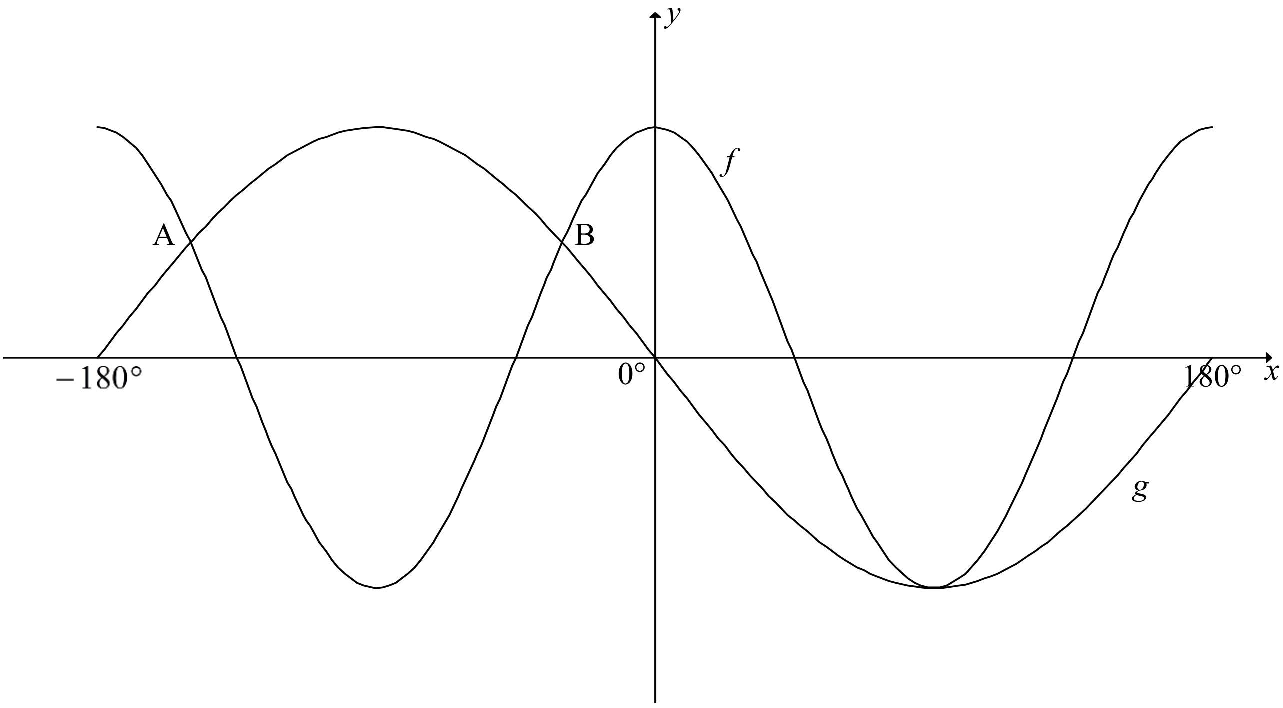
5.1	$\tan(-x).\cos x.\sin(x-180°)-1$ $=-\tan x.\cos x.\sin(-(180°-x))-1$ $=\frac{-\sin x}{\cos x}.\cos x.(-\sin x)-1$ $=\sin^2 x-1$ $=-\cos^2 x$	✓ $-\tan x$ ✓ $-\sin x$ ✓ $\frac{-\sin x}{\cos x}$ ✓ $\sin^2 x-1$ ✓ answer (5)
5.2.1	$\cos 215°$ $=-\cos 35°$ $=-m$	✓ reduction ✓ answer (2)
5.2.2	$\sin 20°$ $=\cos 70°$ $=\cos 2(35°)$ $=2\cos^2 35°-1$ $=2m^2-1$ OR $=\sin(55°-35°)$ $=\sin55°\cos35°-\cos55°\sin35°$ $=m.m-\sqrt{1-m^2}.\sqrt{1-m^2}$ $=m^2-(1-m^2)$ $=2m^2-1$	✓ co-function ✓ double angle expansion ✓ answer in terms of m (3) ✓ compound angle expansion ✓ $\cos 55°=\sqrt{1-m^2}$ or $\sin 35°=\sqrt{1-m^2}$ ✓ answer in terms of m (3)
5.3	$\cos 4x.\cos x+\sin 4x.\sin x=-0{,}7$ $\cos(4x-x)=-0{,}7$ ref $\angle=45{,}57\ldots°$ $3x=180°-45{,}57...°+k.360°$ or $3x=180°+45{,}57...°+k.360°$ $3x=134{,}43°+k.360°$ or $3x=225{,}57°+k.360°$ $x=44{,}81°+k.120°$; $k\in Z$ $x=75{,}19°+k.120°$; $k\in Z$	✓ compound angle ✓ $3x$ = 134,43° or 225,57° ✓ x = 44,81° or 75,19° ✓ $+k.120°$; $k\in Z$ (4)

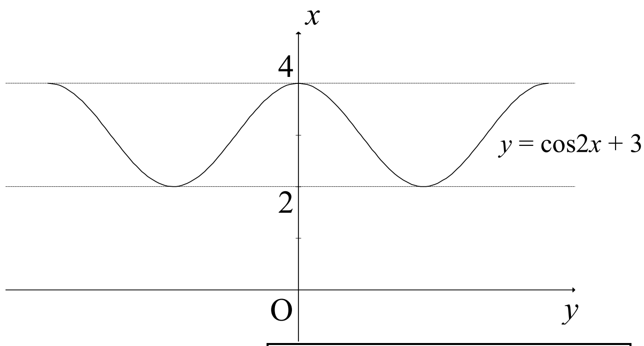
5.4	$\text{RHS} = \cos^2 x - \sin^2 x$ $\text{LHS} = \dfrac{\sin 4x.\cos 2x - 2\cos 4x.\sin x.\cos x}{\tan 2x}$ $= \dfrac{\sin 4x.\cos 2x - \cos 4x.\sin 2x}{\frac{\sin 2x}{\cos 2x}}$ $= \sin(4x - 2x)\left(\dfrac{\cos 2x}{\sin 2x}\right)$ $= \cos 2x$ $= \cos^2 x - \sin^2 x$ LHS = RHS	✓ $\sin 2x$ ✓ $\dfrac{\sin 2x}{\cos 2x}$ ✓ $\sin(4x-2x)$ ✓ $\cos 2x$ (4)
		[18]

QUESTION/VRAAG 6

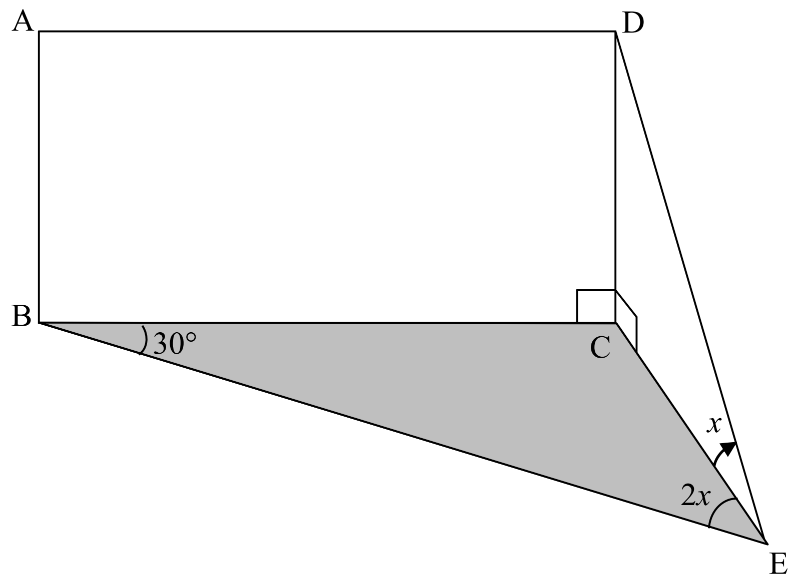
6.1	$1-2\sin^2 x=-\sin x$ $2\sin^2 x-\sin x-1=0$ $(2\sin x+1)(\sin x-1)=0$ $\sin x=-\frac{1}{2}$ or $\sin x=1$ ref $\angle = 30°$ ref $\angle = 90°$ $x=210°+k.360°$ $x=90°+k.360°$ or $x=330°+k.360°$ $x=-150°$ or $x=-30°$ or $x=90°$	✓ identity ✓ factors ✓ $\sin x=-\frac{1}{2}$ ✓ $\sin x=1$ ✓ $-150°$ and $-30°$ ✓ $90°$ (A) (6)
	OR	
	$\cos 2x=-\sin x$ $\cos 2x=-\cos(90°-x)$ $2x=180°-(90°-x)+k.360°$ or $2x=180°+(90°-x)+k.360°$ $2x=90°+x+k.360°$ or $2x=270°-x+k.360°$ $x=90°+k.360°$ $x=90°+k.120°$ $x=-150°$ or $x=-30°$ or $x=90°$	✓ co-functions ✓ $2x$ in quadrant 2 ✓ $2x$ in quadrant 3 ✓ both general solutions ✓ $-150°$ and $-30°$ ✓ $90°$ (A) (6)
	OR	
	$\cos 2x=-\sin x$ $\cos 2x=\cos(90°+x)$ $2x=90°+x+k.360°$ or $2x=360°-(90°+x)+k.360°$ $x=90°+k.360°$ or $3x=270°+k.360°$ $x=90°+k.120°$ $x=-150°$ or $x=-30°$ or $x=90°$	✓ co-functions ✓ $2x$ in quadrant 1 ✓ $2x$ in quadrant 4 ✓ both general solutions ✓ $-150°$ and $-30°$ ✓ $90°$ (A) (6)
	OR	
	$\cos 2x=-\sin x$ $\sin(90°-2x)=-\sin x$ $90°-2x=180+x+k.360°$ or $90°-2x=360°-x+k.360°$ $x=-30°+k.120°$ $x=-270°+k.360°$ $x=-150°$ or $x=-30°$ or $x=90°$	✓ co-functions ✓ $90°–2x$ in quadrant 3 ✓ $90°–2x$ in quadrant 4 ✓ both general solutions ✓ $-150°$ and $-30°$ ✓ $90°$ (A) (6)

6.2



6.2.1	A(–150°; 0,5) B(–30°; 0,5) AB = –30° – (–150°) AB = 120° Answer only: Full marks	✓ AB = –30° – (–150°) ✓ answer (2)
6.2.2	$x \in (0°; 90°)$ or $x \in (90°; 180°)$ **OR** $0° < x < 90°$ or $90° < x < 180°$	✓ $x \in (0°; 90°)$ ✓ $x \in (90°; 180°)$ (2) ✓ $0° < x < 90°$ ✓ $90° < x < 180°$ (2)
6.2.3	$\cos 2x = k - 3$ $k - 3 < -1$ or $k - 3 > 1$ $k < 2$ or $k > 4$ Answer only: Full marks **OR**  $k < 2$ or $k > 4$ Answer only: Full marks	✓ $k - 3 < -1$ or $k - 3 > 1$ ✓ $k < 2$ ✓ $k > 4$ (3) ✓ graph of $y = \cos 2x + 3$ ✓ $k < 2$ ✓ $k > 4$ (3)
		[13]

QUESTION/VRAAG 7

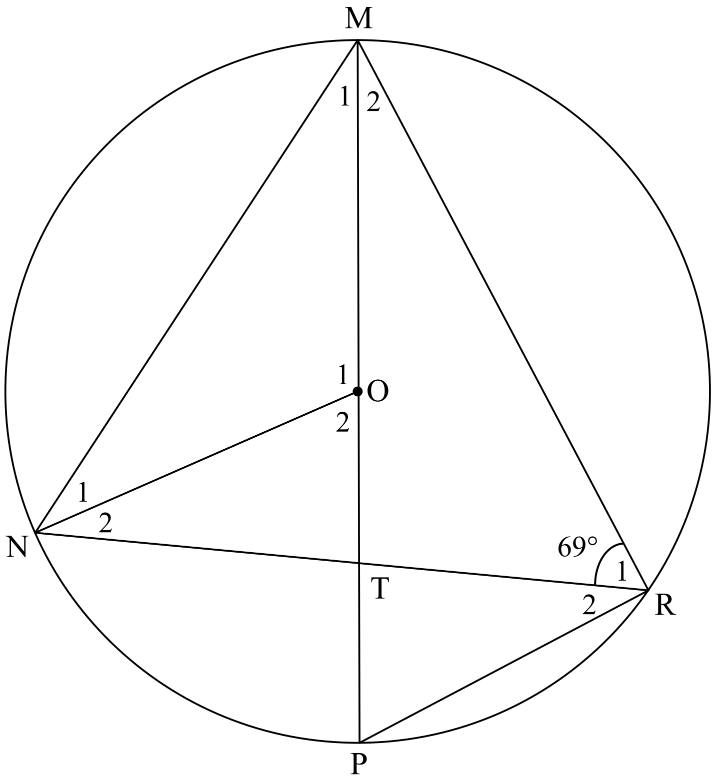


7.1	In ΔBCE: $\dfrac{\text{CE}}{\sin\hat{\text{B}}} = \dfrac{\text{BC}}{\sin\text{B}\hat{\text{E}}\text{C}}$ $\dfrac{\text{CE}}{\sin 30°} = \dfrac{\text{BC}}{\sin 2x}$ $\text{CE} = \dfrac{\text{BC}\sin 30°}{\sin 2x}$ In ΔCDE: $\dfrac{\text{DC}}{\text{CE}} = \tan \text{D}\hat{\text{E}}\text{C}$ $\text{DC} = \dfrac{\text{BC}.\sin 30°}{\sin 2x}(\tan x)$ $\text{DC} = \dfrac{\text{BC}}{4\sin x\cos x}\left(\dfrac{\sin x}{\cos x}\right)$ $\text{DC} = \dfrac{\text{BC}}{4\cos^2 x}$	✓ correct use of sine rule ✓ $\text{CE} = \dfrac{\text{BC}\sin 30°}{\sin 2x}$ ✓ correct trig ratio ✓ Subst CE ✓ $2\sin x\cos x$ ✓ $\dfrac{\sin x}{\cos x}$ (6)

7.2	$\text{DC} = \dfrac{\text{BC}}{4\cos^2 30°}$ $= \dfrac{\text{BC}}{4\left(\dfrac{\sqrt{3}}{2}\right)^2}$ $= \dfrac{\text{BC}}{3}$ $\therefore \text{BC} = 3\text{DC}$ But AB = DC [opp sides of rectangle/*teenoorst. sye v reghoek*] $\therefore \text{BC} = 3\text{AB}$ Area of rectangle = (AB)(BC) = (AB)(3AB) $=3\text{AB}^2$	✓ $\text{DC} == \dfrac{\text{BC}}{3}$ ✓ $\text{BC} = 3\text{AB}$ ✓ substitution into area formula (3)
		[9]

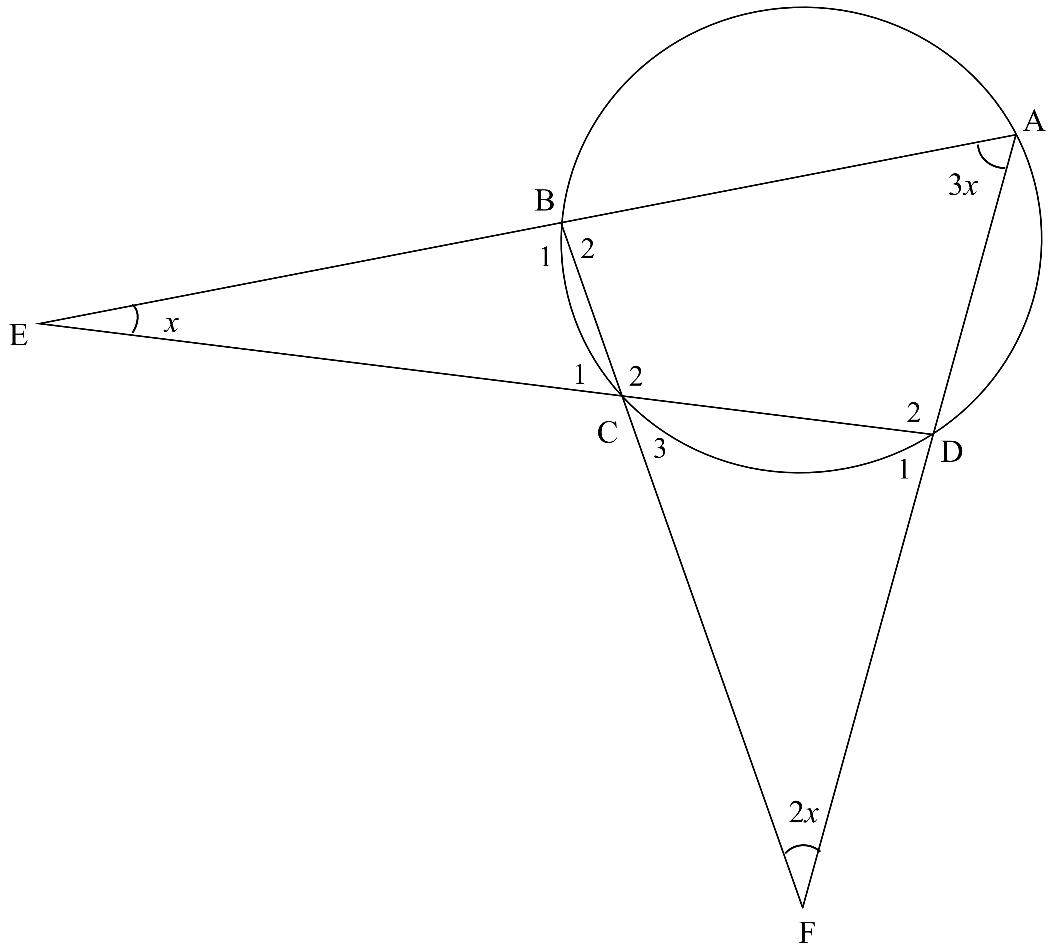
QUESTION/VRAAG 8

8.1



8.1.1	$M\hat{R}P = 90°$ [$\angle$ in semi circle/$\angle$ *in halwe sirkel*] $\hat{R}_2 = 21°$	✓ R ✓ S (2)
8.1.2	$\hat{O}_1 = 138°$ [$\angle$ at centre = 2 ×$\angle$ at circumference/ *midpts.* $\angle$ = 2× *omtreks* $\angle$]	✓ S ✓R (2)
8.1.3	$\hat{M}_1 = 21°$ [$\angle$s in the same segment/$\angle$*e in dieselfde sirkel segment*] **OR** $\hat{M}_1 + N_1 = 180° - 138°$ [sum of $\angle$s in Δ/$\angle$*e v* Δ] $\therefore \hat{M}_1 = 21°$ [$\angle$s opp equal sides/$\angle$*e teenoor gelyke sye*]	✓ S ✓R (2) ✓ S ✓R (2)
8.1.4	$\hat{O}_2 = 42°$ [$\angle$s on a str line/$\angle$*e op 'n reguitlyn*] $\hat{P} = 42°$ [alt $\angle$s; NO || PR/*Verw.* $\angle$*e,* NO || PR] $\hat{M}_2 = 48°$ [sum of $\angle$s in Δ/$\angle$*e v* Δ] **OR** $\hat{N}_2 = \hat{R}_2 = 21°$ [alt $\angle$s; NO || PR/*Verw.* $\angle$*e,* NO || PR] $\hat{N}_1 = \hat{M}_1 = 21°$ [$\angle$s opposite equal sides/$\angle$*e teenoor gelyke sye*] $\hat{M}_2 = 48°$ [sum of $\angle$s of Δ NMR//$\angle$*e v* ΔNMR]	✓ S ✓ S ✓R ✓ S (4) ✓ S ✓R ✓ S ✓ S (4)

8.2

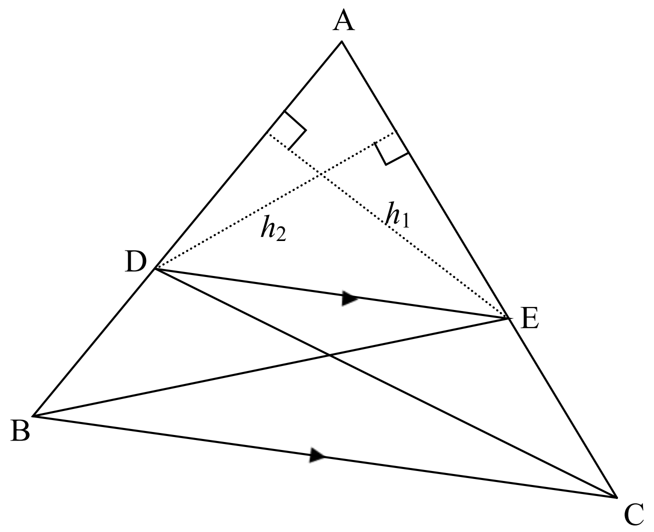


8.2	$\hat{D}_1 = 4x$ $\hat{D}_2 = 180° - 4x$ $\hat{B}_1 = 5x$ $\hat{B}_1 = \hat{D}_2$ $180° - 4x = 5x$ $9x = 180°$ $x = 20°$ **OR** $\hat{C}_1 = 3x$ $\hat{B}_2 = 4x$ $\hat{C}_1 = \hat{C}_3 = 3x$ $\hat{D}_2 = 5x$ $4x + 5x = 180°$ $x = 20°$	[ext ∠ of Δ/*buite ∠ v* Δ] [∠s on a str line/*∠e op ′n reguitlyn*] [ext ∠ of Δ/*buite ∠ v* Δ] [ext ∠ of cyclic quad/*buite ∠ v kvh*] [ext ∠ of cyclic quad/*buite ∠ v kvh*] [ext ∠ of Δ/*buite ∠ v* Δ] [vert opp ∠s] [ext ∠ of Δ/*buite ∠ v* Δ] [opp ∠ of cyclic quad/*teenoorst. ∠e v kvh*]	✓ S/R ✓ S ✓ S ✓S ✓R ✓ answer (6) ✓ S ✓ R ✓ S ✓ S ✓S/R ✓ answer (6)

	OR $\hat{C}_3 = 3x$ [ext ∠ of cyclic quad/*buite ∠ v kvh*] $\hat{D}_1 = 4x$ [ext ∠ of Δ/*buite ∠ v* Δ] $2x + 3x + 4x = 180°$ [sum of ∠s in Δ/∠*e v* Δ] $9x = 180°$ $x = 20°$	✓ S ✓R ✓ S ✓ S ✓R ✓ answer (6)
		[16]

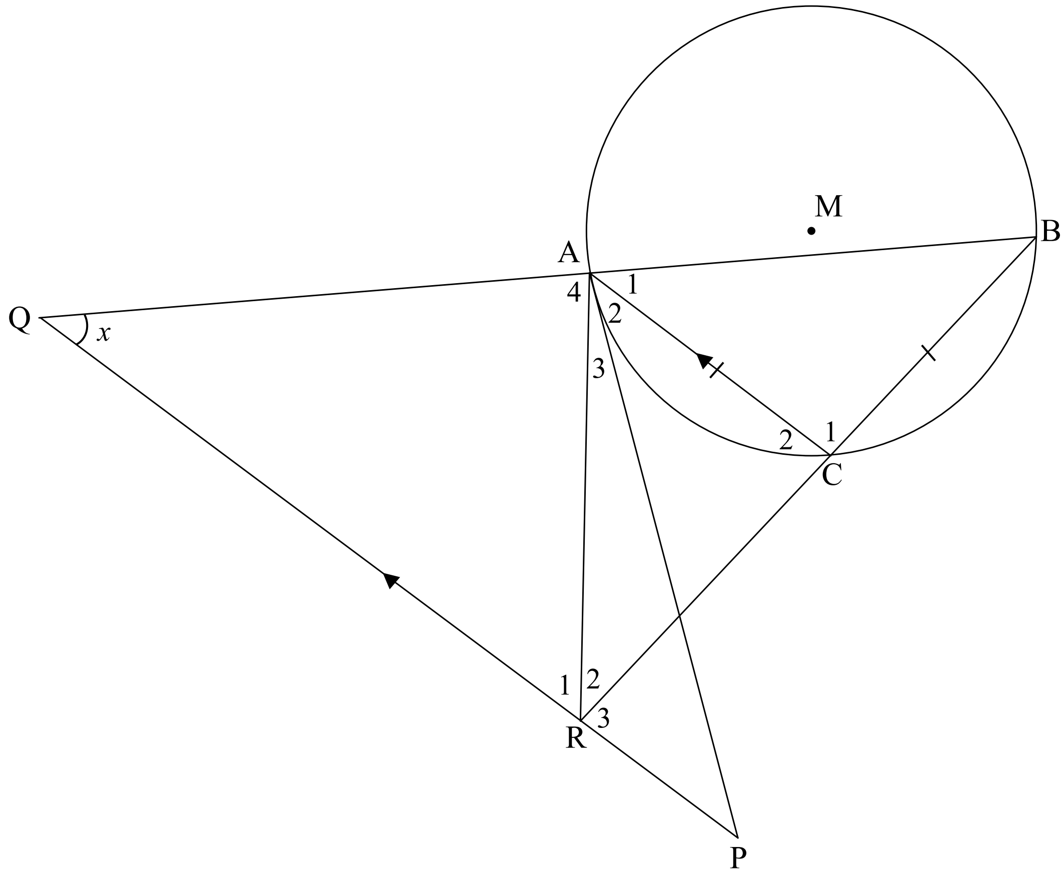
QUESTION/VRAAG 9

9.1



9.1	Constr: Join BE and CD and draw h_1 from E $\perp$ AD and h_2 from D $\perp$ AE *Konstr: Verbind BE en CD en trek* h_1 *vanaf* E $\perp$ AD *en* h_2 *vanaf* D $\perp$ AE Proof/*Bewys*: $\frac{\text{area }\Delta\text{ADE}}{\text{area }\Delta\text{BDE}} = \frac{\frac{1}{2}\text{AD}\times h_1}{\frac{1}{2}\text{BD}\times h_1} = \frac{\text{AD}}{\text{BD}}$ $\frac{\text{area }\Delta\text{ADE}}{\text{area }\Delta\text{DEC}} = \frac{\frac{1}{2}\text{AE}\times h_2}{\frac{1}{2}\text{EC}\times h_2} = \frac{\text{AE}}{\text{EC}}$ area ΔADE = area ΔADE [common/*gemeenskaplik*] But area ΔBDE = area ΔDEC [same base & height ; DE || BC/ *dies basis & hoogte ;* DE || BC] $\therefore \frac{\text{area }\Delta\text{ADE}}{\text{area }\Delta\text{BDE}} = \frac{\text{area }\Delta\text{ADE}}{\text{area }\Delta\text{DEC}}$ $\therefore \frac{\text{AD}}{\text{BD}} = \frac{\text{AE}}{\text{EC}}$	✓ constr/*konstr* ✓ $\frac{\text{area }\Delta\text{ADE}}{\text{area }\Delta\text{BDE}}$ ✓ $\frac{\frac{1}{2}\text{AD}\times h_1}{\frac{1}{2}\text{BD}\times h_1}$ **or R** ✓ $\frac{\text{area }\Delta\text{ADE}}{\text{area }\Delta\text{DEC}} = \frac{\text{AE}}{\text{EC}}$ ✓ S ✓R (6)

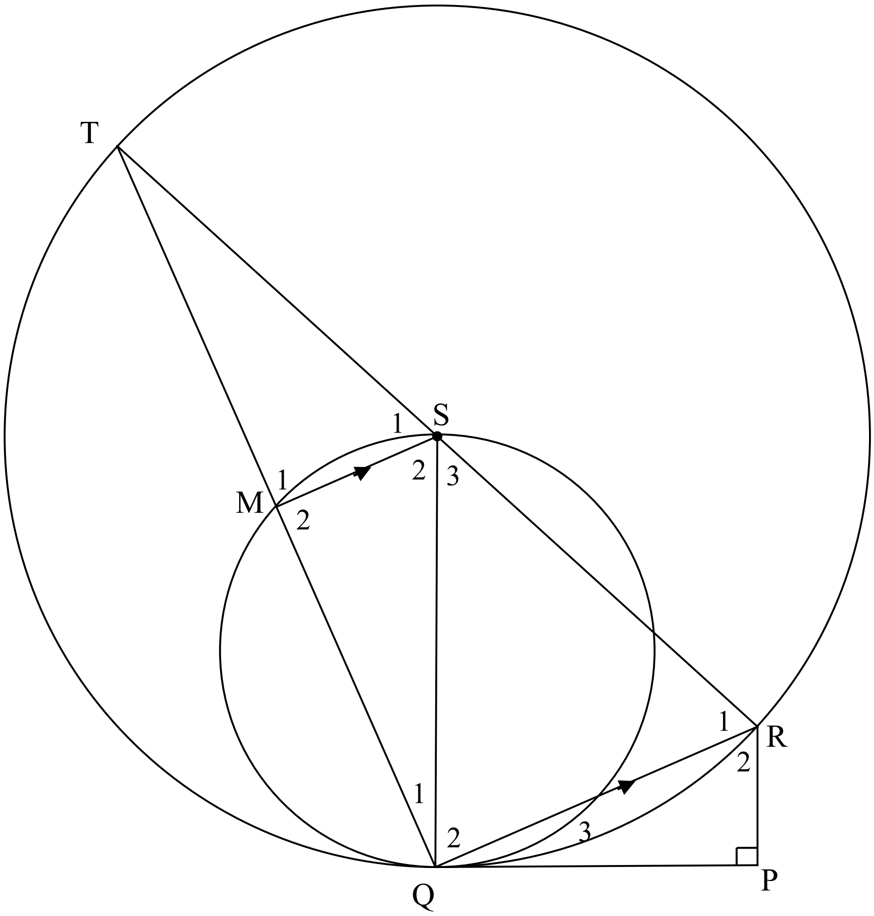
9.2



9.2.1	$\hat{A}_1 = x$ [corresp ∠s; PQ ‖ CA/*ooreenkomstige ∠e,* PQ ‖ CA] $\hat{B} = x$ [∠s opp equal sides/*∠e teenoor gelyke sye*] $\hat{A}_2 = x$ [tan-chord theorem/*∠ tussen raaklyn en koord*] $\hat{P} = x$ [alt ∠s; PQ ‖ CA/*verw. ∠e,* PQ ‖ CA]	✓ S ✓ R ✓ S/R ✓ S ✓ R ✓ S/R (6)
9.2.2	$\hat{B} = \hat{P}$ [proved in 9.2.1/*bewys in 9.2.1*] $\therefore$ A, B, P and R are concyclic $\therefore$ ABPR is a cyclic quadrilateral [conv ∠s in the same segment/ *koord onderspan gelyke omtreks ∠e*]	✓ S ✓ R (2)
9.2.3	$\frac{BA}{BQ} = \frac{BC}{BR}$ [prop th; AC ‖ QP] **OR** [line ‖ one side Δ/*lyn ‖ een syn v Δ*] But QR = BR [sides opp = ∠s/*sye teenoor = ∠e*] $\therefore \frac{BA}{BQ} = \frac{BC}{QR}$	✓ S ✓ R ✓ S (3)

	OR In Δ ABC and Δ BQR: $\hat{A}_1 = \hat{B} = x$ [proved in 9.2.1] $\hat{B} = \hat{Q} = x$ [proved in 9.2.1] $\hat{C}_1 = B\hat{R}Q = 180° - 2x$ [sum of $\angle$s of Δ] $\therefore \Delta ABC \mid\mid\mid \Delta BQR$ $\therefore \frac{BA}{BQ} = \frac{BC}{QR}$	✓ S ✓ S ✓ S (3)
	OR In Δ ABC and Δ BQR: $\hat{A}_1 = \hat{B} = x$ [proved in 9.2.1] $\hat{B} = \hat{Q} = x$ [proved in 9.2.1] $\hat{C}_1 = B\hat{R}Q = 180° - 2x$ [sum of $\angle$s of Δ] $\therefore \Delta ABC \mid\mid\mid \Delta BQR$ [$\angle\angle\angle$] $\therefore \frac{BA}{BQ} = \frac{BC}{QR}$	✓ S ✓ S ✓ R (3)
	OR In Δ ABC and Δ QBR: $\hat{B}$ is common $\hat{A}_1 = \hat{Q} = x$ [corres $\angle$s; PQ $\parallel$ CA] $\hat{C}_1 = B\hat{R}Q = 180° - 2x$ [sum of $\angle$s of Δ] $\therefore \Delta ABC \mid\mid\mid \Delta QBR$ [$\angle\angle\angle$] But QR = BR [sides opp = $\angle$s/*sye teenoor* = $\angle$*e*] $\therefore \frac{BA}{BQ} = \frac{BC}{QR}$	✓ S ✓ S ✓ S (3)
		[17]

QUESTION/VRAAG 10



10.1.1	$\hat{Q}_1 + \hat{Q}_2 = 90°$ [∠ in semi circle/∠ *in halwe sirkel*] $\therefore \hat{M}_2 = 90°$ [co-interior ∠, MS ‖ QR/*ko-binne ∠e,* MS ‖ QR] ∴ SQ is a diameter [converse: ∠ in semi circle/ *Omgekeerde: ∠ in halwe sirkel*] **OR** MS ‖ QR $\frac{TS}{SR} = \frac{TM}{MQ} = \frac{1}{1}$ [prop theorem; SM ‖ QR] **OR** [line ‖ one side of Δ]/*lyn ‖ een sy v*Δ $\therefore TM = MQ$ $\therefore \hat{M}_2 = 90°$ [Line from centre bisects chord/*midpt. sirkel; midpt koord*] ∴ SQ is a diameter [converse: ∠ in semi circle/ *Omgekeerde: ∠ in halwe sirkel*] **OR** SQ ⊥ QP [tan ⊥ rad/*raaklyn ⊥ radius*] ∴ SQ is a diameter [converse: tan ⊥ rad/*Omgekeerde: raaklyn ⊥ radius*]	✓ S/R ✓ S/R ✓ R (3) ✓ S/R ✓ S/R ✓ R (3) ✓ S ✓ R ✓ R (3)

10.1.2	In ΔRTQ and ΔRQP $\hat{T} = \hat{Q}_3$ [tan-chord theorem/$\angle$ *tussen raaklyn en koord*] $\hat{Q}_1 + \hat{Q}_2 = 90°$ [co-interior $\angle$s, MS ∥ QR/*ko-binne* $\angle$*e,* MS ∥ QR] **or** [$\angle$ in semi circle/$\angle$ *in halwe sirkel*] $\therefore \hat{Q}_1 + \hat{Q}_2 = \hat{P} = 90°$ $\hat{R}_1 = \hat{R}_2$ [$\angle$s of Δ/$\angle$*e van Δ*] $\Delta RTQ \,\|\|\, \Delta RQP$ $\frac{RT}{RQ} = \frac{RQ}{RP}$ $RT = \frac{RQ^2}{RP}$ **OR** In ΔRTQ and ΔRQP $\hat{T} = \hat{Q}_3$ [tan-chord theorem $\angle$ *tussen raaklyn en koord*] $\hat{Q}_1 + \hat{Q}_2 = 90°$ [co-interior $\angle$s, MS ∥ QR/*ko-binne* $\angle$*e,* MS ∥ QR] **or** [$\angle$ in semi circle/$\angle$ *in halwe sirkel*] $\therefore \hat{Q}_1 + \hat{Q}_2 = \hat{P} = 90°$ $\Delta RTQ \,\|\|\, \Delta RQP$ [$\angle,\angle,\angle$] $\frac{RT}{RQ} = \frac{RQ}{RP}$ $RT = \frac{RQ^2}{RP}$	✓ S ✓ R ✓ S ✓ S ✓ S ✓ ratio (6) ✓ S ✓ R ✓ S ✓ S ✓ R ✓ ratio (6)
10.2	QR = 28 units [midpoint theorem/*midpt. stelling*] $RP^2 = 28^2 - \left(\sqrt{640}\right)^2$ [Pythagoras/*Pythagoras*] RP = 12 units $RT = \frac{RQ^2}{RP}$ $RT = \frac{28^2}{12}$ $RT = \frac{196}{3}$ Radius = $\frac{98}{3}$ units	✓ S ✓ R ✓ S ✓ RP = 12 ✓ RT ✓ answer (6)
		[15]

TOTAL/TOTAAL: 150

basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

SENIOR CERTIFICATE/*SENIOR SERTIFIKAAT*
NATIONAL SENIOR CERTIFICATE/
NASIONALE SENIOR SERTIFIKAAT

GRADE/*GRAAD* 12

MATHEMATICS P2/*WISKUNDE V2*

NOVEMBER 2020

MARKING GUIDELINES/*NASIENRIGLYNE*

MARKS/*PUNTE*: 150

These marking guidelines consist of 27 pages.
Hierdie nasienriglyne bestaan uit 27 bladsye.

NOTE:

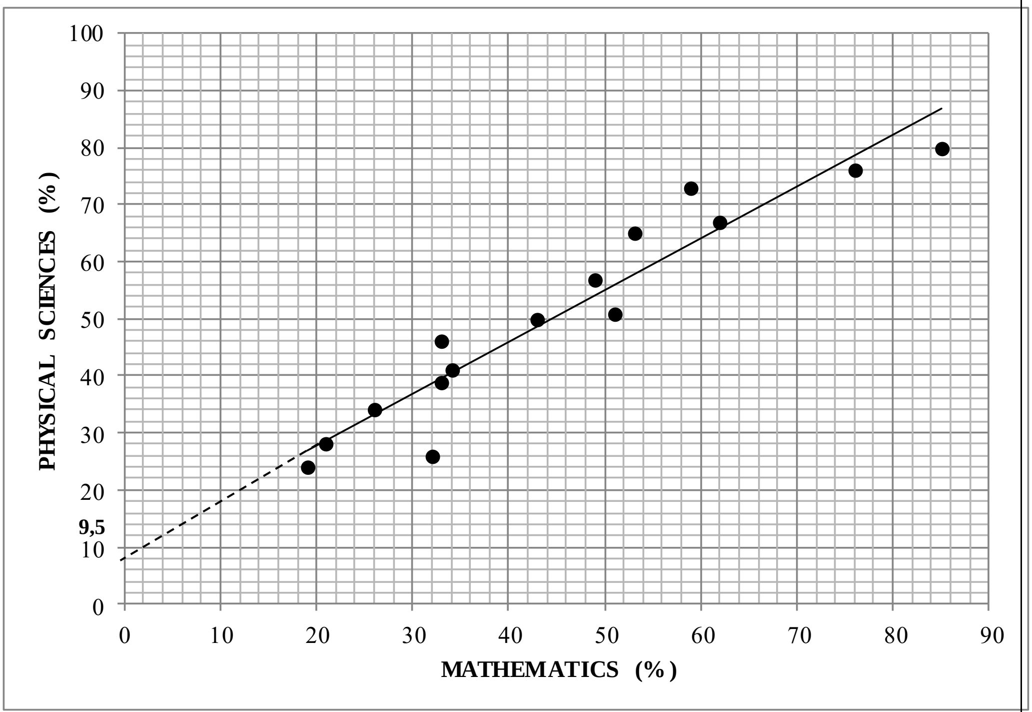
- If a candidate answers a question TWICE, only mark the FIRST attempt.
- If a candidate has crossed out an attempt of a question and not redone the question, mark the crossed out version.
- Consistent accuracy applies in ALL aspects of the marking memorandum. Stop marking at the second calculation error.
- Assuming answers/values in order to solve a problem is NOT acceptable.

LET WEL:

- *As 'n kandidaat 'n vraag TWEE KEER beantwoord, sien slegs die EERSTE poging na.*
- *As 'n kandidaat 'n antwoord van 'n vraag doodtrek en nie oordoen nie, sien die doodgetrekte poging na.*
- *Volgehoue akkuraatheid word in ALLE aspekte van die memorandum toegepas. Hou op nasien by die tweede berekeningsfout.*
- *Aanvaar van antwoorde/waardes om 'n probleem op te los, word NIE toegelaat nie.*

GEOMETRY	
S	**A mark for a correct statement** **(A statement mark is independent of a reason)**
	'n Punt vir 'n korrekte bewering ***('n Punt vir 'n bewering is onafhanklik van die rede)***
R	**A mark for the correct reason** **(A reason mark may only be awarded if the statement is correct)**
	'n Punt vir 'n korrekte rede ***('n Punt word slegs vir die rede toegeken as die bewering korrek is)***
S/R	**Award a mark if statement AND reason are both correct**
	Ken 'n punt toe as die bewering EN rede beide korrek is

QUESTION/*VRAAG* 1

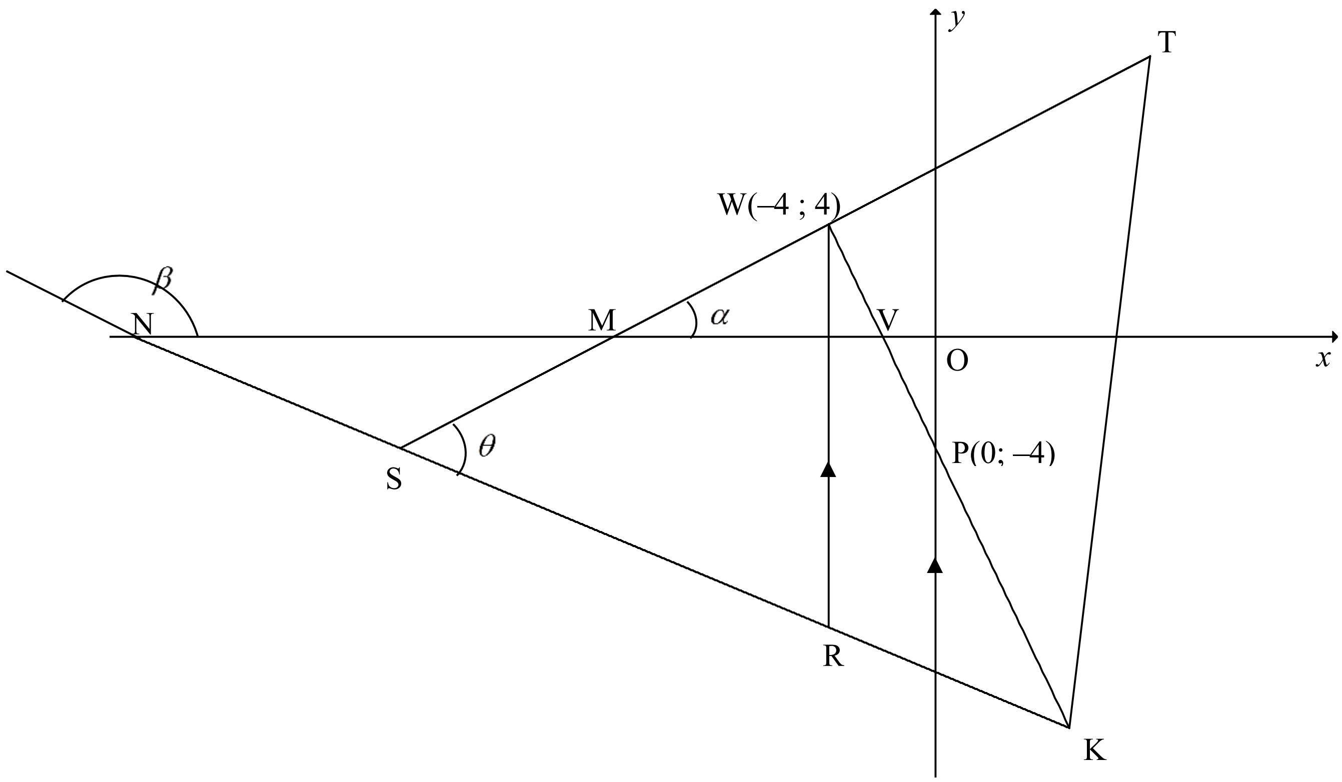
1.1	$a = 9{,}5$ $b = 0{,}909.. = 0{,}91$ $\hat{y} = 9{,}5 + 0{,}91x$	✓ $a = 9{,}5$ ✓ $b = 0{,}91$ ✓ equation (3)
1.2		✓✓ correct slope going through 2 points: (50 ; 55) or (40 ; 46) or (60 ; 64) or (0 ; 9,5) or (45 ; 50) (2)
1.3	Final exam mark ≈ 72,22% (calculator) **OR** $\hat{y} = 9{,}5 + 0{,}91(69)$ $\approx 72{,}29\%$	✓✓ answer (2) ✓ substitution ✓ answer (2)
1.4	$r = 0{,}95$	✓ answer(A) (1)
1.5	There is a **very strong positive** correlation between the Mathematics and Physical Sciences mark. *Daar is 'n baie sterk positiewe korrelasie tussen die Wiskunde en Fisiese Wetenskappunte.*	✓ **strong/ *sterk*** (1)
1.6	The teacher concludes that the higher the learners' Mathematics marks, the higher the learners' Physical Sciences marks. *Die onderwyser het waargeneem dat hoe hoër die wiskunde punte is, hoe hoër is die Fisiese Wetenskappunte.*	✓ answer (1)
		[10]

QUESTION/*VRAAG* 2

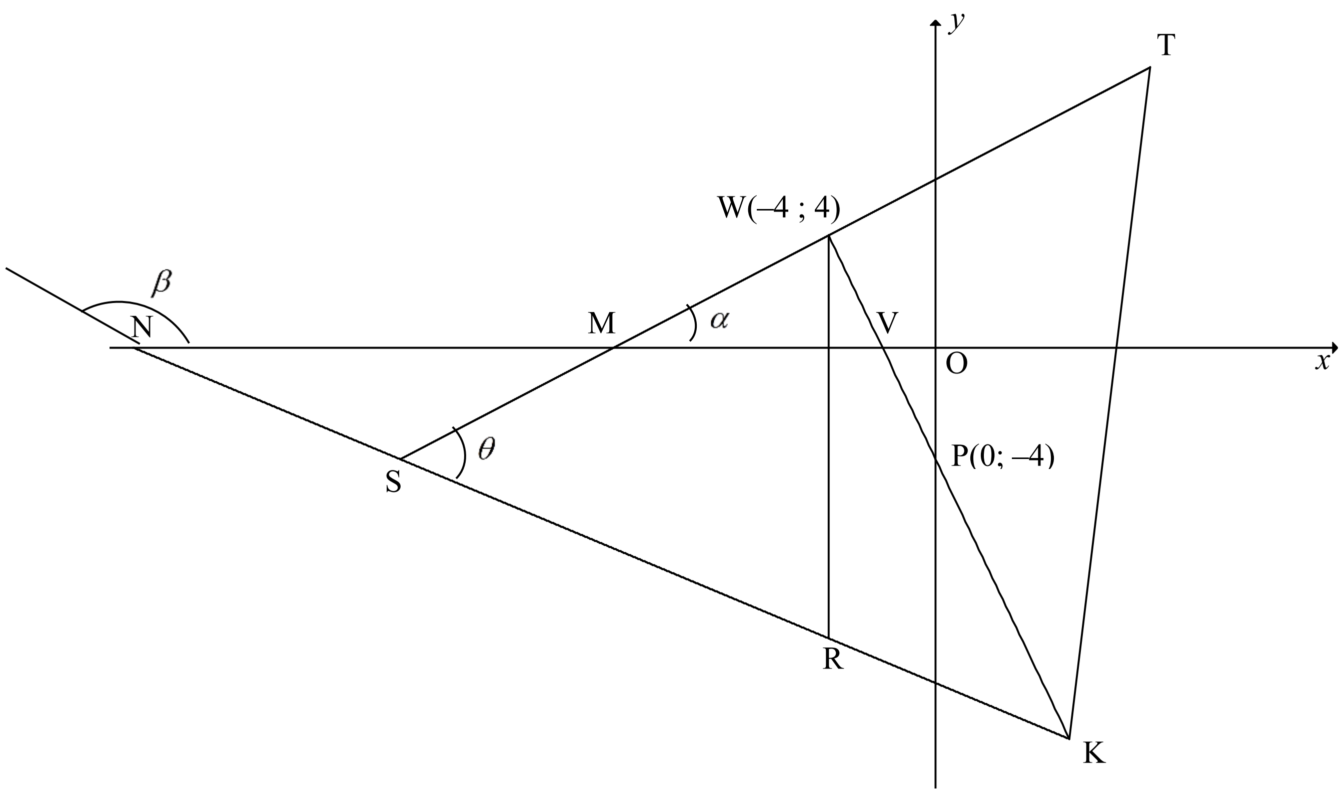
2 018	**2 175**	**2 182**	**2 215**	**2 254**	**2 263**	**2 267**	**2 271**	**2 293**	**2 323**	**2 334**	**2 346**

2.1	July / *Julie*	✓ answer (1)
2.2	$\bar{x} = \frac{26941}{12}$ $= 2\ 245{,}083.. \approx 2\ 245{,}08$ aircraft landings Answer only: Full marks	✓ 26 941 ✓ answer (2)
2.3	Standard deviation for landings at the King Shaka International airport: $\sigma = 86{,}30$	✓✓ answer (2)
2.4	$(\bar{x}-\sigma\,;\bar{x}+\sigma) = (2\ 245{,}08 - 86{,}30\ ;\ 2\ 245{,}08 + 86{,}30)$ limit $= (2\ 158{,}78\ ;\ 2\ 331{,}38)$ There were **9** months when the aircraft arrivals at the King Shaka International airport were within one standard deviation of the mean.	✓ $\bar{x}-\sigma$ ✓ $\bar{x}+\sigma$ ✓ answer (3)
2.5	The standard deviation of the number of landings at the Port Elizabeth Airport will be higher than the standard deviation of the number of arrivals at the King Shaka International Airport **OR C.**	✓ answer (1)
		[9]

QUESTION/*VRAAG* 3



3.1	$m_{WP} = \frac{4-(-4)}{-4-0} = \frac{8}{-4}$ $m_{WP} = -2$	✓ substitution of W and P ✓ m_{WP} (2)
3.2	$m_{ST} = \frac{1}{2}$ (given) $(m_{WP})(m_{ST}) = (-2)\left(\frac{1}{2}\right)$ $= -1$ $\therefore ST \perp WP$	✓ $(m_{WP})(m_{ST})$ ✓ $(m_{WP})(m_{ST}) = -1$ (2)
3.3	$5y + 2x + 60 = 0$ $\therefore y = -\frac{2}{5}x - 12$ $-\frac{2}{5}x - 12 = \frac{1}{2}x + 6$ $-4x - 120 = 5x + 60$ $9x = -180$ $x = -20$ $\therefore y = -\frac{2}{5}(-20) - 12$ $\therefore y = -4$ $\therefore$ S(–20; –4) **OR**	✓ equating ✓ x value ✓ substitution ✓ y value (4)

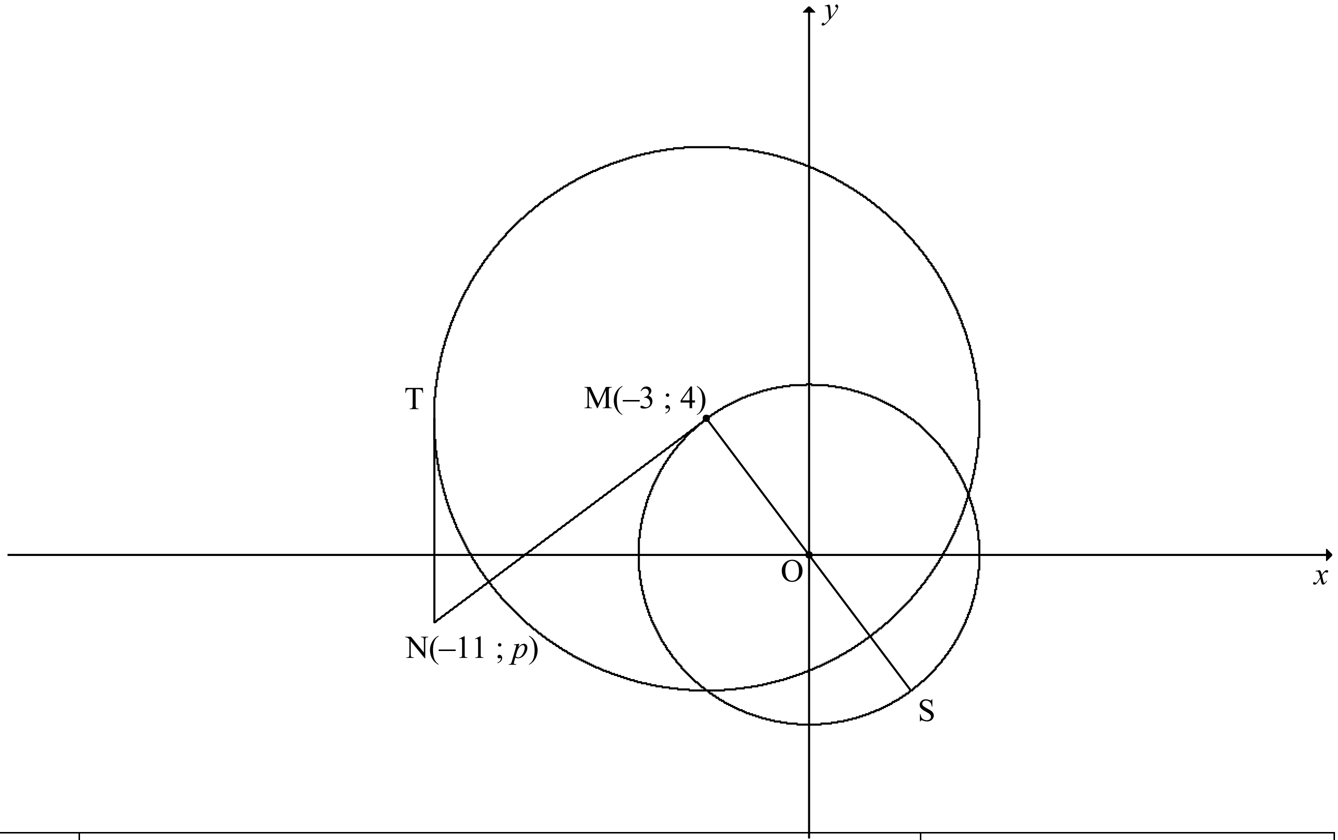


	$5y+2x+60=0$ $5\left(\frac{1}{2}x+6\right)+2x+60=0$ $\frac{5}{2}x+30+2x+60=0$ $\frac{9}{2}x=-90 \qquad \therefore x=-20$ $\therefore y=-\frac{2}{5}(-20)-12$ $\therefore y=-4$ $\therefore$ S(–20; –4) **OR** $5y+2x=-60$ ....(1) $2y-x=12$ ....(2) $(1)+2(2):\ 9y=-36$ $y=-4$ $2(-4)-x=12$ $x=-20$	✓ substitution ✓ x value ✓ substitution ✓ y value (4) ✓ adding ✓ y value ✓ substitution ✓ x value (4)

3.4	$y=-\frac{2}{5}(-4)-12$ OR $5y+2(-4)+60=0$ $y=-\frac{52}{5}$ $\therefore \text{R}\left(-4;-\frac{52}{5}\right)$ OR R(–4 ; –10,4) $\therefore \text{WR}=4-\left(-\frac{52}{5}\right)$ OR $\text{WR}=\sqrt{(-4-(-4))^2+(4-\left(-\frac{52}{5}\right))^2}$ $\therefore \text{WR}=\frac{72}{5}$ units or $\text{WR}=14\frac{2}{5}$ units **OR** $\text{WR}=\text{ST}-\text{SK}$ $=\frac{1}{2}x+6-\left(-\frac{2}{5}x-12\right)$ $=\frac{9}{10}x+18$ $=\frac{9}{10}(-4)+18$ $=14{,}4$ units	✓ substitution ✓ y value ✓ method or subst into distance formula ✓ answer (4) ✓ substitution ✓ simplification ✓ subst $x=-4$ ✓ answer (4)
3.5	$m_{\text{SK}}=-\frac{2}{5}$ $\beta=158{,}19...^\circ$ (Ref. $\angle$ =21, 801…°) $\text{M}\hat{\text{N}}\text{S}=21{,}80...^\circ$ $m_{\text{ST}}=\frac{1}{2}$ $\text{N}\hat{\text{M}}\text{S}=26{,}56...^\circ$ $\theta=21{,}80...^\circ+26{,}56...^\circ$ [ext $\angle$ of Δ] $\theta=48{,}366...^\circ=48{,}37^\circ$	✓ m_{SK} ✓ size of β ✓ size of $\text{N}\hat{\text{M}}\text{S}$ ✓ method ✓ answer (5)
3.6	In ΔSRW: $\perp h=-4-(-20)$ $\perp h=16$units $\text{Area }\Delta\text{SRW}=\frac{1}{2}(\perp h)(\text{WR})$ $=\frac{1}{2}(16)\left(\frac{72}{5}\right)$ $=115{,}2$ square units Area SWRL = 2Area ΔSRW =2(115,2) = 230,4square units **OR**	✓ $\perp h$ ✓ substitution ✓ area Δ ✓ answer (4)

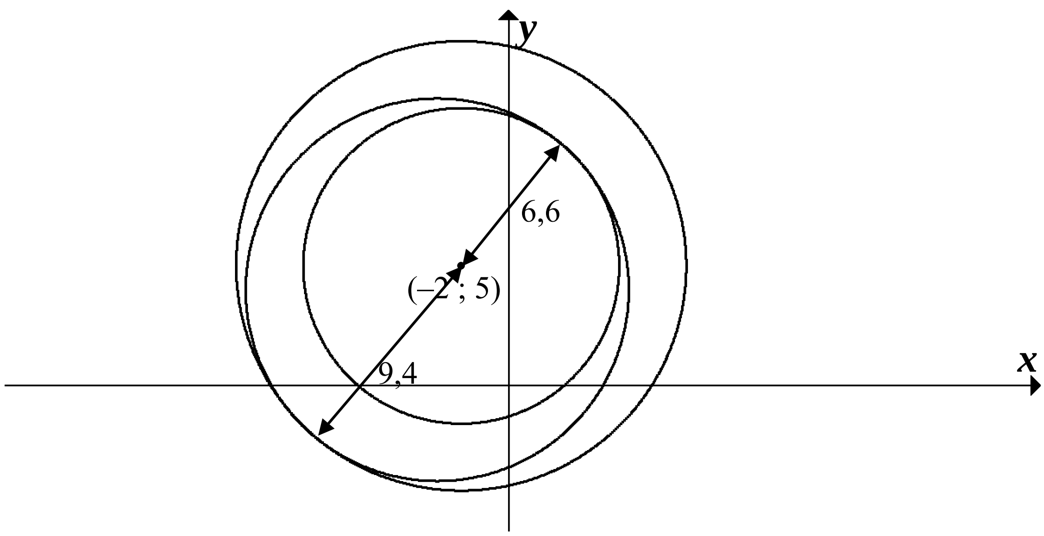
	In ΔSRW: $\perp h = -4 - (-20)$ $\perp h = 16$units Area SWRL $= 16 \times \frac{72}{5}$ $= 230{,}40$ square units	✓ $\perp h$ ✓ ✓ substitution ✓ answer (4)
	OR $\text{SW} = \sqrt{(-20+4)^2 + (-4-4)^2} = 8\sqrt{5} = 17{,}89$ $\text{SR} = \sqrt{(-20+4)^2 + \left(-4+10\frac{2}{5}\right)^2} = \frac{16\sqrt{29}}{5} = 17{,}23$ Area SWRL = $2 \times$ Area ΔSRW $= 2\left(\frac{1}{2}\text{SW} \times \text{SR} \sin\theta\right)$ $= 2\left(\frac{1}{2} 8\sqrt{5} \times \frac{16\sqrt{29}}{5} \sin 48{,}37°\right)$ $= 230{,}41$square units	✓$\text{SW} = 8\sqrt{5}$ ✓$\text{SR} = \frac{16\sqrt{29}}{5}$ ✓substitution ✓answer (4)
		[21]

QUESTION/*VRAAG* 4

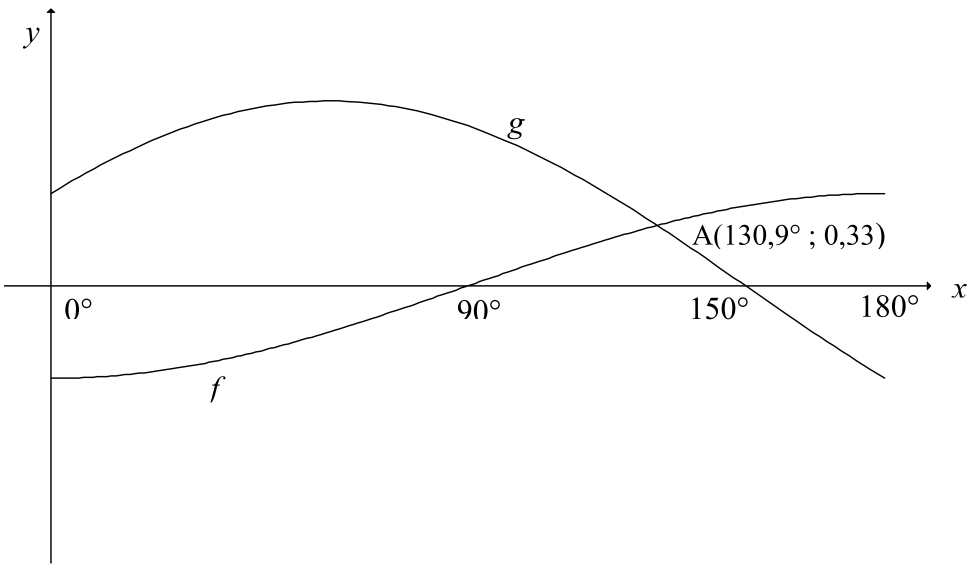


4.1	$x^2 + y^2 = r^2$ $\therefore r^2 = (-3)^2 + (4)^2 = 25$ $x^2 + y^2 = 25$	✓ substitution ✓ answer (2)
4.2	TM ⊥ TN [tangent ⊥ radius] T(–11 ; 4) $r = -3 - (-11) = 8$ $(x+3)^2 + (y-4)^2 = 64$	✓ $x_T = -11$ ✓ LHS ✓ RHS (3)
4.3	O (0 ; 0) and M(–3; 4) $m_{OM} = \frac{4-0}{-3-0} = -\frac{4}{3}$ OR $\frac{0-4}{0-(-3)} = -\frac{4}{3}$ $m_{NM} = \frac{3}{4}$ $y - 4 = \frac{3}{4}(x-(-3))$ **OR** $y = \frac{3}{4}x + c$ $y - 4 = \frac{3}{4}x + \frac{9}{4}$ $4 = \frac{3}{4}(-3) + c$ $\therefore y = \frac{3}{4}x + \frac{25}{4}$ $c = \frac{25}{4}$ $y = \frac{3}{4}x + \frac{25}{4}$	✓ $m_{OM} = -\frac{4}{3}$ ✓ $m_{NM} = \frac{3}{4}$ ✓ substitution of m and M ✓ equation (4)

4.4	N(–11 ; p) $y=\frac{3}{4}x+\frac{25}{4}$ $p=\frac{3}{4}(-11)+\frac{25}{4}$ OR $\frac{4-p}{-3-(-11)}=\frac{3}{4}$ $p=-2$ $\therefore \text{N}(-11;-2)$ $\frac{-3+x_S}{2}=0$ and $\frac{4+y_S}{2}=0$ $\therefore \text{S}(3;-4)$ $\text{SN}=\sqrt{(-11-3)^2+(-2-(-4))^2}$ $=10\sqrt{2}$ units or 14,14 units	✓ subst $x=-11$ into eq or gradient ✓ $p=-2$ ✓ x_S ✓ y_S ✓ answer (CA) (5)
4.5	$\text{B}(-2;\ 5)$ $\text{BM}=\sqrt{2}$ units Radius of circle centred at M = 8 units $k=8-\sqrt{2}$ or $k=8+\sqrt{2}$ = 6,59 units = 9,41 units = 6,6 units = 9,4 units	✓ $\sqrt{2}$ ✓✓ $k=6,6$ ✓✓ $k=9,4$ (5)
		[19]

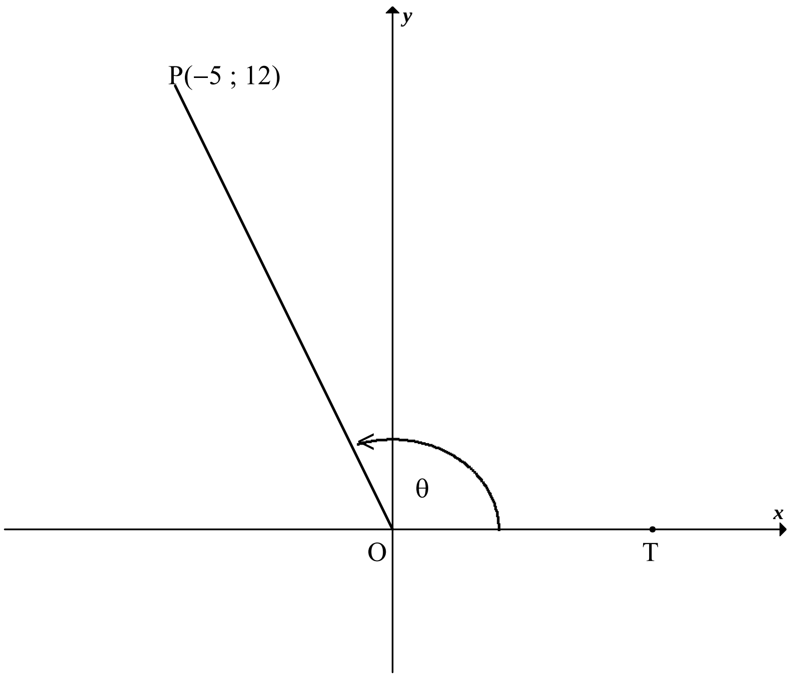


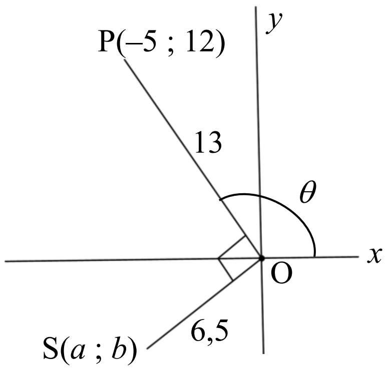
QUESTION/*VRAAG* 5



5.1	Period of $g = 360°$	✓ answer (1)
5.2	Amplitude of $f = \frac{1}{2}$	✓ answer (A) (1)
5.3	$f(180°) - g(180°)$ $= \frac{1}{2} - \left(-\frac{1}{2}\right)$ $= 1$	✓ 1 (1)
5.4.1	$x = 140,9°$	✓ $x = 140,9°$ (1)
5.4.2	$\sqrt{3}\sin x + \cos x \geq 1$ $\frac{\sqrt{3}}{2}\sin x + \frac{1}{2}\cos x \geq \frac{1}{2}$ $\sin x \cos 30° + \cos x \sin 30° \geq \frac{1}{2}$ $\sin(x + 30°) \geq \frac{1}{2}$ $\sin(x + 30°) = \frac{1}{2}$ at $x = 0°$ or $x = 120°$ $\therefore x \in [0°;120°]$ **OR** $0° \leq x \leq 120°$	✓ dividing by 2 ✓ $\cos 30°$; $\sin 30°$ ✓ $\sin(x + 30°) \geq \frac{1}{2}$ ✓ interval (4)
		[8]

QUESTION/*VRAAG* 6

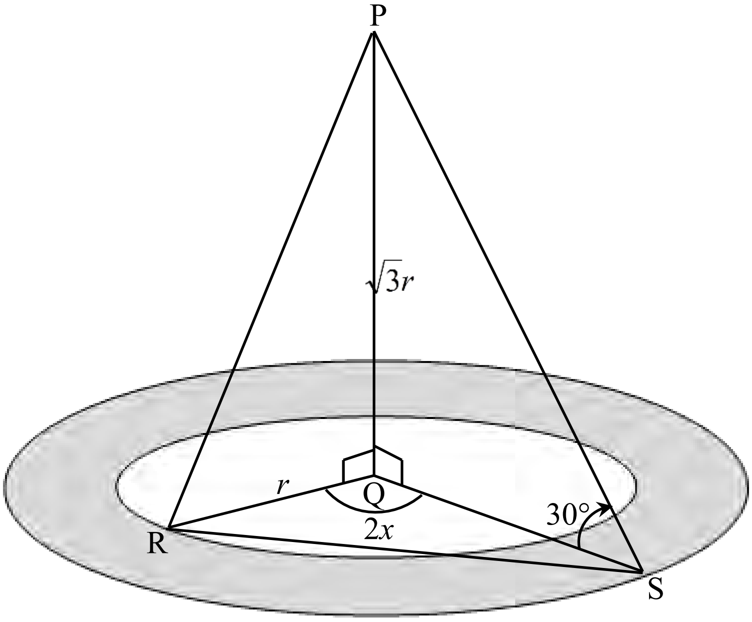


6.1.1	$\tan\theta = -\frac{12}{5}$ or $-2\frac{2}{5}$	✓answer (1)
6.1.2	$(OP)^2 = (-5)^2 + (12)^2$ $OP = 13$ $\cos\theta = -\frac{5}{13}$	✓ Pythagoras ✓ OP ✓ answer (3)
6.1.3	$\sin(\theta + 90°) = \frac{b}{6,5}$ $\cos\theta = \frac{b}{6,5}$ $\frac{-5}{13} = \frac{b}{6,5}$ $b = -\frac{5}{2}$  **OR** $\cos(90° + \theta) = \frac{a}{6,5}$ $-\sin\theta = \frac{a}{6,5}$ $-\frac{12}{13} = \frac{a}{6,5}$ $\therefore a = -6$ $b = \sqrt{(6,5)^2 - (-6)^2} = -\frac{5}{2}$	✓ $\sin(\theta + 90°) = \frac{b}{6,5}$ ✓ $\cos\theta$ ✓ $\frac{-5}{13} = \frac{b}{6,5}$ ✓ value of b (4) ✓ $\cos(\theta + 90°) = \frac{a}{6,5}$ ✓ $-\sin\theta$ ✓ value of a ✓ value of b (4)

6.2	$\dfrac{\sin 2x.\cos(-x)+\cos 2x.\sin(360°-x)}{\sin(180°+x)}$ $=\dfrac{\sin 2x\ \cos x\ +\ \cos 2x\left(-\sin x\right)}{-\sin x}$ $=\dfrac{\sin(2x-x)}{-\sin x}$ $=\dfrac{\sin x}{-\sin x}$ $=-1$	✓ $\cos(-x)=\cos x$ ✓ $\sin(360°-x)=-\sin x$ ✓ $\sin(180°+x)=-\sin x$ ✓ numerator = $\sin x$ ✓ answer (5)
6.3	$6\sin^2 x+7\cos x-3=0$ $6(1-\cos^2 x)+7\cos x-3=0$ $6-6\cos^2 x+7\cos x-3=0$ $6\cos^2 x-7\cos x-3=0$ $(3\cos x+1)(2\cos x-3)=0$ $\cos x=-\dfrac{1}{3}$ or $\cos x=\dfrac{3}{2}$ (N/A) $\therefore\ x=109{,}47°+k.360°;\ k\in Z$ or $x=250{,}53°+k.360°;\ k\in Z$	✓ identity ✓ standard form ✓ factors ✓ both solutions of $\cos x$ ✓ $x=109{,}47°\ \&\ 250{,}53°$ ✓ $+k.360°;\ k\in Z$ (6)
6.4	$x+\dfrac{1}{x}=3\cos\text{A}$ $\left(3\cos\text{A}\right)^2=\left(x+\dfrac{1}{x}\right)^2$ $9\cos^2\text{A}=x^2+\dfrac{1}{x^2}+2$ $9\cos^2\text{A}=2+2$ $\cos^2\text{A}=\dfrac{4}{9}$ $\cos 2\text{A}=2\cos^2\text{A}-1$ $=2\left(\dfrac{4}{9}\right)-1$ $=-\dfrac{1}{9}$ **OR**	✓squaring both sides ✓ $9\cos^2\text{A}=x^2+\dfrac{1}{x^2}+2$ ✓ $\cos^2\text{A}=\dfrac{4}{9}$ ✓ $\cos 2\text{A}=2\cos^2\text{A}-1$ ✓ answer (5)

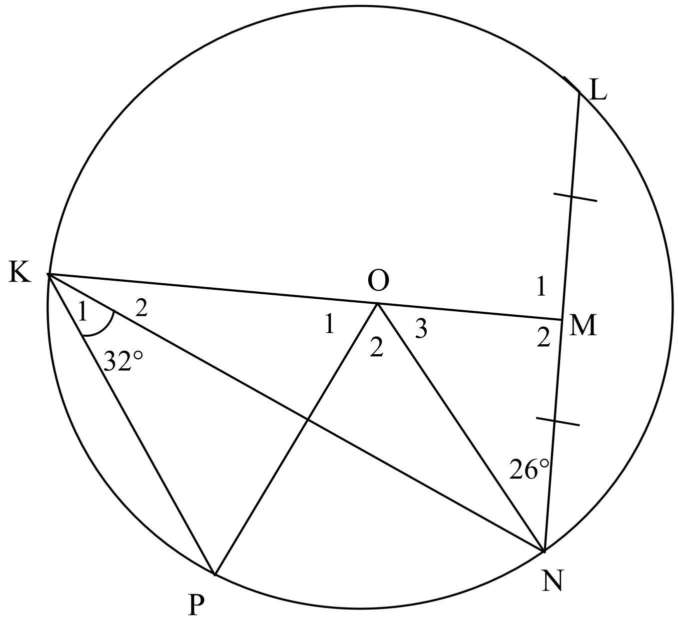
	$x^2 - 2 + \frac{1}{x^2} = 0$ $\left(x - \frac{1}{x}\right)^2 = 0$ $x^2 = 1$ $x = \pm 1$ $3\cos A = 2$ or $3\cos A = -2$ $\cos A = \frac{2}{3}$ or $\cos A = -\frac{2}{3}$ $\cos 2A = 2\cos^2 A - 1$ $= 2\left(\pm\frac{2}{3}\right)^2 - 1$ $= -\frac{1}{9}$	✓ $x = \pm 1$ ✓ $\cos A = \frac{2}{3}$ ✓ $\cos A = -\frac{2}{3}$ ✓double angle identity ✓ answer (5)
		[24]

QUESTION/*VRAAG* 7



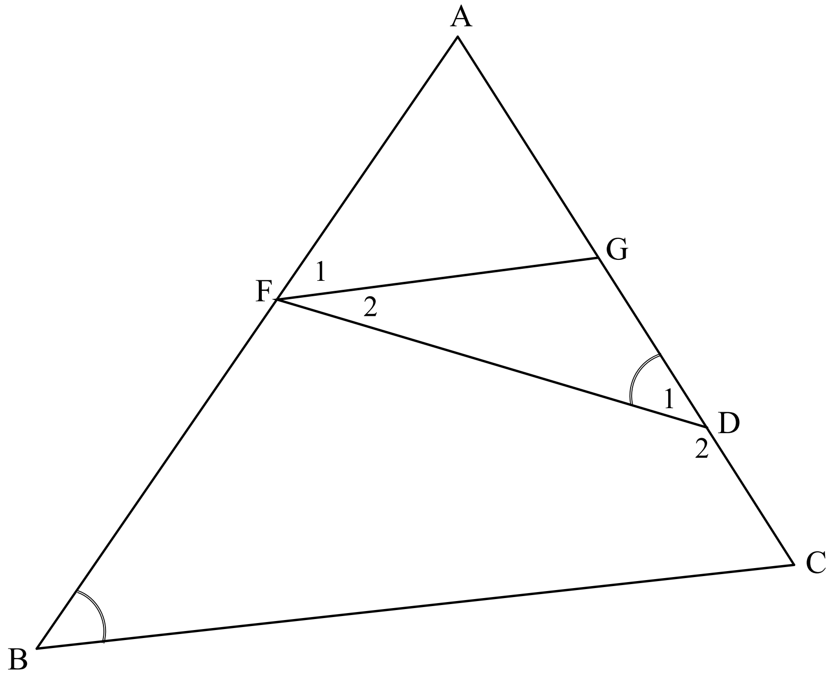
7.1	$\tan 30° = \frac{\sqrt{3}\,r}{QS}$ **OR** $\tan 60° = \frac{QS}{\sqrt{3}r}$ $QS = \frac{\sqrt{3}r}{\tan 30°}$ $\sqrt{3} = \frac{QS}{\sqrt{3}r}$ $= \frac{\sqrt{3}r}{\frac{1}{\sqrt{3}}}$ *or* $\frac{\sqrt{3}r}{\frac{\sqrt{3}}{3}}$ $QS = 3r$ $= 3r$	✓✓ trig ratio ✓ QS subject (3)
7.2	Area of flower garden $= \pi(3r)^2 - \pi r^2$ $= 9\pi r^2 - \pi r^2$ $= 8\pi r^2$	✓ substitution into difference of areas ✓ answer (2)
7.3	$RS^2 = r^2 + (3r)^2 - 2(r)(3r)\cos 2x$ $= r^2 + 9r^2 - 6r^2 \cos 2x$ $= 10r^2 - 6r^2 \cos 2x$ $= r^2(10 - 6\cos 2x)$ $RS = r\sqrt{10 - 6\cos 2x}$	✓substitution into cosine rule correctly ✓ $10r^2 - 6r^2 \cos 2x$ ✓ $r^2(10 - 6\cos 2x)$ (3)
7.4	$RS = 10\sqrt{10 - 6\cos 2(56)}$ $= 34{,}9966...$ ≈ 35 m	✓substitution ✓ answer (2)
		[10]

QUESTION/*VRAAG* 8



8.1.1(a)	$\hat{O}_2 = 64°$ [$\angle$ at centre = 2 ×$\angle$ at circumference/ *Middelpts $\angle$ = 2 ×$\angle$ omtreks$\angle$*]	✓ S ✓ R (2)
8.1.1(b)	$\hat{M}_2 = 90°$ [Line from centre to midpt of chord/*lyn v midpt na midpt v koord*] $K\hat{O}N = 90° + 26° = 116°$ [ext $\angle$ of Δ/*buite $\angle$ van Δ*] $\hat{O}_1 = 116° - 64° = 52°$ **OR** $\hat{M}_2 = 90°$ [Line from centre to midpt of chord/*lyn v midpt na midpt v koord*] $\hat{O}_3 = 64°$ [sum of $\angle$s in Δ] $\hat{O}_1 = 52°$ [$\angle$s on straight line/*op 'n reguitlyn*]	✓ S ✓ R ✓ S ✓ answer (4) ✓ S ✓ R ✓ S ✓ answer (4)
8.1.2	$P\hat{K}O + \hat{P} = 128°$ [sum of $\angle$s in Δ/*som $\angle$e van Δ*] $P\hat{K}O = \hat{P}$ [$\angle$s opp = sides/*$\angle$e teenoor = sye*] $= 64°$ $\therefore \hat{K}_2 = 32°$ *or* $\hat{K}_2 = \hat{K}_1$ $\therefore$ KN bisects/*halveer* $O\hat{K}P$ **OR** $\hat{K}_2 = K\hat{N}O$ [$\angle$s opp = sides/*$\angle$e teenoor = sye*] $\hat{K}_2 + K\hat{N}O = 64°$ [sum of $\angle$s in Δ/*som $\angle$e van Δ*] $\therefore \hat{K}_2 = 32°$ *or* $\hat{K}_2 = \hat{K}_1$ $\therefore$ KN bisects/*halveer* $O\hat{K}P$	✓ S ✓ S ✓ S (3) ✓ S ✓ S ✓ S (3)

8.2

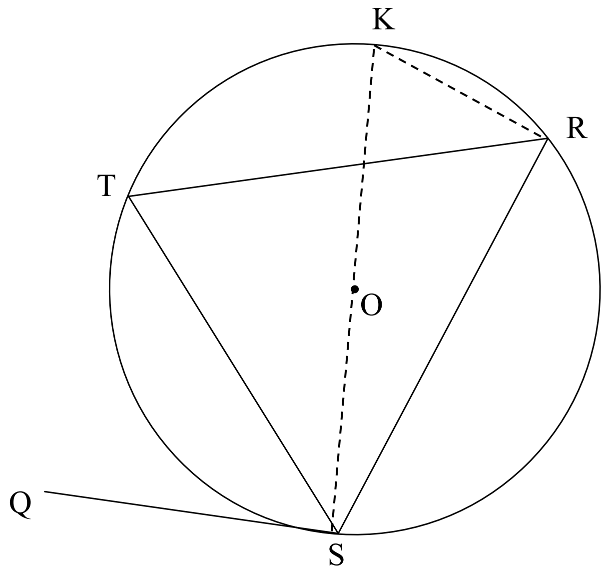


8.2.1	$\hat{F}_1 = \hat{D}_1$ [tan chord theorem/*raaklyn koordst*] $\hat{D}_1 = \hat{B}$ [Given/*Gegee*] $\therefore \hat{F}_1 = \hat{B}$ $\therefore FG \parallel BC$ [corresp ∠s =/*Ooreenkomstige∠e* =]	✓ S ✓ R ✓ $\hat{F}_1 = \hat{B}$ ✓ R (4)
8.2.2	$\frac{GC}{AC} = \frac{FB}{AB}$ [line ∥ one side of Δ/*lyn* ∥ *een sy v Δ*] $\frac{x+9}{2x-6} = \frac{5}{7}$ $7x + 63 = 10x - 30$ $3x = 93$ $x = 31$ **OR** $AG = 2x - 6 - (x+9) = x - 15$ $\frac{AG}{GC} = \frac{AF}{FB}$ [line ∥ one side of Δ/*lyn* ∥ *een sy v Δ*] $\frac{x-15}{x+9} = \frac{2}{5}$ $5x - 75 = 2x + 18$ $3x = 93$ $x = 31$ **OR**	✓ S ✓ R ✓ substitution ✓ answer (4) ✓ S ✓ R ✓ substitution ✓ answer (4)

	$\frac{\text{AF}}{\text{AB}}=\frac{\text{AG}}{\text{AC}}$ [line ∥ one side of Δ /lyn ∥ een sy v Δ] $\frac{2}{7}=\frac{x-15}{2x-6}$ $7x-105=4x-12$ $3x=93$ $x=31$	✓ S ✓ R ✓ substitution ✓ answer (4)
		[17]

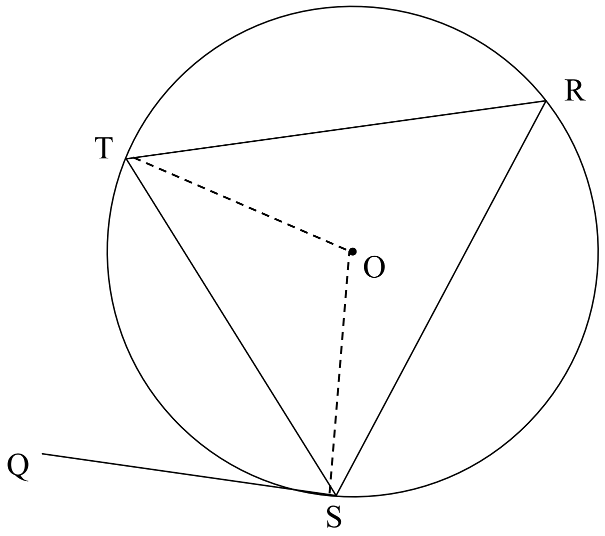
QUESTION/*VRAAG* 9

9.1



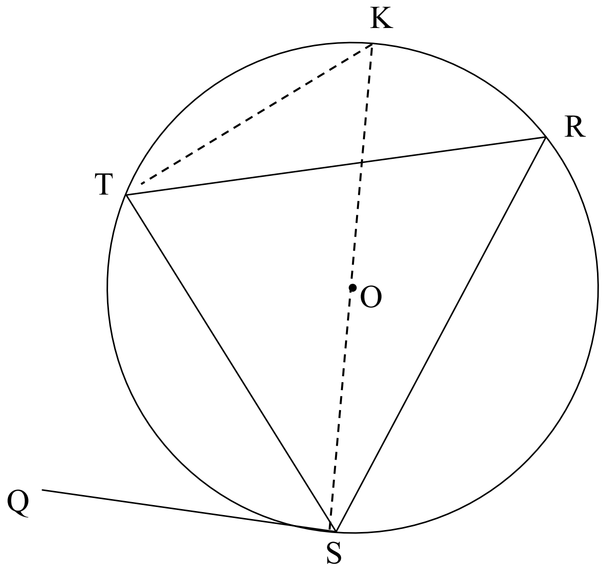
9.1	Construction: Draw diameter KS and draw KR *Konstruksie: Trek middellyn KS en verbind KR* $\text{Q}\hat{\text{S}}\text{T} = 90° - \text{T}\hat{\text{S}}\text{K}$ [radius ⊥ tangent/*raaklyn*] $\text{S}\hat{\text{R}}\text{K} = 90°$ [∠ in semi circle/*halfsirkel*] $\therefore \text{S}\hat{\text{R}}\text{T} = 90° - \text{K}\hat{\text{R}}\text{T}$ $\text{T}\hat{\text{S}}\text{K} = \text{T}\hat{\text{R}}\text{K}$ [∠s same segment/*∠e dieselfde segment*] $\therefore \text{Q}\hat{\text{S}}\text{T} = \hat{\text{R}}$	✓ construction ✓ S/R ✓ S/R ✓ S ✓ S/R (5)

OR



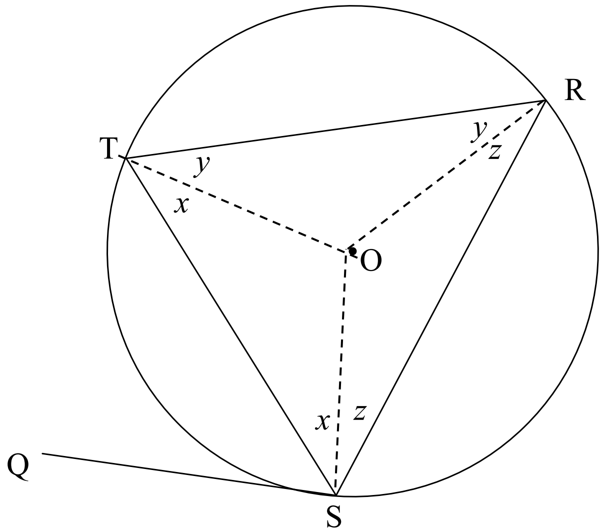
9.1	Construction: Draw radii OS and OT *Konstruksie: Trek radii OS en OT* $\hat{QST} = 90° - \hat{OST}$ [radius ⊥ tangent/*raaklyn*] $\hat{OST} = \hat{STO}$ [∠s opp = sides/*∠e teenoor = sye*] $\therefore \hat{SOT} = 180° - 2\hat{OST}$ [∠s of Δ/*∠e van Δ*] $\hat{R} = 90° - \hat{OST}$ [∠ at centre = 2×∠ circumf/ *midpts∠ = 2× omtreks∠*] $\therefore \hat{QST} = \hat{R}$	✓ construction ✓ S/R ✓ S/R ✓ S ✓ S/R (5)

OR



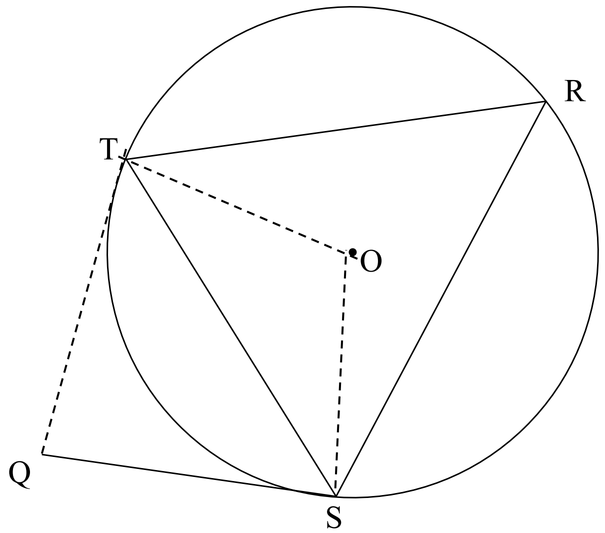
9.1	Construction: Draw diameter KS and join K to T. *Konstruksie: Trek middellyn KS en verbind K tot T* K $\hat{QST} = 90° - \hat{TSK}$ [radius ⊥ tangent/*raaklyn*] $\hat{STK} = 90°$ [∠ in semi circle/*halfsirkel*] $\therefore \hat{K} = 90° - \hat{TSK}$ $\therefore \hat{QST} = \hat{K}$ but $\hat{R} = \hat{K}$ [∠s same segment/*∠e dieselfde segment*] $\therefore \hat{QST} = \hat{R}$	✓ construction ✓ S/R ✓ S/R ✓ S ✓ S/R (5)

OR



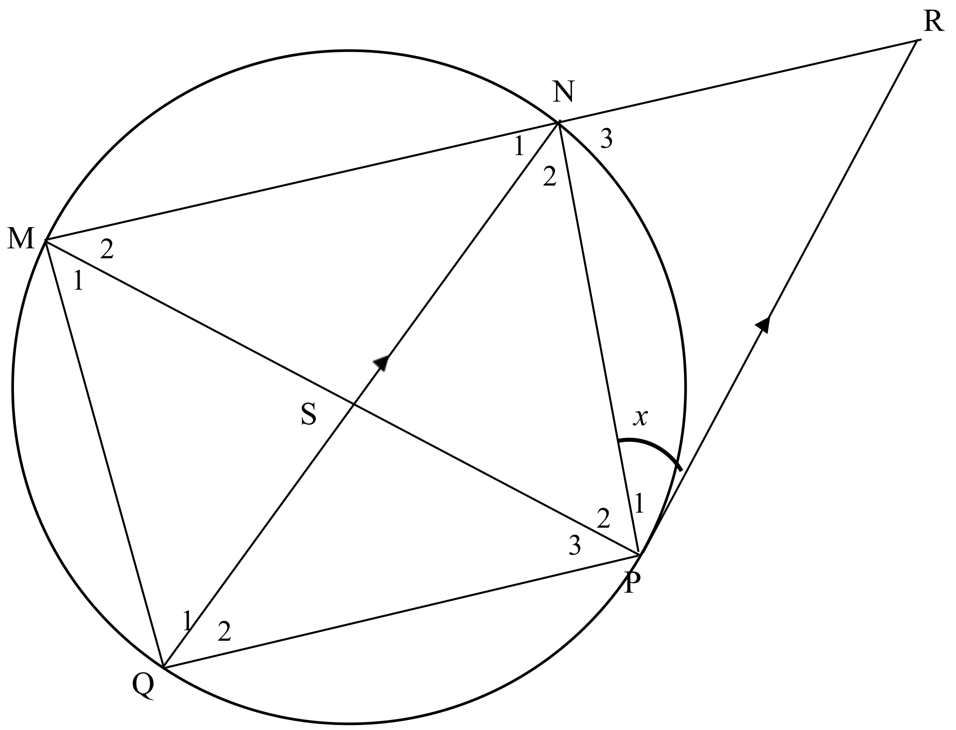
9.1	Construction: Draw radii OT, OR and OS *Konstruksie: Trek radiuse OT, OR en OS* $\hat{OST} = \hat{OTS}$ [∠s opp = radii/*∠e teenoor = radiuse*] Also: $\hat{OTR} = \hat{ORT}$ and $\hat{ORS} = \hat{OSR}$ $2x + 2y + 2z = 180°$ [∠s of Δ] $x + y + z = 90°$ $y + z = 90° - x$ $\hat{OSQ} = 90°$ [radius ⊥ tangent/*raaklyn*] $\therefore\ \hat{TSQ} = 90° - x$ $\therefore\ \hat{TSQ} = y + z = \hat{R}$	✓ construction ✓ S/R ✓ S ✓ S/R ✓ S (5)

OR



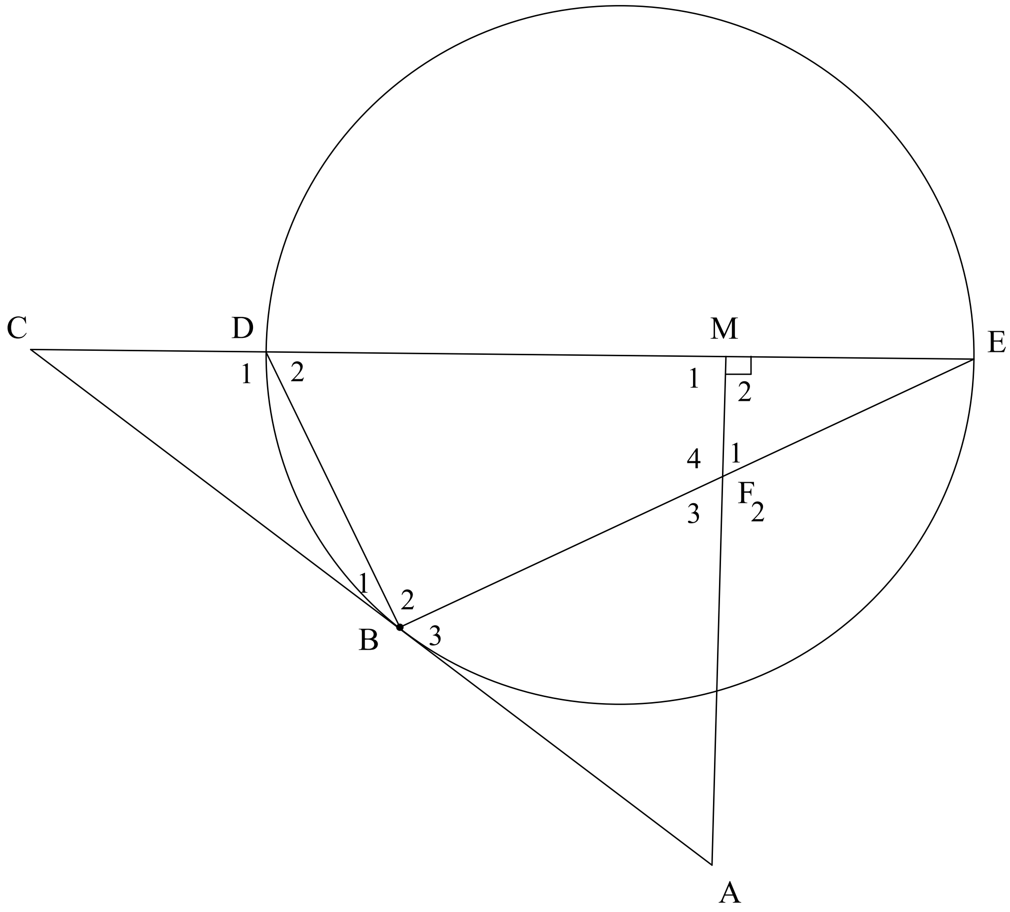
9.1	Construction: Draw radii OT and OS, tangent QT *Konstruksie: Trek radiuse OT en OS, raaklyn QT* $\hat{OSQ} = 90°$ [radius ⊥ tangent/*raaklyn*] $\therefore\ \hat{TSQ} = 90° - \hat{TSO}$ $\therefore\ \hat{TSO} = \hat{STO}$ [∠s opp = radii/*∠e teenoor = radiuse*] $\hat{TOS} = 180° - 2\hat{TSO}$ [∠s of Δ] $\hat{R} = 90° - \hat{TSO}$ [∠ at centre = 2×∠ circumf/ *midpts∠ = 2× omtreks∠*] $\therefore\ \hat{TSQ} = \hat{R}$	✓ construction ✓ S/R ✓ S ✓ S ✓ S/R (5)

9.2



9.2.1(a)	$\hat{N}_2 = x$	[alt ∠s; PR ∥ NQ/*verw. ∠e;* PR ∥ NQ]	✓ S ✓ R (2)
9.2.1(b)	$\hat{Q}_2 = x$ OR $M_2 = x$ $\hat{Q}_2 = x$	[tan chord theorem/*raaklyn koordstelling*] [tan chord theorem/*raaklyn koordstelling*] [∠s in same segment/*∠e in dieselfde segm*]	✓ S ✓ R (2) ✓ S/R ✓ S/R (2)
9.2.2	$\frac{MN}{NR} = \frac{MS}{SP}$ $\hat{N}_1 = \hat{N}_2 = x$ $\hat{P}_3 = x$ $\hat{P}_3 = \hat{Q}_2$ SQ = PS $\frac{MN}{NR} = \frac{MS}{SQ}$	[QN ∥ PR; Prop Th] [given] [∠s in same segment/*∠e in dieselfde segm*] [= x] [sides opp = ∠/*sye teenoor = ∠e*]	✓ S ✓ R ✓ S ✓ S ✓ R ✓ R (6)
			[15]

QUESTION/*VRAAG* 10



10.1.1	$\hat{DBE} = 90°$ [$\angle$ in semi-circle/$\angle$ *in halfsirkel*] $\therefore \hat{DMA} = 90°$ [$AM \perp DE$] $\therefore$ FBDM is a cyclic quadrilateral/*koordevh* [converse opp $\angle$s cyclic quad/*omgek teenoorst* $\angle$*e kvh*] **OR** $\hat{DBE} = 90°$ [$\angle$ in semi-circle/$\angle$ *in halfsirkel*] $\hat{M}_2 = \hat{DBE} = 90°$ $\therefore$ FBDM is a cyclic quadrilateral/*koordevh* [converse ext$\angle$ of cyclic quad/*omgek buite*$\angle$ *van kvh*]	✓S ✓R ✓R (3) ✓S ✓R ✓R (3)

10.1.2	$\hat{B}_3 = \hat{D}_2$ [tangent chord th/*raaklyn koordst*] $\hat{F}_1 = \hat{D}_2$ [ext ∠ cyc quad/*buite ∠ koordevh*] $\therefore \hat{B}_3 = \hat{F}_1$ **OR** $\hat{B}_1 = \hat{E} = x$ [tangent chord th/*raaklyn koordst*] $\hat{F}_1 = 90° - x$ [∠ sum in Δ/∠ *van* Δ] $\hat{D}_2 = 90° - x$ [∠ sum in Δ/∠ *van* Δ] $\therefore \hat{F}_1 = D_2$ $\hat{B}_3 = \hat{D}_2$ [tangent chord th/*raaklyn koordst*] $\therefore \hat{B}_3 = \hat{F}_1$ **OR** $\hat{B}_1 = \hat{E} = x$ [tangent chord th/*raaklyn koordst*] $\hat{B}_3 = 90° - x$ [straight line/*reguitlyn*] $\hat{F}_1 = 90° - x$ [sum of ∠s Δ/ *som van ∠e van* Δ] $\therefore \hat{B}_3 = \hat{F}_1$	✓ S ✓ R ✓ S ✓ R (4) ✓ S ✓ R ✓ $\hat{F}_1 = 90° - x$ $= \hat{D}_2$ ✓ R (4) ✓ S ✓ R ✓ S ✓ S (4)
10.1.3	In ΔCDB and ΔCBE $\hat{C} = \hat{C}$ [common ∠/*gemeenskaplike* ∠] $C\hat{B}D = C\hat{E}B$ [tangent chord th/*raaklyn koordst*] $C\hat{D}B = C\hat{B}E$ [∠ sum in Δ/∠ *van* Δ] ΔCDB ||| ΔCBE **OR** In ΔCDB and ΔCBE $C\hat{B}D = C\hat{E}B$ [tangent chord th/*raaklyn koordst*] $\hat{C} = \hat{C}$ [common ∠/*gemeenskaplike* ∠] ΔCDB ||| ΔCBE [∠, ∠, ∠]	✓ S ✓ S/R ✓ R (3) ✓ S/R ✓ S ✓ R (3)
10.2.1	$\frac{BC}{EC} = \frac{DC}{BC}$ [||| Δs] $BC^2 = EC \times DC$ $= 8 \times 2$ $= 16$ $BC = 4$	✓ ratio ✓ substitution ✓answer (3)

10.2.2	$\frac{BC}{EC}=\frac{DB}{BE}$ [$\parallel\!\mid$ Δs] $\frac{DB}{BE}=\frac{4}{8}=\frac{1}{2}$ $BE = 2DB$ $DB^2+BE^2=DE^2$ [Pyth theorem] $DB^2+(2DB)^2=36$ $5DB^2=36$ $DB^2=\frac{36}{5}$ $DB=\frac{6}{\sqrt{5}}=2{,}68\text{ units}$	✓ BE = 2DB ✓ substitution into Pyth theorem ✓ $DB^2=\frac{36}{5}$ ✓answer (4)
		[17]

TOTAL/*TOTAAL*: 150

basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

NATIONAL
SENIOR CERTIFICATE/
NASIONALE
SENIOR SERTIFIKAAT

GRADE/*GRAAD* 12

MATHEMATICS P2/*WISKUNDE V2*

NOVEMBER 2019

MARKING GUIDELINES/*NASIENRIGLYNE*

MARKS/*PUNTE*: 150

These marking guidelines consist of 26pages.
Hierdie nasienriglyne bestaan uit 26 bladsye.

NOTE:

- If a candidate answers a question TWICE, only mark the FIRST attempt.
- If a candidate has crossed out an attempt of a question and not redone the question, mark the crossed out version.
- Consistent accuracy applies in ALL aspects of the marking memorandum. Stop marking at the second calculation error.
- Assuming answers/values in order to solve a problem is NOT acceptable.

NOTA:

- *As 'n kandidaat 'n vraag TWEE KEER beantwoord, sien slegs die EERSTE poging na.*
- *As 'n kandidaat 'n antwoord van 'n vraag doodtrek en nie oordoen nie, sien die doodgetrekte poging na.*
- *Volgehoue akkuraatheid word in ALLE aspekte van die nasienriglyne toegepas. Hou op nasien by die tweede berekeningsfout.*
- *Om antwoorde/waardes te aanvaar om 'n probleem op te los, word NIE toegelaat NIE.*

GEOMETRY • *MEETKUNDE*	
S	**A mark for a correct statement** **(A statement mark is independent of a reason)**
	'n Punt vir 'n korrekte bewering ***('n Punt vir 'n bewering is onafhanklik van die rede)***
R	**A mark for the correct reason** **(A reason mark may only be awarded if the statement is correct)**
	'n Punt vir 'n korrekte rede ***('n Punt word slegs vir die rede toegeken as die bewering korrek is)***
S/R	**Award a mark if statement AND reason are both correct**
	Ken 'n punt toe as die bewering EN rede beide korrek is

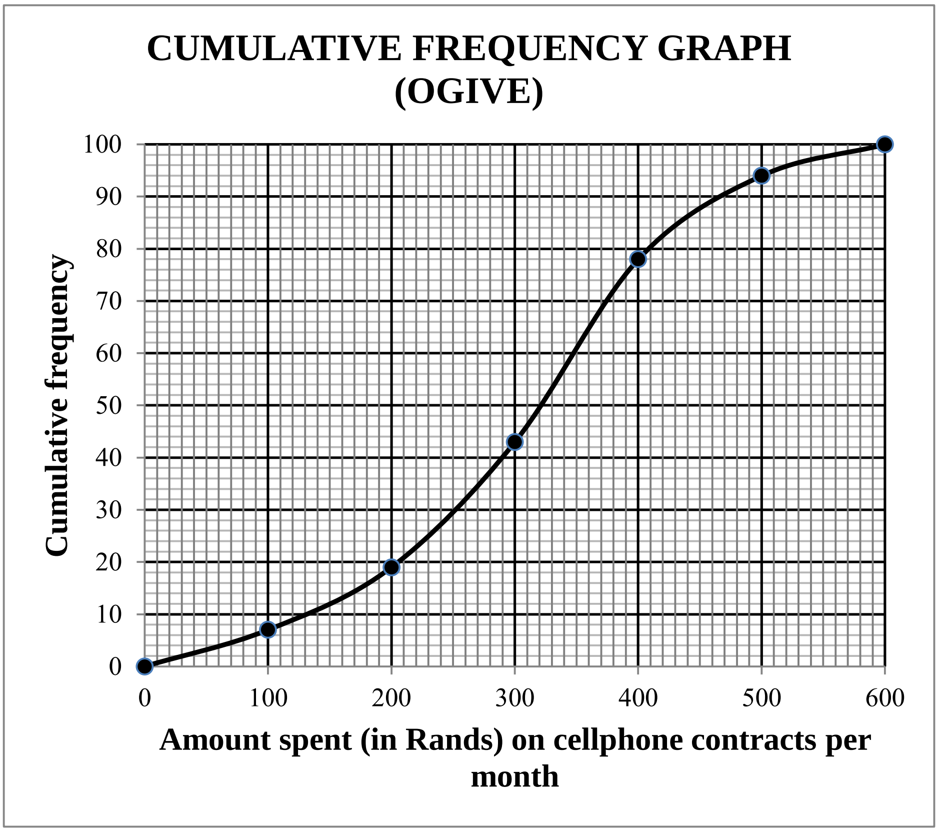
QUESTION/*VRAAG* 1

Monthly income (in rands) Maandelikse inkomste (in rand)	9 000	13 500	15 000	16 500	17 000	20 000
Monthly repayment (in rands) Maandelikse paaiement (in rand)	2 000	3 000	3 500	5 200	5 500	6 000

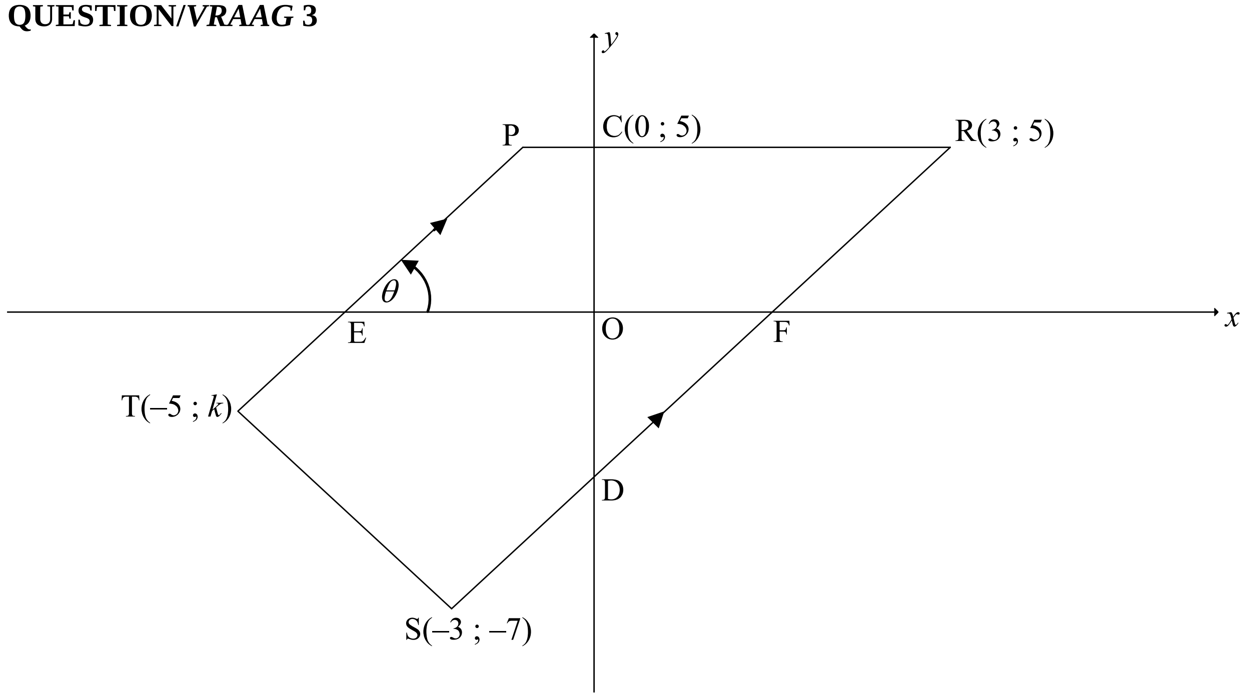
1.1	$a = -1946,875... = -1946,88$ $b = 0,41$ $\hat{y} = -1946,88 + 0,41x$ Answer only: Full marks	✓ $a = -1946,88$ ✓ $b = 0,41$ ✓ equation (3)
1.2	Monthly repayment $\approx$ R3 727,16 (calculator) *Maandelikse paaiement* $\approx$ R3 727,16 **OR** $\hat{y} = -1946,88 + 0,41(14000)$ $\approx$ R3 793,12	✓✓ answer (2) ✓ substitution ✓ answer (2)
1.3	$r = 0,946 \ldots\ldots \approx 0,95$	✓ answer (1)
1.4	Not to spend R9 000 per month because the point (18 000 ; 9 000) lies very far from the least squares regression line. **OR** D *Spandeer nie R9 000 per maand nie, want die punt (18 000 ;9 000) lê baie ver van die kleinste-kwadrate regressielyn.* ***OF*** *D*	✓✓ answer (2)
		[8]

QUESTION/*VRAAG* 2

2.1	Number people paid R200 or less = 19 *Aantal mense wat R200 of minder betaal het = 19*	✓ answer (1)
2.2	$7+12+a+35+b+6=100$ $a=40-b$ $309=\frac{(50\times7)+(150\times12)+(250\times a)+(350\times35)+(450\times b)+(550\times6)}{100}$ $309=\frac{(50\times7)+(150\times12)+(250\times(40-b))+(350\times35)+(450\times b)+(550\times6)}{100}$ $350 + 1800 + 10000 - 250b + 12250 + 450b + 3300 = 30900$ $200b = 3200$ $b = 16$ $a = 24$ **OR/*OF*** $7+12+a+35+b+6=100$ $b=40-a$ $309=\frac{(50\times7)+(150\times12)+(250\times a)+(350\times35)+(450\times b)+(550\times6)}{100}$ $309=\frac{(50\times7)+(150\times12)+(250\times a)+(350\times35)+(450\times(40-a))+(550\times6)}{100}$ $350 + 1800 + 250a + 12250 + 1800 - 450a = 30900$ $200a = 4\ 800$ $a = 24$ $b = 16$	✓ $\sum x=100$ ✓ $a=40-b$ ✓ $\sum fX$ ✓ $\sum\frac{fX}{n}=309$ ✓ $200b = 3200$ (5) ✓ $\sum x=100$ ✓ $b=40-a$ ✓ $\sum fX$ ✓ $\sum\frac{fX}{n}=309$ ✓ $200a = 4\ 800$ (5)
2.3	Modal class/*modale klas:* $300 < x \leq 400$	✓ answer (1)

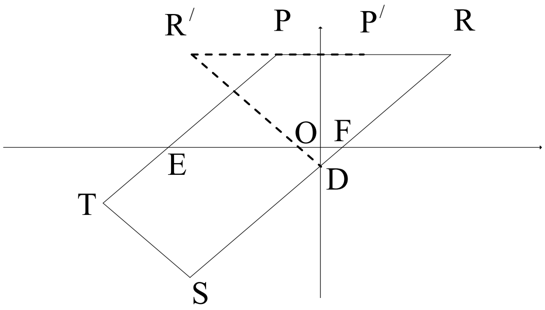
2.4		✓ grounded at (0 ; 0) ✓ (600 ; 100) ✓ cumulative frequencies for y-coordinates ✓ smooth shape (4)
2.5	Number of people/*Aantal mense* = 100 – 82 [accept 80 – 84 people] 18 people paid more than R420 per month/. [accept 16 – 20 people] *18 mense betaal meer as R420 per maand* Answer only: Full marks	✓ 82 ✓ answer (2)
		[13]

QUESTION/*VRAAG* 3

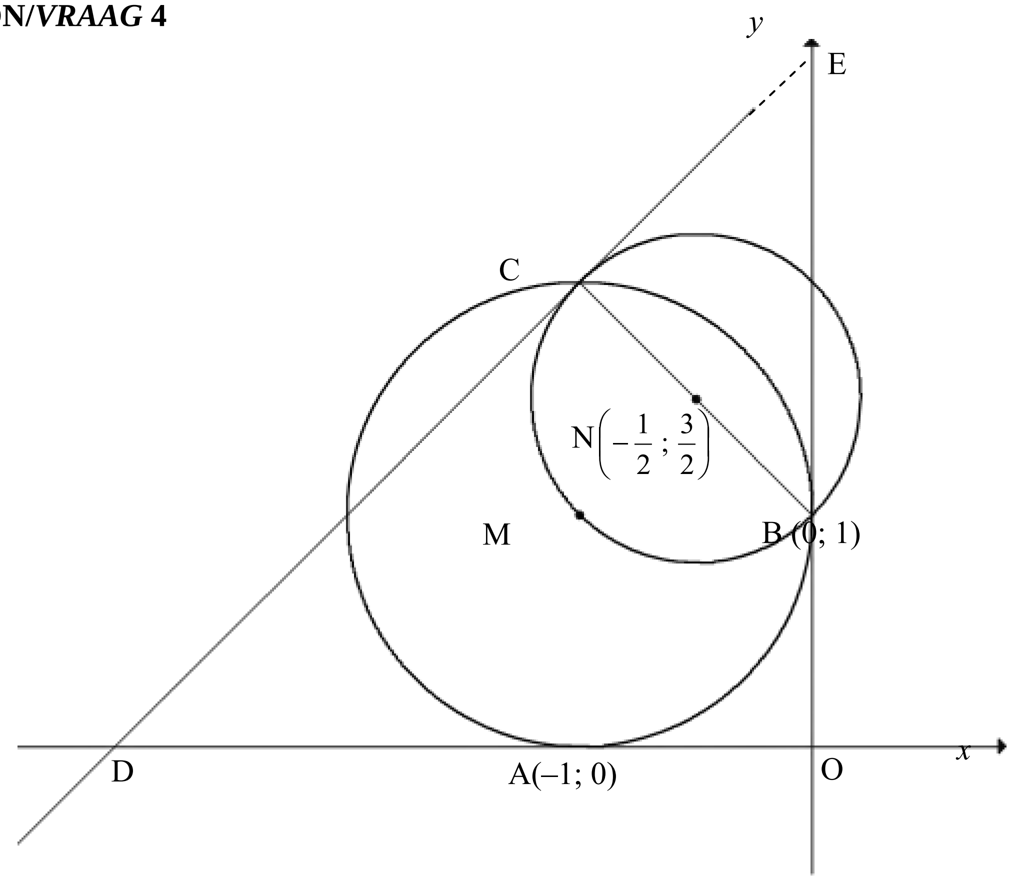


3.1	Equation of PR: $y = 5$	✓ answer (1)
3.2.1	$m_{\text{RS}} = \dfrac{y_2 - y_1}{x_2 - x_1}$ $m_{\text{RS}} = \dfrac{5-(-7)}{3-(-3)} = \dfrac{12}{6}$ $= 2$ Answer only: Full marks	✓ substitution of R & S into gradient formula ✓ answer (2)
3.2.2	$m_{\text{RS}} = m_{\text{PT}}$ [PT ‖ RS] $\tan\theta = 2$ $\theta = 63{,}43°$	✓ $m_{\text{RS}} = m_{\text{PT}}$ ✓ $\tan\theta = 2$ ✓ $\theta = 63{,}43°$ (3)
3.2.3	Equation of RS: $y-5 = 2(x-3)$ or $y-(-7) = 2(x-(-3))$ or $5 = 2(3)+c$ $y-5 = 2x-6$ $\quad$ $y+7 = 2x+6$ $\quad$ $c = -1$ $y = 2x-1$ $\quad$ $y = 2x-1$ $\quad$ $y = 2x-1$ $\therefore$ D(0 ; –1) **OR/OF** $m_{RS} = m_{RD} = m_{DS}$ Answer only: Full marks $2 = \dfrac{5-y}{3-0} = \dfrac{y+7}{0-(-3)}$ $\therefore y = -1$ $\therefore$ D(0 ; –1)	✓ substitution ✓ equation of RS ✓ coordinates of D (3) ✓ equating gradients ✓ value of y ✓ coordinates of D (3)

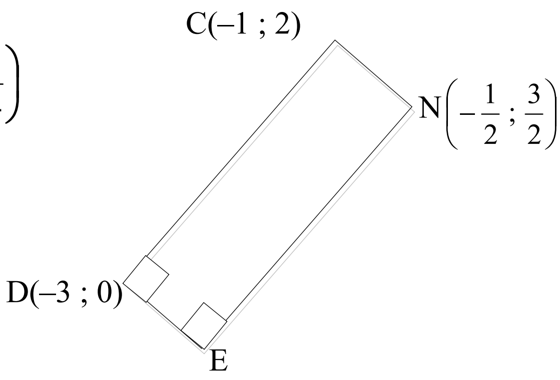
3.3	$ST = 2\sqrt{5} = \sqrt{[-5-(-3)]^2 + (k-(-7))^2}$ $20 = 4 + (k+7)^2$ $(k+7)^2 = 16$ $k + 7 = \pm 4$ $k = -11$ or $k = -3$ $\therefore k = -3$ **OR** $ST = 2\sqrt{5} = \sqrt{[-5-(-3)]^2 + (k-(-7))^2}$ $20 = 4 + k^2 + 14k + 49$ $k^2 + 14k + 33 = 0$ $(k+11)(k+3) = 0$ $k = -11$ or $k = -3$ $\therefore k = -3$	✓ substitute S and T into distance formula ✓ isolate square ✓ square root both sides ✓ answer (4) ✓ substitute S and T into distance formula ✓ standard form ✓ factors ✓ answer (4)
3.4	Method: translation T→S: $(x\,;y) \rightarrow (x+2\,;y-4)$ $\therefore$ by symmetry: D→N: D(0 ; –1) → N(0 + 2 ; –1 – 4) $\therefore$ N(2 ; –5) Answer only: Full marks **OR** Midpoint of TN = Midpoint of SD $\frac{x+(-5)}{2} = \frac{-3+0}{2}$ and $\frac{y+(-3)}{2} = -\frac{-7+(-1)}{2}$ $x = 2$ and $y = -5$ $\therefore$ N(2 ; – 5) Answer only: Full marks	✓ method ✓ x-coordinate ✓ y-coordinate (3) ✓ method: midpoint of diagonals ✓ x-coordinate ✓ y-coordinate (3)

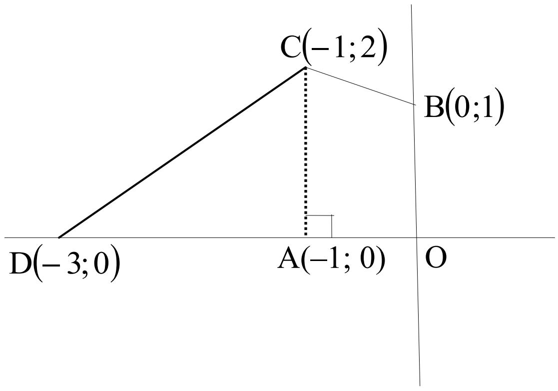
3.5		
	β is the inclination of RS $\therefore\ \beta = 63{,}434...°$	✓ $\beta = 63{,}43°$
	$\hat{OFD} = 63{,}434...°$ [vert opp $\angle$s]	
	$\hat{ODF} = 90° - 63{,}434...° = 26{,}565...°$	✓ $\hat{ODF} = 26{,}57°$
	$\hat{RDR^{/}} = 2(26{,}565...°) = 53{,}13°$	✓ answer (3)
	OR	
	PEFR is a ||m [both pairs of opp sides ||]	
	$\therefore \hat{R} = \theta = 63{,}434...°$ [opp $\angle$s of ||m]	✓ $\hat{R} = 63{,}43°$
	$R\hat{R}^{/}D = 63{,}434...°$ [$\angle$s opp = sides:RD = R$^{/}$D]	✓ $R\hat{R}^{/}D = 63{,}43°$
	$\hat{RDR^{/}} = 180° - (63{,}43° + 63{,}43°)$ [sum of $\angle$s in Δ]	
	$\hat{RDR^{/}} = 53{,}13°$	✓ answer (3)
	OR	
	$\tan \hat{ODF} = \frac{3}{6}$	✓ trig ratio
	$\hat{ODF} = 26{,}565..°$	✓ $\hat{ODF} = 26{,}565..°$
	$\hat{RDR^{/}} = 2(26{,}565...°) = 53{,}13°$	✓ answer (3)
	OR	
	$R^{/}(-3\,;5)$ [reflection of R(3 ; 5) about the *y*-axis]	
	$RD = \sqrt{(3-0)^2 + (5-(-1)^2}$	✓ $R^{/}(-3\,;5)$ **OR** $RD = \sqrt{45} = R^{/}D$
	$RD = \sqrt{45} = R^{/}D$ or $3\sqrt{5}$ or 6,71	
	$(RR^{/})^2 = (\sqrt{45})^2 + (\sqrt{45})^2 - 2(\sqrt{45})(\sqrt{45})(\cos \hat{RDR^{/}})$	✓ substitution into cosine rule
	$6^2 = 45 + 45 - 2(45)(\cos \hat{RDR^{/}})$	
	$\cos \hat{RDR^{/}} = \frac{45+45-36}{2(45)}$	
	$\cos \hat{RDR^{/}} = \frac{3}{5}$	
	$\therefore\ \hat{RDR^{/}} = 53{,}13°$	✓ answer (3)
		[19]

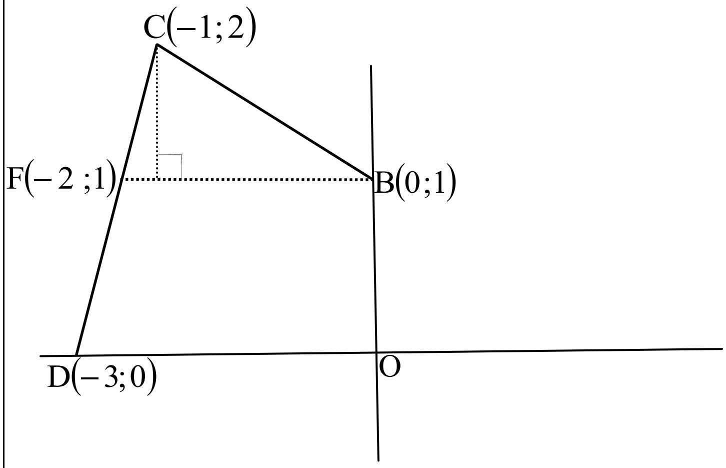
QUESTION/*VRAAG* 4



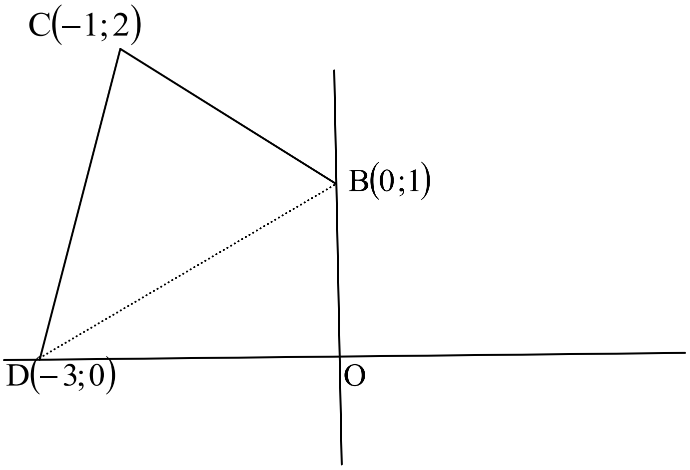
4.1	M$(-1;1)$ $(x+1)^2+(y-1)^2=1$ Answer only: Full marks	✓ M$(-1;1)$ ✓LHS ✓ RHS (3)
4.2	Midpoint of CB, N: (– 0,5 ; 1,5) $\therefore \frac{x_C+0}{2}=-\frac{1}{2}$ and $\frac{y_C+1}{2}=\frac{3}{2}$ Answer only: Full marks $\therefore$ C(– 1 ; 2) **OR** B→N: $(x;y)\rightarrow(x-0,5;y+0,5)$ N→C: Answer only: Full marks $(x;y)\rightarrow(x-0,5;y+0,5)$ $\therefore$ C(– 0,5 – 0,5 ; 1,5 + 0,5) $\therefore$ C(– 1 ; 2)	✓x value ✓ y value (2) ✓x value ✓ y value (2)

4.3	$m_{\text{radius}}=\dfrac{2-1}{-1-0}$ OR $\dfrac{2-(-\frac{1}{2})}{-1-\frac{3}{2}}$ OR $\dfrac{0-(-\frac{1}{2})}{1-\frac{3}{2}}$ $=-1$ $\therefore m_{\text{tangent}}=1$ $y=mx+c$ $y=x+c$ $2=1(-1)+c$ $c=3$ $\therefore\ y=x+3$ $y-x=3$	✓ m_{radius} ✓ m_{tangent} ✓ substitute (– 1 ; 2) and m ✓ simplification (4)
	OR $m_{\text{radius}}=\dfrac{2-1}{-1-0}$ $=-1$ $\therefore m_{\text{tangent}}=1$ $y-y_1=m(x-x_1)$ $y-y_1=1(x-x_1)$ $y-2=1(x-(-1))$ $y-2=x+1$ $\therefore\ y=x+3$ $y-x=3$	✓ m_{radius} ✓ m_{tangent} ✓ substitute (– 1 ; 2) and m ✓ simplification (4)
4.4	Tangents to circle: $y=x+3$ and $y=x+1$ $\therefore t>3$ or $t<1$ Answers only: Full marks	✓ $y=x+1$ ✓ $t>3$ ✓ $t<1$ (3)
4.5	Draw rectangle CNED:  Midpt of DN $\left(-\dfrac{7}{4};\dfrac{3}{4}\right)$ $\therefore$ E$(-\dfrac{5}{2};-\dfrac{1}{2})$	✓ midpt of DN ✓x value ✓ y value (3)
	OR/*OF* D(– 3 ; 0) C→N: $(x;y)\to(x+0{,}5;y-0{,}5)$ D→E: $\text{D}(x;y)\to\text{E}(x+0{,}5;y-0{,}5)$ $\therefore$ E(– 3 + 0,5 ; 0 – 0,5) $\therefore$ E(– 2,5 ; – 0,5) Answer only: Full marks	✓ coordinates of D ✓x value ✓ y value (3)

4.6		
	area of trapezium AOBC $= \frac{1}{2}(1+2)(1)$	✓ substitution into area of trapezium form
	$= 1\frac{1}{2}$ square units	✓ area of trapezium
	area of ΔACD $= \frac{1}{2}(2)(2)$	
	$= 2$ square units	✓ area of triangle
	area of quadrilateral OBCD $= 3\frac{1}{2}$ square units	✓ area of OBCD
	$\therefore 2a^2 = \frac{7}{2}$	✓ equating area OBCD to $2a^2$
	$a^2 = \frac{7}{4}$	
	$a = \frac{\sqrt{7}}{2}$	(5)
	OR	



	BM produced cuts the tangent at F. area of ΔCFB = $\frac{1}{2}(2)(1)$ = 1 square unit area of trapezium BFDO = $\frac{1}{2}(2+3)(1)$ $= 2\frac{1}{2}$ square units area of quadrilateral OBCD = $3\frac{1}{2}$ square units $\therefore 2a^2 = \frac{7}{2}$ $a^2 = \frac{7}{4}$ $a = \frac{\sqrt{7}}{2}$ **OR**	✓ area of triangle ✓ substitution into area of trapezium ✓ area of trapezium ✓ area of OBCD ✓ equating area OBCD to $2a^2$ (5)



Join DB

area of $\Delta ODB = \frac{1}{2}(3)(1)$

$= \frac{3}{2}$ square unit ✓ area of Δ

area of $\Delta DCB = \frac{1}{2}\left(2\sqrt{2}\right)\left(\sqrt{2}\right)$ ✓ subst into area of Δ

$= 2$ square unit ✓ area of Δ

$\therefore$ area of OBCD = $\frac{3}{2} + 2$ = square units ✓ area of OBCD

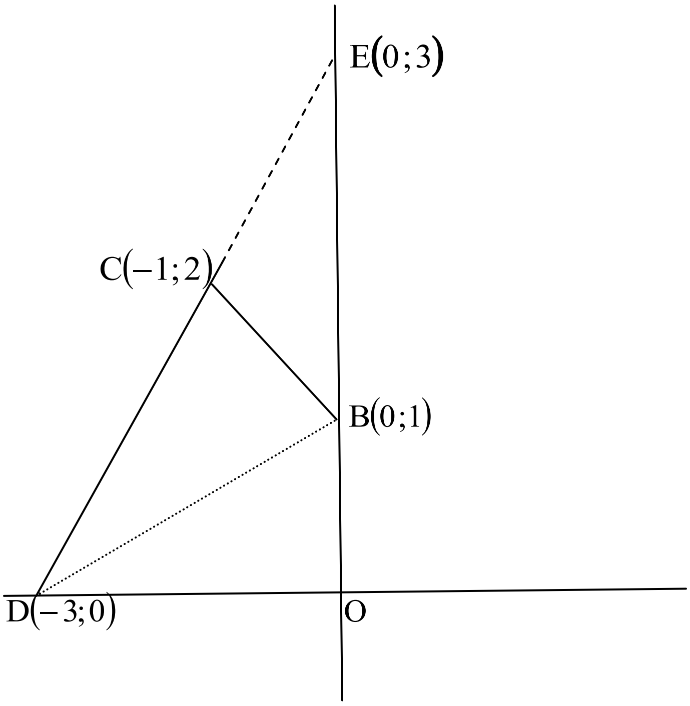
$2a^2 = \frac{7}{2}$ ✓ equating area OBCD to $2a^2$

$a^2 = \frac{7}{4}$

$a = \frac{\sqrt{7}}{2}$

(5)

OR



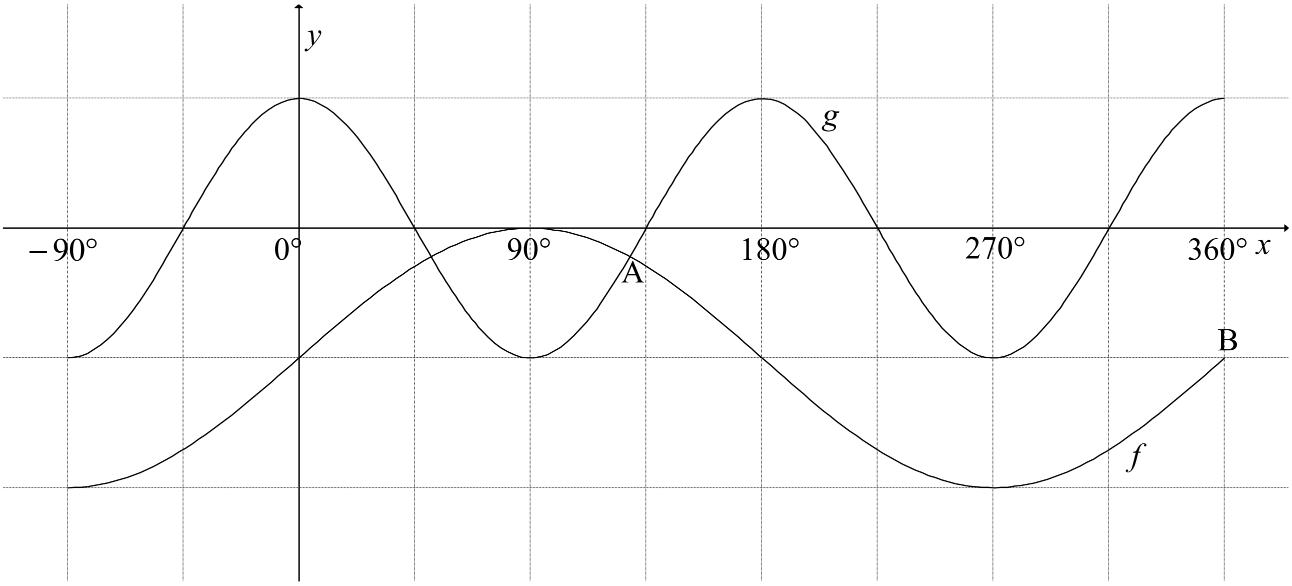
	Let E be the point of intersection of DC with the positive y–axis.	
	area of ΔDEO $= \frac{1}{2}(3)(3)$	
	$= \frac{9}{2}$ square unit	✓ area of Δ
	area of ΔECB $= \frac{1}{2}(2)(1)$ or $\frac{1}{2}(\sqrt{2})(\sqrt{2})$	✓ subst into area of Δ
	$= 1$ square unit	✓ area of Δ
	area of quadrilateral OBCD $= \frac{9}{2} - 1 = 3\frac{1}{2}$ square units	✓ area of OBCD
	$\therefore 2a^2 = \frac{7}{2}$ $a^2 = \frac{7}{4}$ $a = \frac{\sqrt{7}}{2}$	✓ equating area OBCD to $2a^2$
		(5)
		[20]

QUESTION/*VRAAG* 5

5.1	$\dfrac{\sin x}{\cos x.\tan x}+\sin(180°+x)\cos(90°-x)$ $=\dfrac{\sin x}{\cos x.\dfrac{\sin x}{\cos x}}+(-\sin x)\sin x$ $=1-\sin^2 x$ $=\cos^2 x$	✓ $-\sin x$ ✓ $\sin x$ ✓ $\tan x=\dfrac{\sin x}{\cos x}$ ✓ $1-\sin^2 x$ ✓ $\cos^2 x$ (5)
5.2	$\dfrac{\sin^2 35°-\cos^2 35°}{4\sin 10°\cos 10°}$ $=\dfrac{-\left(\cos^2 35°-\sin^2 35°\right)}{2(2\sin 10°\cos 10°)}$ $=\dfrac{-\cos 70°}{2\sin 20°}$ $=\dfrac{-\cos 70°}{2\cos 70°}$ **OR** $=\dfrac{-\sin 20°}{2\sin 20°}$ $=-\dfrac{1}{2}$	✓ $-\left(\cos^2 35°-\sin^2 35°\right)$ ✓ $-\cos 70°$ ✓ $2\sin 20°$ ✓ answer (4)
5.3	$2\sin^2 77°=2[\sin(90°-13°)]^2$ $=2\cos^2 13°$ $=2\cos^2 13°-1+1$ $=\cos 26°+1$ $=m+1$ **OR** $1-2\sin^2 77°=\cos 154°$ $2\sin^2 77°=1-\cos 154°$ $=1-(-\cos 26°)$ $=1+m$	✓ using co-ratio ✓ reduction ✓ $2\cos^2 13°-1=\cos 26°$ ✓ answer (4) ✓ $1-2\sin^2 77°=\cos 154°$ ✓ $2\sin^2 77°=1-\cos 154°$ ✓ reduction ✓ answer (4)
5.4.1	$\sin(x+25°)\cos 15°-\cos(x+25°)\sin 15°=\tan 165°$ $\sin(x+25°-15°)=-0{,}2679...\ OR\ -2+\sqrt{3}$ $\sin(x+10°)=-0{,}2679...OR\ -2+\sqrt{3}$ $x+10°=195{,}54°+k.360°$ or $x+10°=344{,}46°+k.360°$ $x=185{,}54°+k.360°;\ k\in Z$ or $x=334{,}46°+k.360°;k\in Z$ **OR/*OF***	✓✓ $\sin(x+10°)$ ✓ $-0{,}2679...$ ✓ $195{,}54°$ & $344{,}46°$ ✓ $185{,}54°$ & $334{,}46°$ ✓ $+k.360°;\ k\in Z$ (6)

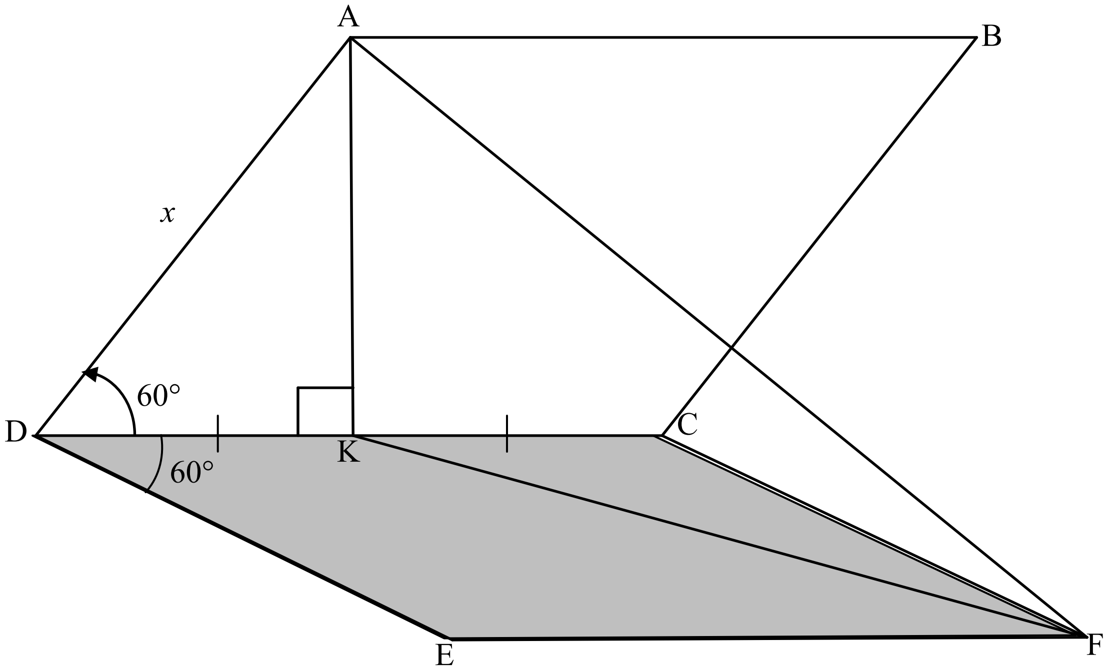
	$\sin(x+25°)\sin 75° - \cos(x+25°)\cos 75° = \tan 165°$ $-(\cos(x+25°)\cos 75° - \sin(x+25°)\sin 75°) = -0,2679...$ $\cos(x+100°) = 0,2679...$ ref. $\angle = 74.4577...°$ $x+100° = 74,46° + k.360°$ *or* $x+100° = 285,54° + k.360°$ $x = -25,54° + k.360°;\ k \in Z$ or $x = 185,54° + k.360°;\ k \in Z$	✓✓ $\cos(x+100°)$ ✓ $-0,2679...$ ✓ $74,46°$ & $285,54°$ ✓ $-25,54°$ & $185,54°$ ✓ $+k.360°;\ k \in Z$ (6)
5.4.2	$f(x) = \sin(x+10°)$ Answers only: Full marks For minimum value of $\sin x$: $x = 270°$ For minimum value of $\sin(x+10°)$: $x = 260°$	✓ $f(x) = \sin(x+10°)$ ✓ $270°$ ✓ answer (3)
		[22]

QUESTION/*VRAAG* 6



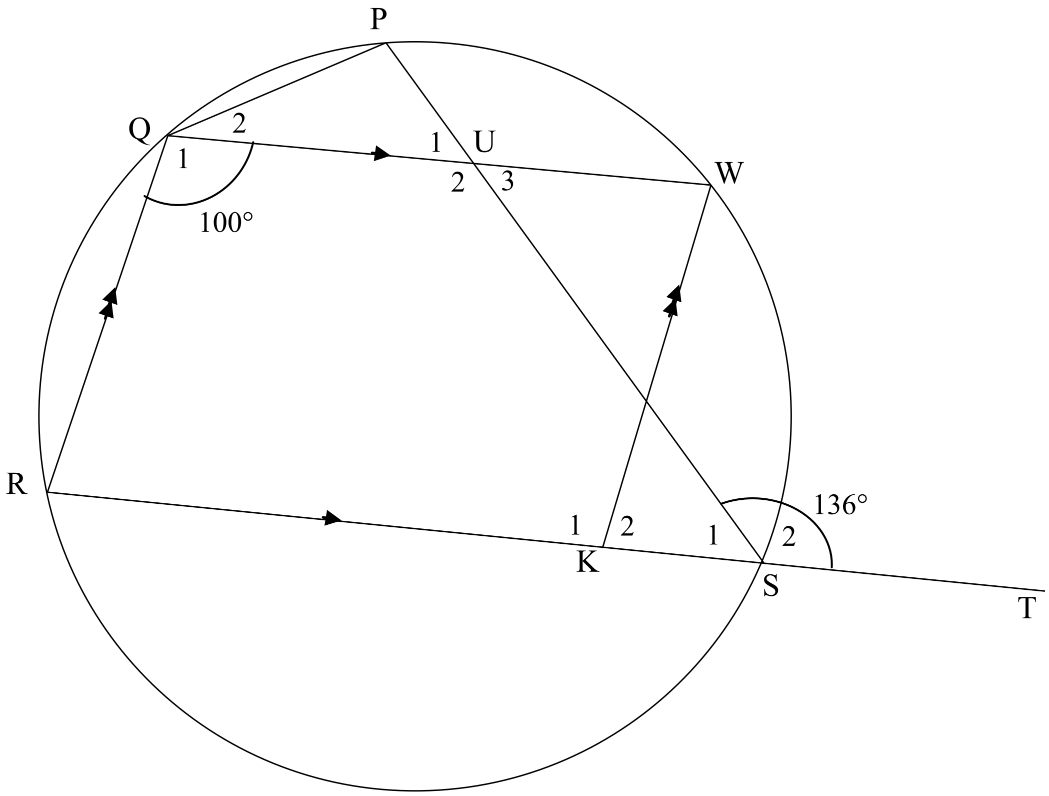
6.1	Range of f: $y \in [-2\ ;0]$ **OR** $-2 \le y \le 0$	✓ critical values ✓ notation (2)
6.2	$x \in (90°\ ;\ 270°)$ OR $x \in [90°\ ;\ 270°]$	✓ critical values ✓ notation (2)
6.3	$\text{PQ} = \cos 2x - (\sin x - 1)$ $= 1 - 2\sin^2 x - \sin x + 1$ $= -2\sin^2 x - \sin x + 2$ $\sin x = -\dfrac{b}{2a}$ $= \dfrac{-(-1)}{2(-2)}$ $\sin x = -\dfrac{1}{4}$ $\therefore\ x = 194{,}48°$ or $x = 345{,}52°$	✓ $\text{PQ} = \cos 2x - (\sin x - 1)$ ✓ $\cos 2x = 1 - 2\sin^2 x$ ✓substitution into formula ✓ $\sin x = -\dfrac{1}{4}$ ✓ 194,48° ✓ 345,52° (6)
		[10]

QUESTION/*VRAAG* 7



7.1	$\sin 60° = \frac{AK}{x}$ $AK = x\sin 60°$ or $\frac{\sqrt{3}}{2}x$ or $0{,}866x$	✓ trig ratio ✓ answer (2)
7.2	$K\hat{C}F = 120°$	✓ answer (1)
7.3	$KF^2 = CF^2 + CK^2 - 2CF.CK\cos K\hat{C}F$ $= x^2 + \left(\frac{x}{2}\right)^2 - 2x\left(\frac{x}{2}\right)\cos 120°$ $= x^2 + \frac{x^2}{4} - x^2\left(-\frac{1}{2}\right)$ $= \frac{7x^2}{4}$ $KF = \frac{\sqrt{7}x}{2}$ $A\hat{K}F = y$ Area $\Delta AKF = \frac{1}{2}.AK.KF\sin A\hat{K}F$ $= \frac{1}{2}.\frac{\sqrt{3}x}{2}.\frac{\sqrt{7}x}{2}\sin y$ $= \frac{x^2\sqrt{21}\sin y}{8}$	✓ correct use of cosine rule ✓substitution ✓ $\cos 120° = -\frac{1}{2}$ ✓ $KF = \frac{\sqrt{7}x}{2}$ ✓ correct use of area rule ✓substitution ✓ answer in terms of x and y (7)
		[10]

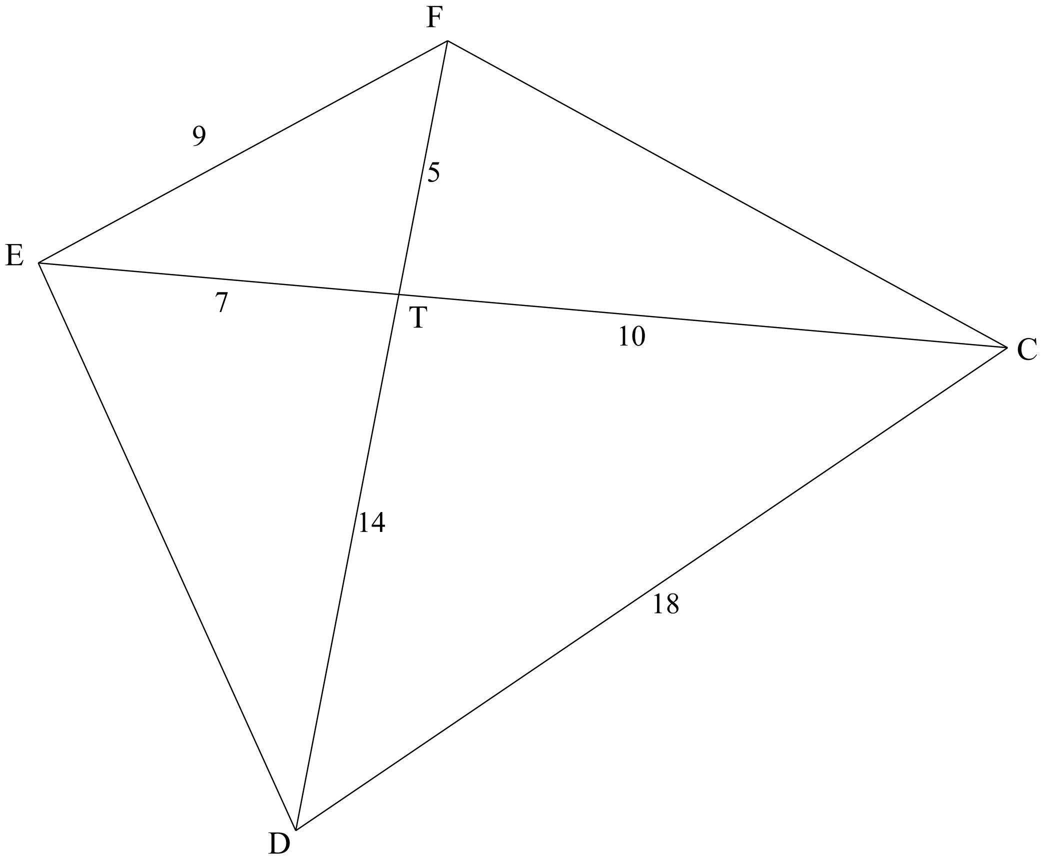
QUESTION/*VRAAG* 8



8.1.1	$\hat{R} = 80°$ [co-int ∠s/*ko-binne ∠e*; QW ‖ RK]	✓S ✓R (2)
8.1.2	$\hat{P} = 100°$ [opp ∠s of cyclic quad/*teenoorst ∠e v koordevh*]	✓S ✓R (2)
8.1.3	$P\hat{Q}R = 136°$ [ext ∠ of cyclic quad/*buite ∠ v koordevh*] $\hat{Q}_2 = 36°$ **OR** $P\hat{U}W = \hat{S}_2 = 136°$ [corresp ∠s/*ooreenkomstige ∠e*; QW ‖ RK] $P\hat{Q}W + \hat{P} = P\hat{U}W$ [ext ∠s of/*buite ∠ van* Δ QPU] $P\hat{Q}W + 100° = 136°$ $P\hat{Q}W = 36°$ **OR** $\hat{U}_3 = 180° - 136° = 44°$ [co-int ∠s/*ko-binne ∠e*; QW ‖ RK] $\hat{U}_1 = \hat{U}_3 = 44°$ [vert opp ∠s/*regoorstaande ∠e*] $P\hat{Q}W = 180° - (100 + 44°)$ [sum of ∠s in Δ/*som ∠e van* Δ] $P\hat{Q}W = 36°$	✓S ✓R ✓S (3) ✓S ✓R ✓S (3) ✓S ✓R ✓S (3)

8.1.4	$\hat{U}_2 = \hat{S}_2 = 136°$ [alt ∠s/*verwiss ∠e* ; QW ‖ RK] **OR** $\hat{U}_2 = 100° + 36°$ $= 136°$ [ext ∠s of/*buite ∠ van* ΔQPU] **OR** $\hat{U}_2 = P\hat{U}W = 136°$ [vert opp ∠s/*regoorstaande ∠e*] **OR** $\hat{U}_2 = 180° - \hat{U}_3$ $= 180° - 44°$ $= 136°$ [∠s on a str line/*∠e op reguitlyn*]	✓S ✓R (2) ✓S ✓R (2) ✓S ✓R (2) ✓S ✓R (2)

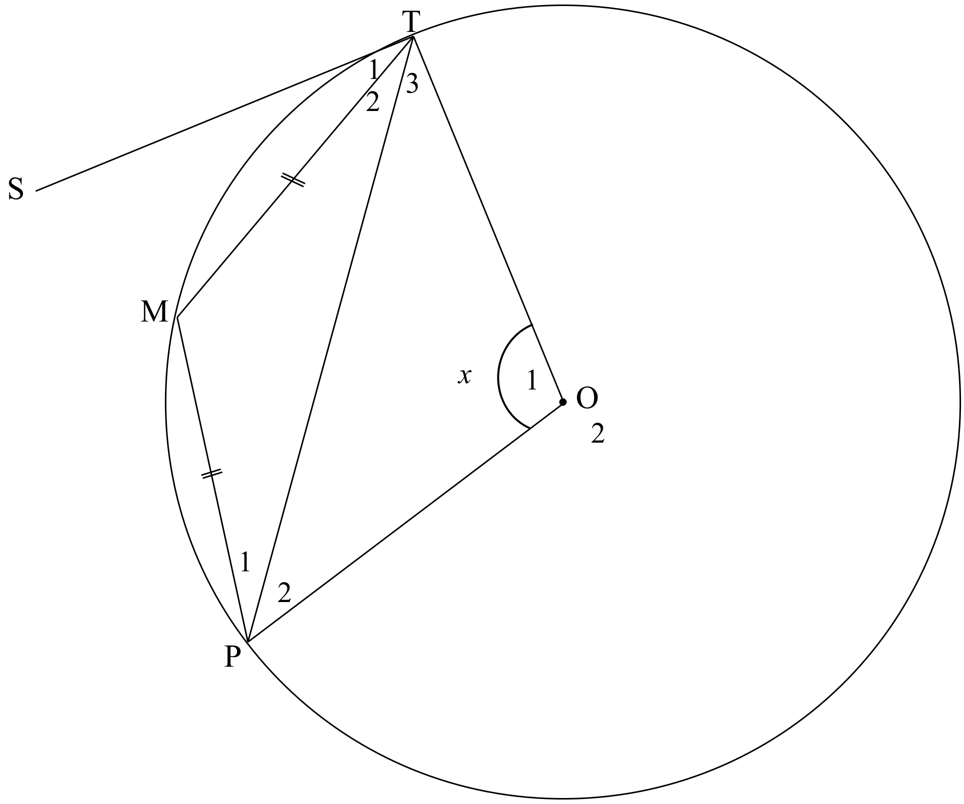
8.2



8.2.1	In ΔEFT and ΔDCT: $\frac{EF}{CD}=\frac{9}{18}=\frac{1}{2}$ $\frac{FT}{TC}=\frac{5}{10}=\frac{1}{2}$ $\frac{ET}{TD}=\frac{7}{14}=\frac{1}{2}$ $\therefore \Delta EFT \parallel\!\mid \Delta DCT$ [Sides of Δ in prop/ *sye van Δ in dieselfde verh*] $\therefore E\hat{F}D = E\hat{C}D$ **OR** In ΔFET: $49 = 25 + 81 - 2(5)(9)\cos\hat{F}$ $\cos\hat{F}=\frac{19}{30}$ $\hat{F}=50{,}7°$ In ΔTDC: $196 = 100 + 256 - 2(10)(18)\cos\hat{C}$ $\cos\hat{C}=\frac{19}{30}$ $\hat{C}=50{,}7°$	✓✓ all 3 ratios = $\frac{1}{2}$ ✓ $\Delta EFT \parallel\!\mid \Delta DCT$ ✓ R (4) ✓✓ $\hat{F}=50{,}7°$ ✓✓ $\hat{C}=50{,}7°$ (4)

8.2.2	$\hat{EFD} = \hat{ECD}$ [proved in 8.2.1] E, F, C and D are concyclic EFCD is a cyclic quad [converse ∠s in the same segment/ *omgekeerde ∠e in dies segment*] $\therefore \hat{DFC} = \hat{DEC}$ [∠s in the same segment/*∠e in dies segment*]	✓S ✓R ✓ R (3)
		[16]

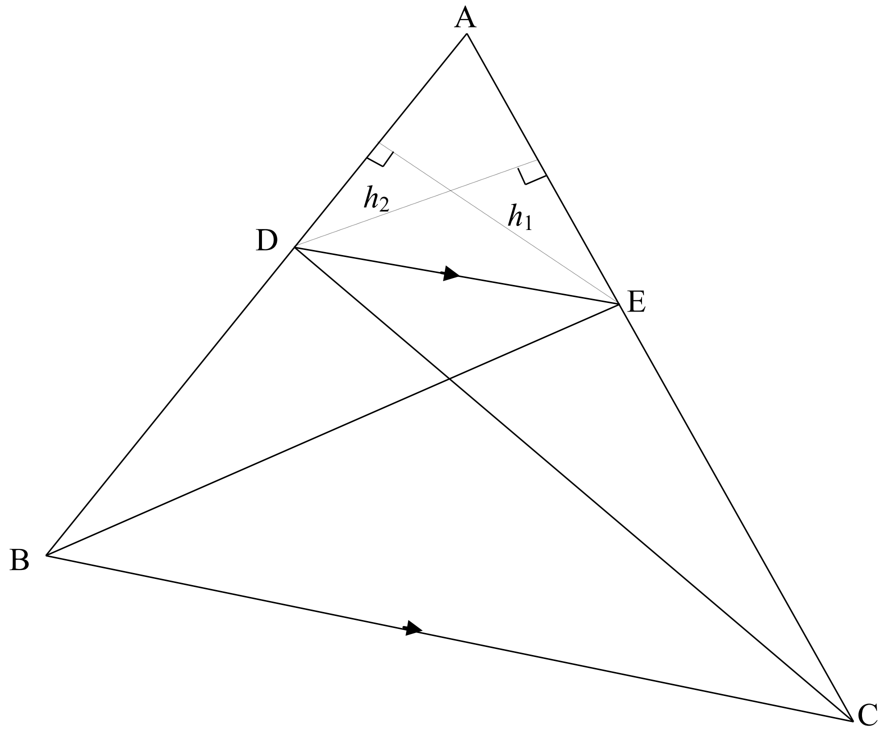
QUESTION/*VRAAG* 9



	$\hat{O}_2 = 360° - x$ [∠s round a pt/*∠e om 'n punt*] $\therefore \hat{M} = 180° - \frac{1}{2}x$ [∠ at centre = 2 ×∠ at circumference/ *middelpunts∠* = 2 × *omtreks∠*] $\therefore \hat{T}_2 + \hat{P}_1 = \frac{1}{2}x$ [sum of ∠s in Δ/*som ∠e van* Δ] $\therefore \hat{T}_2 = \hat{P}_1 = \frac{1}{4}x$ [∠s opp equal sides/*∠e teenoor gelyke sye*] $\therefore S\hat{T}M = \hat{P}_1 = \frac{1}{4}x$ [tan chord theorem/*raaklyn koordstelling*] **OR/*OF*** $\hat{O}_2 = 360° - x$ [∠s round a pt/*∠e om 'n punt*] $\therefore \hat{M} = \frac{1}{2}\hat{O}_2$ [∠ at centre = 2 ×∠ at circumference] $\therefore \hat{T}_2 + \hat{P}_1 = 180° - \hat{M}$ [sum of ∠s in Δ/*som ∠e van* Δ] $\therefore \hat{T}_2 = \hat{P}_1$ [∠s opp equal sides/*∠e teenoor gelyke sye*] $= \frac{180° - \hat{M}}{2} = \frac{180° - \frac{1}{2}\hat{O}_2}{2} = \frac{180° - \frac{1}{2}(360° - x)}{2} = \frac{1}{4}x$ $\therefore S\hat{T}M = \frac{1}{4}x$ [tan chord theorem/*raaklyn koordstelling*]	✓ $\hat{O}_2 = 360° - x$ ✓ $\hat{M} = 180° - \frac{1}{2}x$ ✓R ✓ $\hat{T}_2 + \hat{P}_1 = \frac{1}{2}x$ ✓ $\hat{P}_1 = \frac{1}{4}x$ ✓R ✓R (7) ✓ $\hat{O}_2 = 360° - x$ ✓ S ✓R ✓S ✓R ✓ S ✓R (7)
		[7]

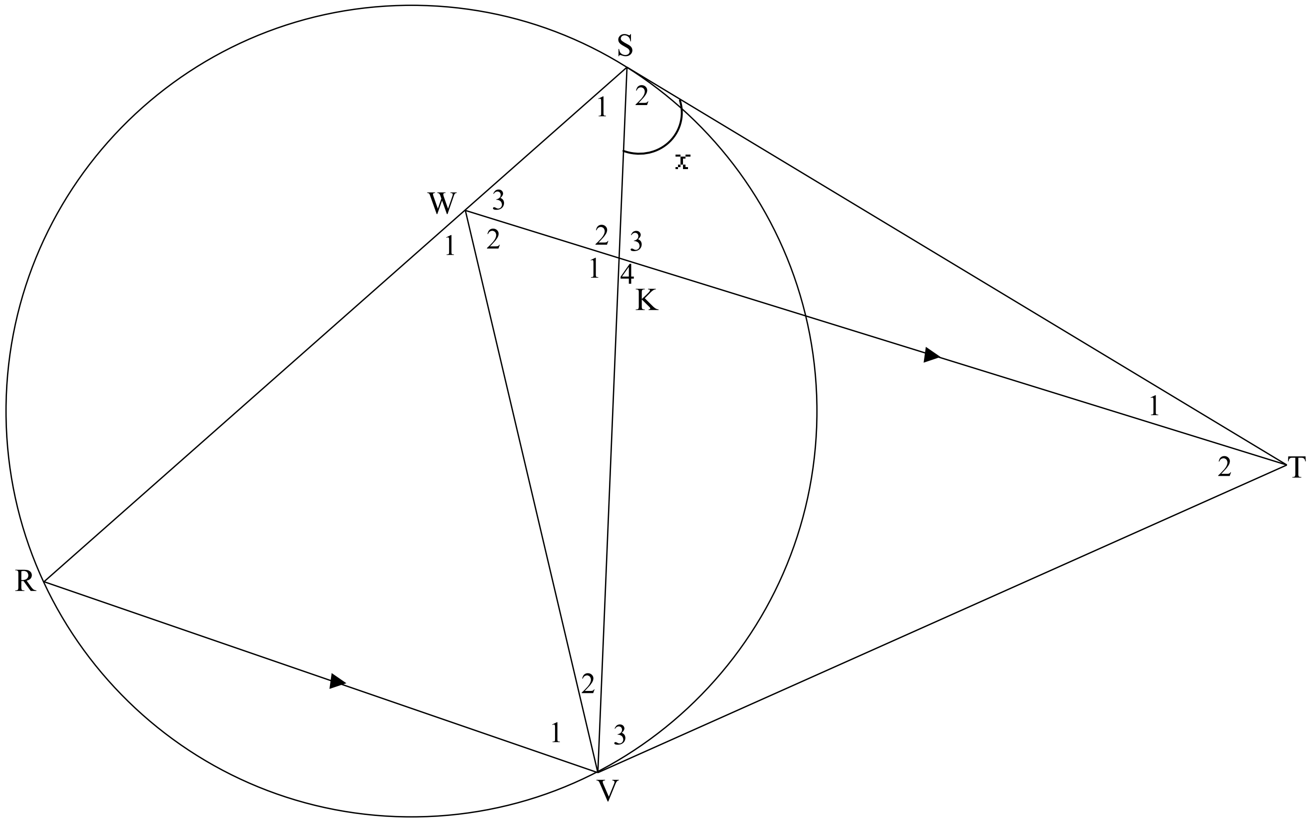
QUESTION/*VRAAG* 10

10.1



10.1	Constr: Draw h_1 from E $\perp$ AD and h_2 from D $\perp$ AE *Konstr: Trek* h_1 *vanaf* E $\perp$ AD *en* h_2 *vanaf* D $\perp$ AE Proof/*Bewys*: $\dfrac{\text{area } \Delta\text{ADE}}{\text{area } \Delta\text{BDE}} = \dfrac{\frac{1}{2}\text{AD} \times h_1}{\frac{1}{2}\text{DB} \times h_1} = \dfrac{\text{AD}}{\text{DB}}$ $\dfrac{\text{area } \Delta\text{ADE}}{\text{area } \Delta\text{DEC}} = \dfrac{\frac{1}{2}\text{AE} \times h_2}{\frac{1}{2}\text{EC} \times h_2} = \dfrac{\text{AE}}{\text{EC}}$ But area ΔBDE = area ΔDEC [same base & height or DE || BC/ *dies basis & hoogte*; of DE || BC] $\therefore \dfrac{\text{area } \Delta\text{ADE}}{\text{area } \Delta\text{BDE}} = \dfrac{\text{area } \Delta\text{ADE}}{\text{area } \Delta\text{DEC}}$ $\therefore \dfrac{\text{AD}}{\text{DB}} = \dfrac{\text{AE}}{\text{EC}}$	✓ constr/*konstr* **OR reason: common vertex or same height** ✓ $\dfrac{\text{area } \Delta\text{ADE}}{\text{area } \Delta\text{BDE}} = \dfrac{\frac{1}{2}\text{AD} \times h_1}{\frac{1}{2}\text{DB} \times h_1}$ ✓ $\dfrac{\text{area } \Delta\text{ADE}}{\text{area } \Delta\text{DEC}} = \dfrac{\text{AE}}{\text{EC}}$ ✓ S ✓R ✓ S (6)

10.2



10.2.1	$\hat{V}_3 = x$ [Tans from same point/*raaklyne vanaf dieselfde pt*] $\hat{R} = x$ [tan chord theorem/*raaklyn koordstelling*] $\hat{W}_3 = x$ [corresp ∠s/*ooreenkomstige ∠e*; WT ‖ RV]	✓ S ✓ R ✓ S ✓ R ✓ S ✓ R (6)
10.2.2(a)	$\hat{V}_3 = \hat{W}_3 = x$ [proved in 10.2.1] W, S, T and V are concyclic/*is konsiklies* WSTV is a cyclic quad [converse ∠s in the same segment/ *Omgekeerde ∠e in dieselfde segment*]	✓ S ✓ R (2)
10.2.2(b)	$\hat{W}_2 = \hat{S}_2 = x$ [∠s in the same segment/∠e *in dies segment*] $\hat{V}_1 = \hat{W}_2 = x$ [alt ∠s/*verwiss* ∠e ; WT ‖ RV] But $\hat{R} = x$ [proved in 10.2.1] $\therefore \hat{R} = \hat{V}_1 = x$ ∴ WR = WV [sides opp equal ∠s/*sye teenoor gelyke ∠e*] ΔWRV is isosceles/*is gelykbenig* **OR/*OF***	✓ S ✓R ✓ S/ R ✓ S (4)

	$\hat{S}_2 = \hat{W}_2 = x$ [$\angle$s in the same segment] $\hat{W}_2 = \hat{W}_3 = x$ $\hat{W}_2 + \hat{W}_3 = \hat{R} + \hat{V}_1$ [ext $\angle$ of Δ] $\therefore\ \hat{V}_1 = x = \hat{R}$ $\therefore$ WR = WV [sides opp equal $\angle$s/*sye teenoor gelyke* $\angle$*e*] ΔWRV is isosceles/*is gelykbenig*	✓ S ✓R ✓ S/ R ✓ S (4)
10.2.2(c)	In ΔWRV and/*en* ΔTSV $\hat{R} = \hat{S}_2 = x$ [proved **OR** tan chord theorem] $\hat{V}_1 = \hat{V}_3 = x$ [proved] $\therefore \Delta WRV \parallel\!\mid \Delta TSV$ [$\angle, \angle, \angle$] **OR/*OF*** In ΔWRV and/*en* ΔTSV $\hat{R} = \hat{S}_2 = x$ [proved **OR** tan chord theorem] $\hat{V}_1 = \hat{V}_3 = x$ [proved] $\hat{W}_1 = S\hat{T}V = x$ [sum of $\angle$s in Δ/$\angle$*e van* Δ] $\therefore \Delta WRV \parallel\!\mid \Delta TSV$	✓ S ✓ S ✓ R (3) ✓ S ✓ S ✓ S (3)
10.2.2(d)	$\frac{RV}{SV} = \frac{WR}{TS}$ [$\Delta WRV \parallel\!\mid \Delta TSV$] $\therefore WR \times SV = RV \times TS$ $\frac{WR}{SR} = \frac{KV}{SV}$ [prop theorem/*eweredighst*; WT $\parallel$ RV] $\therefore WR \times SV = KV \times SR$ $\therefore RV \times TS = KV \times SR$ $\therefore \frac{RV}{SR} = \frac{KV}{TS}$ **OR/*OF*** In ΔRVS and/*en* ΔVKT $S\hat{V}R = \hat{K}_4$ [alt $\angle$s, WT $\parallel$ RV] $S\hat{R}V = \hat{V}_3$ [proven] $\Delta RVS \parallel\!\mid \Delta VKT$ [$\angle, \angle, \angle$] $\therefore \frac{RV}{SR} = \frac{KV}{VT}$ but VT = ST [tans from same point] $\therefore \frac{RV}{SR} = \frac{KV}{TS}$	✓ correct ratios ✓ $\frac{WR}{SR} = \frac{KV}{SV}$ ✓ R ✓ equating $WR \times SV$ (4) ✓ identifying correct Δs ✓ proving $\parallel\!\mid$ ✓ correct ratio ✓ S (4)
		[25]

TOTAL/*TOTAAL*: 150

basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

SENIOR CERTIFICATE EXAMINATIONS/
SENIORSERTIFIKAAT-EKSAMEN
NATIONAL SENIOR CERTIFICATE EXAMINATIONS/
NASIONALE SENIORSERTIFIKAAT-EKSAMEN

MATHEMATICS P2/
WISKUNDE V2

MARKING GUIDELINES/*NASIENRIGLYNE*

2019

MARKS: 150
PUNTE: 150

These marking guidelines consist of 20 pages.
Hierdie nasienriglyne bestaan uit 20 bladsye..

NOTE:

- If a candidate answers a question TWICE, only mark the FIRST attempt.
- If a candidate has crossed out an attempt of a question and not redone the question, mark the crossed out version.
- Consistent accuracy applies in ALL aspects of the marking memorandum. Stop marking at the second calculation error.
- Assuming answers/values in order to solve a problem is NOT acceptable.

NOTA:

- *As 'n kandidaat 'n vraag TWEE KEER beantwoord, sien slegs die EERSTE poging na.*
- *As 'n kandidaat 'n antwoord van 'n vraag doodtrek en nie oordoen nie, sien die doodgetrekte poging na.*
- *Volgehoue akkuraatheid word in ALLE aspekte van die nasienriglyne toegepas. Hou op nasien by die tweede berekeningsfout.*
- *Om antwoorde/waardes te aanvaar om 'n probleem op te los, word NIE toegelaat NIE.*

GEOMETRY • *MEETKUNDE*	
S	**A mark for a correct statement** **(A statement mark is independent of a reason)**
	'n Punt vir 'n korrekte bewering ***('n Punt vir 'n bewering is onafhanklik van die rede)***
R	**A mark for the correct reason** **(A reason mark may only be awarded if the statement is correct)**
	'n Punt vir 'n korrekte rede ***('n Punt word slegs vir die rede toegeken as die bewering korrek is)***
S/R	**Award a mark if statement AND reason are both correct**
	Ken 'n punt toe as die bewering EN rede beide korrek is

QUESTION/*VRAAG* 1

1.1 45 children

✓ answer (1)

1.2

$$\bar{x} = \frac{\sum fx}{n} = \frac{(4\times 2)+(8\times 10)+(12\times 9)+(16\times 7)+(20\times 8)+(24\times 7)+(28\times 2)}{45}$$

$\bar{x} = \dfrac{692}{45}$ OR $\bar{x} = 15{,}38$ minutes

Answer only: full marks

✓ 692
✓ answer (2)

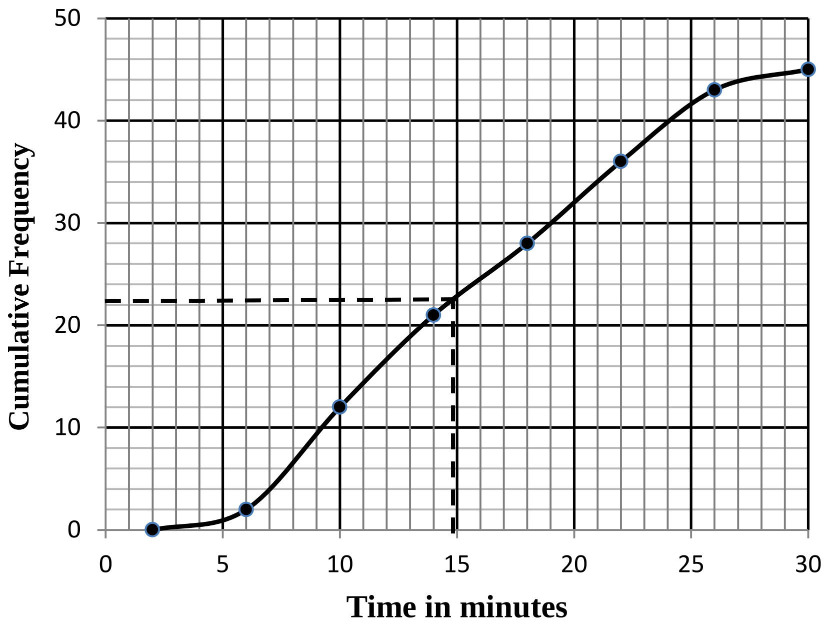
1.3

Time taken (t) (in minutes)	Number of children	Cumulative frequency
$2 < t \leq 6$	2	2
$6 < t \leq 10$	10	12
$10 < t \leq 14$	9	21
$14 < t \leq 18$	7	28
$18 < t \leq 22$	8	36
$22 < t \leq 26$	7	43
$26 < t \leq 30$	2	45

✓ first 4 cum freq correct
✓ last 3 cum freq correct (2)

1.4

CUMULATIVE FREQUENCY GRAPH (OGIVE)



✓ plotting cum freq at upper limits correctly (all points)
✓ shape (smooth)
✓ grounding (2;0) (3)

1.5 On graph at the y-value of 22,5 or 23
Median = $\pm$ 15 minutes.

Answer only: full marks

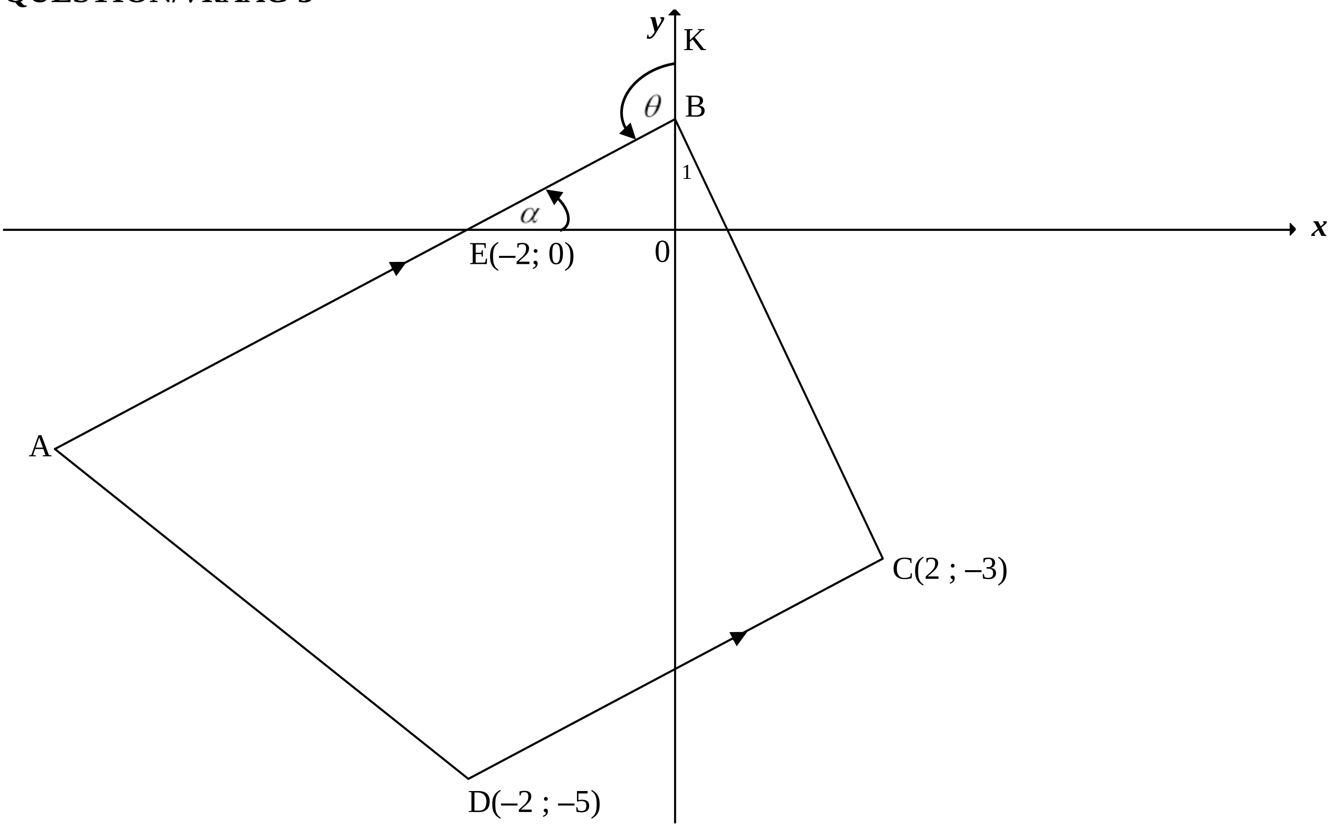
✓ graph
✓ answer (2)

[10]

QUESTION/*VRAAG* 2

2.1	$a = 12{,}44$ $b = 0{,}98$ $y = 12{,}44 + 0{,}98x$ Answer only: full marks	✓ value of a ✓ value of b ✓ equation (3)
2.2.1	Percentage $= \frac{15}{50} \times 100$ $= 30\%$	✓ answer (1)
2.2.2	$\hat{y} = 12{,}44 + 0{,}98x$ $\hat{y} = 12{,}44 + 0{,}98(30)$ $\hat{y} = 41{,}84$ $= 42$ Answer only: full marks **OR** $\hat{y} = 41{,}87$ (if using calculator) $\hat{y} = 42$ **OR** $\hat{y} = \frac{21}{50}$	✓ substitution of 30 ✓ answer as integer (2) ✓ value of y ✓ answer as integer (2) ✓ ✓ answer (2)
2.3.1	standard deviation $=13{,}88$	✓ ✓ answer (2)
2.3.2	$x = 50{,}67 - 45{,}67$ $= 5\%$ Answer only: full marks	✓ $50{,}67 - 45{,}67$ ✓ answer (2)
		[10]

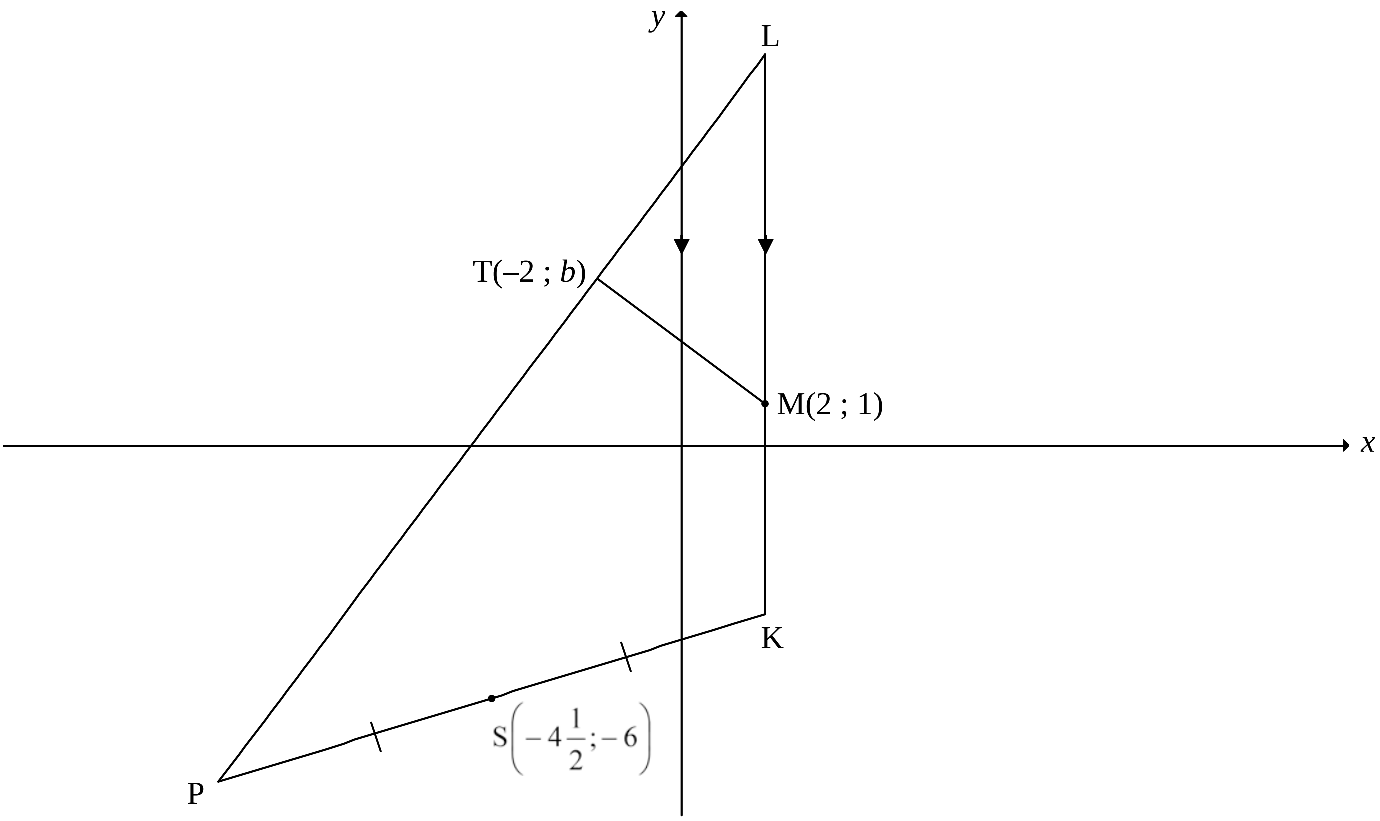
QUESTION/*VRAAG* 3



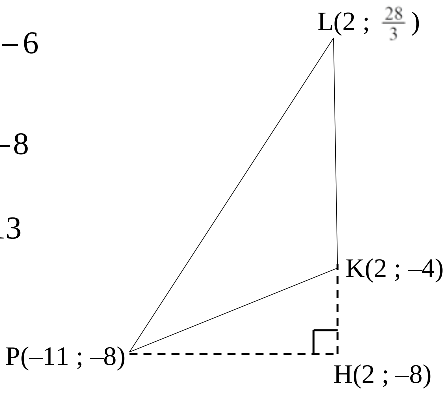
3.1.1	Midpoint of EC: $=\left(\frac{-2+2}{2}\ ;\ \frac{0+(-3)}{2}\right)=\left(0\ ;\ \frac{-3}{2}\right)$	✓ x value ✓ y value (2)
3.1.2	$m_{DC}=\frac{-3-(-5)}{2-(-2)}$ OR $\frac{-5-(-3)}{-2-2}$ $=\frac{2}{4}=\frac{1}{2}$ Answer only: full marks	✓ substitution ✓ answer (2)
3.1.3	$m_{AB}=\frac{1}{2}$ [AB‖DC] $y=\frac{1}{2}x+c$ $0=\frac{1}{2}(-2)+c$ $c=1$ $\therefore y=\frac{1}{2}x+1$ **OR** $y-y_1=\frac{1}{2}(x-x_1)$ $y-0=\frac{1}{2}(x-(-2))$	✓ $m_{AB}=\frac{1}{2}$ ✓ substitution of $(-2;0)$ ✓equation (3)
3.1.4	$\tan\alpha=m_{AB}=\frac{1}{2}$ $\alpha=26{,}57°$ $\theta=90°+26{,}57°$ [ext ∠of Δ] $=116{,}57°$	✓ $\tan\alpha=\frac{1}{2}$ ✓ value of α ✓ value of θ (3)

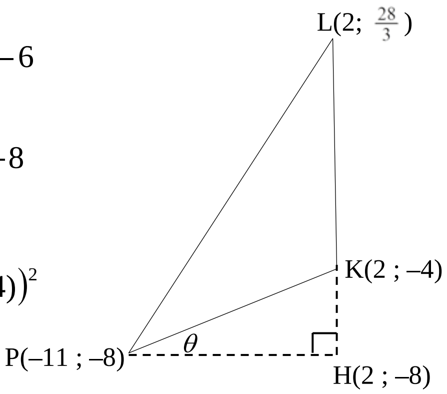
3.2	B(0 ; 1) $m_{BC} = \frac{1-(-3)}{0-2}$ **OR** $m_{BC} = \frac{(-3)-1}{2-0}$ $= -2$ $= -2$ $m_{AB} \times m_{BC} = \frac{1}{2} \times -2$ $= -1$ $\therefore AB \perp BC$	✓ coordinates of B ✓ $m_{BC} = -2$ ✓ product of gradients = –1 (3)
3.3.1	$A\hat{B}C = 90°$ $\therefore$ EC is diameter [converse: $\angle$ in semi circle] $\therefore$ centre of circle $= \left(0\ ;\ -\frac{3}{2}\right)$	✓ answer (1)
3.3.2	$(x-0)^2 + \left(y+\frac{3}{2}\right)^2 = r^2$ $(-2-0)^2 + \left(0+\frac{3}{2}\right)^2 = r^2$ **OR** $(2-0)^2 + \left(-3-\left(\frac{-3}{2}\right)\right)^2 = r^2$ **OR** $(0-0)^2 + \left(1-\left(\frac{-3}{2}\right)\right)^2 = r^2$ **OR** $r = \frac{EC}{2} = \frac{\sqrt{(-2-2)^2 + (0-(-3))^2}}{2}$ **OR** $r = 1 - (-\frac{3}{2})$ $\therefore\ r^2 = \frac{25}{4}$ or $r = \frac{5}{2}$ $x^2 + \left(y+\frac{3}{2}\right)^2 = \frac{25}{4}$	✓ substitution of centre ✓ correct substitution of E(–1 ; 0), B(0 ; 1) or C(2 ; –3) to calculate r^2 or r ✓ value of r^2 or r ✓ equation (4)
		[18]

QUESTION/*VRAAG* 4



4.1	$(x-2)^2+(y-1)^2=25$ $(-2-2)^2+(b-1)^2=25$ $(b-1)^2=9$ $b-1=\pm 3$ $\therefore b=4$ or $b\neq -2$ **OF** $(x-2)^2+(y-1)^2=25$ $(-2-2)^2+(b-1)^2=25$ $16+b^2-2b+1=25$ $b^2-2b-8=0$ $\therefore b=4$ or $b\neq -2$	✓ equation of the circle ✓substitution of point T ✓simplification ✓ answer (4)
4.2.1	K(2 ; 1 – 5) $\therefore$ K(2 ; –4) Answer only: full marks	✓ x value ✓ y value (2)
4.2.2	$m_{\text{MT}}=\frac{4-1}{-2-2}=-\frac{3}{4}$ $m_{\text{PL}}=\frac{4}{3}$ [radius $\perp$ tangent] $y=\frac{4}{3}x+c$ $4=\frac{4}{3}(-2)+c$ $c=\frac{20}{3}$ $y=\frac{4}{3}x+\frac{20}{3}$	✓ m_{MT} ✓ $m_{\text{PL}}=\frac{4}{3}$ ✓ substitution of m_{PL} and the point T ✓ equation (4)

	OR $m_{\text{MT}} = \dfrac{4-1}{-2-2} = -\dfrac{3}{4}$ $m_{\text{PL}} = \dfrac{4}{3}$ [radius ⊥ tangent] $y - y_1 = \dfrac{4}{3}(x - x_1)$ $y - 4 = \dfrac{4}{3}(x+2)$ $y = \dfrac{4}{3}x + \dfrac{20}{3}$ **OR** P(–11 ; –8) $m_{\text{PL}} = \dfrac{4-(-8)}{-2-(-11)}$ $= \dfrac{4}{3}$ $y = \dfrac{4}{3}x + c$ $-8 = \dfrac{4}{3}(-11) + c$ $c = \dfrac{20}{3}$ $y = \dfrac{4}{3}x + \dfrac{20}{3}$	✓ m_{MT} ✓ $m_{\text{PL}} = \dfrac{4}{3}$ ✓ substitution of m_{PL} and the point T ✓ equation (4) ✓ coordinates of P ✓ $m_{\text{PL}} = \dfrac{4}{3}$ ✓ substitution of m_{PL} and the point P or T ✓ equation (4)
4.2.3	$y_{\text{L}} = \dfrac{4}{3}(2) + \dfrac{20}{3} = \dfrac{28}{3}$ $\text{L}\left(2\ ;\ \dfrac{28}{3}\right)$ and $\text{K}(2\ ;-4)$: $\text{LK} = \dfrac{28}{3} - (-4) = \dfrac{40}{3}$ Coordinates of P: $\dfrac{x+2}{2} = -4\dfrac{1}{2}$ and $\dfrac{y-4}{2} = -6$ $\therefore\ x = -11$ $y = -8$ $\therefore \text{P}(-11; -8)$ $\perp$height $(\text{PH}) = 2-(-11) = 13$ $\text{Area}\,\Delta\,\text{PKL} = \dfrac{1}{2}(\text{LK})(\text{PH})$ $= \dfrac{1}{2}\left(\dfrac{40}{3}\right)(13)$ $= \dfrac{260}{3}$ **OR** 86,67 square units  	✓ $y_{\text{L}} = \dfrac{28}{3}$ ✓ length of LK ✓ x_{P} ✓ y_{P} ✓ length of ⊥ height ✓ substitution into the area formula ✓ answer (7)

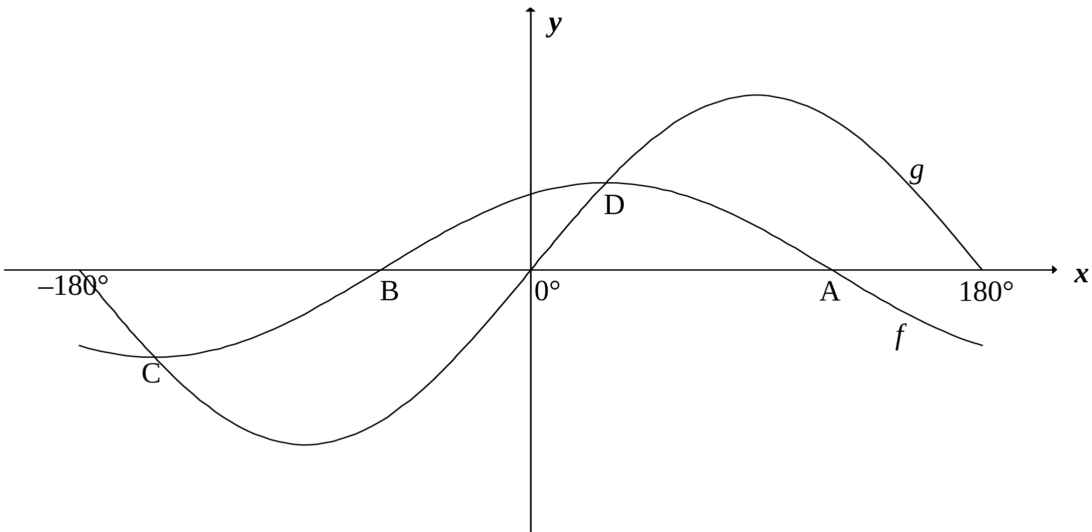
4.2.3	**OR** $y_L = \frac{4}{3}(2) + \frac{20}{3} = \frac{28}{3}$ $L\left(2\ ;\ \frac{28}{3}\right)$ and $K(2\ ;-4)$: $LK = \frac{28}{3} - (-4) = \frac{40}{3}$ <u>Coordinates of P:</u> $\frac{x+2}{2} = -4\frac{1}{2}$ and $\frac{y-4}{2} = -6$ $\therefore\ x = -11$ $y = -8$ $\therefore P(-11; -8)$  $PK^2 = (-11-2)^2 + (-8-(-4))^2$ $PK = \sqrt{185}$ units $m_{PK} = \frac{-8-(-4)}{-11-2} = \frac{4}{13}$ $\tan\theta = \frac{4}{13}$ $\therefore \theta = 17{,}1027...°$ $\therefore P\hat{K}L = 90° + 17{,}1027...° = 107{,}1°$ Area $\Delta PKL = \frac{1}{2}(PK)(LK).\sin P\hat{K}L$ $= \frac{1}{2}\left(\sqrt{185}\right)\left(\frac{40}{3}\right)\sin 107{,}10°$ = 86,67 square units	✓ $y_L = \frac{28}{3}$ ✓ length of LK ✓ x_P ✓ y_P ✓ $P\hat{K}L$ ✓ substitution into the area rule ✓ answer (7)
4.3	The centres of the two circles lie on the same vertical line $x = 2$. and the sum of the radii = 10 $n - 1 = 10$ or $1 - n = 10$ $n = 11$ $n = -9$ Answer only: full marks	✓ correct method ✓ sum of radii = 10 ✓ $n = 11$ ✓ $n = -9$ (4)
		[21]

QUESTION/*VRAAG* 5

5.1.1	$\sin 191°$ $= -\sin 11°$	✓ $-\sin 11°$ (1)
5.1.2	$\cos 22°$ $= \cos(2 \times 11°)$ $= 1 - 2\sin^2 11°$	✓ answer (1)
5.2	$\cos(x - 180°) + \sqrt{2}\sin(x + 45°)$ $= -\cos x + \sqrt{2}(\sin x \cos 45° + \cos x \sin 45°)$ $= -\cos x + \sqrt{2}\left(\sin x\left(\frac{1}{\sqrt{2}}\right) + \cos x\left(\frac{1}{\sqrt{2}}\right)\right)$ $= -\cos x + \sin x + \cos x$ $= \sin x$ **OR** $\cos(x - 180°) + \sqrt{2}\sin(x + 45°)$ $= -\cos x + \sqrt{2}(\sin x \cos 45° + \cos x \sin 45°)$ $= -\cos x + \sqrt{2}\left(\sin x\left(\frac{\sqrt{2}}{2}\right) + \cos x\left(\frac{\sqrt{2}}{2}\right)\right)$ $= -\cos x + \sin x + \cos x$ $= \sin x$	✓ $-\cos x$ ✓ expansion ✓ special angle ratios ✓simplification of last 2 terms ✓answer (5) ✓ $-\cos x$ ✓ expansion ✓ special angle ratios ✓simplification of last 2 terms ✓answer (5)
5.3	$\sin P + \sin Q = \sin P + \cos P$ $(\sin P + \cos P)^2 = \left(\frac{7}{5}\right)^2$ $\sin^2 P + 2\sin P \cos P + \cos^2 P = \frac{49}{25}$ $2\sin P \cos P = \frac{49}{25} - 1$ $\sin 2P = \left(\frac{49}{25} - \frac{25}{25}\right)$ $= \frac{24}{25}$	✓ $\sin Q = \cos P$ ✓ squaring ✓ expansion ✓ $\sin^2 P + \cos^2 P = 1$ ✓answer (5)
		[12]

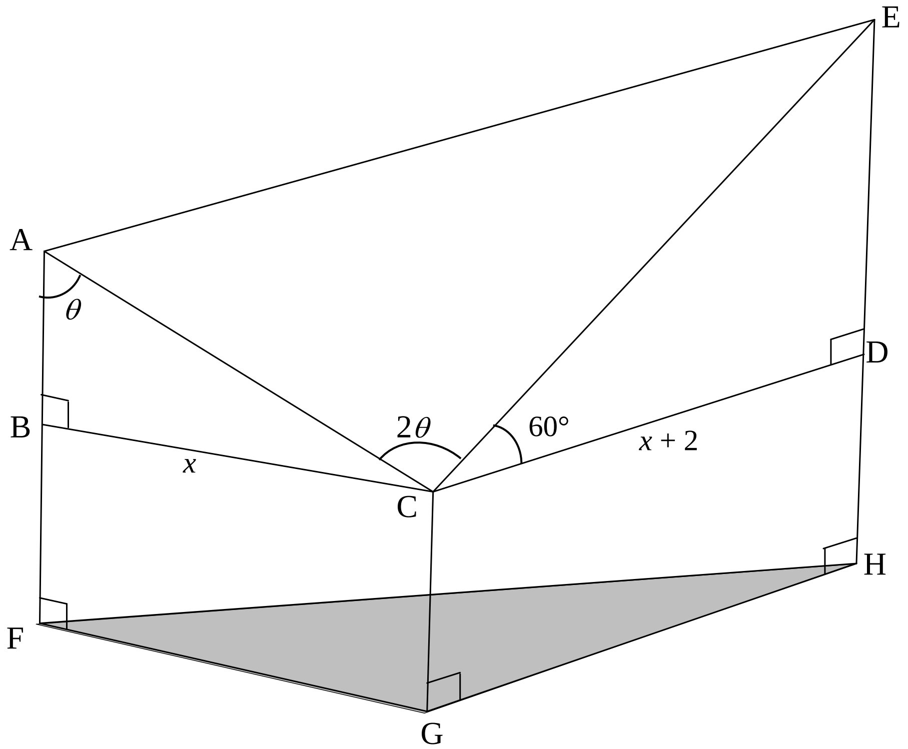
QUESTION/*VRAAG* 6

6.1	$\cos(x-30°) = 2\sin x$ $\cos x\cos 30° + \sin x\sin 30° = 2\sin x$ $\frac{\sqrt{3}}{2}\cos x + \frac{1}{2}\sin x = 2\sin x$ $\frac{\sqrt{3}}{2}\cos x = \frac{3}{2}\sin x$ $\tan x = \frac{\sqrt{3}}{3}$ $x = 30° + k.180°;\quad k \in Z$ **OR** $x = 30° + k.360°$ or $x = 210° + k.360°;\quad k \in Z$	✓ expansion ✓ special $\angle$s ✓ simplification ✓ equation in tan ✓ 30° ✓ $k.180°; k \in Z$ **OR** ✓ 30° and 210° ✓ $k.360°;\quad k \in Z$ (6)



6.2.1(a)	$A(120° ; 0)$	✓ answer (1)
6.2.1(b)	$C(-150° ; -1)$	✓ x value ✓ y value (2)
6.2.2(a)	$x \in (-90° ; 30°)$ OR $-90° < x < 30°$	✓ endpoints ✓ correct interval (2)
6.2.2(b)	$x \in (-160° ; 20°)$ OR $-160° < x < 20°$	✓endpoints ✓correct interval (2)
6.2.3	$y = 2^{2\sin x+3}$ Range of $y = 2\sin x$: $y \in [-2 ; 2]$ **OR** $-2 \le y \le 2$ Range of $y = 2\sin x + 3$: $y \in [1 ; 5]$ **OR** $1 \le y \le 5$ Range: $y = 2^{2\sin x+3}$: $y \in [2 ; 32]$ **OR** $2 \le y \le 32$ Answer only: full marks	✓ 1 ✓ 5 ✓ 2 ✓ 32 ✓ correct interval (5)
		[18]

QUESTION/*VRAAG* 7

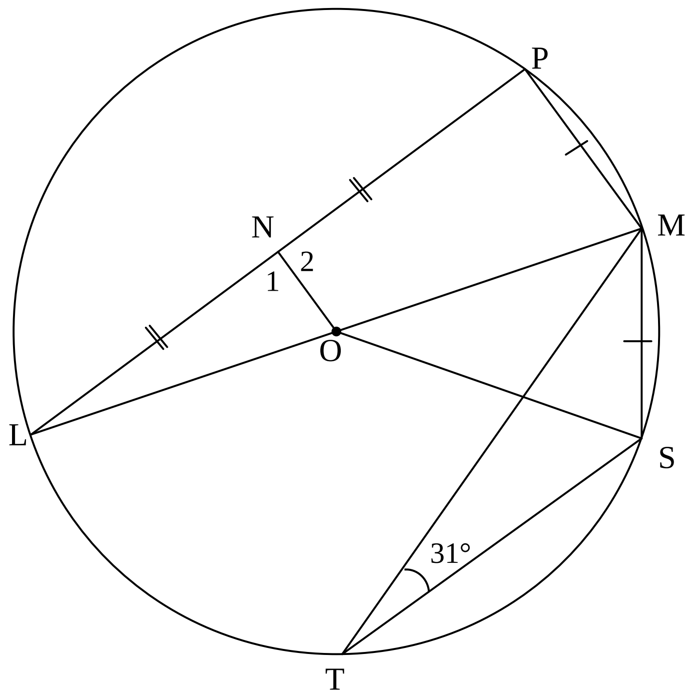


7.1.1	$\sin\theta = \frac{x}{AC}$ **OR** $\frac{\sin\theta}{x} = \frac{\sin 90°}{AC}$ $AC = \frac{x}{\sin\theta}$ $AC = \frac{x}{\sin\theta}$	✓ trig ratio ✓ simplification (2)
7.1.2	$\cos 60° = \frac{x+2}{CE}$ **OR** $\frac{\sin 30}{x+2} = \frac{\sin 90°}{CE}$ $CE = \frac{x+2}{\cos 60°}$ $CE = \frac{x+2}{\sin 30°}$ $= \frac{x+2}{\frac{1}{2}} = 2(x+2)$ $= 2(x+2)$	✓ trig ratio ✓ making CE the subject (2)
7.2	$\text{Area } \Delta ACE = \frac{1}{2} AC.EC.\sin A\hat{C}E$ $= \frac{1}{2}\left(\frac{x}{\sin\theta}\right)(2(x+2))\sin 2\theta$ $= \frac{x(x+2)\times 2\sin\theta\cos\theta}{\sin\theta}$ $= 2x(x+2)\cos\theta$	✓ use area rule correctly ✓ substitution of $\frac{x}{\sin\theta}(2(x+2))$ ✓ substitution of $\sin 2\theta$ (3)

7.3	EC = 2(12 + 2) = 28 $AE^2 = AC^2 + EC^2 - 2(AC)(EC)\cos A\hat{C}E$ $= \left(\frac{12}{\sin 55°}\right)^2 + 28^2 - 2\left(\frac{12}{\sin 55°}\right)(28)\cos 110°$ $AE = 35{,}77m$	✓ EC ✓ use cosine rule correctly ✓ substitution ✓ answer (4)
		[11]

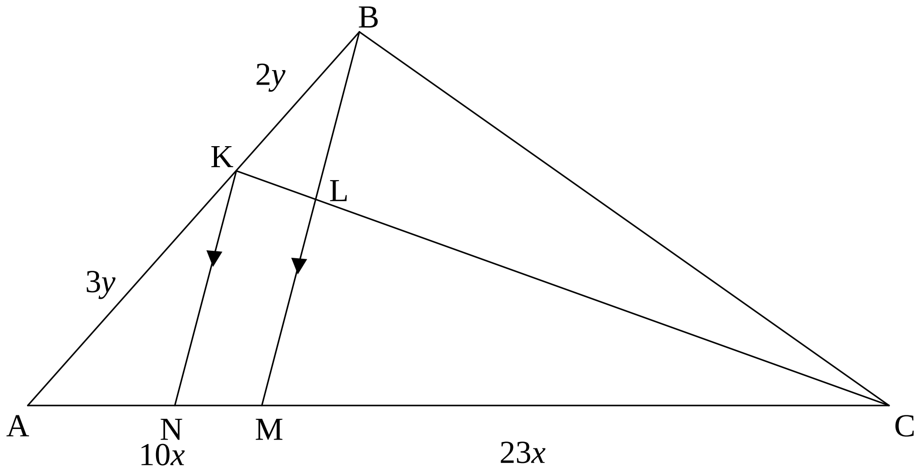
QUESTION/*VRAAG* 8

8.1



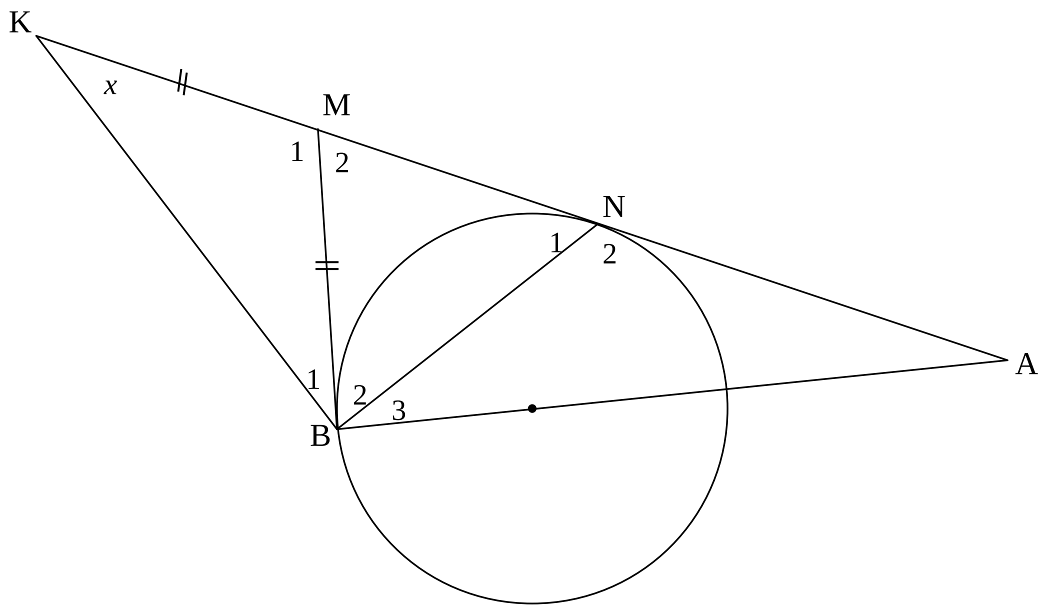
8.1.1(a)	$\hat{MOS} = 62°$ [∠ at centre = 2× ∠ at circumf/*middelpts∠ =2omtreks∠*]	✓ S ✓ R (2)
8.1.1(b)	$\hat{L} = 31°$ [equal chords; equal ∠s /= *koorde; = ∠e*]	✓ S ✓ R (2)
8.1.2	LN = NP and LO = OM $\therefore$ ON = $\frac{1}{2}$PM [midpoint theorem/*middelpuntstelling*] $\therefore$ ON = $\frac{1}{2}$MS [PM = MS] **OR** $\hat{N}_1 = 90°$ [line from centre to midpt chord/*lyn v midpt na midpt kd*] $\hat{P} = 90°$ [∠ in semi-circle/∠ *in halfsirkel*] $\hat{L}$ is common/*gemeen* $\therefore$ ΔNLO ||| Δ PLM (∠∠∠) $\frac{NL}{PL} = \frac{NO}{PM} = \frac{1}{2}$ $\therefore$ ON = $\frac{1}{2}$PM $\therefore$ ON = $\frac{1}{2}$MS [PM = MS]	✓ LO = OM ✓ S ✓ R ✓ S (4) ✓ S R ✓ S/R ✓ S ✓ S (4)

8.2



8.2.1	$\frac{AN}{AM}=\frac{AK}{AB}$ [line || one side of Δ **OR** prop theorem; KN ||BM/ *lyn || sy van Δ **OR** eweredigheidst; KN||BM*] $\frac{AN}{AM}=\frac{3y}{5y}=\frac{3}{5}$	✓ R ✓ S (2)
8.2.2	$\frac{AM}{MC}=\frac{10x}{23x}$ [given] $AM = 5y = 10x \quad \therefore \quad y = 2x$ $\frac{LC}{KL}=\frac{MC}{NM}$ [line || one side of Δ **OR** prop theorem; KN ||LM/ *lyn || sy van Δ **OR** eweredigheidst; KN||BM*] $=\frac{23x}{2y}=\frac{23x}{4x}=\frac{23}{4}$ **OR** $\frac{AM}{MC}=\frac{10x}{23x}$ [given] $\frac{AN}{MN}=\frac{3y}{2y}=\frac{6x}{4x}$ $\frac{LC}{KL}=\frac{MC}{NM}$ [line || one side of Δ **OR** prop theorem; KN ||LM/ *lyn || sy van Δ **OR** eweredigheidst; KN||BM*] $=\frac{23x}{2y}=\frac{23x}{4x}=\frac{23}{4}$	 ✓ S ✓ R ✓ S (3) ✓ S ✓ R ✓ S (3)
		[13]

QUESTION/*VRAAG* 9

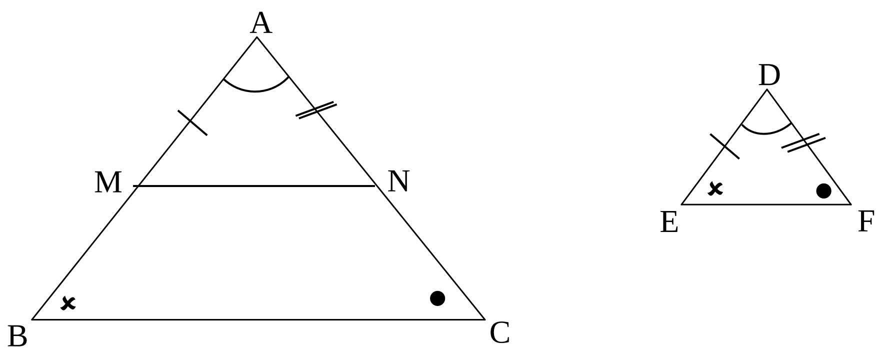


9.1	$\hat{B}_1 = x$ [$\angle$'s opp = sides/$\angle$*e teenoor = sye*] $\hat{M}_2 = 2x$ [ext $\angle$ of Δ] **OR** $\hat{M}_1 = 180° - 2x$ [$\angle$s of Δ] BM = MN [2 tans from a common point/*raaklyne vanuit dieselfde punt*] $\hat{N}_1 = \frac{180° - 2x}{2} = 90° - x$ [$\angle$'s opp = sides/$\angle$*e teenoor = sye*]	✓S ✓S ✓R ✓S ✓R ✓answer (6)
	OR NM = BM [2 tans from a common point/*raaklyne vanuit dieselfde punt*] $\hat{B}_2 = \hat{N}_1$ [$\angle$'s opp = sides/$\angle$*e teenoor = sye*] $\hat{B}_1 = x$ [$\angle$'s opp = sides/$\angle$*e teenoor = sye*] In Δ KBN: $x + x + \hat{B}_2 + \hat{N}_1 = 180°$ [sum of $\angle$'s of Δ] $2x + 2\hat{N}_1 = 180°$ $x + \hat{N}_1 = 90°$ $\hat{N}_1 = 90° - x$	✓S ✓R ✓S ✓R ✓S ✓answer (6)
9.2	$M\hat{B}A = \hat{B}_2 + \hat{B}_3 = 90°$ [tangent$\perp$diameter/*raaklyn$\perp$middellyn*] $\hat{B}_3 = 90° - \hat{B}_2$ $= 90° - (90° - x) = x$ $\hat{B}_3 = \hat{K} = x$ $\therefore$ AB is a tangent/*raaklyn* converse tan-chord theorem/ *omgekeerde raakl koordst*]]	✓S ✓ R ✓ S ✓ S ✓ R (5)

	OR $\hat{B}_2 = \hat{N}_1$ $\hat{B}_1 + \hat{B}_2 = x + (90° - x) = 90°$ $\therefore$ KN is diameter/*middellyn* [converse $\angle$ in semi-circle/ *omgekeerde* $\angle$ *in halfsirkel*] $M\hat{B}A = \hat{B}_2 + \hat{B}_3 = 90°$ [tangent $\perp$ diameter] $\therefore$ AB is a tangent/*raaklyn* converse tan-chord theorem/ *omgekeerde raakl koordst*]]	✓S ✓ R ✓ S ✓ R ✓ R (5)
		[11]

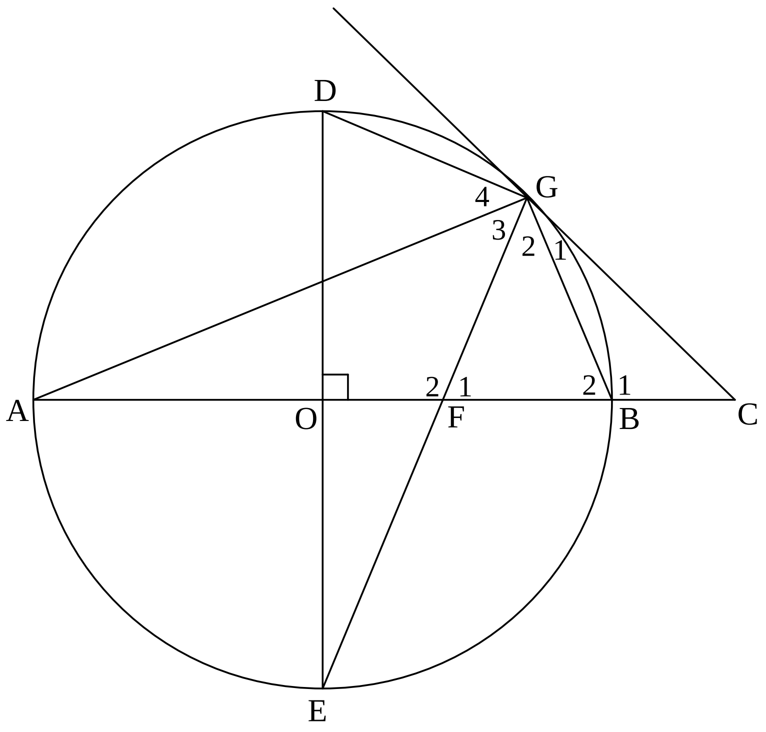
QUESTION/*VRAAG* 10

10.1



10.1	Constr: Let M and N lie on AB and AC respectively such that AM = DE and AN = DF. Draw MN. *Konst: Merk M en N op AB en AC onderskeidelik af sodanig dat AM = DE en AN = DF. Verbind MN.* Proof: In Δ AMN and Δ DEF AM = DE [Constr] AN = DF [Constr] $\hat{A} = \hat{D}$ [Given] $\therefore\ \Delta AMN \equiv \Delta DEF (SAS)$ $\therefore A\hat{M}N = \hat{E} = \hat{B}$ MN ∥ BC [corresp ∠'s are equal/*ooreenkomstige ∠e =*] $\frac{AB}{AM} = \frac{AC}{AN}$ [line ∥ one side of Δ **OR** prop theorem; MN ∥BC] $\therefore\ \frac{AB}{DE} = \frac{AC}{DF}$ [$AM = DE$ and $AN = DF$]	✓Constr / *Konstr* ✓ $\Delta AMN \equiv \Delta DEF$ ✓SAS ✓MN ∥ BC and R ✓ $\frac{AB}{AM} = \frac{AC}{AN}$ ✓R (6)

10.2



10.2.1(a)	$\hat{DOB}=90°$ $\hat{DGF}=\hat{G}_3+\hat{G}_4=90°$ [∠ in semi-circle/∠ *in halfsirkel*] $\hat{DOB}+\hat{DGF}=180°$ $\therefore$ DGFO is a cyclic quad. [converse: opp ∠s of cyclic quad/ *omgekeerde teenoorst ∠e v koordevh*] OR ∠s of quad = 180°/*∠e van koordevh = 180°*] **OR** $\hat{EOB}=90°$ $\hat{DGF}=\hat{G}_3+\hat{G}_4=90°$ [∠ in semi-circle/∠ *in halfsirkel*] $\hat{EOB}=\hat{DGF}$ $\therefore$ DGFO is a cyclic quad. . [converse: ext ∠ = opp int ∠/ *omgekeerde buite∠ = teenoorst ∠*] OR ext∠ of quad = opp int ∠/*buite∠ v vh = teenoorst ∠*]	✓ S ✓R ✓ R (3) ✓ S ✓R ✓ R (3)
10.2.1(b)	$\hat{F}_1=\hat{D}$ [ext ∠of cyclic quad/buite∠ v koordevh] $\hat{G}_1+\hat{G}_2=\hat{D}$ [tan-chord theorem/raakl koordst] $\therefore \hat{F}_1=\hat{G}_1+\hat{G}_2$ $\therefore$ GC = CF [sides opp equal ∠s/sye teenoor = ∠e]	✓ S ✓ R ✓ S ✓ R ✓ R (5)

10.2.2(a)	AB = DE = 14 [diameters/*middellyne*] $\therefore$ OB = 7 units $\therefore$ BC = OC – OB = 11 – 7 = 4 units Answer only: full marks	✓ S ✓ S ✓ S (3)
10.2.2(b)	In ΔCGB and ΔCAG $\hat{G}_1 = \hat{A} = x$ [tan-chord theorem/*raakl koordst*] $\hat{C} = \hat{C}$ [common] ΔCGB ||| ΔCAG [$\angle, \angle, \angle$] $\frac{CG}{CA} = \frac{CB}{CG}$ $\frac{CG}{18} = \frac{4}{CG}$ $CG^2 = 72$ $CG = \sqrt{72}$ or $6\sqrt{2}$ *or* 8,49 units	✓ S/R ✓ S ✓ S ✓ CA = 18 ✓answer (5)
10.2.2(c)	OF = OC – FC $= 11 - \sqrt{72}$ $\tan E = \frac{OF}{OE}$ $= \frac{11-\sqrt{72}}{7} = 0{,}36$ $\hat{E} = 19{,}76°$ **OR** OF = OC – FC $= 11 - \sqrt{72}$ $FE^2 = OE^2 + OF^2$ $= 7^2 + (11-\sqrt{72})^2$ FE = 7,437.. = 7,44 $\cos E = \frac{OE}{FE}$ *OR* $\sin E = \frac{OF}{FE}$ $= \frac{7}{7{,}44} = 0{,}94$ $= \frac{11-\sqrt{72}}{7{,}44} = 0{,}338$ $\hat{E} = 19{,}76°$ $\hat{E} = 19{,}76°$	✓ OF ✓ trig ratio ✓ substitution ✓ answer (4) ✓ OF ✓ trig ratio ✓ substitution ✓ answer (4)
		[26]

	TOTAL/*TOTAAL*:	**150**

basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE/ *NASIONALE SENIOR SERTIFIKAAT*

GRADE/*GRAAD* 12

MATHEMATICS P2/*WISKUNDE V2*

NOVEMBER 2018

MARKING GUIDELINES/*NASIENRIGLYNE*

MARKS/*PUNTE*: 150

These marking guidelines consist of 24pages.
Hierdie nasienriglyne bestaan uit 24 bladsye.

NOTE:

- If a candidate answers a question TWICE, only mark the FIRST attempt.
- If a candidate has crossed out an attempt of a question and not redone the question, mark the crossed out version.
- Consistent accuracy applies in ALL aspects of the marking memorandum. Stop marking at the second calculation error.
- Assuming answers/values in order to solve a problem is NOT acceptable.

NOTA:

- *As 'n kandidaat 'n vraag TWEE KEER beantwoord, sien slegs die EERSTE poging na.*
- *As 'n kandidaat 'n antwoord van 'n vraag doodtrek en nie oordoen nie, sien die doodgetrekte poging na.*
- *Volgehoue akkuraatheid word in ALLE aspekte van die nasienriglyne toegepas. Hou op nasien by die tweede berekeningsfout.*
- *Om antwoorde/waardes te aanvaar om 'n probleem op te los, word NIE toegelaat NIE.*

GEOMETRY • *MEETKUNDE*	
S	**A mark for a correct statement (A statement mark is independent of a reason)**
	'n Punt vir 'n korrekte bewering ('n Punt vir 'n bewering is onafhanklik van die rede)
R	**A mark for the correct reason (A reason mark may only be awarded if the statement is correct)**
	'n Punt vir 'n korrekte rede ('n Punt word slegs vir die rede toegeken as die bewering korrek is)
S/R	**Award a mark if statement AND reason are both correct**
	Ken 'n punt toe as die bewering EN rede beide korrek is

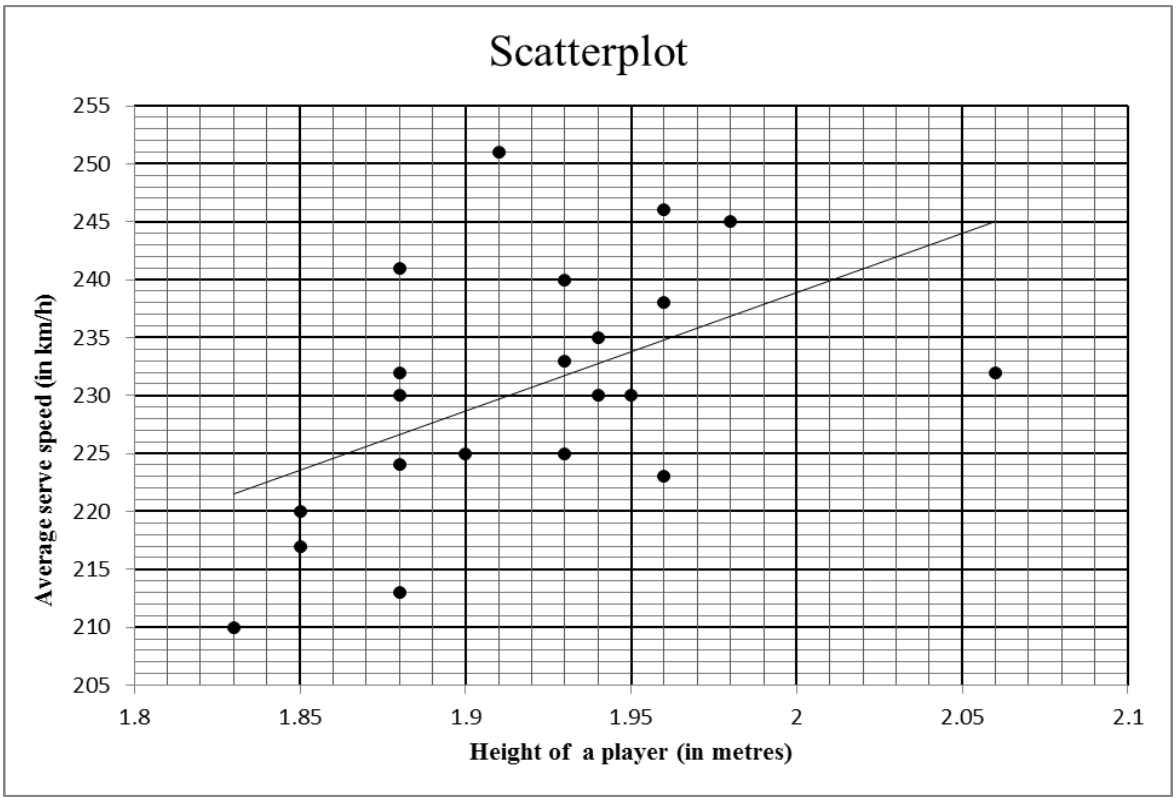
QUESTION/*VRAAG* 1

1.1.1	140 items	✓ answer (1)
1.1.2	Modal class/*modale klas:* $20 < x \leq 30$ minutes **OR/*OF*** $20 \leq x < 30$ minutes	✓ answer (1) ✓ answer (1)
1.1.3	Number of minutes taken = 20 minutes	✓ answer (1)
1.1.4	140 – 126 [Accept: 124 to 128] 14 orders (12 to 16) Answer only: Full marks	✓ 126 ✓ answer (2)
1.1.5	75th percentile is at 105 items =37 minutes [accept 36 – 38 minutes] Answer only: Full marks	✓ 105 ✓ answer (2)
1.1.6	Lower quartile is at 35 items =21,5 min [accept 21 – 23 min] IQR = 37 – 21,5 = 15,5 min [accept 13 – 17 min] Answer only: Full marks	✓ lower quartile (Q_1) ✓ answer (2)

35	70	75	80	80
90	100	100	105	105
110	110	115	120	125

1.2.1(a)	$\bar{x} = \frac{1420}{15}$ $= \text{R}94{,}666.. = R94{,}67$ Answer only: Full marks	✓ 1420 ✓ answer (2)
1.2.1(b)	$\sigma = R22{,}691... = \text{R}22{,}69$	✓✓ answer (2)
1.2.2(a)	They both collected the **same (equal) amount** in tips, i.e. R1 420 over the 15-day period. *Hulle albei het **dieselfde bedrag** met fooitjies ontvang, nl. R1 420 oor die 15 dae-tydperk*	✓ answer (1)
1.2.2(b)	Mary's standard deviation is smaller than Reggie's which suggests that there was **greater variation in the amount of tips that Reggie collected** each day compared to the number of tips that Mary collected each day. *Marie se standaardafwyking is kleiner as Reggie s'n wat beteken dat daar **groter variasie/verspreiding in die fooitjies was wat Reggie** elke dag **ontvang** het in vergelyking met die getal fooitjies wat Marie elke dag ontvang het.*	✓ explanation (1)
		[15]

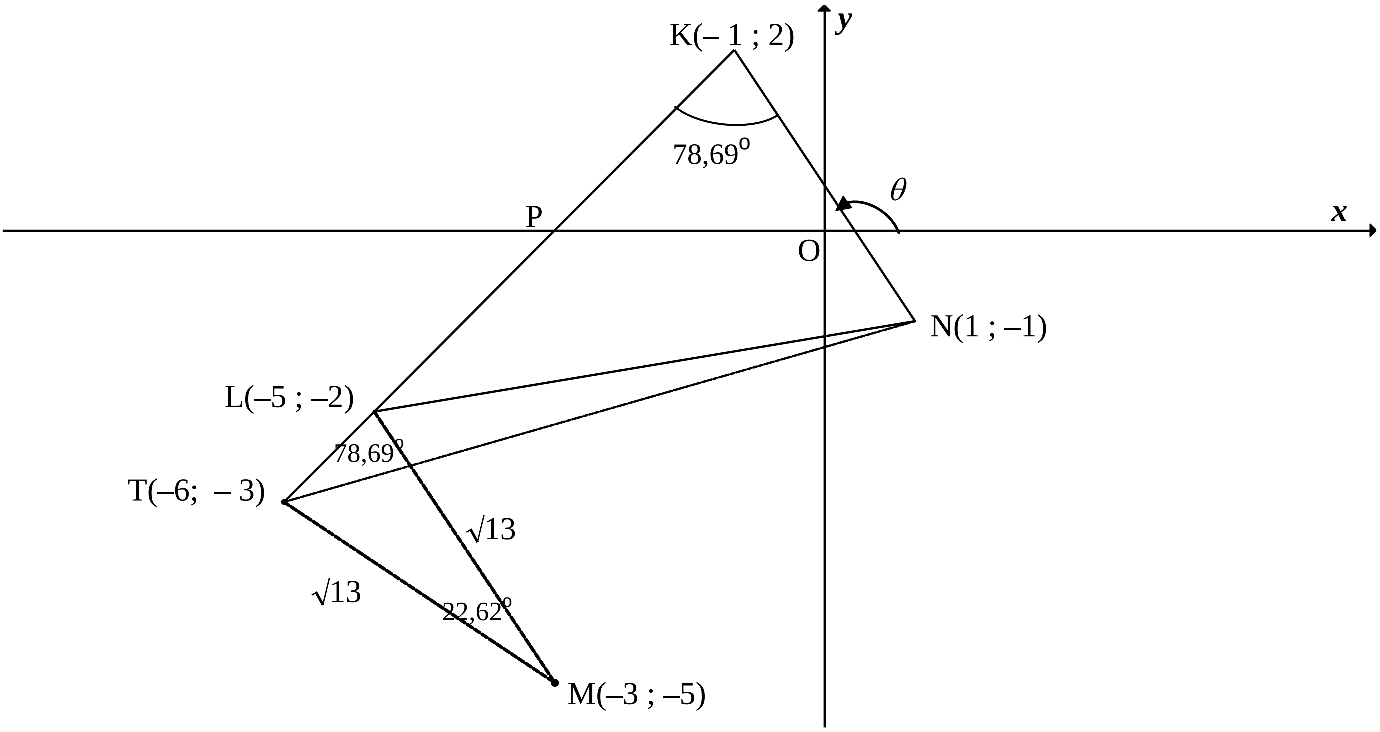
QUESTION/*VRAAG* 2



2.1	251 km/h	✓ answer (1)
2.2.1	$r = 0{,}52$ **OR** C	✓ answer (1)
2.2.2	The points are **fairly scattered** and the least squares regression line is increasing. *Die punte is **redelik verspreid** en die kleinstekwadrate-regressielyn neem toe.*	✓ reason (1)
2.3	There is a weak positive relation hence the height could have an influence *Daar is 'n swak positiewe verband, tog kan die lengte 'n invloed hê.* **OR/*OF*** There is no conclusive evidence that the height of a player will influence his/her tennis serve speed. *Daar is geen duidelike bewys dat die lengte van die speler sy/haar afslaanspoed kan beïnvloed nie.* **OR/*OF*** There is no conclusive evidence that a taller person will serve faster than a shorter person. *Daar is geen duidelike bewys dat 'n langer speler vinniger sal afslaan as 'n korter een nie.*	✓ answer (1) ✓ answer (1) ✓ answer (1)

2.4	For (0 ; 27,07), it means that the player has a height of 0 m but can serve at a speed of 27,07 km/h. **It is impossible for a person to have a height of 0 m**. *(0 ; 27,07) beteken dat 'n speler 'n lengte van 0* m *kan hê en teen 'n spoed van 27,07* km/h *kan afslaan.* ***Dit is onmoontlik om 'n lengte van 0* m *te hê.*** **OR/OF** This means that the **player does not exist and therefore cannot serve and have a serve speed.** *Dit beteken dat die* ***speler nie bestaan nie en daarom nie kan afslaan en 'n afslaanspoed hê nie.***	✓ explanation (1) ✓ explanation (1)
		[5]

QUESTION/*VRAAG* 3



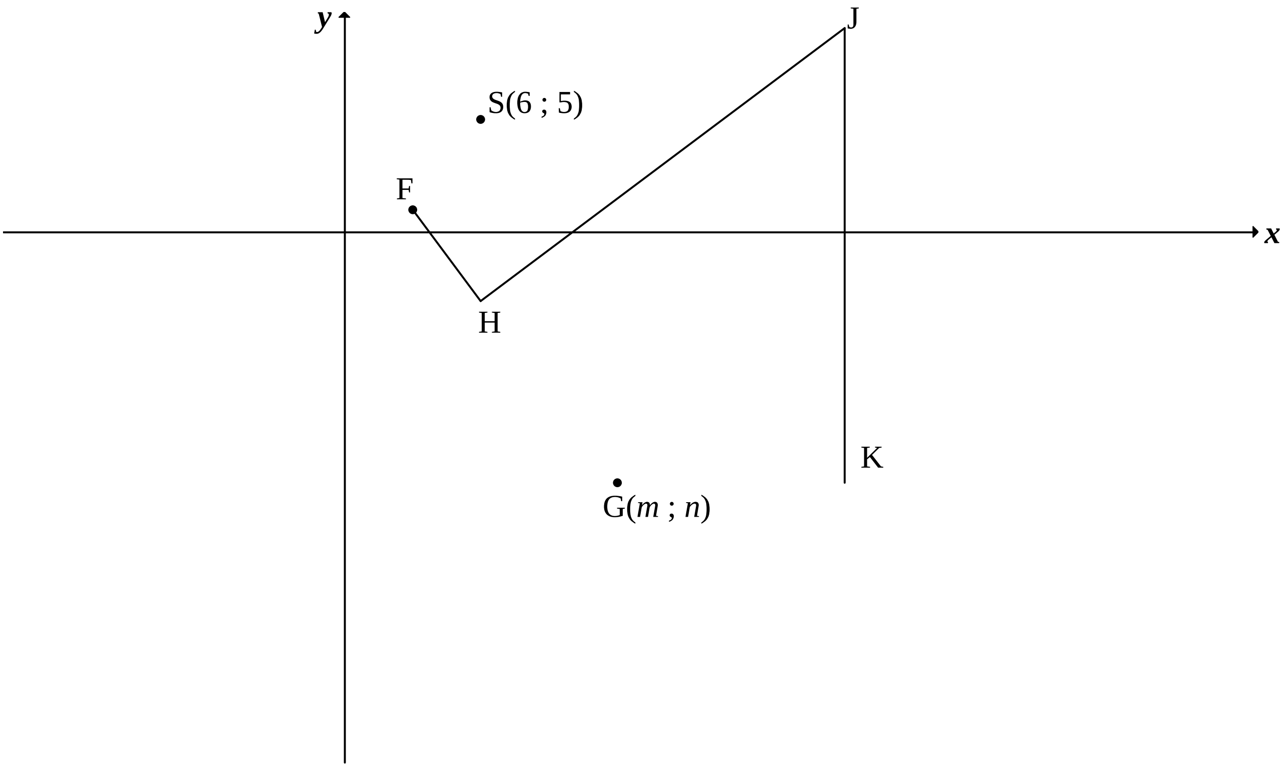
3.1.1	$m_{KN} = \frac{y_2 - y_1}{x_2 - x_1}$ $m_{KN} = \frac{2-(-1)}{-1-1}$ $= -\frac{3}{2}$ Answer only: Full marks	✓ correct substitution ✓ answer (2)
3.1.2	$\tan\theta = m_{KN} = -\frac{3}{2}$ $\theta = 180° - 56{,}31°$ $\theta = 123{,}69°$ Answer only: Full marks	✓ $\tan\theta = m_{KN} = -\frac{3}{2}$ ✓ answer (2)
3.2	Inclination KL = $123{,}69° - 78{,}69° = 45°$ [ext $\angle\Delta$] $\tan 45° = m_{KL} = 1$	✓ S ✓ $\tan 45° = m_{KL} = 1$ (2)
3.3	$y = x + c$ $2 = -1 + c$ $c = 3$ $y = x + 3$ **OR/*OF*** $y - y_1 = 1(x - x_1)$ $y - 2 = 1(x-(-1))$ $y = x + 3$	✓ substitute (– 1 ; 2) and m ✓ equation (2) ✓ substitute (– 1 ; 2) and m ✓ equation (2)

3.4	$\text{KN} = \sqrt{(1+1)^2 + (-1-2)^2}$ $\text{KN} = \sqrt{13}$ or 3,61 Answer only: Full marks	✓ substitute K and N into distance formula ✓ answer (2)
3.5.1	$(x+3)^2 + (y+5)^2 = 13$ ...(1) L is a point on KL $y = x + 3$ …(2) (2) in (1): $(x+3)^2 + (x+3+5)^2 = 13$ $x^2 + 6x + 9 + x^2 + 16x + 64 = 13$ $2x^2 + 22x + 60 = 0$ $x^2 + 11x + 30 = 0$ $(x+5)(x+6) = 0$ $x = -5$ or $x = -6$ $y = -2$ or $y = -3$ L(–5 ; –2) or (–6 ; –3)	✓ equation (1) ✓ substituting eq (2) ✓ standard form ✓ x-values ✓ y-values (5)
	OR/OF	
	$(x+3)^2 + (y+5)^2 = 13$ ...(1) L is a point on KL $y = x + 3 \quad \therefore x = y - 3$ …(2) (2) in (1): $(y-3+3)^2 + (y+5)^2 = 13$ $y^2 + y^2 + 10y + 25 = 13$ $2y^2 + 10y + 12 = 0$ $y^2 + 5y + 6 = 0$ $(y+2)(y+3) = 0$ $y = -2$ or $y = -3$ $x = -5$ or $x = -6$ L(–5 ; –2) or (–6 ; –3)	✓ equation (1) ✓ substituting eq (2) ✓ standard form ✓ y-values (both) ✓ x-values (both) (5)
3.5.2	Midpoint of KM: (– 2 ; – 1,5) $\therefore \frac{x_L + 1}{2} = -2$ and $\frac{y_L - 1}{2} = -\frac{3}{2}$ $\therefore$ L(– 5 ; – 2)	✓ midpoint of KM ✓x value ✓ y value (3)
	OR/*OF* $m_{KN} = m_{LM}$ $\frac{y-(-5)}{x-(-3)} = -\frac{3}{2}$ $2(x+3+5) = -3(x+3)$ $2x + 16 = -3x - 9$ $5x = -25$ $x = -5$ $\therefore$ L(– 5 ; – 2) Answer only: Full marks	✓ $m_{LM} = m_{KN}$ ✓x value ✓ y value (3)

	OR/*OF* N→M: $(x\,;y)\rightarrow(x-4\,;y-4)$ $\therefore$ L(– 1 – 4 ; 2 – 4) $\therefore$ L(– 5 ; – 2) **OR/OF** N→K: $(x\,;y)\rightarrow(x-2\,;y+3)$ $\therefore$ L(– 3 – 2 ; – 5 + 3) $\therefore$ L(– 5 ; – 2)	
	OR/*OF* N→M: $(x\,;y)\rightarrow(x-4\,;y-4)$ $\therefore$ L(– 1 – 4 ; 2 – 4) $\therefore$ L(– 5 ; – 2) **OR/OF** N→K: $(x\,;y)\rightarrow(x-2\,;y+3)$ $\therefore$ L(– 3 – 2 ; – 5 + 3) $\therefore$ L(– 5 ; – 2)	
	OR/*OF* N→M: $(x\,;y)\rightarrow(x-4\,;y-4)$ $\therefore$ L(– 1 – 4 ; 2 – 4) $\therefore$ L(– 5 ; – 2) **OR/OF** N→K: $(x\,;y)\rightarrow(x-2\,;y+3)$ $\therefore$ L(– 3 – 2 ; – 5 + 3) $\therefore$ L(– 5 ; – 2)	
	OR/*OF* N→M: $(x\,;y)\rightarrow(x-4\,;y-4)$ $\therefore$ L(– 1 – 4 ; 2 – 4) $\therefore$ L(– 5 ; – 2) **OR/OF** N→K: $(x\,;y)\rightarrow(x-2\,;y+3)$ $\therefore$ L(– 3 – 2 ; – 5 + 3) $\therefore$ L(– 5 ; – 2)	
	OR/*OF* N→M: $(x\,;y)\rightarrow(x-4\,;y-4)$ $\therefore$ L(– 1 – 4 ; 2 – 4) $\therefore$ L(– 5 ; – 2) **OR/OF** N→K: $(x\,;y)\rightarrow(x-2\,;y+3)$ $\therefore$ L(– 3 – 2 ; – 5 + 3) $\therefore$ L(– 5 ; – 2)	✓ transformation ✓*x* value ✓ *y* value (3)
3.6	T(–6 ; –3) (from Question 3.5.1) $\text{KT}=\sqrt{(-1-(-6))^2+(2-(-3))^2}$ $=\sqrt{50}$ $\text{KN}=\sqrt{13}$ (CA from 3.4) $\text{Area of }\Delta\text{KTN}=\frac{1}{2}\text{KT.KN}\sin\text{L}\hat{\text{K}}\text{N}$ $=\frac{1}{2}\sqrt{50}.\sqrt{13}\sin 78{,}69°$ = 12,50 square units	✓ coordinates of T ✓ length of KT ✓ substitution into area rule ✓ answer (4)

	OR/*OF*	
	In ΔKLM:	
	$\dfrac{\text{TL}}{\sin 22{,}62°} = \dfrac{\sqrt{13}}{\sin 78{,}69°}$	
	$\text{TL} = 1{,}414..$	✓ length of TL
	$\text{KL} = \sqrt{(-1-(-5))^2 + (2-(-2))^2}$	
	$= \sqrt{32}$	
	$\therefore$ KT = 7,0708…	✓ length of KT
	Area of $\Delta\text{KTN} = \dfrac{1}{2}\text{KT.KN}\sin \text{L}\hat{\text{K}}\text{N}$	
	$= \dfrac{1}{2}(7{,}0708).\sqrt{13}\sin 78{,}69°$	✓ substitution into area rule
	= 12,50 square units	✓ answer
		(4)
		[22]

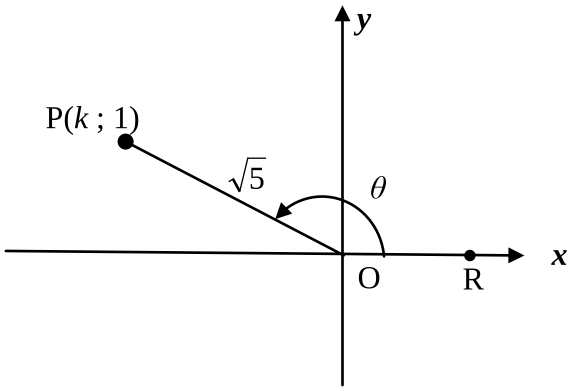
QUESTION/*VRAAG* 4



4.1	$F(3;1)$	✓x value ✓ y value (2)
4.2	$FS = \sqrt{(6-3)^2 + (5-1)^2}$ FS = 5	✓ substitution of F & S ✓ answer (2)
4.3	FH(FS) : HG = 1 : 2 $\therefore$ HG = 2 FH = 10	✓ HG = 10 (1)
4.4	Tangents from common/same point / *Raaklyne vanaf gemeenskaplike of dieselfde punt*	✓ answer (1)
4.5.1	$F\hat{H}J = 90°$ [tan $\perp$ radius / *rkl* $\perp$ *radius*] $FJ^2 = 20^2 + 5^2$ [Pyth theorem/*stelling*] FJ = $\sqrt{425}$ or $5\sqrt{17}$ or 20,62	✓ S ✓ R ✓ S ✓ answer (4)
4.5.2	$(x-m)^2 + (y-n)^2 = 100$	✓ answer (1)

4.5.3	K(22; n) [radius ⊥ tangent] GK = HG = 10 [radii] FH = FS = 5 [radii] $m = 22 - 10$ $m = 12$ F, H and G are collinear [HJ is a common tangent] *F, H en G is saamlynig* *[HJ is 'n gemeemskaplike raaklyn]* $FG^2 = (12-3)^2 + (n-1)^2$ $15^2 = 81 + (n-1)^2$ $(n-1)^2 = 144$ $n - 1 = \pm 12$ $n \neq 13$ or $n = -11$ $\therefore$ G(12; –11) **OR/*OF*** $n^2 - 2n - 143 = 0$ $(n+11)(n-13) = 0$ $n = -11$ or $n \neq 13$	✓ K(22; n) ✓ value of m ✓ subst. of F and G in distance formula ✓ FG = 15 ✓ simplification/ standard form ✓ value of n ✓ coordinates of G (7)
	OR/*OF*	
	K(22; n) [radius ⊥ tangent] GK = HG = 10 [radii] FH = FS = 5 [radii] $m = 22 - 10$ $m = 12$ Let J(22 ; y): $FJ^2 = (22-3)^2 + (y-1)^2$ $425 = 361 + y^2 - 2y + 1$ $0 = y^2 - 2y - 63$ $0 = (y-9)(y+7)$ $\therefore$ $y = 9$ or/of $y \neq -7$ $\therefore$ $n = 9 - 20 = -11$ $\therefore$ G(12; –11)	✓ K(22; n) ✓ value of m ✓ subst. of F and J in distance formula ✓ FJ = $\sqrt{425}$ ✓ standard form ✓ value of n ✓ coordinates of G (7)
		[18]

QUESTION/*VRAAG* 5



5.1.1	$k^2 = (\sqrt{5})^2 - 1^2$ $= 4$ $k = -2$ Answer only: full marks	✓ substitution into theorem of Pythagoras ✓ answer (2)
5.1.2(a)	$\tan\theta = -\frac{1}{2}$	✓answer (1)
5.1.2(b)	$\cos(180° + \theta) = -\cos\theta$ $= \frac{2}{\sqrt{5}}$ Answer only: full marks	✓ reduction ✓ answer (2)
5.1.2(c)	$\sin(\theta + 60°) = \frac{a+b}{\sqrt{20}}$ LHS$=\sin\theta\cos 60° + \cos\theta\sin 60°$ $= \left(\frac{1}{\sqrt{5}}\right)\left(\frac{1}{2}\right) + \left(-\frac{2}{\sqrt{5}}\right)\left(\frac{\sqrt{3}}{2}\right)$ $= \frac{1-2\sqrt{3}}{2\sqrt{5}}$ $= \frac{1-2\sqrt{3}}{\sqrt{20}}$	✓ expansion ✓ subst of sin θ ✓ subst of cos θ ✓ both special ∠s ✓ $\frac{1-2\sqrt{3}}{2\sqrt{5}}$ (5)
5.1.3	$\tan\theta = -\frac{1}{2}$ $\therefore \theta = 180° - 26{,}57°$ $\therefore \theta = 153{,}43°$ $\tan(2\theta - 40°) = \tan[(2 \times 153{,}43°) - 40°]$ $= \tan 266{,}87°$ $=18{,}3$	✓ θ ✓ substitution ✓ answer (3)

5.2	LHS $= \dfrac{\cos x+\sin x}{\cos x-\sin x} - \dfrac{\cos x-\sin x}{\cos x+\sin x}$ RHS $= 2\tan 2x$ $= \dfrac{(\cos x+\sin x)^2 - (\cos x-\sin x)^2}{(\cos x-\sin x)(\cos x+\sin x)}$ $= \dfrac{\cos^2 x+2\sin x\cos x+\sin^2 x-\cos^2 x+2\sin x\cos x-\sin^2 x}{\cos^2 x-\sin^2 x}$ $= \dfrac{2(2\sin x\cos x)}{\cos^2 x-\sin^2 x}$ $= \dfrac{2\sin 2x}{\cos 2x}$ $= 2\tan 2x$ $=$ RHS	✓ single fraction ✓ expansion ✓ simplification (both) ✓ double ∠ identity ✓ double ∠ identity (5)
	OR/*OF* LHS $= \dfrac{\cos x+\sin x}{\cos x-\sin x} - \dfrac{\cos x-\sin x}{\cos x+\sin x}$ RHS $= 2\tan 2x$ $= \dfrac{(\cos x+\sin x)^2 - (\cos x-\sin x)^2}{(\cos x-\sin x)(\cos x+\sin x)}$ $= \dfrac{(\cos x+\sin x+\cos x-\sin x)(\cos x+\sin x-\cos x+\sin x)}{\cos^2 x-\sin^2 x}$ $= \dfrac{(2\cos x)(2\sin x)}{\cos^2 x-\sin^2 x}$ $= \dfrac{2(2\sin x\cos x)}{\cos^2 x-\sin^2 x}$ $= \dfrac{2\sin 2x}{\cos 2x}$ $= 2\tan 2x$ $=$ RHS	✓ single fraction ✓ difference of two squares ✓ simplification (both) ✓ double ∠ identity ✓ double ∠ identity (5)
	OR/*OF* RHS $= 2\tan 2x$ $= \dfrac{2\sin 2x}{\cos 2x}$ $= \dfrac{2(2\sin x.\cos x)}{\cos^2 x-\sin^2 x}$ $= \dfrac{4\sin x.\cos x}{\cos^2 x-\sin^2 x}$ $= \dfrac{1+2\sin x.\cos x-(1-2\sin x.\cos x)}{\cos^2 x-\sin^2 x}$ $= \dfrac{(\cos x+\sin x)^2-(\cos x-\sin x)^2}{(\cos x+\sin x)(\cos x-\sin x)}$ $= \dfrac{(\cos x+\sin x)^2}{(\cos x+\sin x)(\cos x-\sin x)} - \dfrac{(\cos x-\sin x)^2}{(\cos x+\sin x)(\cos x-\sin x)}$ $= \dfrac{\cos x+\sin x}{\cos x-\sin x} - \dfrac{\cos x-\sin x}{\cos x+\sin x} =$ LHS	✓ double ∠ identity ✓ double ∠ identity ✓ identity & method ✓ factorising numerator and denominator ✓ writing as 2 terms (5)

<table>
<tr><td>5.3</td><td>

$\sum_{A=38^\circ}^{52^\circ} \cos^2 A$
$= \cos^2 38° + \cos^2 39° + \cos^2 40° + ... + \cos^2 51° + \cos^2 52°$
$= \sin^2 52° + \sin^2 51° + \sin^2 50° + ... + \cos^2 51° + \cos^2 52°$
$= 7(1) + \cos^2 45°$
$= 7 + \left(\frac{\sqrt{2}}{2}\right)^2$ or $= 7 + \left(\frac{1}{\sqrt{2}}\right)^2$
$= 7\frac{1}{2}$
OR/*OF*
$\sum_{A=38^\circ}^{52^\circ} \cos^2 A$
$= \cos^2 38° + \cos^2 39° + \cos^2 40° + ... + \cos^2 51° + \cos^2 52°$
$= (\cos^2 38° + \sin^2 52°) + (\cos^2 39° + \sin^2 51°)... + \cos^2 45°$
$= 7(1) + \cos^2 45°$
$= 7 + \left(\frac{\sqrt{2}}{2}\right)^2$ or $= 7 + \left(\frac{1}{\sqrt{2}}\right)^2$
$= 7\frac{1}{2}$

</td><td>

✓ expansion
✓ co ratio
✓ $\cos^2 45°$
✓ $7 \times$ identity
✓ answer
(5)
✓ expansion
✓ pairing
✓ $\cos^2 45°$
✓ $7 \times$ identity
✓ answer
(5)

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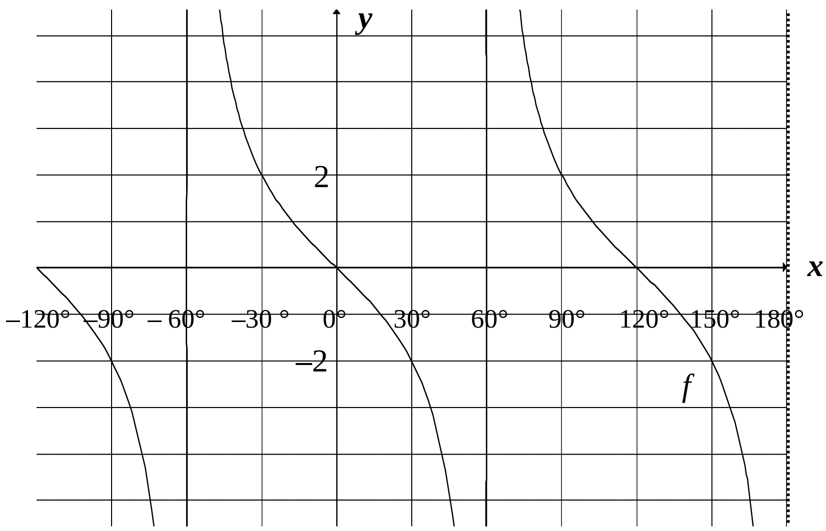
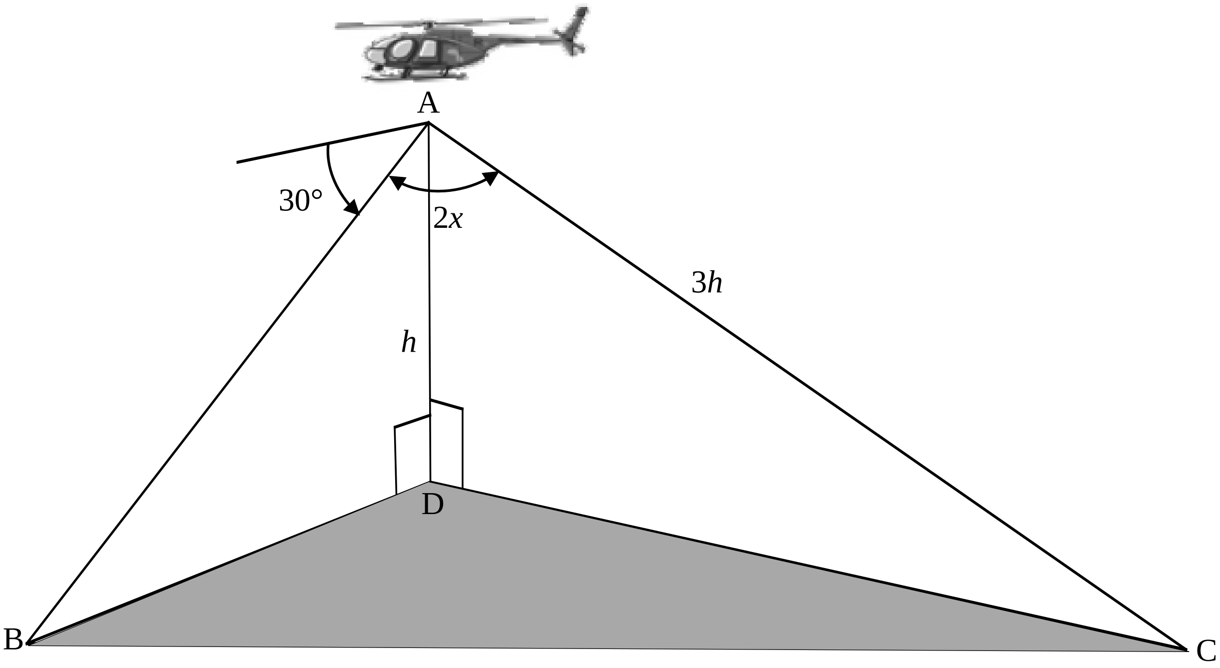
<tr><td></td><td></td><td>**[23]**</td></tr>
</table>

QUESTION/*VRAAG* 6

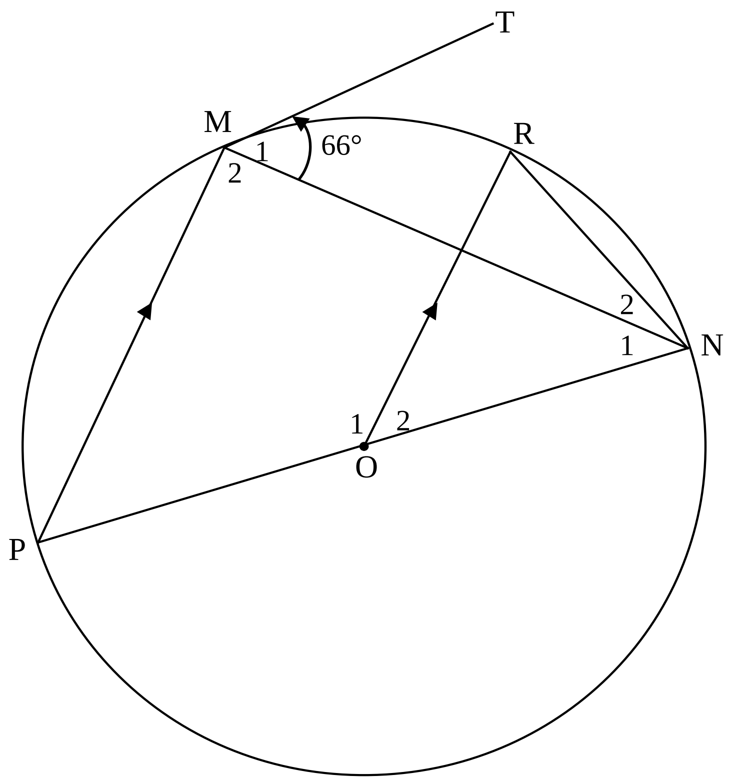
6.1	Period = 120°	✓ answer (1)
6.2	$2 = -2\tan\frac{3}{2}x$ $\tan\left(\frac{3}{2}t\right) = -1$ $\frac{3}{2}t = 135° + k.180°$ **OR/*OF*** $\frac{3}{2}t = -45° + k.180°$ $t = 90° + k.120°\ ;\ k \in Z$ $t = -30° + k.120°\ ;\ k \in Z$ **OR/*OF*** $2 = -2\tan\frac{3}{2}x$ $\tan\left(\frac{3}{2}t\right) = -1$ $\frac{3}{2}t = 135° + k.360°$ **o**r/of $\frac{3}{2}t = 315° + k.360°$ $t = 90° + k.240°$ **o**r.of $t = 210° + k.240°\ ;\ k \in Z$	✓ equating ✓ general solution of $\frac{3}{2}t$ ✓ general solution of t (3) ✓ equating ✓ general solution of $\frac{3}{2}t$ ✓ general solution of t (3)
6.3		✓asymptotes: $x = \pm 60°$; $x = 180°$ ✓x-intercepts 0°; ±120° ✓ negative shape ✓ (90° ; 2) or (− 30° ; 2) or (30° ; − 2) or (− 90° ; − 2) (4)
6.4	$x \in (-60°\ ;\ -30°]$ or $(60°\ ;\ 90°]$ **OR/*OF*** $-60° < x \le -30°$ or $60° < x \le 90°$	✓ interval ✓ interval ✓ notation (3) ✓ interval ✓ interval ✓ notation (3)
6.5	$g(x) = -2\tan\left[\frac{3}{2}(x+40°)\right] = f(x+40°)$ Translation of 40° to the left / skuif met 40° links	✓ Translation of 40° ✓ to the left (2)
		[13]

QUESTION/*VRAAG* 7



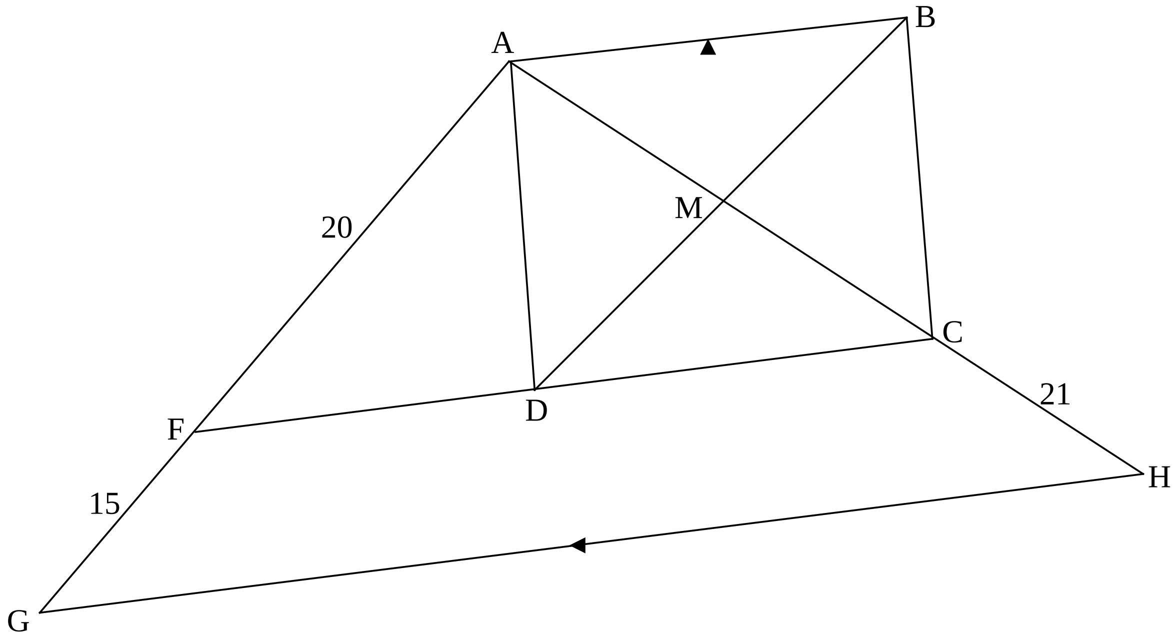
7.1	$A\hat{B}D = 30°$ $\sin 30° = \dfrac{h}{AB}$ $AB = \dfrac{h}{\sin 30°}$ **OR** $AB = \dfrac{h}{\frac{1}{2}}$ **OR** $AB = 2h$ **OR/*OF*** $B\hat{A}D = 60°$ $\cos 60° = \dfrac{h}{AB}$ $AB = \dfrac{h}{\cos 60°}$ **OR** $AB = \dfrac{h}{\frac{1}{2}}$ **OR** $AB = 2h$	✓ $A\hat{B}D = 30°$ ✓ answer (2) ✓ $B\hat{A}D = 60°$ ✓ answer (2)
7.2	$BC^2 = AB^2 + AC^2 - 2AB.AC\cos B\hat{A}C$ $= (2h)^2 + (3h)^2 - 2(2h)(3h)\cos 2x$ $= 13h^2 - 12h^2(2\cos^2 x - 1)$ $= 13h^2 - 24h^2\cos^2 x + 12h^2$ $= 25h^2 - 24h^2\cos^2 x$ $BC = h\sqrt{25 - 24\cos^2 x}$	✓ use of cosine rule in ΔABC ✓ substitution ✓ double angle identity ✓ $25h^2 - 24h^2\cos^2 x$ (4)
		[6]

QUESTION/*VRAAG* 8



8.1.1	$\hat{P} = \hat{M}_1 = 66°$ [tan chord theorem/*raaklyn koordst*]	✓S ✓R (2)
8.1.2	$\hat{M}_2 = 90°$ [$\angle$ in semi circle/$\angle$ *in halfsirkel*]	✓S ✓R (2)
8.1.3	$\hat{N}_1 = 180° - (90° + 66°)$ [sum of $\angle$s of /*som van* $\angle$*e* ΔMNP] $= 24°$	✓S (1)
8.1.4	$\hat{O}_2 = \hat{P} = 66°$ [corres. $\angle$s;/*ooreenk* $\angle$*e*, PM || OR]	✓S ✓R (2)
8.1.5	$\hat{R} + \hat{N}_1 + \hat{N}_2 = 180° - 66°$ [sum of $\angle$s of/*som van* $\angle$*e* ΔRNO] $= 114°$ $\hat{R} = \hat{N}_1 + \hat{N}_2 = 57°$ [$\angle$s opposite = radii/$\angle$*e teenoor = radii*] $\therefore \hat{N}_2 = 33°$ **OR/*OF*** $P\hat{O}R = 114°$ [$\angle$s on straight line/$\angle$*e op reguitlyn*] $P\hat{N}R = 57°$ [$\angle$ at centre = twice $\angle$ at circumference/ *midpts$\angle$ = 2 × omtreks$\angle$*] $\therefore \hat{N}_2 = 33°$	✓S ✓S/R ✓S (3) ✓S ✓S/R ✓S (3)

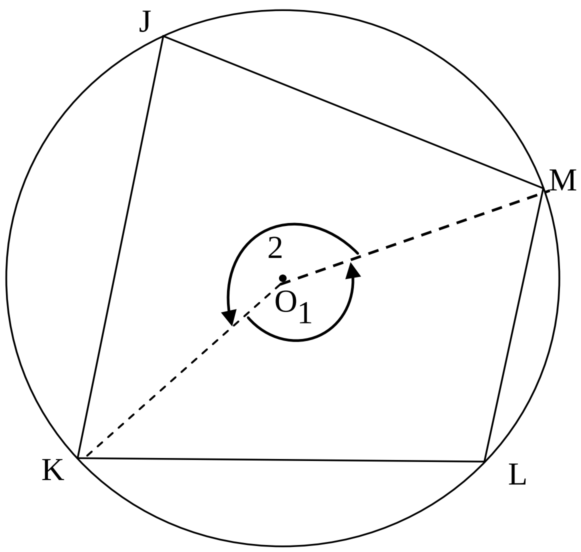
8.2



8.2.1	FC ∥ AB ∥ GH [opp sides of **rectangle**/*teenoorst sye v reghoek*]	✓ R (1)
8.2.2	$\frac{AC}{CH} = \frac{AF}{FG}$ [line ∥ one side of Δ] **OR** [prop theorem; FC ∥ GH] *[lyn ∥ een sy van Δ]* ***OF*** *[eweredighst; FC ∥ GH]* $\frac{AC}{21} = \frac{20}{15}$ $AC = \frac{20 \times 21}{15}$ $= 28$ DB = AC = 28 [diags of rectangle =/*hoeklyne v reghoek* =] $DM = \frac{1}{2}DB = 14$ [diags of rectangle bisect/*hoekl v reghoek halveer*]	✓S ✓R ✓ AC ✓ S ✓ S (5)
		[16]

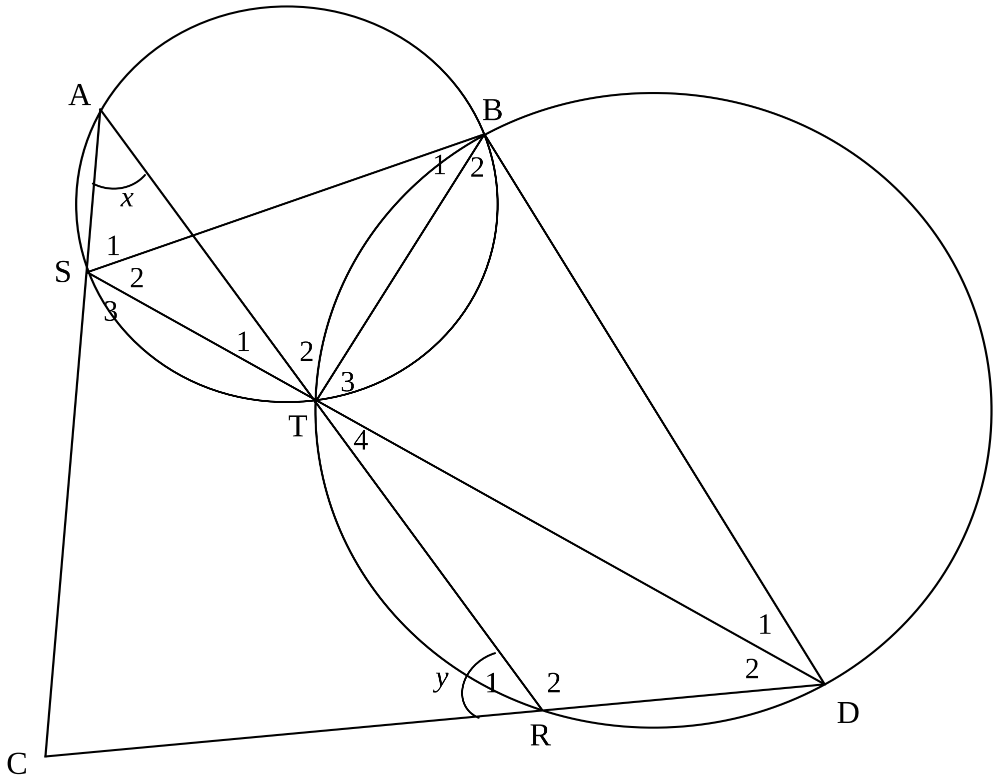
QUESTION/*VRAAG* 9

9.1



9.1	Constr/*Konstr*.: Draw KO and MO/*Trek KO en MO* Proof:		✓ construction
	$\hat{O}_1 = 2\hat{J}$	[∠ at centre = twice ∠ at circumference] *[midpts∠ = 2 × omtreks∠]*	✓ S/R
	$\hat{O}_2 = 2\hat{L}$	[∠ at centre = twice ∠ at circumference]	✓ S
	$\hat{O}_1 + \hat{O}_2 = 360°$	[∠s around a point /*∠e om 'n punt*]	✓ S/R ✓ S
	$\therefore 2\hat{J} + 2\hat{L} = 360°$ $\therefore 2(\hat{J} + \hat{L}) = 360°$ $\therefore \hat{J} + \hat{L} = 180°$		(5)
	OR/*OF*		
	Constr/*Konstr*.: Draw KO and MO/*Trek KO en MO* Proof:		✓ construction
	Let $\hat{J} = x$ $\hat{O}_1 = 2x$	[∠ at centre = twice ∠ at circumference] *[midpts∠ = 2 × omtreks∠]*	✓ S ✓ R
	$\hat{O}_2 = 360° - 2x$	[∠s around a point /*∠e om 'n punt*]	✓ S/R
	$\therefore \hat{L} = 180° - x$	[∠ at centre = twice ∠ at circumference]	✓ S
	$\therefore \hat{J} + \hat{L} = 180°$		(5)

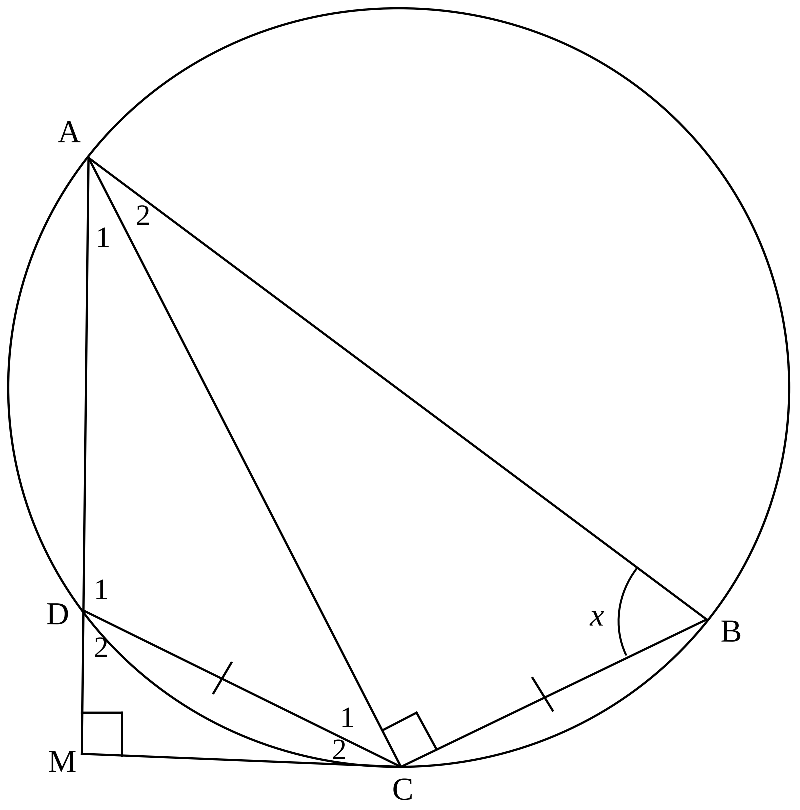
9.2



9.2.1(a)	$\hat{B}_1 = x$ [$\angle$s in same seg/$\angle$*e in dieselfde segm*]	✓ S ✓ R (2)
9.2.1(b)	$\hat{B}_2 = y$ [ext $\angle$ of cyclic quad/*buite*$\angle$ *koordevh*]	✓ S ✓ R (2)
9.2.2	$\hat{C} = 180° - (x + y)$ [sum of $\angle$s of/*som v* $\angle$*e*, Δ ACR] $S\hat{B}D + \hat{C} = x + y + 180° - (x + y)$ $S\hat{B}D + \hat{C} = 180°$ SCDB is a cyclic quad [converse opp angles of cyclic quad] [*omgekeerde teenoorst* $\angle$*e koordevh*] **OR/*OF*** $\hat{S}_1 = \hat{T}_2$ [$\angle$s in same segment/$\angle$*e in dies. segment*] $\hat{T}_2 = \hat{D}_1 + \hat{D}_2 = B\hat{D}R$ [ext $\angle$ of cyc quad/*buite*$\angle$ *koordevh*] $\therefore\ \hat{S}_1 = B\hat{D}R$ $\therefore$ SCDB is cyc quad [ext $\angle$ of quad = opp $\angle$/*buite*$\angle$ = *tos* $\angle$]	✓ S ✓ S ✓ R (3) ✓ S ✓ S ✓ R (3)

9.2.3	$\hat{T}_4 = y - 30°$ [ext $\angle$ of/*buite* $\angle$ ΔTDR] $\hat{T}_1 = y - 30°$ [vert opp $\angle$s =/*regoorst* $\angle$*e* =] $y - 30° + x + 100° = 180°$ [sum of $\angle$s of/*som v* $\angle$*e*, ΔAST] $\therefore x + y = 110°$ $S\hat{B}D = 110°$ $\therefore$ SD not diameter [line does not subtend 90° $\angle$] *SD nie 'n middellyn [lyn onderspan nie 90°$\angle$]* **OR/*OF*** $A\hat{S}T = \hat{C} + \hat{D}_2$ [ext $\angle$ of/*buite* $\angle$ ΔSCD] $\hat{C} = 100° - 30° = 70°$ $S\hat{B}D = 180° - 70°$ [opp $\angle$s cyclic quad/ teenoorst $\angle$e kdvh] $= 110°$ $\therefore$ SD not diameter [line does not subtend 90° $\angle$] *SD nie 'n middellyn [lyn onderspan nie 90°$\angle$]*	✓ S ✓ S ✓ S ✓ R (4) ✓ S ✓ S ✓ S ✓ R (4)
		[16]

QUESTION/*VRAAG* 10



10.1.1	$\hat{A}_2 = \hat{A}_1 = 90° - x$ [= chords subtend = ∠s = *kde onderspan* =∠*e*] $\hat{D}_2 = x$ [exterior angle of cyclic quad/*buite∠ koordevh.*] $\therefore \hat{C}_2 = 90° - x$ [sum of ∠s of/*som v ∠e*, ΔDCM] $\therefore \hat{C}_2 = \hat{A}_1 = 90° - x$ ∴ MC is a tangent to the circle at C [converse: tan chord th] *MC is 'n raaklyn by C [omgekeerde raakl koordst]* **OR/*OF*** $\hat{A}_2 = \hat{A}_1 = 90° - x$ [= chords subtend = ∠s/ = *kde onderspan* =∠*e*] $\hat{C}_1 + \hat{C}_2 = x$ [sum of ∠s of/*som v ∠e*, ΔACM] $\therefore \hat{C}_1 + \hat{C}_2 = \hat{B} = x$ ∴ MC is a tangent to the circle at C [converse : tan chord th] *MC is 'n raaklyn by C [omgekeerde raakl koordst]* **OR/*OF*** In ΔAMC and ΔACB: $\hat{A}_2 = \hat{A}_1 = 90° - x$ [= chords subtend = ∠s/ = *kde onderspan* =∠*e*] $A\hat{M}C = A\hat{C}B = 90°$ [given] $\therefore \hat{C}_1 + \hat{C}_2 = \hat{B} = x$	✓ S ✓R ✓ S/R ✓ $\hat{C}_2 = 90° - x$ ✓ R (5) ✓ S ✓ R ✓✓ $\hat{C}_1 + \hat{C}_2 = x$ ✓ R (5) ✓ S ✓ R ✓✓ $\hat{C}_1 + \hat{C}_2 = x$

	$\therefore$ MC is a tangent to the circle at C [converse : tan chord th] *MC is 'n raaklyn by C [omgekeerde raakl koordst]*	✓ R (5)
	In ΔACB and/*en* ΔCMD $\hat{B} = \hat{D}_2 = x$ [proved **OR** exterior $\angle$ of cyclic quad.] *[bewys **OF** buite $\angle$v koordevh]* $\hat{A}_2 = \hat{C}_2 = 90° - x$ [proved **OR** sum of $\angle$s in Δ] [*Bewys **OF** som v $\angle$e in Δ*] $\Delta ACB \parallel\mid \Delta CMD$ [$\angle, \angle, \angle$] **OR/*OF*** In ΔACB and/*en* ΔCMD $\hat{B} = \hat{D}_2 = x$ [proved **OR** exterior $\angle$ of cyclic quad.] *[bewys **OF** buite $\angle$v koordevh]* $A\hat{C}B = A\hat{M}C = 90°$ [given/*gegee*] $\Delta ACB \parallel\mid \Delta CMD$ [$\angle, \angle, \angle$] **OR/*OF*** In ΔACB and/*en* ΔCMD $\hat{B} = \hat{D}_2 = x$ [proved **OR** exterior $\angle$ of cyclic quad] *[bewys **OF** buite $\angle$v koordevh]* $\hat{A}_2 = \hat{C}_2 = 90° - x$ [proved **OR** sum of $\angle$s in Δ] *[Bewys **OF** som v $\angle$e in Δ]* $A\hat{C}B = A\hat{M}C = 90°$ [given **OR** sum of $\angle$s in Δ] *[gegee **OF** som v $\angle$e in Δ]* $\Delta ACB \parallel\mid \Delta CMD$	✓ S ✓ S ✓ R (3) ✓ S ✓ S ✓ R (3) ✓ S ✓ S ✓ S (3)
10.2.1	$\frac{BC}{MD} = \frac{AB}{DC}$ [$\Delta ACB \parallel\mid \Delta CMD$] $\frac{DC}{MD} = \frac{AB}{DC}$ [BC = DC] $\therefore DC^2 = AB \times MD$ In ΔAMC and/*en* ΔCMD $\hat{M}$ is common/*gemeen* $\hat{A}_1 = \hat{C}_2$ [tan chord th /*raaklyn koordst*] **OR/*OF*** $\hat{C}_1 + \hat{C}_2 = \hat{B} = \hat{D} = x$ [tan chord th /*raaklyn koordst* **OR/*OF*** exterior $\angle$ of cyclic quad/ *buite* $\angle$*v kdvh*] $\Delta AMC \parallel\mid \Delta CMD$ [$\angle, \angle, \angle$] $\frac{AM}{CM} = \frac{CM}{MD}$ $\therefore CM^2 = AM \times MD$ $\therefore \frac{CM^2}{DC^2} = \frac{AM \times MD}{AB \times MD}$ $= \frac{AM}{AB}$	✓ $\frac{BC}{MD} = \frac{AB}{DC}$ ✓ $DC^2 = AB \times MD$ ✓ S ✓ S ✓ $CM^2 = AM \times MD$ ✓ $\frac{AM \times MD}{AB \times MD}$ (6)

	OR/*OF* $\frac{AC}{MC}=\frac{AB}{DC}$ [$\Delta ACB \parallel\parallel \Delta CMD$] $\therefore CM\times AB = AC\times DC$ In ΔAMC and/*en* ΔACB $\hat{C}=\hat{M}=90°$ [given] $\hat{A}_1=\hat{A}_2$ [proven] **OR/*OF*** $A\hat{C}M=\hat{B}=x$ [proven] $\Delta AMC \parallel\parallel \Delta ACB$ [$\angle, \angle, \angle$] $\frac{AC}{AM}=\frac{BC}{MC}$ $\therefore AC\times MC = AM\times BC$ $\therefore AC=\frac{BC.AM}{MC}$ $CM\times AB=\frac{BC.AM}{MC}\times DC$ $CM^2=\frac{DC.AM}{AB}\times DC$ [$BC=DC$] $\frac{CM^2}{DC^2}=\frac{AM}{AB}$	✓ $\frac{AC}{MC}=\frac{AB}{DC}$ ✓ S ✓ S ✓ $AC.MC=AM.BC$ ✓ equating ✓ S (6)
10.2.2	In ΔDMC: $\frac{CM}{DC}=\sin x$ $\frac{CM^2}{DC^2}=\sin^2 x$ $\frac{AC}{AB}=\frac{CM}{DC}$ $\therefore \frac{AM}{AB}=\sin^2 x$ **OR/*OF*** In ΔABC: $\sin x=\frac{AC}{AB}$ In ΔAMC: $\sin x=\frac{AM}{AC}$ $\sin x.\sin x=\frac{AC}{AB}\times\frac{AM}{AC}=\frac{AM}{AB}$	✓ trig ratio ✓ square both sides (2) ✓ 2 equations for $\sin x$ ✓ product (2)
		[16]

TOTAL/TOTAAL: 150

basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE/ *NASIONALE SENIOR SERTIFIKAAT*

GRADE/*GRAAD* 12

MATHEMATICS P2/*WISKUNDE V2*

FEBRUARY/MARCH/*FEBRUARIE/MAART* 2018

MARKING GUIDELINES/*NASIENRIGLYNE*

MARKS/*PUNTE*: 150

These marking guidelines consist of 22 pages./
Hierdie nasienriglyne bestaan uit 20 bladsye.

NOTE:

- If a candidate answers a question TWICE, only mark the FIRST attempt.
- If a candidate has crossed out an attempt of a question and not redone the question, mark the crossed out version.
- Consistent accuracy applies in ALL aspects of the marking guidelines. Stop marking at the second calculation error.
- Assuming answers/values in order to solve a problem is NOT acceptable.

NOTA:

- *As 'n kandidaat 'n vraag TWEE KEER beantwoord, merk slegs die EERSTE poging.*
- *As 'n kandidaat 'n antwoord van 'n vraag doodtrek en nie oordoen nie, merk die doodgetrekte poging.*
- *Volgehoue akkuraatheid word in ALLE aspekte van die nasienriglyne toegepas. Hou op nasien by die tweede berekeningsfout.*

Aanvaar van antwoorde/waardes om 'n probleem op te los, word NIE toegelaat nie.

GEOMETRY	
S	**A mark for a correct statement** **(A statement mark is independent of a reason.)**
	'n Punt vir 'n korrekte bewering **('n Punt vir 'n bewering is onafhanklik van die rede.)**
R	**A mark for a correct reason** **(A reason mark may only be awarded if the statement is correct.)**
	'n Punt vir 'n korrekte rede **('n Punt word slegs vir die rede toegeken as die bewering korrek is.)**
S/R	**Award a mark if the statement AND reason are both correct.**
	Ken 'n punt toe as beide die bewering EN rede korrek is.

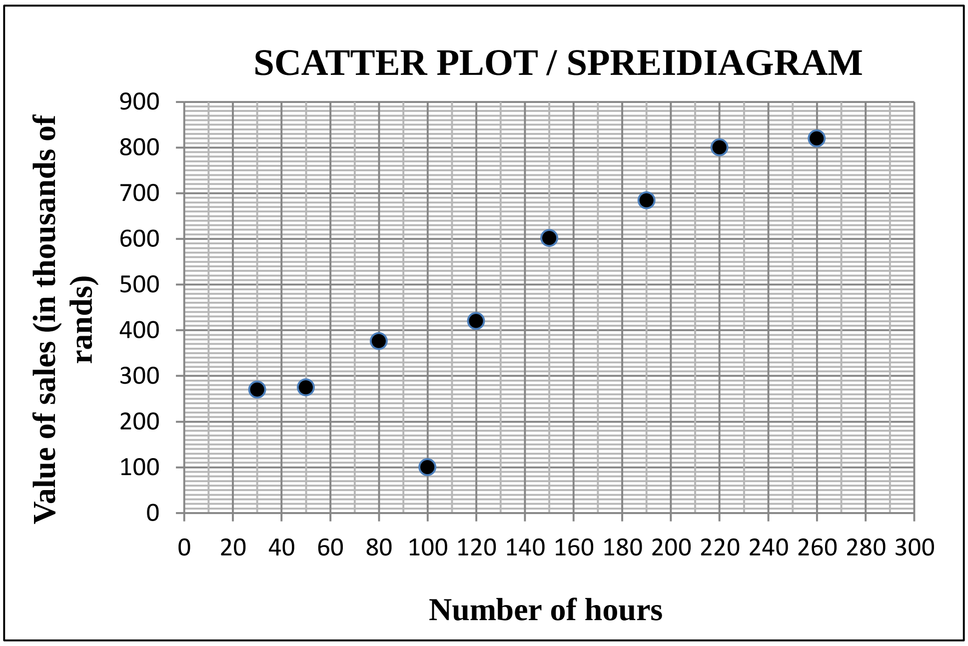
QUESTION/*VRAAG* 1

Days/*Dae*	1	2	3	4	5	6	7	8	9	10
Units of blood/ *Eenhede bloed*	45	59	65	73	79	82	91	99	101	106

1.1.1	$\bar{x} = \frac{800}{10}$ $= 80$ Answer only: full marks	✓ 800 (addition of units) ✓ answer (CA if ÷ 10) (2)
1.1.2	$\sigma = 18{,}83$ No penalty for rounding	✓✓ answer (A) (2)
1.1.3	$(61{,}17\,;98{,}83)$ Days 1, 2, 8, 9 and 10 lie outside 1 standard deviation from the mean $\therefore$ 5 days Correct answer only: full marks provided that 1.1.1. & 1.1.2 both correct	✓ mean – 1 SD ✓ mean + 1 SD ✓ answer (3)
1.2.1	Skewed to the left or negatively skewed/ *Skeef na links of negatief skeef*	✓ answer (1)
1.2.2	A = 65 B = 99 Answers without labelling: 1/2	✓ answer ✓ answer (2)
1.3	New total = 95 × 10 = 950 $\therefore$ Units not counted = 950 – 800 = 150	✓ answer (CA from 1.1.1) (1) **[11]**

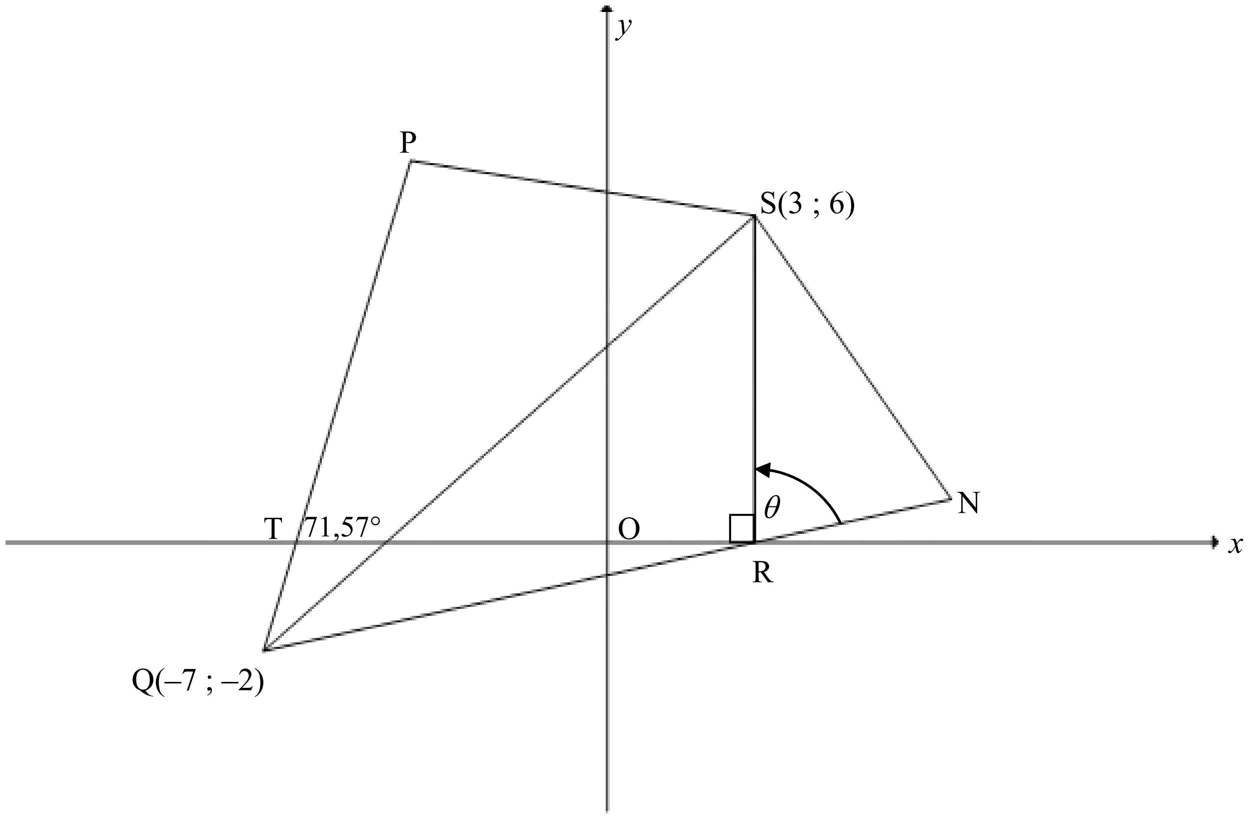
QUESTION/*VRAAG* 2

Number of hours Aantal uur	30	50	80	100	120	150	190	220	260
Value of sales (in thousands of rands) Waarde van verkope (in duisend rand)	270	275	376	100	420	602	684	800	820

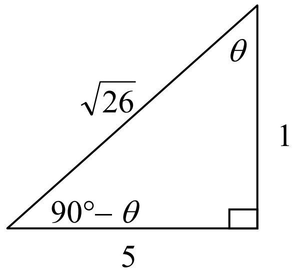
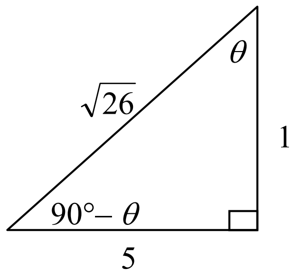


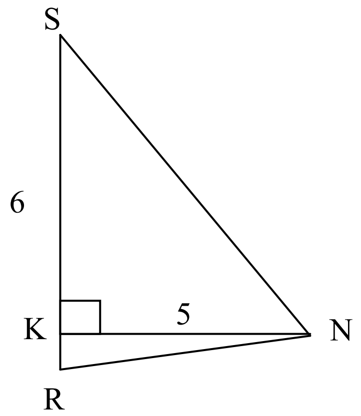
2.1	Outlier/*Uitskieter*: (100 ; 100) accept: 100 as answer	✓ answer (1)
2.2	$a = 94{,}50273\ldots$ $b = 2{,}913729\ldots$ $\hat{y} = 94{,}50 + 2{,}91x$ Integral values: max 2/3 Swopped *a* and *b*: 2/3	✓ value of *a* ✓ value of *b* ✓ equation (3)
2.3	$\hat{y} = 2{,}91(240) + 94{,}50$ (CA from 2.1) $= 792{,}90$ Value = R793 000 **OR/*OF*** $\hat{y} = 793{,}7978142$ (calculator) Value = R794 000 Penalise 1 mark if answer not in thousands of Rands	✓ substitution ✓ answer in thousands of Rands (2) ✓✓ answer in thousands of Rands (2)
2.4	$b = 2{,}913729\ldots$ $\therefore$ R2 914 **OR/*OF*** R2 910 (calculator) Answer only: full marks	✓ value of *b* ✓ answer (2) **[8]**

QUESTION/*VRAAG* 3

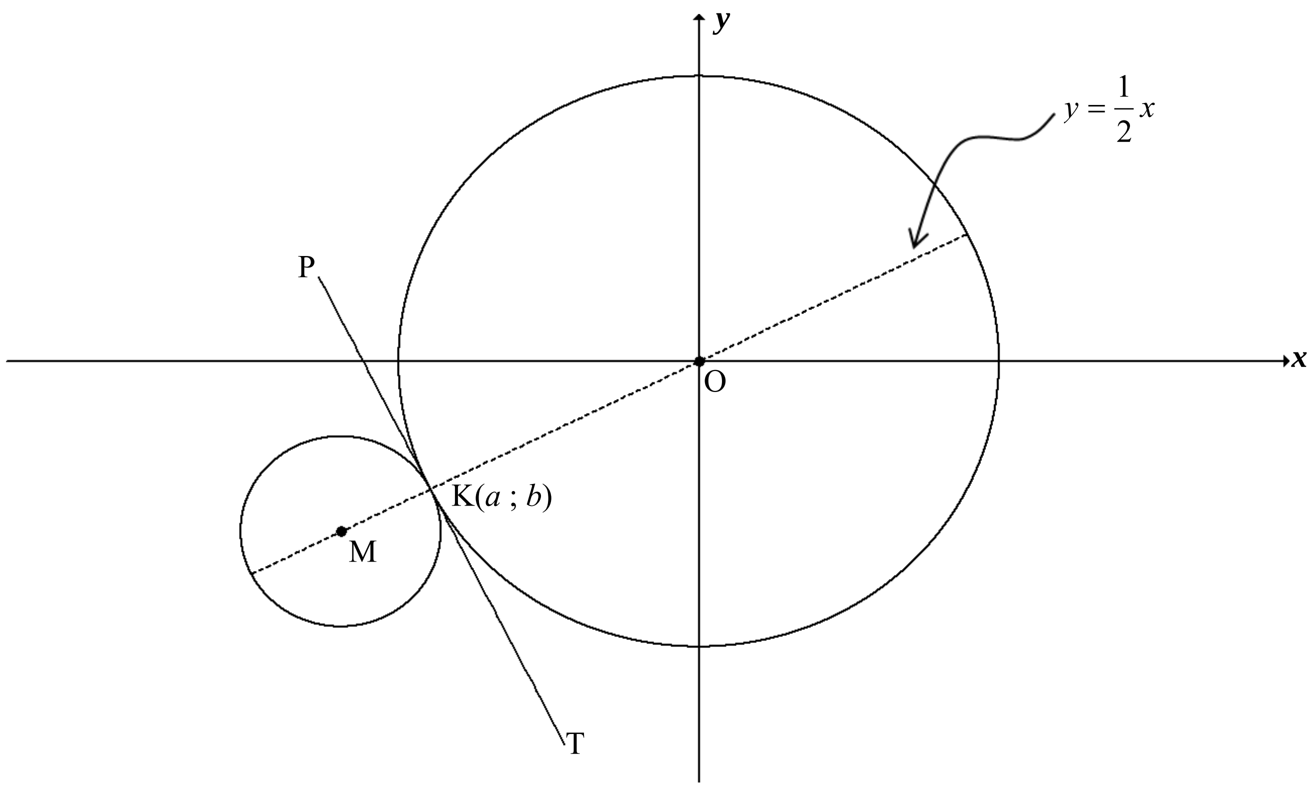


3.1	$x = 3$	✓ answer (1)
3.2	$m_{QP} = \tan 71{,}57°$ $= 3$ Answer only: full marks	✓ $m_{QP} = \tan 71{,}57°$ ✓ answer (2)
3.3	$y = mx + c$ $-2 = 3(-7) + c$ $y = 3x + 19$ or $y - y_1 = m(x - x_1)$ $y + 2 = 3(x + 7)$	(m CA from 3.2 if > 0) ✓ substitution of m & Q ✓ equation (2)
3.4	R(3 ; 0) $QR = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $= \sqrt{(-7-3)^2 + (-2-0)^2}$ $= \sqrt{104}$ or $2\sqrt{26}$	(wrong R: CA if x > 0) ✓ substitution ✓ answer (in surd form) (2)

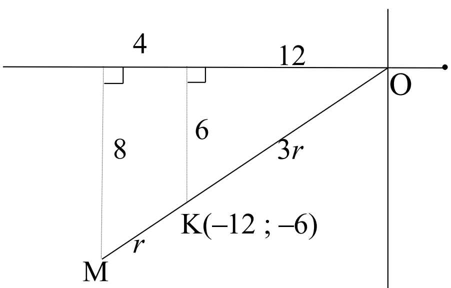
3.5	$\tan(90° - \theta) = m_{QR}$ $= \frac{0-(-2)}{3-(-7)}$ $= \frac{1}{5}$ Answer only: full $\tan\theta = \frac{1}{5}$: 1/3	(wrong R: CA if $x > 0$) ✓ gradient of QR/RN/QN ✓ substitution of Q & R ✓ answer (3)
3.6 using calculator: max 4 marks	$RN = \frac{1}{2}.2\sqrt{26} = \sqrt{26}$ SR = 6  Area $\Delta RSN = \frac{1}{2}$ SR . RN. $\sin\theta$ $= \frac{1}{2} \times 6 \times \sqrt{26} \times \frac{5}{\sqrt{26}}$ = 15 square units **OR/*OF*** $RN = \frac{1}{2}.2\sqrt{26} = \sqrt{26}$ SR = 6  Area $\Delta RSN = \frac{1}{2}$ SR . RN. $\sin\theta$ $= \frac{1}{2}(6)(\frac{1}{2}QP).\sin\theta$ $= \frac{3}{2}(\sqrt{104}).\sin\theta$ $= \frac{3}{2}(\sqrt{104})\left(\frac{5}{\sqrt{26}}\right)$ $= 15$ square units	✓ RN ✓ SR ✓ diagram (5 & $\sqrt{26}$) ✓ use of correct area rule ✓ substitution of $\sin\theta$ ✓ answer (6) ✓ RN ✓ SR ✓ diagram ✓ use of correct area rule ✓ substitution of $\sin\theta$ ✓ answer (6)

	OR/*OF* SR = 6 $\perp$height = 5  $A = \frac{1}{2}SR \times \perp h$ $= \frac{1}{2}(6)(5)$ $= 30$ square units **Using A = $\frac{1}{2}$b×⊥h incorrectly: max 1/6**	✓ SR ✓✓ $\perp$ height ✓ use of correct area formula ✓ substitution of $\sin\theta$ ✓ answer (6) **[16]**

QUESTION/*VRAAG* 4

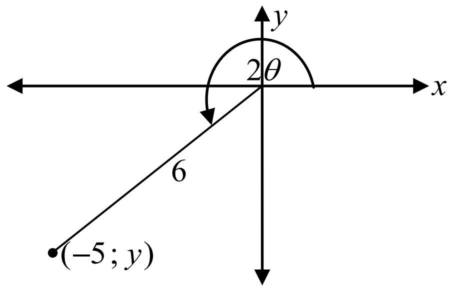


4.1	$\text{OK} = \sqrt{180}$ or $6\sqrt{5}$	✓ answer (1)
4.2	$a^2 + b^2 = 180$ $b = \frac{1}{2}a$ $a^2 + \left(\frac{1}{2}a\right)^2 = 180$ $a^2 + \frac{1}{4}a^2 = 180$ $a^2 = 144 \quad \therefore a = -12$ $b = \frac{1}{2}(-12)$ K (–12 ; – 6) (given) No penalty if x and y are not converted to a and b Error in simplification: max 2/4 **OR/*OF*** $a^2 + b^2 = 180$ $a = 2b$ $(2b)^2 + b^2 = 180$ $5b^2 = 180$ $b^2 = 36 \quad \therefore b = -6$ $a = 2(-6)$ K (–12 ; – 6) (given)	✓ b in terms of a ✓ substitution ✓ $a^2 = 144$ ✓ substitution (4) ✓ a in terms of b ✓ substitution ✓ $b^2 = 36$ ✓ substitution (4)

4.3.1	$m_{OK} = \frac{1}{2}$ $\quad [\ y = \frac{1}{2}x\ ]$ $m_{PT} = -2$ $\quad$ [radius $\perp$ tangent/*raaklyn*] $y = mx + c$ **OR/*OF*** $y - y_1 = m(x - x_1)$ $-6 = -2(-12) + c$ $\quad$ $y - (-6) = -2(x - (-12))$ $c = -30$ $\quad$ $c = -30$ $y = -2x - 30$ Using $m = \frac{1}{2} : 0/3$ Using $m = -\frac{1}{2}$ *or* $2 : 2/3$	✓ $m_{PT} = -2$ ✓ substitution of m & K(–12 ; – 6) ✓ equation (3)
4.3.2	3MK = OK $\Rightarrow$OM = $\frac{4}{3}$ OK $M = \frac{4}{3}(-12\ ;-6)$ $\therefore$M(–16 ; –8) Answer only: 0/6 **OR/*OF*** 3MK = OK $9MK^2 = OK^2 = 180$ $\therefore MK^2 = 20$ Let M(x ; y), then : $(x+12)^2 + (y+6)^2 = 20$ $(x+12)^2 + \left(\frac{1}{2}x + 6\right)^2 = 20$ $x^2 + 24x + 144 + \frac{1}{4}x^2 + 6x + 36 = 20$ $\frac{5}{4}x^2 + 30x + 160 = 0$ $x^2 + 24x + 128 = 0$ $(x+16)(x+8) = 0$ $x = -16$ $\quad x \neq -8$ [since M is outside the large circle] $y = -8$ $M(-16; -8)$ **OR/*OF***  $\therefore$M(–16 ; –8) **OR/*OF***	✓ 3MK = OK ✓ OM = $\frac{4}{3}$ OK ✓✓ $M = \frac{4}{3}(-12\ ;-6)$ ✓ x-coordinate ✓ y-coordinate (6) ✓ 3MK = OK ✓ $MK^2 = 20$ ✓ equation ✓ substitution ✓ x-coordinate ✓ y-coordinate (6) ✓ 3MK = OK ✓ ✓✓ diagram with values **OR** valid explanation ✓ x-coordinate ✓ y-coordinate (6)

	3MK = OK $9\text{MK}^2 = \text{OK}^2 = 180$ $\therefore \text{MK}^2 = 20$ Let M(x ; y), then $y = \frac{1}{2}x$: $(x+12)^2 + (y+6)^2 = 20$ $(x+12)^2 + \left(\frac{1}{2}x+6\right)^2 = 20$ $4(x+12)^2 + (x+12)^2 = 80$ $(x+12)^2 = 16$ $x+12 = \pm 4$ $x = -16 \quad x \neq -8$ [since M is outside the large circle] $y = -8$ $\text{M}(-16;-8)$	✓ 3MK = OK ✓ $\text{MK}^2 = 20$ ✓ equation ✓ substitution ✓ x-coordinate ✓ y-coordinate (6)
4.3.3	$(x-(-16))^2 + (y-(-8))^2 = \left(\frac{1}{3}\sqrt{180}\right)^2$ $(x+16)^2 + (y+8)^2 = 20$	✓ LHS (CA from 4.3.2) ✓ RHS (CA from 4.1) (2)
4.4	$\text{OK} < r < \text{OK} + 2\text{KM}$ $\sqrt{180} < r < \sqrt{180} + \frac{2}{3}\sqrt{180}$ $6\sqrt{5} < r < 10\sqrt{5}$ Answer only: full marks (No need to simplify)	✓✓ values ✓ inequality (3)
4.5	$x^2 + 32x + (16)^2 + y^2 + 16y + (8)^2 = 256 + 64 - 240$ $(x+16)^2 + (y+8)^2 = 80$ New circle/*nuwe sirkel*: Centre/*middelpt* $(-16;-8)$ & $r = 4\sqrt{5}$ Original circle/*oorspronklike sirkel*: $\text{M}(-16;-8)$ & $r = 2\sqrt{5}$ This circle will never cut the circle with centre M as they have the **same centre (concentric circles) but unequal radii**/*Hierdie sirkel sal nooit die sirkel met middelpt M sny nie, want hulle is* ***konsentries, want het dieselfde middelpunt met verskillende radii****.*	✓ equation in centre, radius form ✓ Centre: $(-16;-8)$ ✓ $r = 4\sqrt{5}$ (new) ✓ $r = 2\sqrt{5}$ (original) ✓ conclusion (“concentric” must be stated) (5) **[24]**

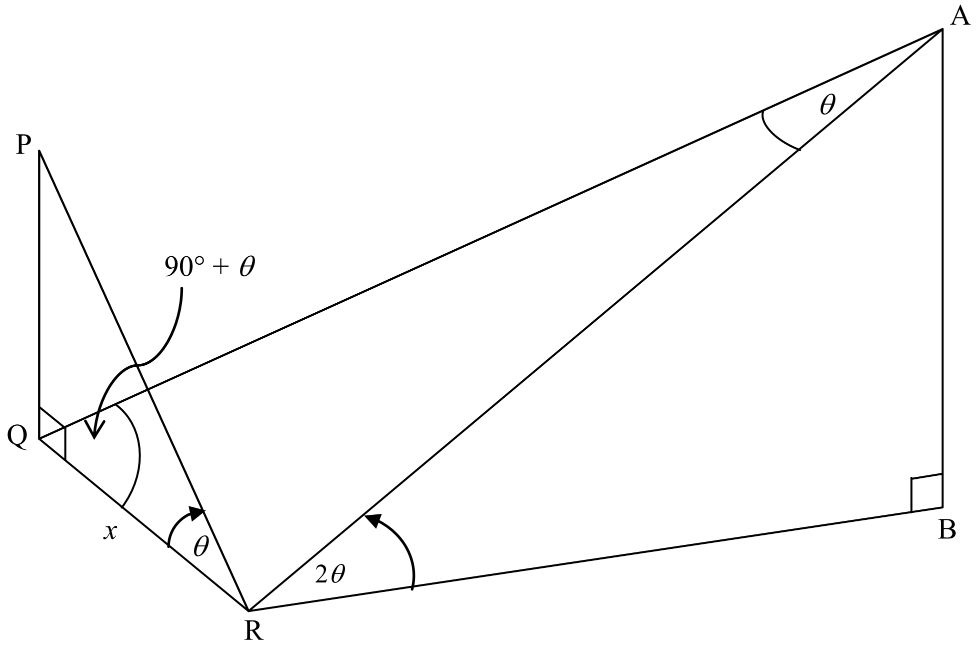
QUESTION/*VRAAG* 5

5.1.1 **no calculator in 5.1**	$\cos 2\theta = -\frac{5}{6}$, where $2\theta \in [180°; 270°]$  $y^2 = 6^2 - (-5)^2$ [Pythagoras] $y = \pm\sqrt{11}$ $(5 ; y)$ is in 3rd quadrant: $\therefore y = -\sqrt{11}$ $\sin 2\theta = -\frac{\sqrt{11}}{6}$ Getting to $\sin 2\theta = \frac{\sqrt{11}}{6}$: 3/4 **OR/*OF*** $\sin^2 2\theta = 1 - \cos^2 2\theta$ $= 1 - \left(-\frac{5}{6}\right)^2$ $= 1 - \frac{25}{36}$ $= \frac{11}{36}$ $\sin 2\theta = -\frac{\sqrt{11}}{6}$	✓ diagram (3rd quadrant only) ✓ using Pythagoras ✓ y – value ✓ answer (4) .✓ $\sin^2 2\theta = 1 - \cos^2 2\theta$ ✓ substitution ✓ value of $\sin^2 2\theta$ ✓ answer (4)
5.1.2	$\cos 2\theta = 1 - 2\sin^2\theta$ $2\sin^2\theta = 1 - \cos 2\theta$ $\sin^2\theta = \frac{1 - \left(-\frac{5}{6}\right)}{2}$ $= \frac{11}{6} \times \frac{1}{2}$ $= \frac{11}{12}$	✓ $\cos 2\theta = 1 - 2\sin^2\theta$ ✓ substitution ✓ answer (3)

5.2	$\sin(180°-x).\cos(-x)+\cos(90°+x).\cos(x-180°)$ $=\sin x.\cos x-\sin x(-\cos x)$ $=2\sin x.\cos x$ $=\sin 2x$ Second line written as $\sin x \cos x + \sin x \cos x$: max 5/6	✓ $\sin x$ ✓ $\cos x$ ✓ $-\sin x$ ✓ $-\cos x$ ✓ simplification ✓ answer (6)
5.3	$\sin 3x.\cos y+\cos 3x.\sin y$ $\sin(3x+y)$ $=\sin 270°$ $=-1$	✓ compound angle ✓ answer (2)
5.4.1	$2\cos x=3\tan x$ $2\cos x=\dfrac{3\sin x}{\cos x}$ $2\cos^2 x=3\sin x$ $2(1-\sin^2 x)=3\sin x$ $2-2\sin^2 x=3\sin x$ $2\sin^2 x+3\sin x-2=0$	✓ $\tan x=\dfrac{\sin x}{\cos x}$ ✓ multiplying by $\cos\theta$ ✓ $\cos^2 x=1-\sin^2 x$ (3)
5.4.2	$2\sin^2 x+3\sin x-2=0$ $(2\sin x-1)(\sin x+2)=0$ $\sin x=\dfrac{1}{2}$ or $\sin x=-2$ (no solution) $x=30°+k.360°$ or $x=150°+k.360°$; $k\in Z$	✓ factors ✓ both values of $\sin x$ ✓ no solution ✓ $30°+k.360°$ ✓ $150°+k.360°$; $k\in Z$ (5)
5.4.3	$5y=30°+k.360°$ or $5y=150°+k.360°$ $y=6°+k.72°$ or $y=30°+k.72°$ $\therefore y=144°+6°$ or $y=144°+30°$ $y=150°$ or $y=174°$ **OR/*OF*** $144°\le y\le 216°$ $720°\le 5y\le 1080°$ $5y=750°$ or $5y=870°$ $y=150°$ or $y=174°$	✓ $y=6°+k.72°$ ✓ $y=30°+k.72°$ ✓ $150°$ ✓ $174°$ (4) ✓ $5y=750°$ ✓ $5y=870°$ ✓ $150°$ ✓ $174°$ (4)
5.5.1	$g(x)=-4\cos(x+30°)$ maximum value = 4	✓ answer (1)

5.5.2	range of/*waardeversameling van* $g(x)$: $-4 \le y \le 4$ **OR/*OF*** $y \in [-4\ ;\ 4]$ $\therefore$ range of/*waardeversameling van* $g(x) + 1$: $-3 \le y \le 5$ **OR/*OF*** $y \in [-3\ ;\ 5]$ Answer only: full marks	✓ range of $g(x)$ ✓ answer (2)
5.5.3	$y = -4\cos(x + 30°)$ shifted to the left/*skuif na links*: $y = -4\cos(x + 30° + 60°)$ $= -4\cos(x + 90°)$ $= 4\sin x$ $\therefore h(x) = -4\sin x$ Answer only: full marks	✓ shift of 60° to the left ✓ reduction ✓ equation of h (3) **[33]**

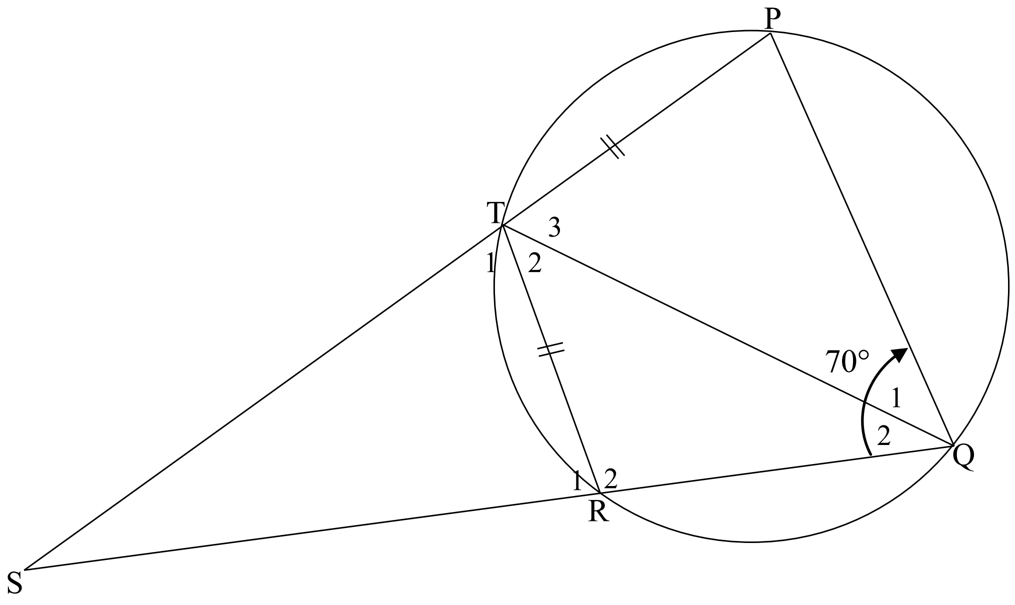
QUESTION/*VRAAG* 6



6.1.1	$\tan\theta = \frac{PQ}{QR} = \frac{PQ}{x}$ $\therefore PQ = x\tan\theta$ Answer only: full marks **OR/*OF*** $\frac{QR}{\sin P} = \frac{PQ}{\sin P\hat{R}Q}$ $\therefore PQ = \frac{x.\sin\theta}{\sin(90° - \theta)}$	✓ trig ratio ✓ answer (2) ✓ trig ratio ✓ answer (2)
6.1.2	$\frac{AR}{\sin A\hat{Q}R} = \frac{QR}{\sin Q\hat{A}R}$ $AR = \frac{x\sin(90° + \theta)}{\sin\theta}$ Answer only: full marks	✓ use of sine rule ✓ substitution into sine rule correctly (2)

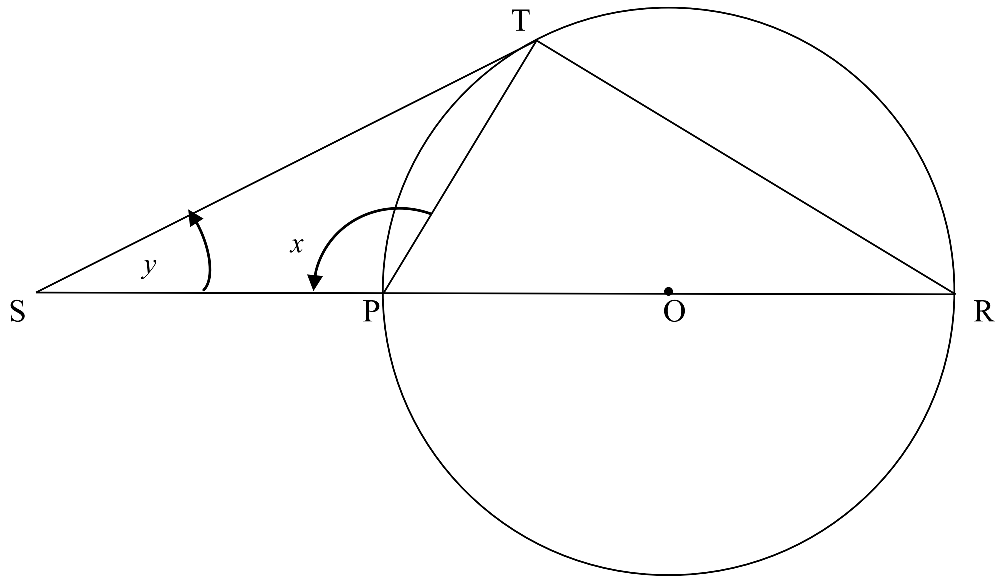
6.2	$\sin 2\theta = \dfrac{AB}{AR}$ $AB = AR\sin 2\theta$ $= \dfrac{x\sin(90° + \theta).\sin 2\theta}{\sin\theta}$ $= \dfrac{x\cos\theta.\sin 2\theta}{\sin\theta}$ $= \dfrac{x\cos\theta.2\sin\theta\cos\theta}{\sin\theta}$ $= 2x\cos^2\theta$	✓ substitution into trig ratio and AB as subject ✓ substitution of AR ✓ co-ratio ✓ $\sin 2\theta = 2\sin\theta\cos\theta$ (4)
6.3	$\dfrac{AB}{QP} = \dfrac{2x\cos^2 12°}{x\tan 12°}$ $= 9$	✓ substitution CA from 6.1.1) ✓ answer (2) **[10]**

QUESTION/*VRAAG* 7



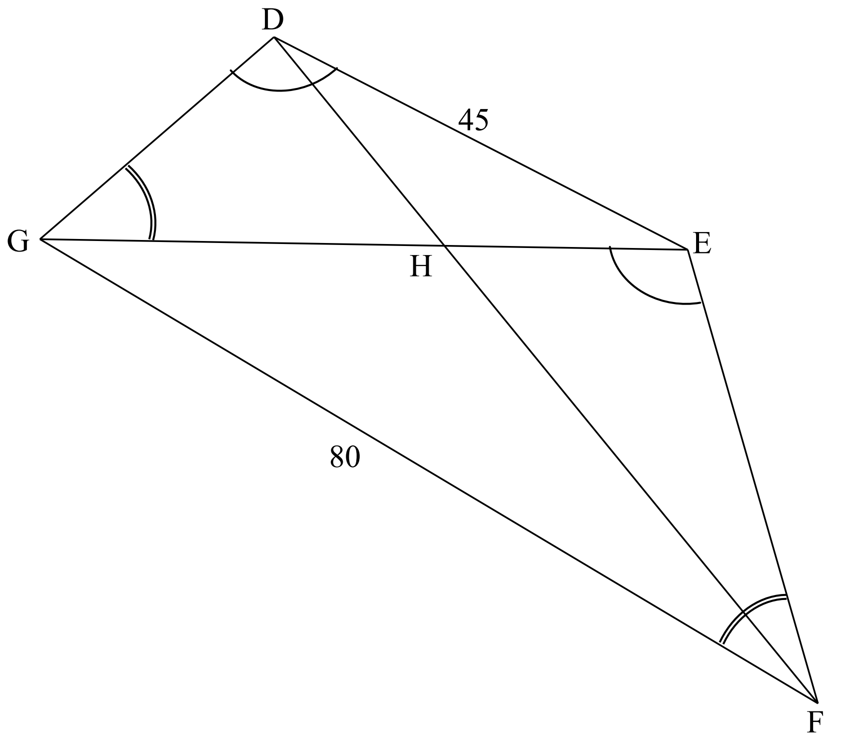
7.1.1	$\hat{T}_1 = 70°$ [ext ∠ of cyclic quad/*buite ∠ van koordevh*]	✓ S ✓ R (2)
7.1.2	$\hat{Q}_1 = \hat{Q}_2 = 35°$ [equal chords;equal∠s/*gelyke koorde; gelyke∠e*]	✓ S ✓ R (2)
7.2.1	$\hat{T}_2 = \hat{Q}_1 = 35°$ [alt ∠s/*verwiss ∠e*; PQ ∥ TR]	✓ S ✓ R (2)
7.2.2	$\frac{PT}{TS} = \frac{QR}{RS}$ [prop theorem/*eweredighst*; PQ ∥ TR] $\therefore \frac{TR}{TS} = \frac{QR}{RS}$ [PT = TR]	✓ S ✓ R (2) **[8]**

QUESTION/*VRAAG* 8



	$\text{P}\hat{\text{T}}\text{R} = 90°$ [$\angle$ in semi-circle/*halfsirkel*] $x = 90° + \hat{\text{R}}$ [ext/*buite* $\angle$ of/*van* Δ] $\therefore \hat{\text{R}} = x - 90°$ $\text{S}\hat{\text{T}}\text{P} = x - 90°$ [tan chord theorem/*raakl koordstelling*] $x + x - 90° + y = 180°$ [sum of/*som van* $\angle$s/*e* in Δ] $\therefore y = 270° - 2x$	✓ S/R ✓ S/R ✓ S ✓R ✓ S ✓ answer **[6]**

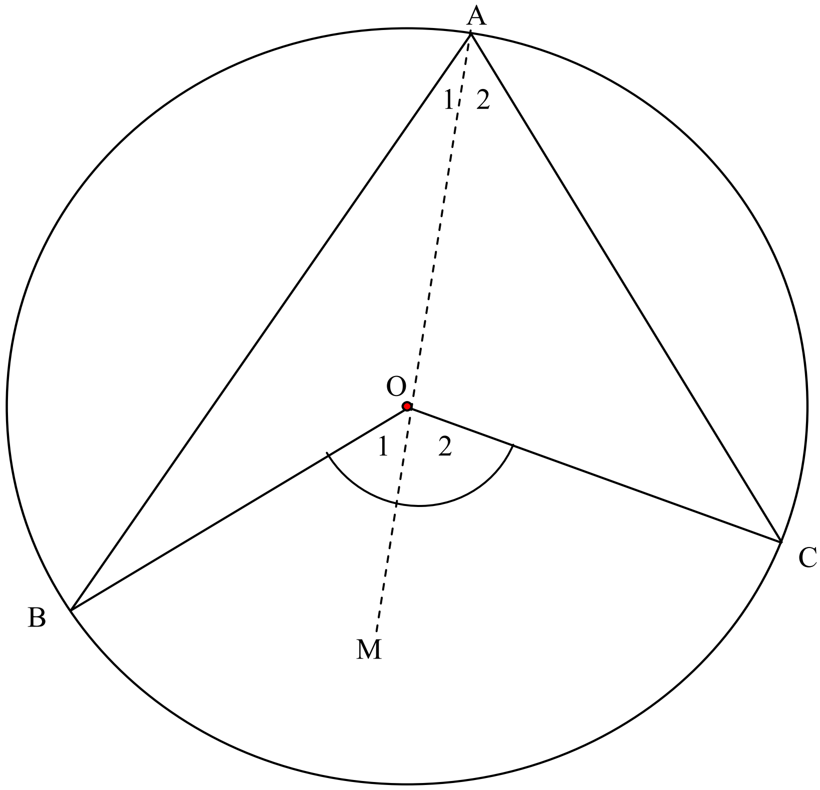
QUESTION/*VRAAG* 9



9.1	equiangular Δs/*gelykhoekige Δe* **OR/*OF*** $(\angle\angle\angle)$		✓ answer (1)
9.2	$\therefore \frac{GE}{GF}=\frac{DE}{GE}$ $GE^2 = 45\times 80$ $GE = 60$	[||| Δs]	✓ proportion ✓ substitution ✓ answer (3)
9.3	In ΔDEH and ΔFGH: $D\hat{H}E = F\hat{H}G$ $D\hat{E}H = F\hat{G}H$ $E\hat{D}H = G\hat{F}H$ $\therefore \Delta DEH \mathbin{\|\|\|} \Delta FGH$	 [vert opp $\angle$s =/*regoorst* $\angle$*e* =] [||| Δs] [sum of/*som van* $\angle$s/*e* in Δ]	 ✓ S/R ✓ S/R ✓ S (3)
	OR/*OF* In ΔDEH and ΔFGH: $D\hat{H}E = F\hat{H}G$ $D\hat{E}H = F\hat{G}H$ $\therefore \Delta DEH \mathbin{\|\|\|} \Delta FGH$	 [vert opp $\angle$s =/*regoorst* $\angle$*e* =] [||| Δs] [$\angle\angle\angle$]	 ✓ S/R ✓ S/R ✓ R (3)

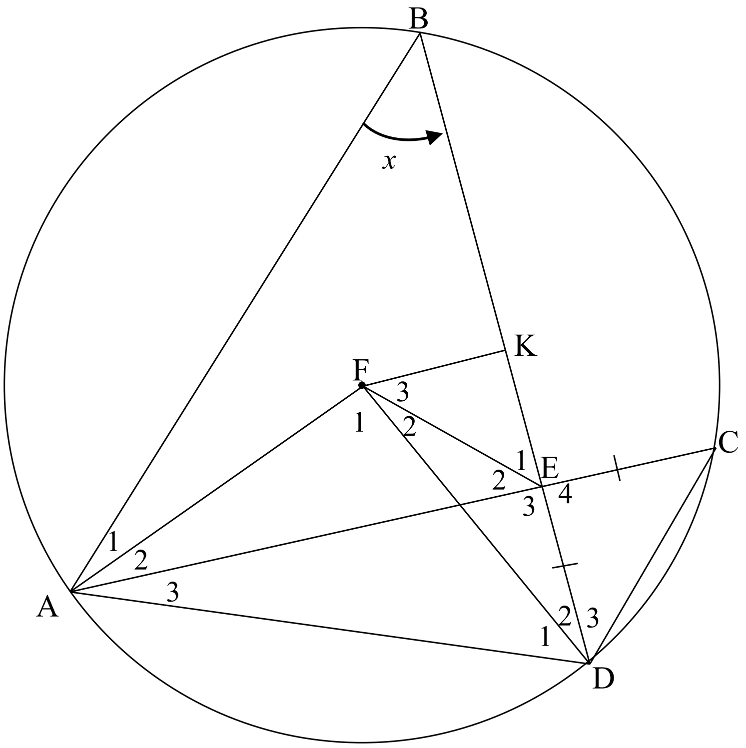
9.4	$\frac{GH}{EH}=\frac{FG}{DE}$ $\frac{GH}{60-GH}=\frac{80}{45}$ 45 GH = 80(60 – GH) 45 GH = 4800 – 80 GH 125 GH = 4800 GH = 38,4	[||| Δs] [EH = 60 – GH]	✓S ✓ substitution ✓ answer (3) **[10]**

QUESTION/*VRAAG* 10



10.1	Construction: AO is drawn and produced to M $\hat{O}_1 = \hat{A}_1 + \hat{B}$ [ext $\angle$ of Δ/*buite $\angle$ van Δ*] But $\hat{A}_1 = \hat{B}$ [$\angle$s opp = radii/$\angle$*e teenoor =radii*] $\therefore \hat{O}_1 = 2\hat{A}_1$ Similarly/*Netso*: $\hat{O}_2 = 2\hat{A}_2$ $\therefore \hat{O}_1 + \hat{O}_2 = 2\hat{A}_1 + 2\hat{A}_2$ $= 2(\hat{A}_1 + \hat{A}_2)$ $B\hat{O}C = 2B\hat{A}C$	✓ Constr ✓ S/R ✓ S/R ✓ S ✓ S (5)

10.2



10.2.1(a)	$\hat{F}_1 = 2x$ [$\angle$ centre = 2$\angle$ at circum/*midpts$\angle$=2omtreks$\angle$*]	✓ S ✓ R (2)
10.2.1(b)	$\hat{C} = x$ [$\angle$s in the same seg/*$\angle$e in dieselfde segment*] **OR/*OF*** $\hat{C} = x$ [$\angle$ centre = 2$\angle$ at circum/*midpts$\angle$=2omtreks$\angle$*]	✓ S ✓ R (2) ✓ S ✓ R (2)
10.2.2	$\hat{D}_3 = x$ [$\angle$s opp equal sides/*$\angle$e teenoor = sye*] $\hat{E}_3 = 2x$ [ext $\angle$ of Δ/*buite $\angle$ van Δ*] $\therefore \hat{F}_1 = \hat{E}_3 = 2x$ $\therefore$ AFED is a cyclic quadrilateral [converse $\angle$s in the same seg]/ *Is 'n koordevierhoek* [*omgekeerde $\angle$e in dieselfde segm*]	✓ S/R ✓ S/R ✓ S ✓ R (4)

10.2.3	$\hat{A}_2 + \hat{A}_3 + \hat{D}_1 + \hat{F}_1 = 180°$ [sum of ∠s in Δ/*som van ∠e in Δ*] $\hat{A}_2 + \hat{A}_3 = D_1$ [∠s opp = sides/*∠e teenoor = sye*] $\therefore \hat{A}_2 + \hat{A}_3 = 90° - x$ $\hat{E}_1 = \hat{A}_2 + \hat{A}_3$ [ext∠of cyclic quad/*buite∠ v koordevh*] $= 90° - x$ $F\hat{K}E = 90°$ [line from centre bisects chord]/ [*lyn van midpt halveer koord*] $\hat{F}_3 = x$ [sum of ∠s in Δ/*som van ∠e in Δ*]	✓ S ✓ S ✓ R ✓ S ✓ S ✓ R (6)
10.2.4	$B\hat{A}C = \hat{D}_3$ [∠s in the same seg/*∠e in dieselfde segm*] $AE = BE$ [sides opp equal ∠s/*sye teenoor =∠e*] $\dfrac{\text{area } \Delta AEB}{\text{area } \Delta DEC} = \dfrac{\frac{1}{2}(BE)(AE).\sin A\hat{E}B}{\frac{1}{2}(EC)(ED).\sin D\hat{E}C}$ $6{,}25 = \dfrac{AE^2}{ED^2}$ $\therefore \dfrac{AE}{ED} = 2{,}5$	✓ S ✓ S ✓ substitution into area rule ✓ simplification of RHS ✓ answer (5) **[24]**

TOTAL/*TOTAAL*: 150

basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

SENIOR CERTIFICATE EXAMINATIONS
SENIORSERTIFIKAAT-EKSAMEN

MATHEMATICS P2/*WISKUNDE V2*

2018

MARKING GUIDELINES/*NASIENRIGLYNE*

MARKS: 150
PUNTE: 150

These marking guidelines consist of 21 pages.
Hierdie nasienriglyne bestaan uit 21 bladsye.

NOTE:

- If a candidate answers a question TWICE, only mark the FIRST attempt.
- If a candidate has crossed out an attempt of a question and not redone the question, mark the crossed out version.
- Consistent accuracy applies in ALL aspects of the marking memorandum. Stop marking at the second calculation error.
- Assuming answers/values in order to solve a problem is NOT acceptable.

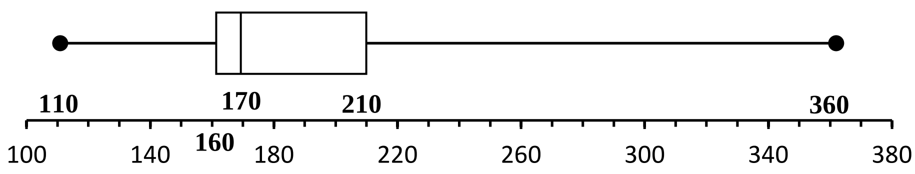
LET WEL:

- *As 'n kandidaat 'n vraag TWEE KEER beantwoord, sien slegs die EERSTE poging na.*
- *As 'n kandidaat 'n antwoord van 'n vraag doodtrek en nie oordoen nie, sien die doodgetrekte poging na.*
- *Volgehoue akkuraatheid word in ALLE aspekte van die nasienriglyne toegepas. Hou op nasien by die tweede berekeningsfout.*
- *Aanvaar van antwoorde/waardes om 'n probleem op te los, word NIE toegelaat nie.*

GEOMETRY	
S	**A mark for a correct statement** **(A statement mark is independent of a reason.)**
	'n Punt vir 'n korrekte bewering **('n Punt vir 'n bewering is onafhanklik van die rede.)**
R	**A mark for a correct reason** **(A reason mark may only be awarded if the statement is correct.)**
	'n Punt vir 'n korrekte rede **('n Punt word slegs vir die rede toegeken as die bewering korrek is.)**
S/R	**Award a mark if the statement AND reason are both correct.**
	Ken 'n punt toe as beide die bewering EN rede korrek is.

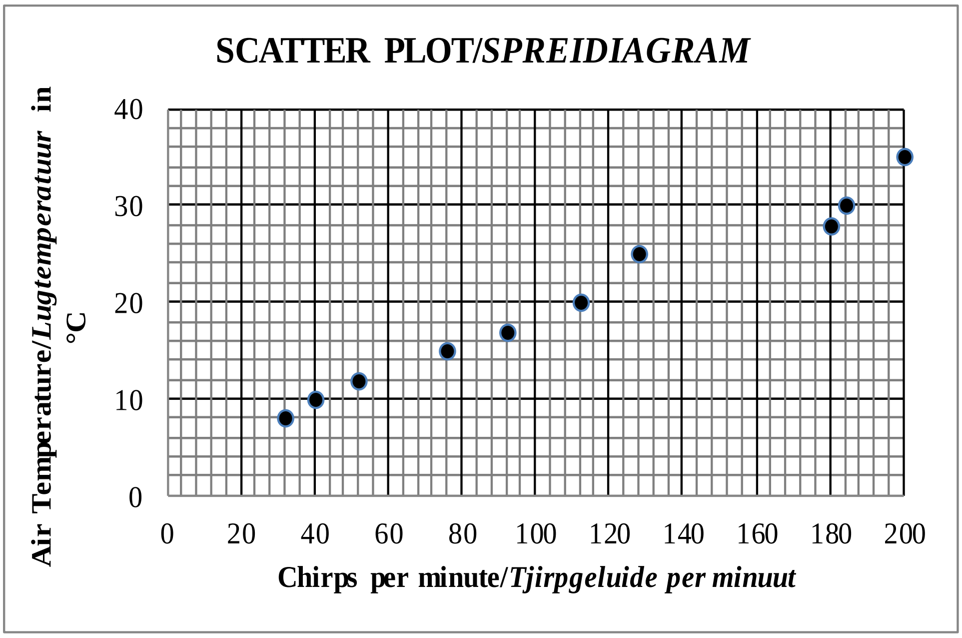
QUESTION/*VRAAG* 1

110	112	156	164	167	169
171	176	192	228	278	360

1.1.1	Mean/*Gemiddelde* $=\frac{2283}{12}$ $=190{,}25$ Mean profit/*Gemiddelde wins* $=$ R190 250,00 or 190,25 thousand rands	✓ sum/*som* ✓ answer ✓ answer in thousands of rands (3)
1.1.2	Median $=\frac{169+171}{2}=170$ thousand rands = R170 000	✓ answer (1)
1.2	 	✓ whiskers ✓ quartiles (2)
1.3	$\text{IQR} = \text{Q}_3 - \text{Q}_1$ = 210 – 160 thousand rands = R50 000	✓ answer (1)
1.4	Skewed to the right or positively skewed.	✓ answer (1)
1.5.1	$\sigma = 67{,}04118759$ thousand rands = R67 041,19	✓ answer (1)
1.5.2	$\bar{x} - \sigma$ = 123,21 thousand rands For **2** months the profit was less than one standard deviation below the mean.	✓ lower limit ✓ answer (2)
		[11]

QUESTION/*VRAAG* 2

CHIRPS/*TJIRPGELUIDE* PER MINUTE/ *PER MINUUT*	**AIR TEMPERATURE/ *LUGTEMPERATUUR* IN °C**
32	8
40	10
52	12
76	15
92	17
112	20
128	25
180	28
184	30
200	35

<table>
<tr><td>2.1</td><td>
</td><td>3 marks:
All points correct

2 marks:
6 – 9 points correct

1 mark:
3 – 5 points correct

(3)</td></tr>
<tr><td>2.2</td><td>The points lie almost in a straight line. This suggests a very strong positive relationship between the number of chirps per minute and the temperature of the air.
Die punte lê amper in 'n reguitlyn, wat beteken dat daar 'n baie sterk positiewe verband tussen die aantal tjirpgeluide per minuut en die lugtemperatuur is.

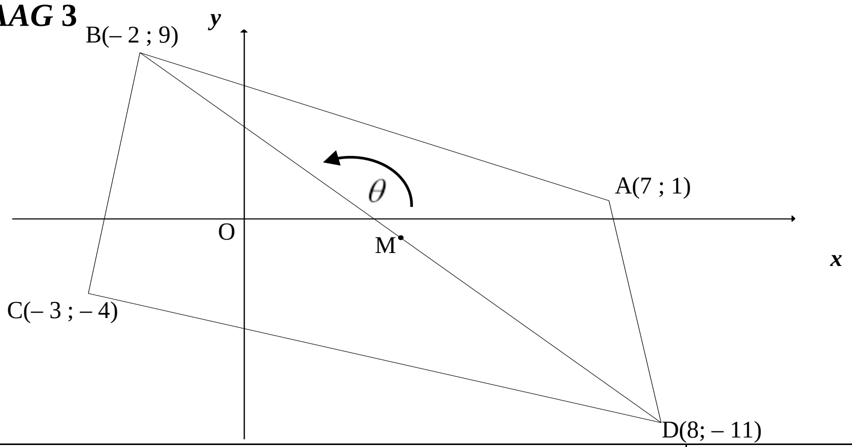
OR/OF

r = 0,99 so there is a very strong positive relationship between the number of chirps per minute and the temperature of the air.
r = 0,99, dus is daar 'n baie sterk positiewe verband tussen die aantal kriekgeluide per minuut en die lugtemperatuur.</td><td>✓ justify with straight line / Motivering mbv reguitlyn
(1)

✓ link with / gebruik r = 0,99 om te motiveer
(1)</td></tr>
</table>

2.3	$a = 3{,}97$ $b = 0{,}15$ $\hat{y} = 3{,}97 + 0{,}15x$	✓ $a = 3{,}97$ ✓ $b = 0{,}15$ ✓ equation (3)
2.4	Air temperature $\approx$ 15,67°C (calculator) **OR** $\hat{y} \approx 3{,}97 + 0{,}15(80)$ $\approx 15{,}97$°C **OR** Air temperature $\approx$ 16°C (graph: Accept between 15°C and 17°C)	✓✓ answer (2) ✓ substitution ✓ answer (2) ✓✓ answer (2)
		[9]

QUESTION/*VRAAG* 3

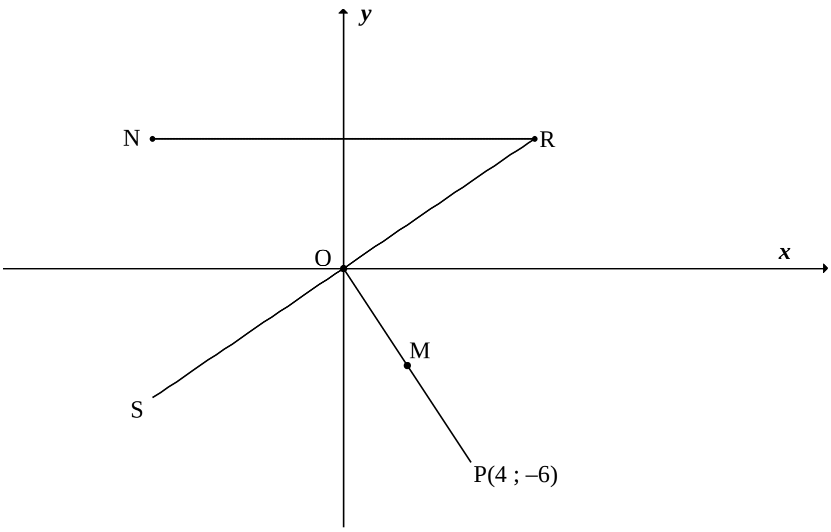


3.1	$m_{\text{AC}} = \dfrac{1-(-4)}{7-(-3)}$ OR $\dfrac{-4-1}{-3-7}$ $= \dfrac{5}{10} = \dfrac{1}{2}$	✓ substitution ✓answer (2)
3.2.1	$y = \frac{1}{2}x + c$ $y - y_1 = \frac{1}{2}(x - x_1)$ $1 = \frac{1}{2}(7) + c$ $y - 1 = \frac{1}{2}(x - 7)$ $c = -\frac{5}{2}$ **OR/*OF*** $y - 1 = \frac{1}{2}x - \frac{7}{2}$ $y = \frac{1}{2}x - 2\frac{1}{2}$ $y = \frac{1}{2}x - 2\frac{1}{2}$ **OR/*OF*** $y = \frac{1}{2}x + c$ $y - y_1 = \frac{1}{2}(x - x_1)$ $-4 = \frac{1}{2}(-3) + c$ $y - (-4) = \frac{1}{2}(x - (-3))$ $c = -\frac{5}{2}$ **OR/*OF*** $y + 4 = \frac{1}{2}x + \frac{3}{2}$ $y = \frac{1}{2}x - 2\frac{1}{2}$ $y = \frac{1}{2}x - 2\frac{1}{2}$	✓ substitution M and A(7 ; 1) ✓equation (2) ✓ substitution M and C(–3 ; –4) ✓equation (2)

3.2.2	$M\left(\frac{-2+8}{2}\ ;\ \frac{9+(-11)}{2}\right)$ $\therefore M(3;-1)$ Equation of AC: $y=\frac{1}{2}x-2\frac{1}{2}$ **OR/OF** $y=\frac{1}{2}x-2\frac{1}{2}$ $y=\frac{1}{2}(3)-2\frac{1}{2}$ $-1=\frac{1}{2}x-2\frac{1}{2}$ $y=-1$ $x=3$ $\therefore$ M lies on AC **OR/OF** $M\left(\frac{-2+8}{2}\ ;\ \frac{9+(-11)}{2}\right)$ $\therefore M(3;-1)$ $m_{CM}=\frac{-4+1}{-3-3}=\frac{1}{2}$ $\therefore$ $m_{CM}=m_{AC}$ and C a common point $\therefore$ M lies on AC	✓x coordinate ✓y coordinate ✓ substitution of x ✓ conclusion (4) ✓ x coordinate ✓ y coordinate ✓ gradient of CM ✓ reasoning & conclusion (4)
3.3	$m_{BD}=\frac{9-(-11)}{-2-8}$ OR $\frac{(-11)-9}{8-(-2)}$ $=-2$ $m_{BD}\times m_{AC}=\frac{1}{2}\times-2$ $=-1$ $\therefore BD\perp AC$	✓ correct substitution ✓ m_{BD} ✓ product of gradients = –1 (3)
3.4.1	$\tan\theta=m_{BD}=-2$ $\therefore\theta=116{,}57°$	✓ $\tan\theta=m_{BD}$ ✓ answer (2)
3.4.2	$\tan\beta=m_{BC}$ $m_{BC}=\frac{9-(-4)}{-2-(-3)}$ *OR* $\frac{-4-9}{-3-(-2)}$ $=13$ $\beta=85{,}6°$ $\therefore C\hat{B}D=116{,}57°-85{,}60°$ [ext $\angle$ of Δ] $=30{,}97°$ **OR/OF** BD = $\sqrt{500}$; BC = $\sqrt{170}$ & CD = $\sqrt{170}$ $CD^2=BD^2+BC^2-2BD.BC.\cos C\hat{B}D$ $170=500+170-2\sqrt{500}.\sqrt{170}.\cos C\hat{B}D$ $\cos C\hat{B}D=\frac{\sqrt{500}}{2\sqrt{170}}=0{,}85749...$ $C\hat{B}D=30{,}96°$	✓ $m_{BC}=13$ ✓ value of β ✓ answer (3) ✓ subst into cos rule ✓ value of $\cos C\hat{B}D$ ✓ answer (3)

3.4.3	$\text{AC} = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$ $= \sqrt{(7-(-3))^2 + (1-(-4))^2}$ *OR* $\sqrt{((-3)-7)^2 + ((-4)-1)^2}$ $= \sqrt{100+25}$ $= \sqrt{125} = 5\sqrt{5} = 11,58$	✓ correct substitution into distance formula ✓ answer (2)
3.4.4	$\text{BM} = \sqrt{((-2)-3)^2 + (9-(-1)^2}$ *OR* $\sqrt{(3-(-2))^2 + ((-1)-9)^2}$ $= \sqrt{125} = 5\sqrt{5}$ Area of $\Delta\text{ABC} = \frac{1}{2}\text{base} \times \perp \text{ height}$ $= \frac{1}{2}(\sqrt{125})(\sqrt{125})$ = 62,5 square units Area of ABCD = 2 × 62,5 = 125 square units	✓ correct substitution into distance formula ✓ BM ✓ substitution into area formula ✓ 62,5 ✓ 2 × ΔABC (5)
		[23]

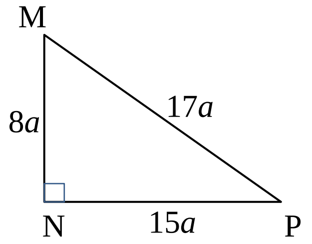
QUESTION/*VRAAG* 4



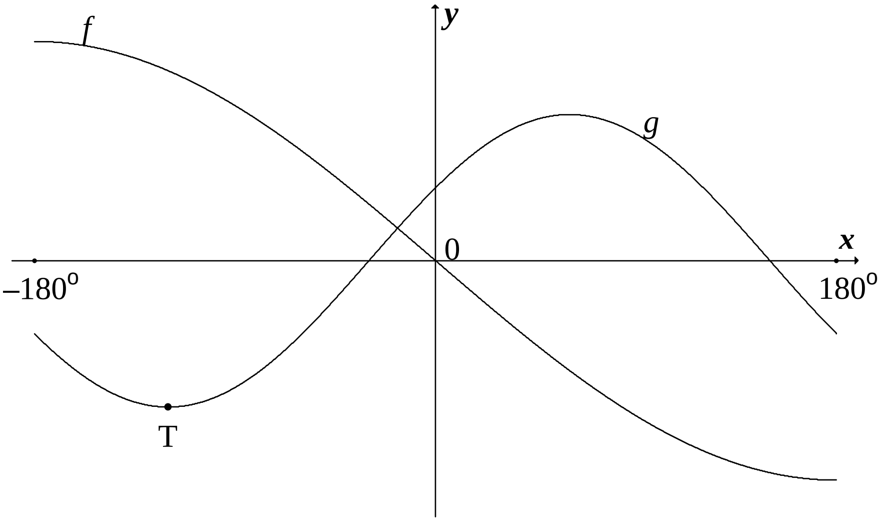
4.1	$M\left(\frac{0+4}{2}\ ;\ \frac{0+(-6)}{2}\right)$ $\therefore M(2\ ;-3)$	✓ 2 ✓ – 3 (2)
4.2.1	$x^2+y^2=4^2+(-6)^2$ $=52$ $\therefore\ x^2+y^2=52$	✓ substitution ✓ equation (2)
4.2.2	$(x-2)^2+(y+3)^2=\left(\frac{\sqrt{52}}{2}\right)^2=13$ $x^2-4x+4+y^2+6y+9-13=0$ $x^2+y^2-4x+6y=0$	✓ substitution of M ✓ substitution of radius = $\frac{\sqrt{52}}{2}$ ✓ answer (3)
4.2.3	$m_{OP}=\frac{-6}{4}=-\frac{3}{2}$ $m_{RS}\times m_{OP}=-1$ [radius $\perp$ tangent / *raaklyn*] $\therefore\ m_{RS}=\frac{2}{3}$ $\therefore\ y=\frac{2}{3}x$	✓ m_{OP} ✓ m_{RS} ✓ equation (3)

4.3	$x^2 + y^2 = 52$ *and* $y = \frac{2}{3}x$ $x^2 + \left(\frac{2}{3}x\right)^2 = 52$ $x^2 + \frac{4}{9}x^2 = 52$ $1\frac{4}{9}x^2 = 52$ $x^2 = 36$ $x = 6$ $\therefore$ R(6 ; 4) and N(–6 ; 4) $\therefore$ NR = 12 units	✓ substitution ✓ simplification ✓ value of *x* ✓ length of NR (4)
4.4	Let T(x ; 0) be the other x intercept of the small circle Then OT is the common chord $\therefore (x-2)^2 + (0+3)^2 = 13$ $(x-2)^2 = 13 - 9 = 4$ $x - 2 = \pm 2$ $x = 2 \pm 2$ $x = 4$ *or* 0 OR $x^2 - 4x + 4 + 9 = 13$ $x^2 - 4x = 0$ $x(x-4) = 0$ $x = 0$ or $x = 4$ $\therefore$ length of common chord = OT = 4 units	✓ $y = 0$ ✓ x-values ✓ answer (3) **[17]**

QUESTION/*VRAAG* 5

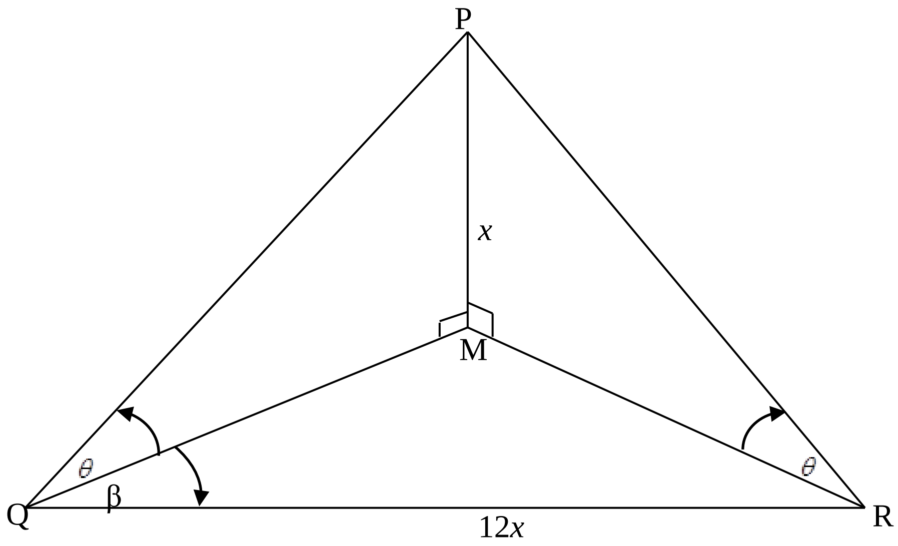
5.1.1	$\text{Given}: \sin \text{M} = \frac{15}{17}$ $\text{MN}^2 = 17^2 - 15^2$ $= 64$ $\text{MN} = 8$ OR  $\therefore \tan \text{M} = \frac{15}{8}$	✓ sketch or Pyth ✓ MN = 8 ✓answer (3)
5.1.2	$\sin \text{M} = \frac{\text{NP}}{\text{MP}}$ $\frac{\text{NP}}{51} = \frac{15a}{17a}$ $\therefore \text{NP} = 45$	✓ equating trig ratios ✓ answer (2)
5.2	$\cos(x - 360°).\sin(90° + x) + \cos^2(-x) - 1$ $= \cos x \,.\, \cos x + \cos^2 x - 1$ $= \cos^2 x + \cos^2 x - 1$ $= 2\cos^2 x - 1$ $= \cos 2x$	✓ $\cos x$ ✓ $\cos x$ ✓ $\cos^2 x$ ✓ identity (4)
5.3.1	$\sin(2x + 40°)\cos(x + 30°) - \cos(2x + 40°)\sin(x + 30°)$ $= \sin[(2x + 40°) - (x + 30°)]$ $= \sin(x + 10°)$	✓ reduction ✓ answer (2)
5.3.2	$\sin(2x + 40°)\cos(x + 30°) - \cos(2x + 40°)\sin(x + 30°) = \cos(2x - 20°)$ $\therefore \cos(2x - 20°) = \sin(x + 10°)$ $\cos(2x - 20°) = \cos[90° - (x + 10°)]$ $2x - 20° = 80° - x + k.360°$ or $2x - 20° = 360° - (80° - x) + k.360°$ $3x = 100° + k.360°$ or $2x - 20° = 280° + x + k.360°$ $x = 33{,}33° + k.120°$ or $x = 300° + k.360°$; $k \in Z$ **OR/*OF*** $\therefore \cos(2x - 20°) = \sin(x + 10°)$ $\sin[90° - (2x - 20°)] = \sin(x + 10°)$ $110° - 2x = x + 10° + k.360°$ or $110° - 2x = 180° - (x + 10°) + k.360°$ $3x = 100° - k.360°$ or $110° - 2x = 170° - x + k.360°$ $x = 33{,}33° - k.120°$ or $x = -60° - k.360°$; $k \in Z$	✓ equating ✓ co ratio ✓ $80° - x$ ✓$280° + x$ ✓ simplification/*vereenv* ✓ $x = 33{,}33° + k.120°$ ✓ $x = 300° + k.360°$; $k \in Z$ (7) ✓ equating ✓ co ratio ✓ $x + 10°$ ✓$170° - x$ ✓ simplification/*vereenv* ✓ $x = 33{,}33° - k.120°$ ✓ $x = -60° - k.360°$; $k \in Z$ (7)
		[18]

QUESTION/*VRAAG* 6



6.1	Period = 720°	✓ answer (1)
6.2	$y \in [-2\ ;\ 2]$ **OR/*OF*** $-2 \leq y \leq 2$	✓✓ answer (2) ✓✓ answer (2)
6.3	$f(-120°) - g(-120°)$ $= -3\sin\left(-\frac{120°}{2}\right) - 2\cos(-120° - 60°)$ $= \frac{4 + 3\sqrt{3}}{2}$ or 4,60 (4,5980...)	✓ $x = -120°$ ✓ substitution ✓ answer (3)
6.4.1	x-intercepts of g at –90° + 60° = –30° and 90° + 60° = 150° $\therefore x \in (-30°\ ;\ 150°)$ **OR/*OF*** x-intercepts of g at –90° + 60° = –30° and 90° + 60° = 150° $-30° < x < 150°$	✓ value ✓ value ✓ answer (3) ✓ value ✓ value ✓ answer (3)
6.4.2	$x \in [-180°\ ;\ -120°) \cup (-30°\ ; 60°) \cup (150°\ ; 180°]$ **OR/*OF*** $-180° \leq x < -120°$ or $-30° < x < 60°$ or $150° < x \leq 180°$	✓ $[-180°\ ;\ -120°)$ ✓ $(-30°\ ; 60°)$ ✓ $(150°\ ; 180°]$ ✓ notation for inclusive in the first/last interval (4) ✓ $-180° \leq x < -120°$ ✓ $-30° < x < 60°$ ✓ $150° < x \leq 180°$ 1 mark: each interval ✓ notation for inclusive in the first/last interval (4)
		[13]

QUESTION/*VRAAG* 7

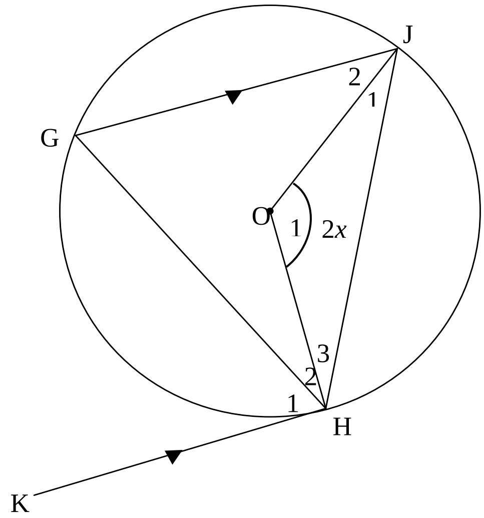


7.1	In PMQ: $\tan\theta = \dfrac{x}{\text{QM}}$ $\therefore \text{QM} = \dfrac{x}{\tan\theta}$ **OR/*OF*** $\dfrac{x}{\sin\theta} = \dfrac{\text{MQ}}{\sin P}$ $MQ = \dfrac{x\sin P}{\sin\theta}$ $= \dfrac{x\cos\theta}{\sin\theta}$ $= \dfrac{x}{\tan\theta}$	✓ trig ratio ✓ answer (2) ✓ sine rule ✓ answer (2)
7.2	In PMR: $\tan\theta = \dfrac{x}{\text{MR}}$ *OR* $\text{PMQ} \equiv \text{PMR}$ [AAS/HHS] $\therefore \text{MR} = \dfrac{x}{\tan\theta} = \text{QM}$ $\text{Q}\hat{\text{M}}\text{R} = 180° - 2\beta$ $\dfrac{\sin\beta}{\text{MR}} = \dfrac{\sin \text{Q}\hat{\text{M}}\text{R}}{12x}$ $\sin\beta \times \dfrac{\tan\theta}{x} = \dfrac{\sin(180° - 2\beta)}{12x}$ $\tan\theta = \dfrac{\sin 2\beta}{12x} \times \dfrac{x}{\sin\beta}$ $\tan\theta = \dfrac{2\sin\beta\cos\beta}{12x} \times \dfrac{x}{\sin\beta}$ $\tan\theta = \dfrac{\cos\beta}{6}$ **OR**	✓ MR = QM ✓ correct substitution into the sine rule in ΔQMR ✓ reduction ✓ double angle (4)

	In PMR : $\tan\theta = \frac{x}{\text{MR}}$ *OR* $\text{PMQ} \equiv \text{PMR}$ [AAS/HHS] $\text{MR}^2 = \text{QM}^2 + \text{QR}^2 - 2\text{QM.QR}\cos\beta$ $\text{MR}^2 = \left(\frac{x}{\tan\theta}\right)^2 + (12x)^2 - 2\left(\frac{x}{\tan\theta}\right)(12x)(\cos\beta)$ $\frac{x^2}{\tan^2\theta} = \frac{x^2}{\tan^2\theta} + 144x^2 - 24\left(\frac{x^2}{\tan\theta}\right)(\cos\beta)$ $24\left(\frac{x^2}{\tan\theta}\right)(\cos\beta) = 144x^2$ $\cos\beta = 6\tan\theta$ $\tan\theta = \frac{\cos\beta}{6}$	✓ correct substitution into the cosine rule in ΔQMR ✓ substitution ✓ MR = QM ✓ simplification (4)
7.3	$\frac{x}{\text{QM}} = \frac{\cos\beta}{6}$ [both equal $\tan\theta$] $x = \frac{60\cos 40}{6}$ x = 7,66 The height of the lighthouse is 8 metres	✓ equating ✓ subst. QM = 60 and $\beta = 40°$ ✓ answer (3)
		[9]

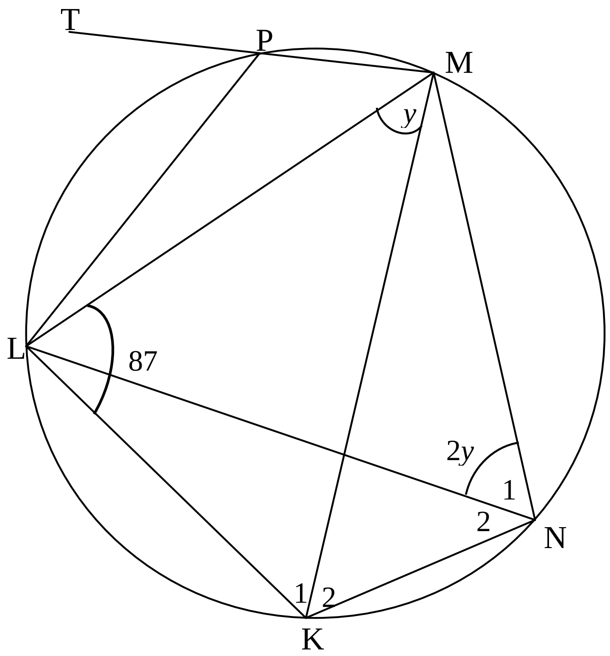
QUESTION/*VRAAG* 8

8.1



8.1.1	$\hat{G} = x$ [$\angle$ centre $= 2\times$ circumference / *midpts*$\angle = 2\times$ *omtreks*$\angle$] $\hat{H}_1 = x$ [alt $\angle$s / *verwiss* $\angle$*e*; KH || GJ] $G\hat{J}H = x$ [tan chord theorem / *raaklyn koordstelling*]	✓S ✓R ✓S ✓S ✓R (5)
8.1.2	$\hat{J}_1 + \hat{H}_3 = 180° - 2x$ [sum of $\angle$s in Δ / *som van* $\angle$*e in* Δ] $\therefore \hat{J}_1 = \hat{H}_3 = 90° - x$ [$\angle$s opp equal sides / $\angle$*e teenoor gelyke sye*] $\therefore x + \hat{H}_2 = 90°$ **OR** [tan $\perp$ radius / *raaklyn* $\perp$ *radius*] $\hat{H}_2 = 90° - x$ $\therefore \hat{H}_2 = \hat{H}_3$	✓S ✓S ✓R (3)

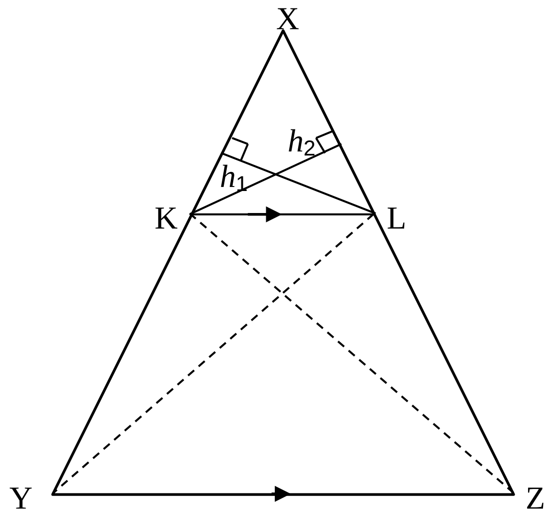
8.2



8.2.1	$\hat{N}_2 = y$ [$\angle$s in the same seg / $\angle$*e in dieselfde segment*]	✓S ✓R (2)
8.2.2(a)	$2y + y + 87° = 180°$ [opp $\angle$s of cyclic quad / *teenoorst* $\angle$*e v kvh*] $3y = 93°$ $y = 31°$	✓S ✓R ✓S (3)
8.2.2(b)	$T\hat{P}L = 62°$ [ext. $\angle$ of cyclic quad / *buite* $\angle$ *v kvh*]	✓S ✓R (2)
		[15]

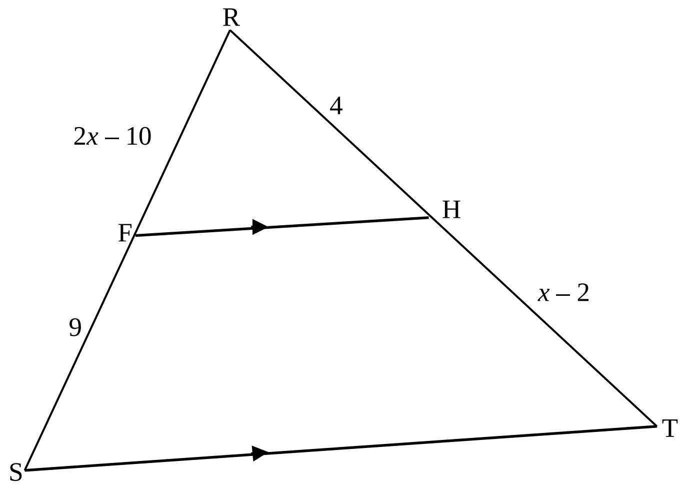
QUESTION/*VRAAG* 9

9.1



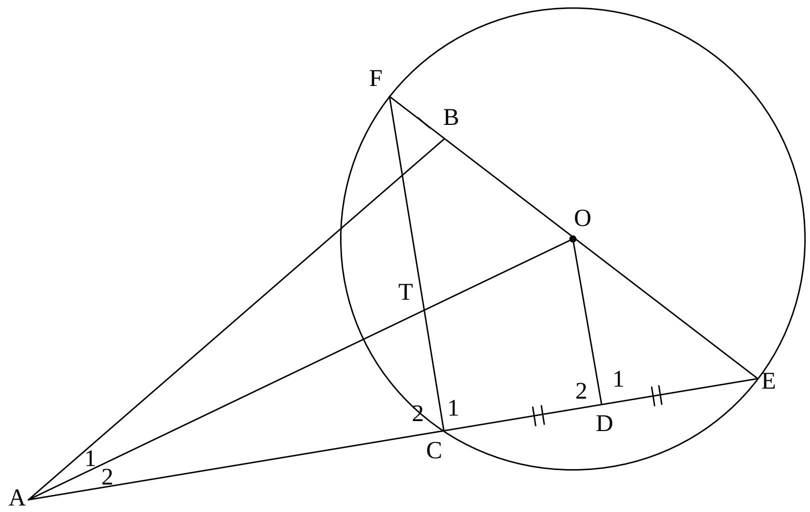
9.1	Constr: Join KZ and LY and draw h_1 from K $\perp$ XL and h_2 from L $\perp$ XK *Konstr: Verbind KZ en LY en trek* h_1 *vanaf* K $\perp$ XL *en* h_2 *vanaf* L $\perp$ XK . Proof / *Bewys*: $\frac{\text{area } \Delta\text{XKL}}{\text{area } \Delta\text{LYK}} = \frac{\frac{1}{2}\text{XK} \times h_1}{\frac{1}{2}\text{KY} \times h_1} = \frac{\text{XK}}{\text{KY}}$ $\frac{\text{area } \Delta\text{XKL}}{\text{area } \Delta\text{KLZ}} = \frac{\frac{1}{2}\text{XL} \times h_2}{\frac{1}{2}\text{LZ} \times h_2} = \frac{\text{XL}}{\text{LZ}}$ area ΔXKL = area ΔXKL [common / *gemeenskaplik*] But area ΔLYK = area ΔKLZ [same base & height ; LK ‖ YZ / *dies basis & hoogte ;* LK ‖ YZ] $\therefore \frac{\text{area } \Delta\text{XKL}}{\text{area } \Delta\text{LYK}} = \frac{\text{area } \Delta\text{XKL}}{\text{area } \Delta\text{KLZ}}$ $\therefore \frac{\text{XK}}{\text{KY}} = \frac{\text{XL}}{\text{LZ}}$	✓ constr / *konstr* ✓ $\frac{\text{area } \Delta\text{XKL}}{\text{area } \Delta\text{LYK}} = \frac{\frac{1}{2}\text{XK} \times h_1}{\frac{1}{2}\text{KY} \times h_1}$ ✓ S ✓R ✓ S (5)

9.2



9.2.1	$\frac{RF}{FS}=\frac{RH}{HT}$ [line || one side of Δ **OR** prop theorem; FH || ST] [*Lyn || een sy van Δ* ***OF*** *eweredigh. st;* FH|| ST] $\frac{2x-10}{9}=\frac{4}{x-2}$ $(2x-10)(x-2)=4\times 9$ $2x^2-14x-16=0$ $x^2-7x-8=0$ $(x-8)(x+1)=0$ $\therefore x=8\ \ (x\neq -1)$ **OR/OF** $\frac{RF}{RS}=\frac{RH}{RT}$ [line || one side of Δ **OR** prop theorem; FH || ST] [*Lyn || een sy van Δ* ***OF*** *eweredigh. st;* FH|| ST] $\frac{2x-10}{2x-1}=\frac{4}{x+2}$ $(2x-10)(x+2)=4(2x-1)$ $2x^2-14x-16=0$ $x^2-7x-8=0$ $(x-8)(x+1)=0$ $\therefore x=8\ \ (x\neq -1)$	✓S/R ✓ substitution ✓ standard form ✓ factors ✓answer with rejection (5) ✓S/R ✓ substitution ✓ standard form ✓ factors ✓answer with rejection (5)
9.2.2	$\frac{\text{area } \Delta RFH}{\text{area } \Delta RST}=\frac{\frac{1}{2}RF\times RH\sin\hat{R}}{\frac{1}{2}RS\times RT\sin\hat{R}}$ $=\frac{\frac{1}{2}\times 6\times 4\times \sin\hat{R}}{\frac{1}{2}\times 15\times 10\times \sin\hat{R}}$ $=\frac{24}{150}=\frac{4}{25}$	✓ numerator/*teller* ✓ denominator/*noemer* ✓ substitution ✓answer (4)
		[14]

QUESTION/*VRAAG* 10



10.1.1	$\hat{C}_1 = 90°$ [$\angle$ in semi circle / $\angle$ *in halfsirkel*] $\hat{D}_1 = 90°$ [line from centre to midpt of chord / *lyn vanaf midpt na midpt van koord*] $\therefore \hat{C}_1 = \hat{D}_1$ $\therefore$ FC ∥ OD [corresp $\angle$s = / *ooreenkomstige* $\angle$*e* =] **OR/OF** FO = OE [radii] CD = DE [given / *gegee*] $\therefore$ FC ∥ OD [midpoint theorem / *middelpuntstelling*]	✓ S ✓ R ✓ S ✓ R ✓ R (5) ✓ S ✓ R ✓ S ✓✓ R (5)
10.1.2	$D\hat{O}E = \hat{F}$ [corresp $\angle$s =; FC ∥ OD] $B\hat{A}E = \hat{F}$ [$\angle$s in the same seg] $\therefore D\hat{O}E = B\hat{A}E$	✓ S ✓ R ✓ S ✓ R (4)
10.1.3	In ΔABE and ΔFCE: $\hat{E}$ is common $B\hat{A}E = \hat{F}$ [proved in 10.1.2] $\therefore A\hat{B}E = \hat{C}_1$ [sum of $\angle$s in Δ] $\therefore \Delta ABE \parallel\!\mid \Delta FCE$ [$\angle\angle\angle$] $\frac{AB}{FC} = \frac{AE}{FE}$ [$\parallel\!\mid$ Δs] $AB \times FE = AE \times FC$ But FE = 2 OF [$d = 2r$] And FC =2 OD [midpoint theorem] $AB \times 2OF = AE \times 2OD$ $\therefore AB \times OF = AE \times OD$	✓ S ✓ S ✓ R ✓ S ✓ S ✓ S/R ✓ S (7)

	OR/*OF* In ΔODE and ΔABE 1. $\hat{E}$ is common 2. $D\hat{O}E = E\hat{A}B$ (proved in 10.1.2) 3. $\hat{D}_1 = A\hat{B}E$ ($\angle$ sum Δ) $\Delta ODE \mid\mid\mid \Delta ABE$ ($\angle\angle\angle$) $\frac{EO}{EA} = \frac{OD}{AB} = \frac{ED}{EB}$ ($\mid\mid\mid$ Δs) $\therefore AB.EO = OD.EA$ but OE = FO (radii) $\therefore AB \times OF = OD \times EA$	✓ S ✓ S ✓ R ✓ S ✓ S ✓ S ✓ R (7)
10.2	$\frac{AT}{TO} = \frac{AC}{CD} = \frac{3}{1}$ [line || one side of Δ **OR** prop theorem; FC || OD] But CD = DE $\frac{AE}{CE} = \frac{5}{2}$ $\therefore$ $AE = \frac{5}{2}CE$ $\frac{BE}{CE} = \frac{AE}{FE}$ [$\mid\mid\mid$ Δs] $\frac{BE}{CE} = \frac{\frac{5}{2}CE}{FE}$ $BE \times FE = \frac{5}{2}CE^2$ $\therefore 5CE^2 = 2BE.FE$	✓ S ✓ R ✓ S ✓ S ✓substitute $AE = \frac{5}{2}CE$ (5)
		[21]

TOTAL/TOTAAL: 150

MATHEMATICS P2: JUNE 2018
MARKING GUIDELINES NOTES

QUESTION 1

1.1.1	If left as 190, 25 then penalise 1 mark.
1.1.2	If the position is used: $\left[\frac{1}{4}(n+1)+\frac{3}{4}(n+1)\right]\div 2$ $=\frac{158+219}{2}$ $=\frac{377}{2}$ $=188,5$

QUESTION 2

2.4	Do not accept estimation from the table.

QUESTION 3

3.1	No ca if $\frac{x_2-x_1}{y_2-y_1}$	
3.3	MD^2+AM^2 $=\left[(3-8)^2+(-1+11)^2\right]+\left[(3-7)^2+(-1-1)^2\right]$ $=125+20$ $=145$ AD^2 $=(7-8)^2+(1+11)^2$ $=145$ $MD^2+AM^2=AD^2$	✓ AM^2+MD^2 ✓ AD^2 ✓ $MD^2+AM^2=AD^2$ (3)

QUESTION 4

4.3	Candidates can use the rotation of P through $90°$ to get to R(6 ; 4) If the candidate assumes that R(4 ; 6) : 1/4 marks

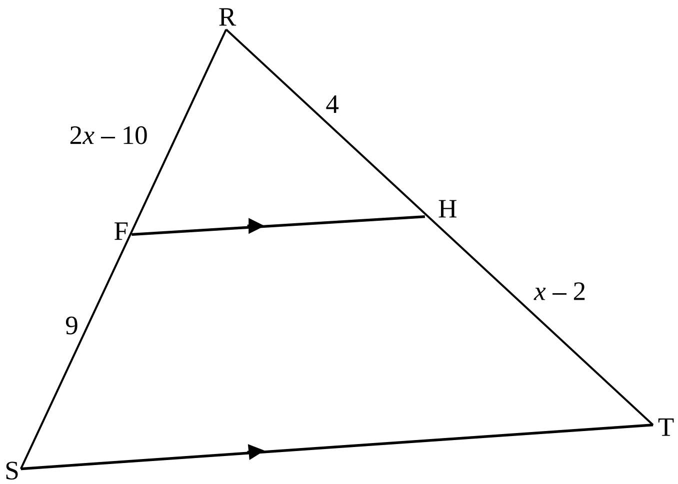
QUESTION 6

6.2	$y\in(-2\ ;\ 2)$ $-2<y<2$	1/2 marks 1/2 marks

QUESTION 7

7.3	There is NO penalty for incorrect rounding.

QUESTION 9



9.2.2	Join FT. area ΔRFH = $\frac{4}{10} \times$ (area ΔRFT) But area ΔRFT = $\frac{6}{15} \times$ (area ΔRST) (common vertex; = heights) area ΔRFH = $\frac{4}{10} \times \frac{6}{15} \times$(area ΔRST) $\frac{\text{area}\Delta\text{RFH}}{\text{area}\Delta\text{RST}} = \frac{4}{25}$

basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

NATIONAL
SENIOR CERTIFICATE
NASIONALE SENIOR
SERTIFIKAAT

GRADE 12/*GRAAD* 12

MATHEMATICS P2/*WISKUNDE V2*

NOVEMBER 2017

MARKING GUIDELINES/*NASIENRIGLYNE*

MARKS/*PUNTE:* *150*

These marking guidelines consist of 29 pages.
Hierdie nasienriglyne bestaan uit 28 bladsye.

NOTE:

- If a candidate answers a question TWICE, only mark the FIRST attempt.
- If a candidate has crossed out an attempt of a question and not redone the question, mark the crossed out version.
- Consistent accuracy applies in ALL aspects of the marking guidelines. Stop marking at the second calculation error.
- Assuming answers/values in order to solve a problem is NOT acceptable.

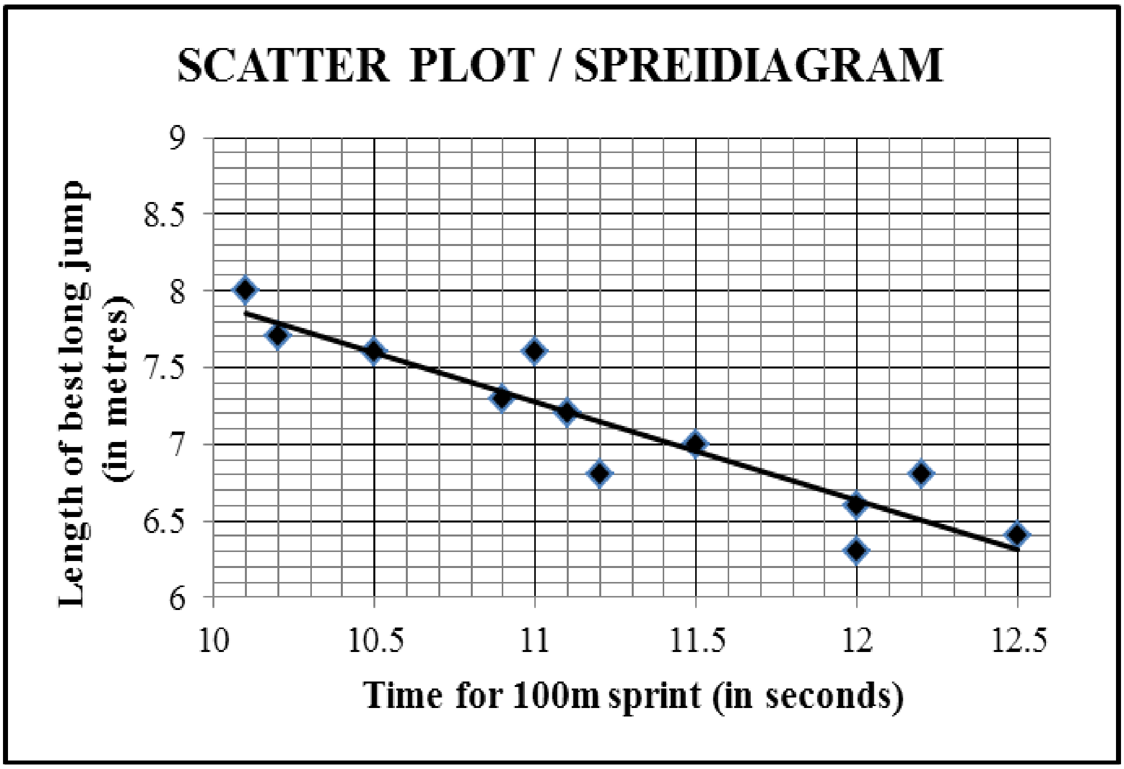
NOTA:

- *As 'n kandidaat 'n vraag TWEE KEER beantwoord, merk slegs die EERSTE poging.*
- *As 'n kandidaat 'n antwoord van 'n vraag doodtrek en nie oordoen nie, merk die doodgetrekte poging.*
- *Volgehoue akkuraatheid word in ALLE aspekte van die nasienriglyne toegepas. Hou op nasien by die tweede berekeningsfout.*
- *Aanvaar van antwoorde/waardes om 'n probleem op te los, word NIE toegelaat nie.*

GEOMETRY	
S	**A mark for a correct statement** **(A statement mark is independent of a reason.)**
	'n Punt vir 'n korrekte bewering **('n Punt vir 'n bewering is onafhanklik van die rede.)**
R	**A mark for a correct reason** **(A reason mark may only be awarded if the statement is correct.)**
	'n Punt vir 'n korrekte rede **('n Punt word slegs vir die rede toegeken as die bewering korrek is.)**
S/R	**Award a mark if the statement AND reason are both correct.**
	Ken 'n punt toe as beide die bewering EN rede korrek is.

QUESTION/*VRAAG* 1

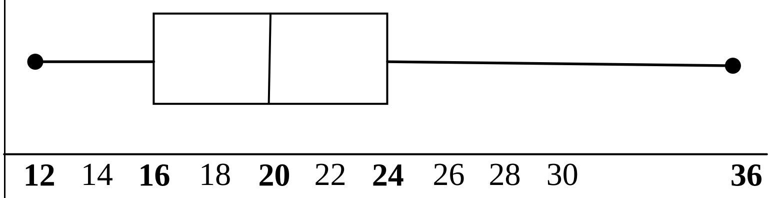
Time for 100 m sprint (in seconds) ***Tyd vir 100 m-naelloop (in sekondes)***	10,1	10,2	10,5	10,9	11	11,1	11,2	11,5	12	12	12,2	12,5
Distance of best long jump (in metres) ***Afstand van beste sprong in verspring (in meter)***	8	7,7	7,6	7,3	7,6	7,2	6,8	7	6,6	6,3	6,8	6,4



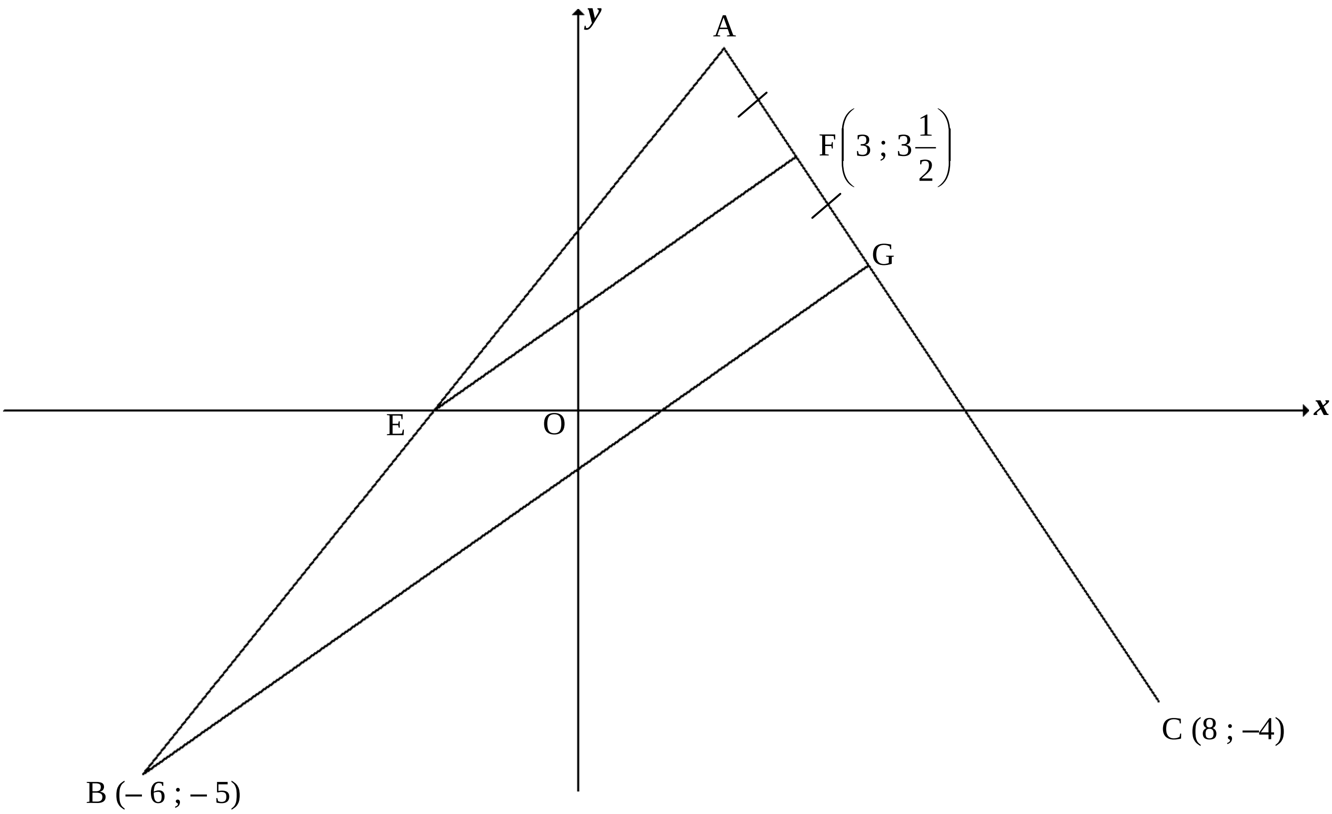
1.1	$a = 14{,}343\ldots = 14{,}34$ $b = -0{,}642\ldots = -0{,}64$	✓✓ value of *a* ✓ value of *b* (3)
1.2	$y = 14{,}34 - 0{,}64(11{,}7)$ $= 6{,}85$ **OR/OF** $y = 6{,}83$ (calculator / *sakrekenaar*)	✓ substitution correctly ✓ answer (2) ✓✓answer (2)
1.3	The gradient increases / *Die gradient neem toe* The point (12,3 ; 7,6) lies some distance above the current data. /*Die punt (12,3 ; 7,6) lê bokant die huidige data.*	✓ increases/*neem toe* ✓ reasoning in words/ *redenasie in woorde* (2) **[7]**

QUESTION/*VRAAG* 2

12	13	13	14	14	16	17	18	18	18	19	20
21	21	22	22	23	24	25	27	29	30	36	

2.1.1	$\bar{x} = \frac{472}{23}$ $\bar{x} = 20{,}52$ seconds / *sekonde*	✓ $\frac{472}{23}$ ✓ answer (2)
2.1.2	$Q_1 = 16$ $Q_3 = 24$ IQR/*IKO* $= Q_3 - Q_1$ $= 24 - 16 = 8$	✓ Q_1 ✓ Q_3 ✓ answer (3)
2.2	$20{,}52 + 5{,}94 = 26{,}46$ $\therefore > 26{,}46$ $\therefore$ 4 girls/*dogters*	✓ 26,46 ✓ answer (2)
2.3		✓ whiskers ending at 12 & 36 ✓ $Q_1 = 16$ & $Q_3 = 24$ (box) ✓ $Q_2 = 20$ (3)
2.4.1	Girls / *Meisies*	✓ answer (1)
2.4.2	Five-number summary of boys: (15 ; 21 ; 23,5 ; 26 ; 38) **None** of the boys / ***Nie een** van die seuns **nie*** 5 girls completed in less than 15 seconds which was the minimum time taken by the boys. *5 meisies voltooi in minder as 15 sekondes, wat die minimumtyd is wat die seuns geneem het.*	✓ answer ✓ reason/*rede* (2) **[13]**

QUESTION/*VRAAG* 3



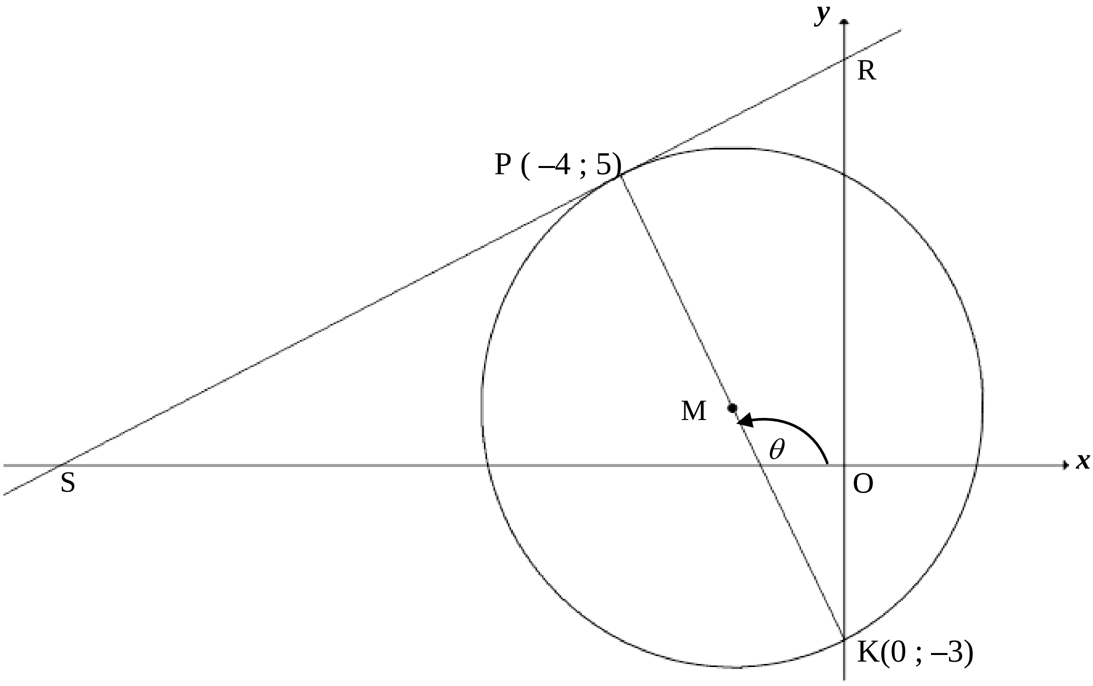
3.1.1	$m_{FC}=\dfrac{y_2-y_1}{x_2-x_1}$ $=\dfrac{3\frac{1}{2}-(-4)}{3-8}$ $=-\dfrac{3}{2}$ $y = mx + c$ $\qquad$ $y-y_1=m(x-x_1)$ $y=-\dfrac{3}{2}x+c$ $-4=-\dfrac{3}{2}(8)+c$ **OR/*OF*** $(y-(-4))=-\dfrac{3}{2}(x-8)$ $c=8$ $\qquad$ $y+4=-\dfrac{3}{2}x+12$ $y=-\dfrac{3}{2}x+8$ $\qquad$ $y=-\dfrac{3}{2}x+8$ **OR/*OF***	✓ substitution of (8 ; –4) & $\left(3;3\frac{1}{2}\right)$ ✓ gradient ✓ substitution of *m* and (8 ; –4) ✓ equation of AC (4)

	$m_{FC} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{(-4)-\left(3\frac{1}{2}\right)}{8-3}$ $= -\frac{3}{2}$ $y = mx + c$ $y - y_1 = m(x - x_1)$ $3\frac{1}{2} = -\frac{3}{2}(3) + c$ $\left(y - 3\frac{1}{2}\right) = -\frac{3}{2}(x-3)$ $c = 8$ **OR/*OF*** $\left(y - 3\frac{1}{2}\right) = -\frac{3}{2}x + \frac{9}{2}$ $y = -\frac{3}{2}x + 8$ $y = -\frac{3}{2}x + 8$	✓ substitution of (8 ; –4) & $\left(3 ; 3\frac{1}{2}\right)$ ✓ gradient ✓ substitution of *m* and $\left(3 ; 3\frac{1}{2}\right)$ ✓ equation of AC (4)
3.1.2	AC: $3x + 2y = 16$ and BG: $7x - 10y = 8$ $15x + 10y = 80$ $\underline{7x - 10y = 8}$ $22x = 88$ $x = 4$ $3(4) + 2y = 16$ $y = 2$ $\therefore G(4 ; 2)$ **OR/*OF*** BG: $7x - 10y = 8$ $\therefore y = \frac{7}{10}x - \frac{8}{10}$ $\therefore \frac{7}{10}x - \frac{8}{10} = -\frac{3}{2}x + 8$ [CA from 3.1.1] $\frac{11}{5}x = \frac{44}{5}$ $x = 4$ $3(4) + 2y = 16$ $y = 2$ $\therefore G(4 ; 2)$	✓ method /*metode*: solving simultaneously / *los gelyktydig op* ✓ *x* coordinate ($x > 0$) ✓ *y* coordinate (3) ✓ method: equating *metode: stel vgls gelyk* ✓ *x* coordinate ($x > 0$) ✓ *y* coordinate (3)
3.2	$\frac{x_A + 4}{2} = 3$ and $\frac{y_A + 2}{2} = 3\frac{1}{2}$ $\therefore A(2 ; 5)$ **OR/*OF*** by translation/*deur translasie*: $x_A = 3 - (4 - 3) = 2$ $y_A = 3\frac{1}{2} + (3\frac{1}{2} - 2) = 5$ $\therefore A(2 ; 5)$	✓ equation ito *x* ✓ equation ito *y* (2) ✓ equation ito *x* ✓ equation ito *y* (2)

3.3	The coordinates of the midpt of AB / *Die koordinaat van midpt van AB is*: $\left(\frac{2+(-6)}{2};\frac{5+(-5)}{2}\right)=(-2\ ;\ 0)$ **But the *y*-coordinate of E is 0** **$\therefore$ E(–2 ; 0) is the midpoint of AB** $\therefore$ EF ‖ BG [midpoint theorem/*middelpuntst* ***OR/OF*** line divides 2 sides of Δ in prop/*lyn verdeel 2 sye van Δ in dies verh*]	✓ subst A & B into midpt formula ✓ *y* coordinate = 0 ✓ E = midpt ✓ Reason (4)
	OR/*OF* The coordinates of the midpt of AB / *Die koordinaat van midpt van AB is*: $\left(\frac{2+(-6)}{2};\frac{5+(-5)}{2}\right)=(-2\ ;\ 0)$ $AE=\sqrt{(-2-2)^2+(0-5)^2}=\sqrt{41}$ $EB=\sqrt{(-2-(-6))^2+(0-(-5))^2}=\sqrt{41}$ $\therefore$ In ΔABG: **AE = EB** and AF = FG $\therefore$ EF ‖ BG [midpoint theorem/*middelpuntst*]	✓ subst A & B into midpt formula ✓ lengths of AE & EB ✓ AE = EB or E = midpt ✓ Reason (4)
	OR/*OF* Equation of AB: $y-(-5)=\left(\frac{5-(-5)}{2-(-6)}\right)(x-(-6))$ $y+5=\frac{10}{8}x+\frac{15}{2}\qquad \therefore y=\frac{5}{4}x+\frac{5}{2}$ x-intercept of AB: $0=\frac{5}{4}x+\frac{5}{2}\quad \therefore x=-2$ $\therefore$ E(–2 ; 0) $m_{EF}=\frac{3\frac{1}{2}-0}{3-(-2)}=\frac{7}{10}$ $m_{EF}=m_{BG}=\frac{7}{10}$ $\therefore$ EF ‖ BG BG: $7x-10y=8$ $\therefore\quad y=\frac{7}{10}x-\frac{8}{10}$ $\therefore\ m_{BG}=\frac{7}{10}$	✓ equation of AB ✓ coordinates of E ✓ gradient of EF ✓ gradient EF = gradient BG (4)

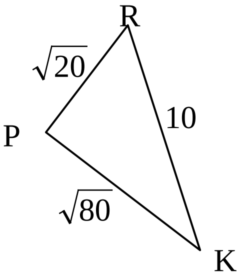
3.4	Midpoint of AC = $\left(5\ ;\ \frac{1}{2}\right)$ $\frac{x_D+(-6)}{2}=5$ and $\frac{y_D+(-5)}{2}=\frac{1}{2}$ $\therefore$ D(16 ; 6)	✓✓ $\left(5\ ;\ \frac{1}{2}\right)$ ✓ x value ✓ y value (4)
	OR/*OF* by translation/*dmv translasie*: D(16 ; 6)	✓ method finding x ✓ method finding y ✓ x value ✓ y value (4)
	OR/*OF* $m_{BC}=\frac{-5-(-4)}{-6-8}=\frac{1}{14}$ and $m_{AB}=\frac{5-(-5)}{2-(-6)}=\frac{5}{4}$ AD: $y-5=\frac{1}{14}(x-2)\Rightarrow y=\frac{1}{14}x+\frac{34}{7}$ CD: $y+4=\frac{5}{4}(x-8)\Rightarrow y=\frac{5}{4}x-14$ $\frac{5}{4}x-14=\frac{1}{14}x+\frac{34}{7}$ $\therefore$ $x=16$ $y=6$	✓✓ equating ✓ x value ✓ y value (4) **[17]**

QUESTION/*VRAAG* 4



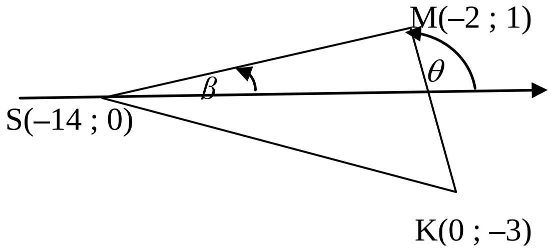
4.1.1	$m_{PK} = \frac{5-(-3)}{-4-0}$ $= -2$ PK $\perp$ SR [radius $\perp$ tangent/*raaklyn*] $\therefore\ m_{PK} \times m_{RS} = -1$ $\therefore m_{RS} = \frac{1}{2}$	✓ substitution P & K into gradient formula ✓ gradient of PK ✓ PK $\perp$ SR **OR** r $\perp$ tangent ✓ answer (4)
4.1.2	$y = \frac{1}{2}x + c$ $5 = \frac{1}{2}(-4) + c$ **OR/*OF*** $(y-5) = \frac{1}{2}(x-(-4))$ $c = 7$ $(y-5) = \frac{1}{2}x + 2$ $y = \frac{1}{2}x + 7$ $y = \frac{1}{2}x + 7$	✓ substitution of *m* and P ✓ equation (2)

4.1.3	$M\left(\frac{-4+0}{2}\ ;\ \frac{5+(-3)}{2}\right)$ $\therefore$ M(–2 ; 1) $r^2=(x_2-x_1)^2+(y_2-y_1)^2$ $r^2=(-2+4)^2+(1-5)^2$ $\therefore r^2=20$ $\therefore (x+2)^2+(y-1)^2=20\ \ or\ (\sqrt{20})^2$	✓ x value of M ✓ y value of M ✓ $r^2=20$ ✓ equation (4)
	OR/*OF*	
	$M\left(\frac{-4+0}{2}\ ;\ \frac{5+(-3)}{2}\right)\quad \therefore$ M(–2 ; 1) $(x+2)^2+(y-1)^2=r^2$ $(-4+2)^2+(5-1)^2=r^2$ $\therefore r^2=20$ $\therefore (x+2)^2+(y-1)^2=20\ \ or\ (\sqrt{20})^2$	✓✓ M (– 2 ; 1) $r^2=20$ ✓ equation (4)
	OR/*OF*	
	$M\left(\frac{-4+0}{2}\ ;\ \frac{5+(-3)}{2}\right)\quad \therefore$ M(–2 ; 1) $PK=\sqrt{(-4-0)^2+(5-(-3))^2}=\sqrt{80}$ $r=\frac{\sqrt{80}}{2}=\sqrt{20}$ $\therefore (x+2)^2+(y-1)^2=20\ \ or\ (\sqrt{20})^2$	✓✓ M (– 2 ; 1) $r^2=20$ ✓ equation (4)

4.1.4	$\tan\theta = m_{PK} = -2$ $\therefore\ \theta = 180° - 63{,}43°$ $= 116{,}57°$ $P\hat{K}R = 116{,}57° - 90°$ [ext $\angle$ of ΔMOK] $= 26{,}57°$	✓ $\tan\theta = -2$ ✓ size of θ ✓ answer (3)
	OR/*OF*  In ΔRPK: $PK = \sqrt{(0-(-4))^2 + (-3-5)^2} = \sqrt{80}$ $PR = \sqrt{(-4-0)^2 + (5-7)^2} = \sqrt{20}$ RK = 10 $\cos P\hat{K}R = \dfrac{PK^2 + KR^2 - PR^2}{2.PK.KR} = \dfrac{(\sqrt{80})^2 + (10)^2 - (\sqrt{20})^2}{2(\sqrt{80})(10)}$ $= \dfrac{2\sqrt{5}}{5}$ $P\hat{K}R = 26{,}57°$	✓ lengths of PK, PR & RK ✓ correct values into cos rule ✓ answer (3)
	OR/*OF* $\sin P\hat{K}R = \dfrac{\sqrt{20}}{10}$ **OR/*OF*** $\cos P\hat{K}R = \dfrac{\sqrt{80}}{10}$ $P\hat{K}R = 26{,}57°$ $P\hat{K}R = 26{,}57°$	✓ lengths of sides ✓ ratio ✓ answer (3)
	OR/*OF* $\tan P\hat{K}R = \dfrac{\sqrt{20}}{\sqrt{80}}$ $P\hat{K}R = 26{,}57°$	✓ lengths of sides ✓ ratio ✓ answer (3)

4.1.5	RS ∥ tangent at K$(0\ ;\ -3)$ $\therefore m_{\text{PS}} = m_{\text{tang}} = \frac{1}{2}$ $\therefore y = \frac{1}{2}x - 3$ **OR/*OF*** $m_{PK} = \frac{1-5}{-2+4} = -2$ $m_{PK} \times m_{\tan g} = -1$ [radius ⊥ tangent/*raaklyn*] $\therefore m_{\tan g} = \frac{1}{2}$ $\therefore\ y = \frac{1}{2}x - 3$	✓ gradient ✓ equation (2) ✓ gradient ✓ equation (2)
4.2	$t \in (-3\ ;\ 7)$ **OR/*OF*** $-3 < t < 7$	✓ -3 (A) ✓ 7 (CA from 4.1.2) ✓ correct inequality (3) ✓ -3 (A) ✓ 7 (CA from 4.1.2) ✓ correct inequality (3)
4.3	RS: $y = \frac{1}{2}x + 7$ $\therefore$ S(–14 ; 0) SP $= \sqrt{(-14-(-4))^2 + (0-5)^2} = \sqrt{100+25} = \sqrt{125}$ Area ΔSMK = $\frac{1}{2}$. MK . SP $= \frac{1}{2}(\sqrt{20})(\sqrt{125})$ = 25 square units	✓ coordinates of S ✓ length of SP ✓ correct base & height into Area rule ✓ correct substitution ✓ answer (5)

OR/OF



Let β = inclination of SM/ *inklinasie van SM*

RS: $y = \frac{1}{2}x + 7 \quad \therefore$ S(–14 ; 0) ✓ coordinates of S

$\text{SM} = \sqrt{(-14-(-2))^2 + (0-1)^2} = \sqrt{145}$ ✓ length of SM

$\tan\beta = \dfrac{1-0}{-2-(-14)} = \dfrac{1}{12} \quad \therefore \ \beta = 4{,}76°$

$\therefore \text{S}\hat{\text{M}}\text{K} = 116{,}57° - 4{,}76°$ [ext ∠ of Δ]

$= 111{,}81°$ ✓ size of/*grootte v* $\text{S}\hat{\text{M}}\text{K}$

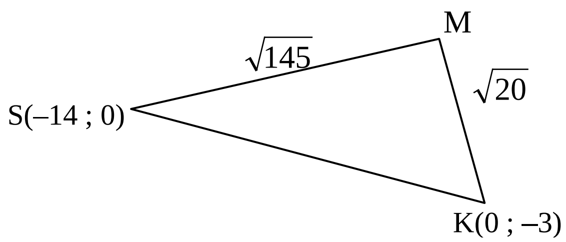
$\text{Area } \Delta\text{SMK} = \frac{1}{2}(\text{SM})(\text{MK}).\sin\text{S}\hat{\text{M}}\text{K}$

$= \frac{1}{2}(\sqrt{145})(\sqrt{20}).\sin 111{,}81°$ ✓ correct substitution into area rule

$= 24{,}9985 = 25$ square units ✓ answer

(5)

OR/*OF*



RS: $y = \frac{1}{2}x + 7 \quad \therefore$ S(–14 ; 0) ✓ coordinates of S

$\text{SK} = \sqrt{(-14-0)^2 + (0+3)^2} = \sqrt{205}$ ✓ length of SK

$\cos\text{S}\hat{\text{M}}\text{K} = \dfrac{(\sqrt{145})^2 + (\sqrt{20})^2 - (\sqrt{205})^2}{2(\sqrt{145})(\sqrt{20})} = -\dfrac{2\sqrt{29}}{29}$

$\text{S}\hat{\text{M}}\text{K} = 111{,}80°$ ✓ size of /*grootte v* $\text{S}\hat{\text{M}}\text{K}$

$\text{Area } \Delta\text{SMK} = \frac{1}{2}(\text{SM})(\text{MK}).\sin\text{S}\hat{\text{M}}\text{K}$

$= \frac{1}{2}(\sqrt{145})(\sqrt{20}).\sin 111{,}81°$ ✓ correct substitution into area rule

$= 24{,}9985 = 25$ square units ✓ answer

(5)

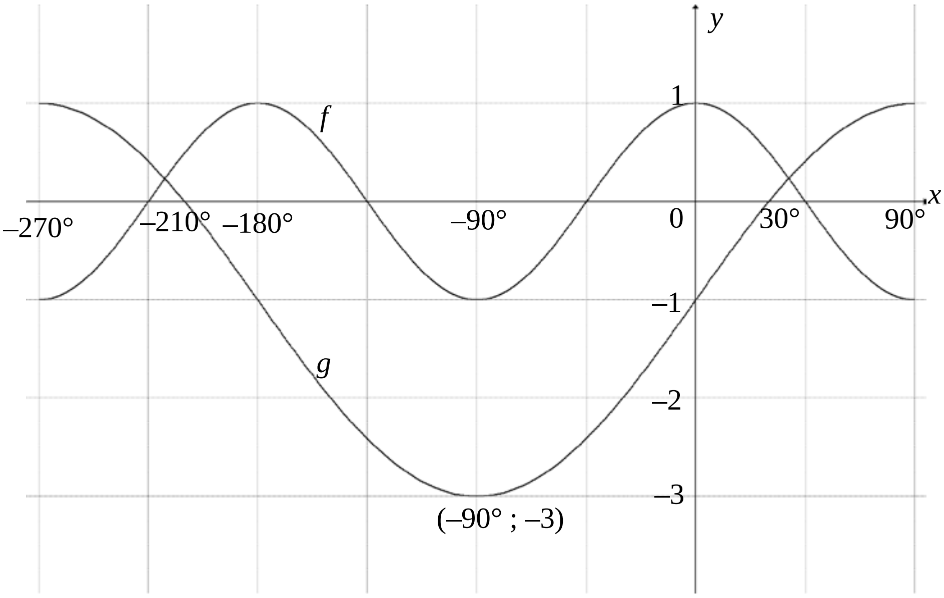
	OR/*OF* Produce KS to T RS: $y=\frac{1}{2}x+7 \quad \therefore$ S(–14 ; 0) $SK=\sqrt{(-14-0)^2+(0+3)^2}=\sqrt{205}$ $SM=\sqrt{(-14-(-2))^2+(0-1)^2}=\sqrt{145}$ $m_{SK}=-\frac{3}{14} \Rightarrow T\hat{S}O=167{,}91°$ $m_{SM}=\frac{1}{12} \Rightarrow M\hat{S}O=4{,}76°$ $M\hat{S}K=180°-167{,}91°+4{,}76°=16{,}85°$ Area $\Delta SMK=\frac{1}{2}(SM)(SK).\sin M\hat{S}K$ $=\frac{1}{2}(\sqrt{145})(\sqrt{205}).\sin 16{,}85°$ $=24{,}9985=25$ square units	✓ coordinates of S ✓ length of SK & SM ✓ size of /*grootte v* $M\hat{S}K$ ✓ correct substitution into area rule ✓ answer (5)

QUESTION/*VRAAG* 5

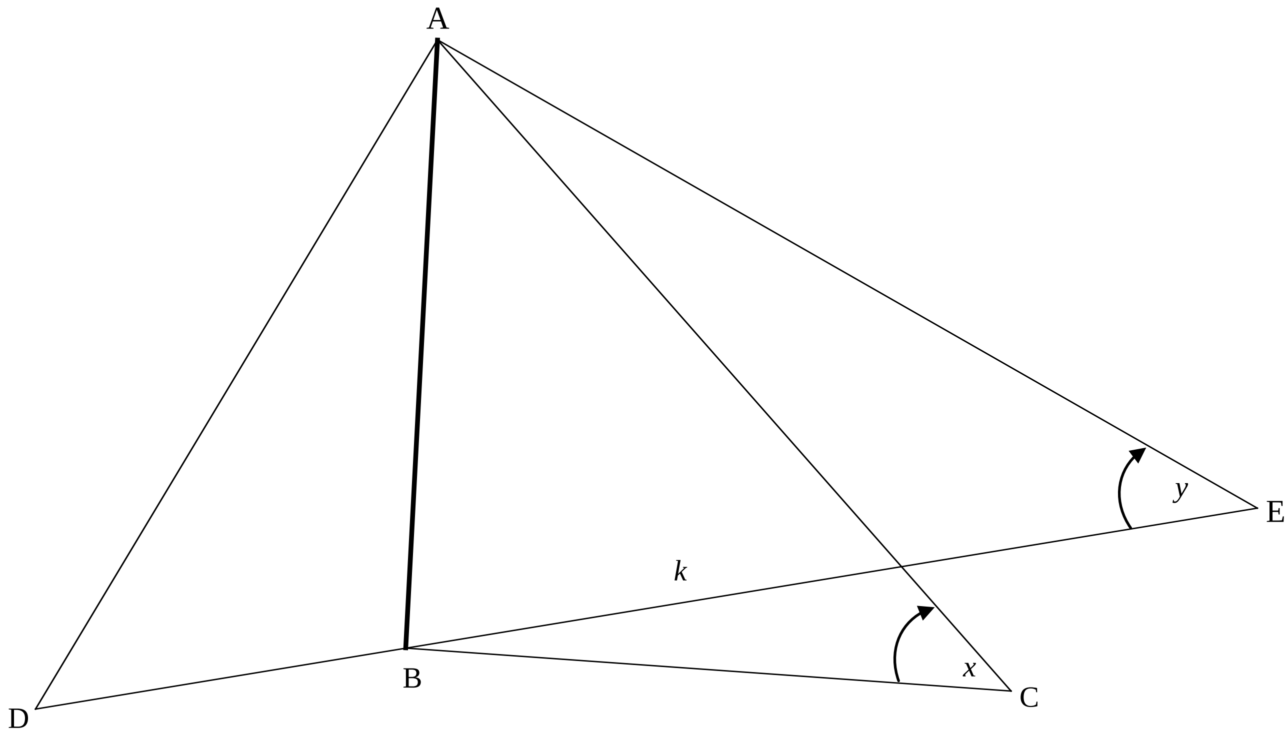
5.1	$\frac{\sin(A-360°).\cos(90°+A)}{\cos(90°-A).\tan(-A)}$ $=\frac{\sin A(-\sin A)}{\sin A(-\tan A)}$ $=\frac{\sin A}{\left(\frac{\sin A}{\cos A}\right)}$ $=\cos A$	✓ sin A ✓–sin A ✓ sin A ✓ –tan A ✓ $\tan A=\frac{\sin A}{\cos A}$ ✓ answer (6)
5.2.1	$t^2=(\sqrt{34})^2-(3)^2$ $\therefore t=-5$	✓ substitution ✓ answer (2)
5.2.2	$\tan\beta=\frac{-5}{3}$	✓ correct ratio (1)
5.2.3	$\cos 2\beta=2\cos^2\beta-1$ $=2\left(\frac{3}{\sqrt{34}}\right)^2-1$ $=2\left(\frac{9}{34}\right)-1$ $=-\frac{16}{34}$ OR $-\frac{8}{17}$ **OR/*OF*** $\cos 2\beta=1-2\sin^2\beta$ $=1-2\left(-\frac{5}{\sqrt{34}}\right)^2$ $=1-2\left(\frac{25}{34}\right)$ $=-\frac{16}{34}$ OR $-\frac{8}{17}$ **OR/*OF*** $\cos 2\beta=\cos^2\beta-\sin^2\beta$ $=\left(\frac{3}{\sqrt{34}}\right)^2-\left(-\frac{5}{\sqrt{34}}\right)^2$ $=\frac{9}{34}-\frac{25}{34}$ $=-\frac{16}{34}$ OR $-\frac{8}{17}$	✓ compound formula ✓ substitution ✓ simplification ✓ answer (4) ✓ compound formula ✓ substitution ✓ simplification ✓ answer (4) ✓ compound formula ✓ substitution ✓ simplification ✓ answer (4)

5.3.1	LHS = sin(A + B) – sin(A – B) = sin A.cos B + cos A.sin B – (sin A. cos B – cos A. sin B) = sin A.cos B + cos A.sin B – sin A. cos B + cos A. sin B = 2cos A.sin B = RHS	✓ compound formula ✓ compound formula (2)
5.3.2	$\sin 77° - \sin 43° = \sin(60° + 17°) - \sin(60° - 17°)$ $= 2\cos 60°.\sin 17°$ $= 2 \times \frac{1}{2} \times \sin 17°$ $= \sin 17°$ **OR/*OF*** $\sin 77° - \sin 43° = \sin(60° + 17°) - \sin(60° - 17°)$ $= (\sin 60° \cos 17° + \cos 60° \sin 17°) -$ $(\sin 60° \cos 17° - \cos 60° \sin 17°)$ $= \frac{\sqrt{3}}{2}\cos 17° + \frac{1}{2}\sin 17° - \frac{\sqrt{3}}{2}\cos 17° + \frac{1}{2}\sin 17°$ $= \sin 17°$	✓ 60° + 17° ✓ 60° – 17° ✓ simplify ✓ $\frac{1}{2}$ (4) ✓ 60° + 17° ✓ 60° – 17° ✓ expansion ✓ $\frac{1}{2}$ (4) **[19]**

QUESTION/*VRAAG* 6

6.1		✓ (–90° ; –3) ✓ (0 ; –1) ✓ x – intercepts: –210° & 30° ✓ shape (4)
6.2	$\cos 2x = 2\sin x - 1$ $1 - 2\sin^2 x = 2\sin x - 1$ $2\sin^2 x + 2\sin x - 2 = 0$ $\sin^2 x + \sin x - 1 = 0$ $\sin x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $= \dfrac{-1 \pm \sqrt{1^2 - 4(1)(-1)}}{2(1)}$ $\sin x = \dfrac{-1+\sqrt{5}}{2}$, since $\sin x = \dfrac{-1-\sqrt{5}}{2} < -1$ has no solution	✓ $\cos 2x = 1 - 2\sin^2 x$ ✓ standard form ✓ using quadratic formula ✓ substitution into quadratic formula (4)
6.3	$\sin x = \dfrac{-1+\sqrt{5}}{2} = 0{,}618...$ Reference $\angle$ = 38,17° $\therefore$ x = 38,17° + k.360° or x = 141,83° + k.360° ; $k \in Z$ $\therefore x = 38{,}17°$ or $-218{,}17°$ $y = 0{,}24$ $\therefore$ Points of intersection/*snypunte*: $(38{,}17°\,;0{,}24)$ and $(-218{,}17°\,;0{,}24)$	✓ 38,17° ✓ 141,83° ✓ –218,17° ✓ 0,24 (4) **[12]**

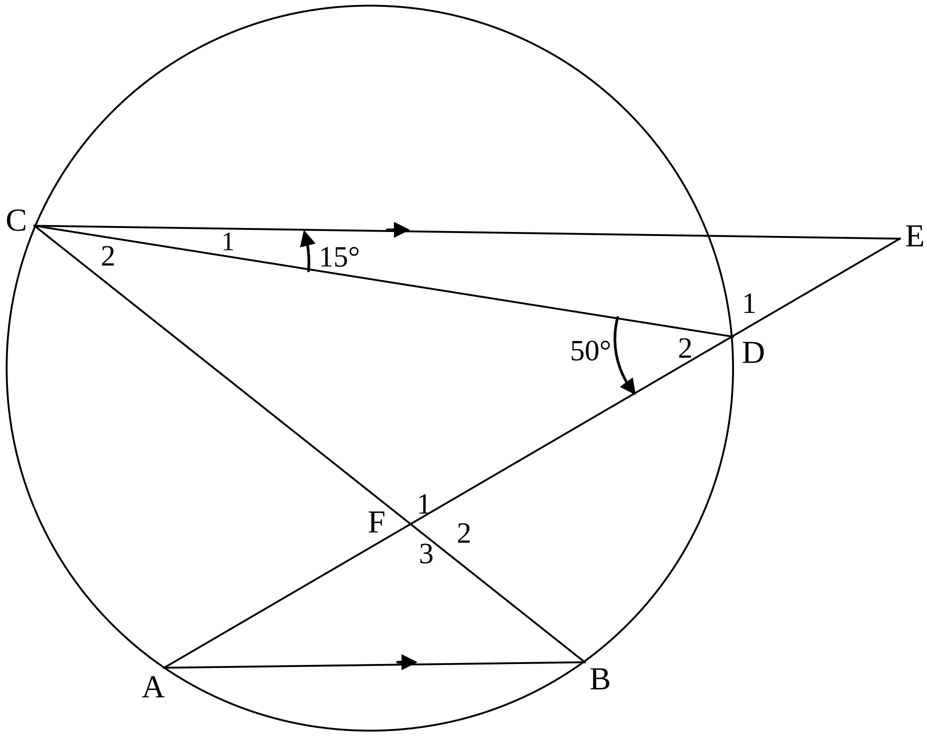
QUESTION/*VRAAG* 7



7.1	$A\hat{B}C = 90°$	✓ answer (1)
7.2	In Δ ABE: $\frac{AB}{BE} = \tan y$ $AB = k \tan y$ In Δ ABC: $\frac{AB}{AC} = \sin x$ $AC = \frac{AB}{\sin x}$ $= \frac{k \tan y}{\sin x}$	✓ correct ratio ✓ value AB ✓ correct ratio ✓ AC as subject and substitution (4)

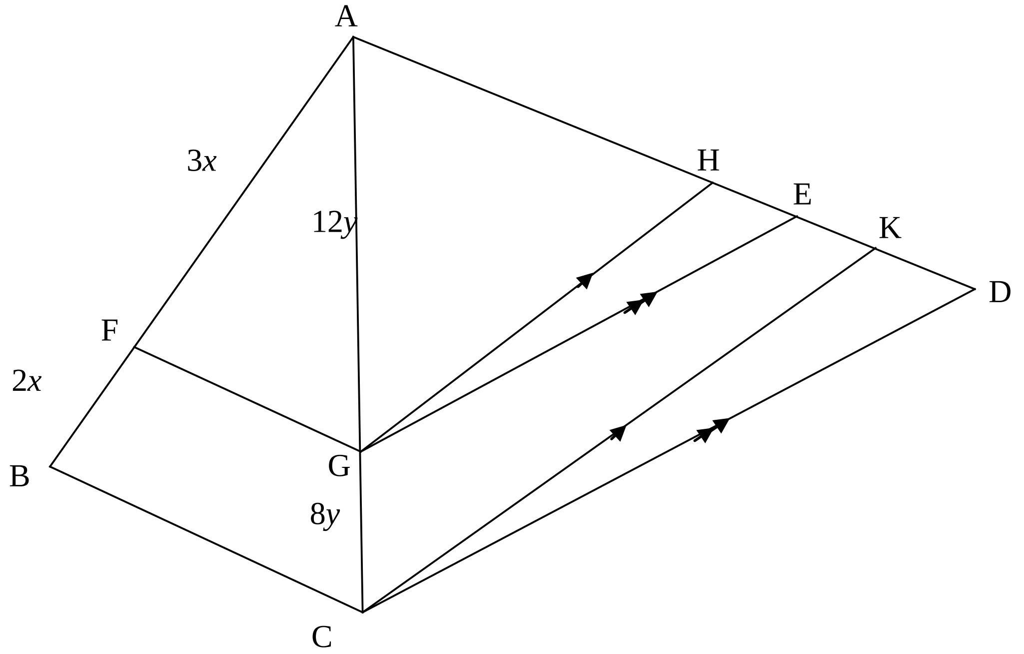
7.3	$\hat{ADC} = \hat{ACD} = \dfrac{180° - 2x}{2} = 90° - x$ $\dfrac{DC}{\sin 2x} = \dfrac{AC}{\sin(90° - x)}$ $\dfrac{DC}{2\sin x\cos x} = \dfrac{AC}{\cos x}$ $DC = \dfrac{AC(2\sin x\cos x)}{\cos x}$ $= \dfrac{k\tan y}{\sin x}.\dfrac{2\sin x\cos x}{\cos x}$ $= 2k\tan y$	✓ $90° - x$ ✓subst into sine rule ✓ $2\sin x\cos x$ ✓ $\cos x$ ✓ substitution (5)
	OR/*OF* $DC^2 = AD^2 + AC^2 - 2AD.AC\cos 2x$ $= AC^2 + AC^2 - 2AC^2\cos 2x$ $= 2AC^2(1 - \cos 2x)$ $= 2AC^2(1 - 1 + \sin^2 x)$ $= 4AC^2\sin^2 x$ $DC = 2AC.\sin x$ $= 2\left(\dfrac{k.\tan y}{\sin x}\right).\sin x$ $= 2k.\tan y$	✓substitution into cos rule ✓ factorisation ✓ $1 - 2\sin^2 x$ ✓ DC ito AC and sin x ✓ substitution (5)
	OR/*OF* $DC^2 = AD^2 + AC^2 - 2AD.AC\cos 2x$ $= 2\left(\dfrac{k\tan y}{\sin x}\right)^2 - 2\left(\dfrac{k\tan y}{\sin x}\right)^2\cos 2x$ $= \dfrac{2k^2\tan^2 y}{\sin^2 x} - \dfrac{2k^2\tan^2 y}{\sin^2 x}(1 - 2\sin^2 x)$ $= \dfrac{2k^2\tan^2 y}{\sin^2 x} - \dfrac{2k^2\tan^2 y}{\sin^2 x} + 4k^2\tan^2 y$ $DC = \sqrt{4k^2\tan^2 y}$ $= 2k\tan y$	✓correct cos rule ✓ substitution ✓ $1 - 2\sin^2 x$ ✓ squaring and multiplication ✓ $\sqrt{4k^2\tan^2 y}$ (5) **[10]**

QUESTION/*VRAAG* 8



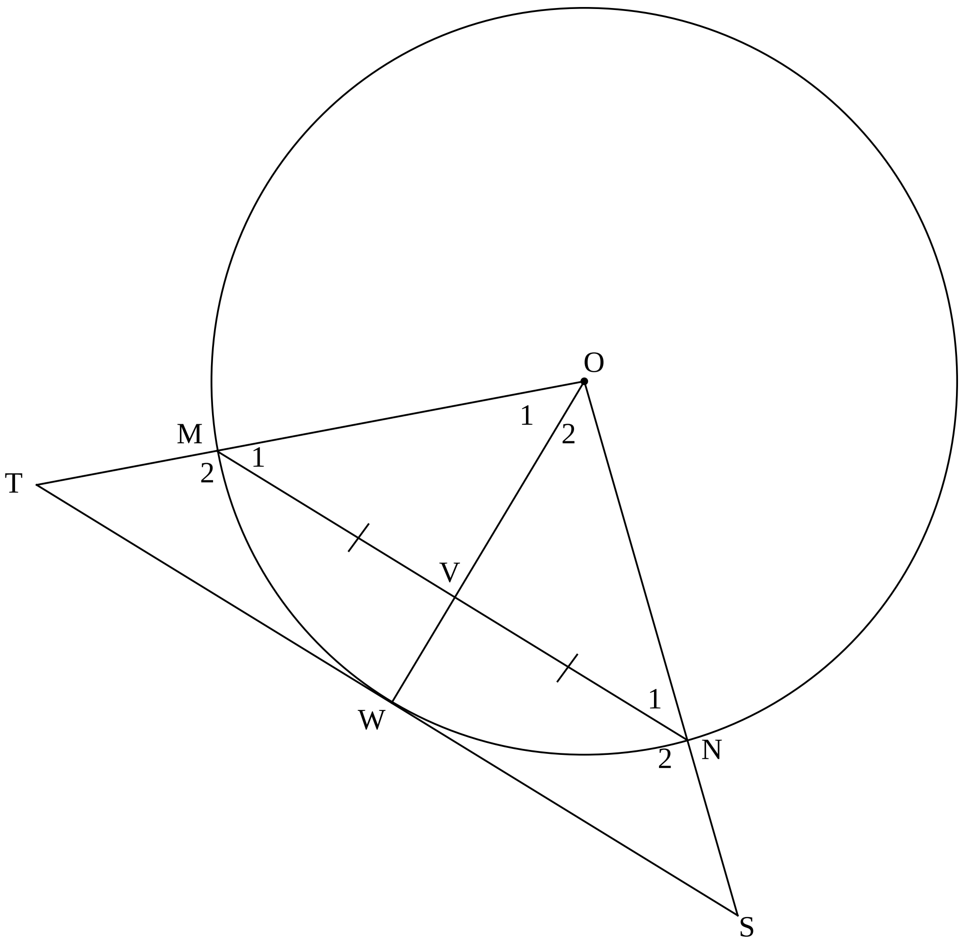
8.1.1	$\hat{E} = 50° - 15° = 35°$ [ext ∠ of Δ/*buite ∠ van Δ*] $\hat{A} = 35°$ [alt ∠s / *verwiss ∠e*; CE || AB] **OR/*OF*** $\hat{E} = 180° - (130° + 15°) = 35°$ [str line; ∠s of Δ/*rt lyn;∠e van Δ*] $\hat{A} = 35°$ [alt ∠s / *verwiss ∠e*; CE || AB] **OR/*OF*** $\hat{B} = 50°$ [∠s in same segment/*∠e in dieselfde segment*] $\hat{C}_2 + 15° = 50°$ [alt ∠s / *verwiss ∠e*; CE || AB] $\therefore \hat{C}_2 = 35°$ $\hat{A} = 35°$ [∠s in same segment/*∠e in dieselfde segment*]	✓ S ✓S ✓ R (3) ✓ S ✓S ✓ R (3) ✓ S ✓S ✓ R (3)
8.1.2	$\hat{C}_2 = 35°$ [∠s in same segment/*∠e in dieselfde segment*]	✓ S ✓ R (2)
8.2	$\hat{C}_2 = \hat{E}$ [from 8.1.1 and 8.1.2] ∴ CF is a tangent to the circle [converse tan chord theorem] ∴ *CF is 'n raaklyn aan die sirkel [omgekeerde raakl koordst]*	✓ S ✓ R (2) **[7]**

QUESTION/*VRAAG* 9



9.1.1	$\frac{AF}{BF}=\frac{3x}{2x}=\frac{3}{2}$ & $\frac{AG}{CG}=\frac{12y}{8y}=\frac{3}{2}$ $\therefore \frac{AF}{BF}=\frac{AG}{CG}$ $\therefore$ FG || BC [conv prop th/*omg eweredigh st.* **OR** line divides 2 sides of Δ in prop/*lyn verdeel 2 sye v Δ in dies verh*]	✓ $\frac{AF}{BF}=\frac{AG}{CG}$ ✓R (2)
9.1.2	$\frac{AG}{GC}=\frac{AH}{HK}$ [prop theorem/*eweredigh st*; GH || CK **OR** line || to 1 side of Δ/*lyn || 1 sy van Δ*] $\frac{AG}{GC}=\frac{AE}{ED}$ [prop theorem/*eweredigh st*; GE || CD] $\therefore \frac{AH}{HK}=\frac{AE}{ED}$	✓S ✓ R ✓ S (3)
9.2	$\frac{AE}{ED}=\frac{3}{2}$ and $\frac{AH}{HK}=\frac{3}{2}$ $\frac{AE}{12}=\frac{3}{2}$ and $\frac{15}{HK}=\frac{3}{2}$ $\therefore$ AE = 18 and HK = 10 $\therefore$ HE = AE – AH = 18 – 15 = 3 $\therefore$ EK = HK – HE = 10 – 3 = 7 **OR/*OF*** AD = 30 KD = AD – AH – HK = 30 – 15 – 10 = 5 EK = ED – KD = 12 – 5 = 7	✓ use of ratios ✓ AE = 18 ✓ HK = 10 ✓ HE = 3 **or** KD = 5 ✓ EK = 7 (5) **[10]**

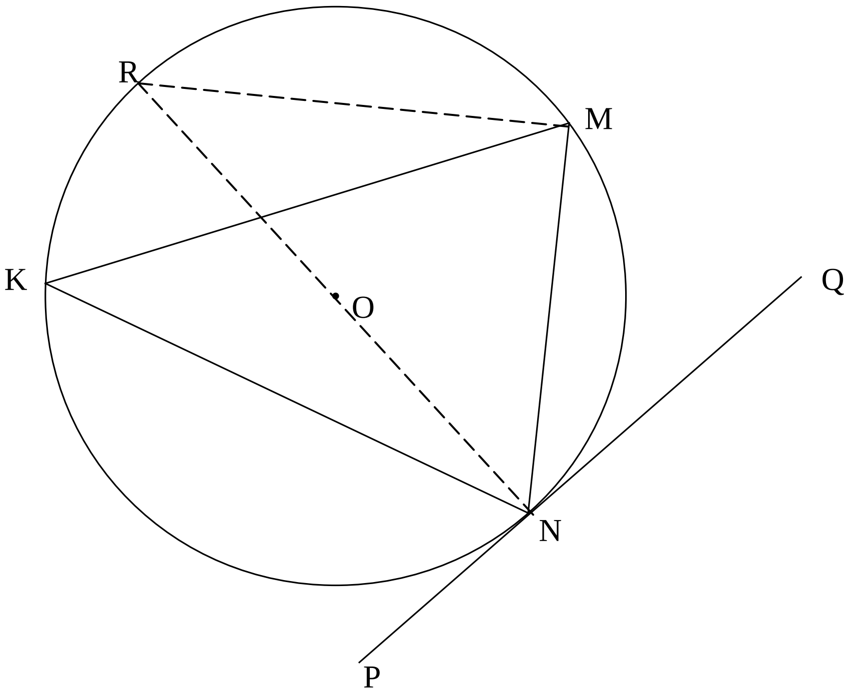
QUESTION/*VRAAG* 10



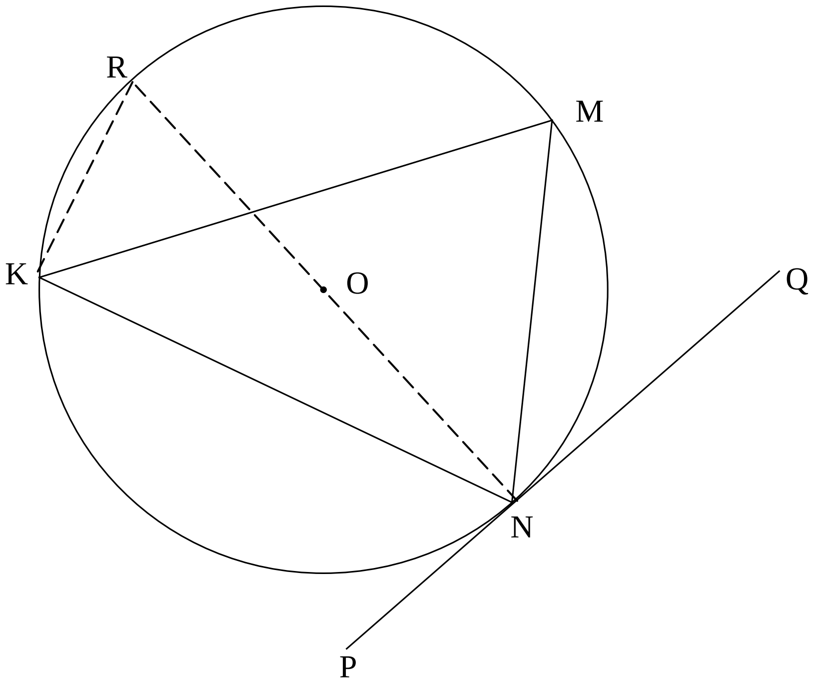
10.1	Line from centre to midpoint of chord/ *lyn vanaf midpt na midpt van koord*	✓ R (1)
10.2.1	$\text{O}\hat{\text{W}}\text{T} = \text{O}\hat{\text{W}}\text{S} = 90°$ [radius ⊥ tangent/*raaklyn*] $\therefore$ MN ∥ TS [corresp ∠s =/*ooreenkomstige ∠e* = **OR** co-int ∠s 180°/*ko-binne∠e 180°* **OR** alternate ∠s/*verwiss ∠e*]	✓ R ✓ R (2)
10.2.2	$\hat{\text{M}}_1 = \hat{\text{N}}_1$ [∠s opp = sides/ *∠e teenoor = sye*] $\hat{\text{M}}_1 = \hat{\text{T}}$ [corresp ∠s/*ooreenk ∠e*; MN ∥ TS] $\therefore \hat{\text{N}}_1 = \hat{\text{T}}$ $\therefore$ TMNS is a cyclic quadrilateral [conv: ext∠ cyclic quad] TMNS *is 'n koordevierhoek* [*omgek: buite∠ kdvh*] **OR/*OF*** $\hat{\text{M}}_1 = \hat{\text{N}}_1$ [∠s opp = sides/*∠e teenoor = sye*] $\hat{\text{N}}_1 = \hat{\text{S}}$ [corresp ∠s/*ooreenk ∠e*; MN ∥ TS] $\therefore \hat{\text{S}} = \hat{\text{M}}_1$ $\therefore$ TMNS is a cyclic quadrilateral [conv: ext∠ cyclic quad] TMNS *is 'n koordevierhoek* [*omgek: buite∠ kdvh*]	✓ S ✓ S ✓ S ✓ R (4) ✓ S ✓ S ✓ S ✓ R (4)

10.2.3	In ΔOVN and ΔOWS $\hat{O}_2 = \hat{O}_2$ [common/*gemeenskaplik*] $O\hat{V}N = O\hat{W}S = 90°$ [from 10.1] $O\hat{N}V = O\hat{S}W$ [sum $\angle$s Δ/*som* $\angle$*e* Δ] $\therefore \Delta OVN \parallel\!\mid \Delta OWS$ [$\angle, \angle, \angle$] $\therefore \frac{VN}{WS} = \frac{ON}{OS}$ But VN = $\frac{1}{2}$MN [given] $\therefore \frac{\frac{1}{2}MN}{WS} = \frac{ON}{OS}$ $\therefore$OS.MN = 2ON. WS	✓ S; S; S **OR** S; S; R ✓$\Delta OVN \parallel\!\mid \Delta OWS$ ✓ $\frac{VN}{WS} = \frac{ON}{OS}$ ✓ VN = $\frac{1}{2}$MN ✓ substitution (5)
	OR/*OF* In ΔOVM and ΔOWS $O\hat{V}M = O\hat{W}S = 90°$ [from 10.1] $O\hat{M}V = O\hat{S}W$ [sum $\angle$s Δ/*som* $\angle$*e* Δ] $\therefore \Delta OVM \parallel\!\mid \Delta OWS$ [$\angle, \angle, \angle$] $\therefore \frac{OM}{OS} = \frac{VM}{WS}$ But VN = $\frac{1}{2}$MN [given] $\therefore \frac{\frac{1}{2}MN}{WS} = \frac{OM}{OS}$ $\therefore$OS.MN = 2ON. WS [VM = VN]	✓ S; S; S **OR** S; S; R ✓$\Delta OVM \parallel\!\mid \Delta OWS$ ✓ $\frac{OM}{OS} = \frac{VM}{WS}$ ✓ VN = $\frac{1}{2}$MN ✓ substitution (5)
	OR/*OF* If any other 2 Δs are used, first need to prove that TW = WS by proving $\Delta OWT \equiv \Delta OWS$ In ΔOVM and ΔOWT $\hat{O}_1 = \hat{O}_1$ [common/*gemeenskaplik*] $O\hat{V}M = O\hat{W}T = 90°$ [from 10.1] $O\hat{M}V = O\hat{T}W$ [sum $\angle$s Δ/*som* $\angle$*e* Δ] $\therefore \Delta OVM \parallel\!\mid \Delta OWT$ [$\angle, \angle, \angle$] $\therefore \frac{VM}{WT} = \frac{OM}{OT}$ But VN = VM = $\frac{1}{2}$MN [given] and WT = WS and OT = OS [$\Delta OWT \equiv \Delta OWS$] $\therefore \frac{\frac{1}{2}MN}{WS} = \frac{ON}{OS}$ $\therefore$OS.MN = 2ON. WS	✓✓ similarity ✓✓ congruency ✓ VN = VM = $\frac{1}{2}$MN (5) **[12]**

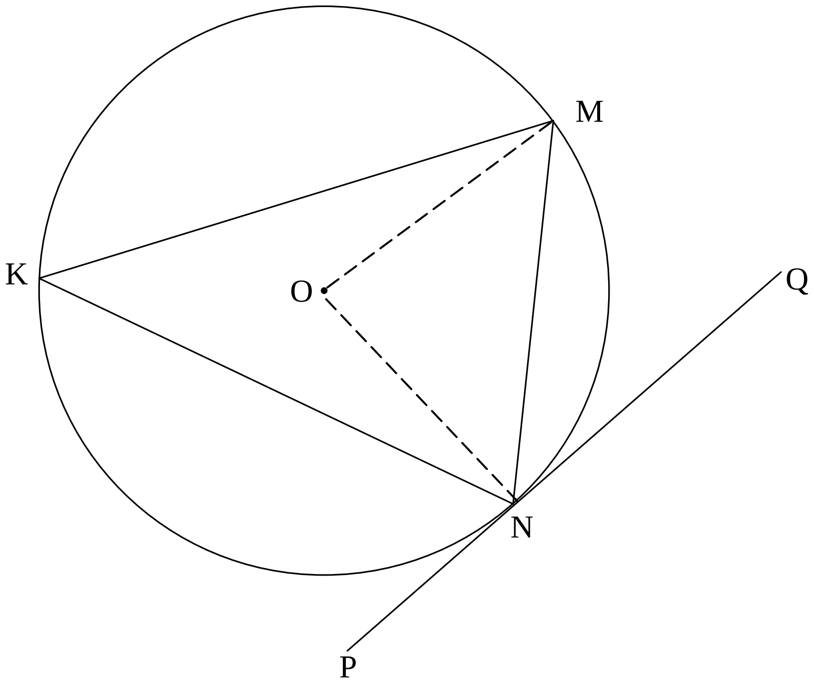
QUESTION/*VRAAG* 11



11.1	Construction: Draw diameter NR and draw RM *Konstruksie: Trek middellyn NR en verbind RM* $O\hat{N}M + M\hat{N}Q = 90°$ [radius ⊥ tangent/*raaklyn*] $N\hat{M}R = 90°$ [∠ in semi circle/*semi-sirkel*] $\therefore M\hat{R}N = 180° - (90° + 90° - M\hat{N}Q)$ [sum ∠s Δ] $= M\hat{N}Q$ but $M\hat{R}N = M\hat{K}N$ [∠s same segment/∠*e dieselfde segment*] $\therefore M\hat{N}Q = \hat{K}$ **OR/*OF***	✓ construction ✓ S /R ✓ S/ R ✓ S ✓ S / R (5)

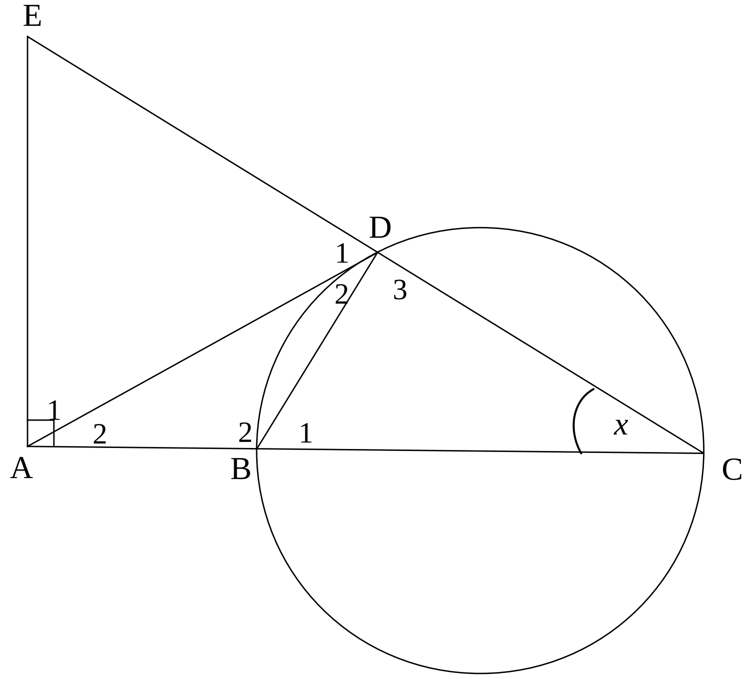


11.1	Construction: Draw diameter NR and draw RK *Konstruksie: Trek middellyn NR en verbind RK* $M\hat{N}Q + R\hat{N}M = 90°$ [radius ⊥ tangent/*raaklyn*] $N\hat{K}R = 90°$ [∠ in semicircle/*semi-sirkel*] $\therefore M\hat{K}N = 90° - R\hat{K}M$ $= 90° - R\hat{N}M$ [∠s same segment/*∠e dieselfde segment*] $\therefore M\hat{N}Q = \hat{K}$	✓ construction ✓ S /R ✓ S/ R ✓ S ✓ S / R (5)



11.1	Construction: Draw radii ON and OM *Konstruksie: Trek radiusse ON en OM* $\hat{MON} = 2\hat{K}$ [∠ at centre = 2∠ at circumf/*midpts∠ = 2 omtreks∠*] $\hat{ONM} + \hat{OMN} = 180° - 2\hat{K}$ [∠s of Δ/*∠e van Δ*] $\hat{ONM} = \hat{OMN} = \frac{180° - 2\hat{K}}{2} = 90° - \hat{K}$ [∠s opp = sides/*∠e teenoor = sye*] $\hat{ONQ} = 90°$ [radius⊥tangent/*radius⊥raaklyn*] $\therefore \hat{MNQ} = \hat{K}$	✓ construction ✓ S /R ✓ S ✓ S/ R ✓ S / R (5)

11.2



11.2.1(a)	Angle in a semi circle/ *Hoek in halfsirkel*	✓ R (1)
11.2.1(b)	Exterior $\angle$of quad = opp interior $\angle$/ *Buite $\angle$ van vierh = teenoorst binne $\angle$* **OR/*OF*** Opp $\angle$s of quad supplementary/ *Teenoorst $\angle$e van vierh is supplementêr*	✓ R (1)
11.2.1(c)	tangent chord theorem/ *raaklyn koord stelling*	✓ R (1)
11.2.2(a)	In ΔAEC $\hat{E} = 180° - (90° + x)$ [sum $\angle$s Δ] $= 90° - x$ $\hat{D}_1 = 180° - (90° + x)$ [$\angle$s on a straight line] $= \hat{E} = 90° - x$ $\therefore$ AD = AE [sides opp = $\angle$s/ *sye teenoor = $\angle$e*]	✓ S ✓ S ✓ R (3)
11.2.2(b)	In ΔADB and ΔACD $\hat{A}_2 = \hat{A}_2$ [common] $\hat{D}_2 = \hat{C}$ [proven] $\hat{B}_2 = \hat{D}_2 + \hat{D}_3$ [sum $\angle^e$ Δ] $\therefore \Delta ADB \parallel\!\mid \Delta ACD$ **OR/*OF*** In ΔADB and ΔACD $\hat{A}_2 = \hat{A}_2$ [common] $\hat{D}_2 = \hat{C}$ [proven] $\therefore \Delta ADB \parallel\!\mid \Delta ACD$ [$\angle, \angle, \angle$]	✓ S ✓ S ✓ S (3) ✓ S ✓ S ✓ R (3)

11.2.3(a)	$\frac{AD}{AC}=\frac{AB}{AD}$ [$\|\|\|$ Δs] $AD^2 = AC \cdot AB$ $= 3r \times r$ $= 3r^2$	✓ ratio ✓ substitution (2)
11.2.3(b)	$AD = AE = \sqrt{3}r$ [from 11.2.2(a) &11.2.3(a)] $AB = r$ and $BC = 2r$ $\therefore AC = 3r$ <u>In ΔACE:</u> $\tan\hat{E} = \frac{AC}{AE}$ $= \frac{3r}{\sqrt{3}r} = \sqrt{3}$ $\therefore\ \hat{E} = 60°$ $\therefore\ \hat{D}_1 = 60°$ [from 11.2.2(a)] $\therefore\ \hat{A}_1 = 60°$ [$\angle$s of $\Delta = 180°$] $\therefore$ ΔADE is equilateral/*is gelyksydig* **OR/*OF*** $\frac{AD}{AC}=\frac{DB}{CD}$ [$\|\|\|$ Δs] $\frac{\sqrt{3}r}{3r}=\frac{DB}{CD}$ $\tan x = \frac{1}{\sqrt{3}}$ $\therefore$ In ΔBDC: $x = 30°$ $\therefore\ \hat{E} = 60°$ $\therefore\ \hat{D}_1 = 60°$ [from 11.2.2(a)] $\therefore\ \hat{A}_1 = 60°$ [$\angle$s of $\Delta = 180°$] $\therefore$ ΔADE is equilateral/*is gelyksydig* **OR/*OF*** $\frac{AD}{AC}=\frac{DB}{CD}$ [$\|\|\|$ Δs] $\frac{\sqrt{3}r}{3r}=\frac{DB}{CD}$ $\therefore BD = \frac{CD}{\sqrt{3}}$ $DC^2 = BC^2 - DB^2$ $= 4r^2 - \frac{CD^2}{3}$ $3DC^2 = 12r^2 - CD^2$ $4CD^2 = 12r^2$ $DC = \sqrt{3}r$	✓ AC ito r ✓ trig ratio ✓ simplification ✓ all 3 $\angle$s = 60° (4) ✓ $\frac{\sqrt{3}r}{3r}=\frac{DB}{CD}$ ✓ $\frac{1}{\sqrt{3}} = \tan x$ ✓ $x = 30°$ ✓ all 3 $\angle$s = 60° (4) ✓ $BD = \frac{CD}{\sqrt{3}}$ ✓ $DC = \sqrt{3}r$

	$EC^2 = EA^2 + AC^2$ $\quad = 3r^2 + 9r^2$ $EC = 2\sqrt{3}r$ $\therefore$ ED = EC – DC $\quad = \sqrt{3}r$ $\therefore$ ED = EA = AD $\therefore$ ΔADE is equilateral/*is gelyksydig*	✓ $EC = 2\sqrt{3}r$ ✓ ED = EA = AD (4) **[20]**

TOTAL/*TOTAAL*: 150

basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

SENIOR CERTIFICATE EXAMNATIONS
SENIORSERTIFIKAAT-EKSAMEN

MATHEMATICS P2/*WISKUNDE V2*

2017

MARKING GUIDELINES/*NASIENRIGLYNE*

MARKS: 150
PUNTE: 150

These marking guidelines consist of 22 pages.
Hierdie nasienriglyne bestaan uit 22 bladsye..

NOTE:

- If a candidate answers a question TWICE, only mark the FIRST attempt.
- If a candidate has crossed out an attempt of a question and not redone the question, mark the crossed out version.
- Consistent accuracy applies in ALL aspects of the marking memorandum. Stop marking at the second calculation error.
- Assuming answers/values in order to solve a problem is NOT acceptable.
- Geometry:
  S = a mark for a correct statement (a statement mark is independent of a reason)
  R = a mark for a correct reason (a reason mark may only be awarded if the statement is correct)
  S/R = award a mark if statement and reason are both correct

NOTA:

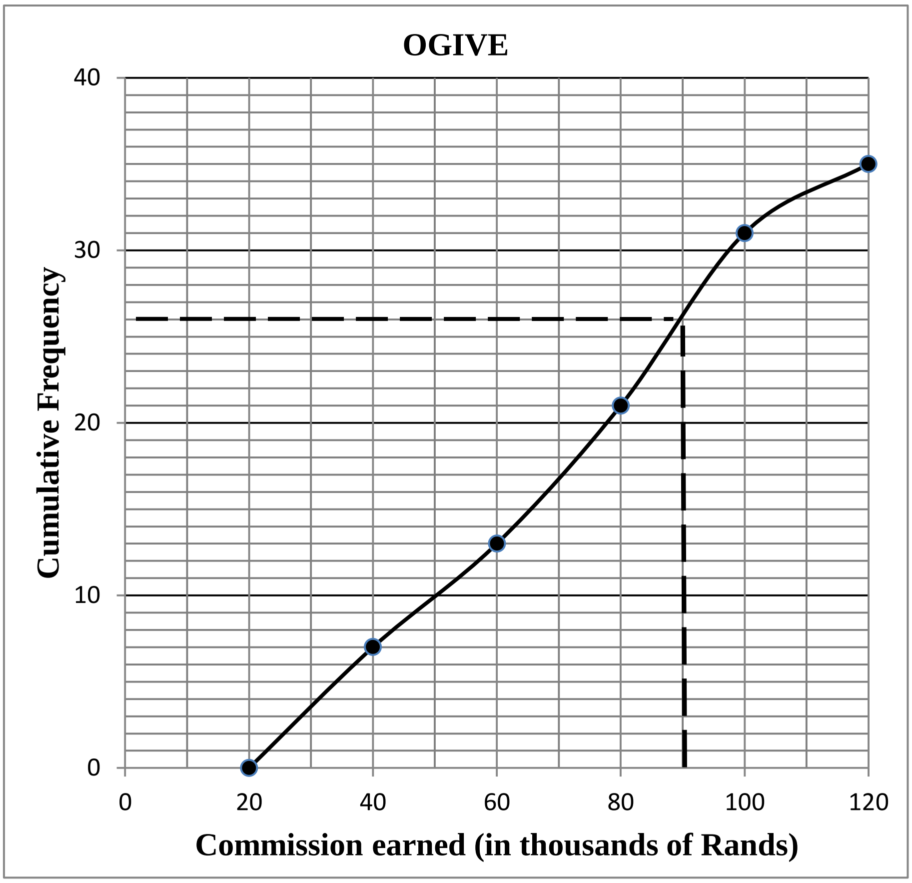
- *As 'n kandidaat 'n vraag TWEEKEER beantwoord, merk slegs die EERSTE poging.*
- *As 'n kandidaat 'n antwoord van 'n vraag doodtrek en nie oordoen nie, merk die doodgetrekte poging.*
- *Volgehoue akkuraatheid word in ALLE aspekte van die memorandum toegepas. Hou op nasien by die tweede berekeningsfout.*
- *Aanvaar van antwoorde/waardes om 'n probleem op te los, word NIE toegelaat nie.*
- *Euklidiese Meetkunde:*
  S = ‘n punt vir ‘n korrekte bewering (‘n beweringspunt is onafhanklik van die rede)
  R = ‘n punt vir ‘n korrekte rede (‘n punt kan slegs vir ‘n rede toegeken word, indien die bewering korrek is
  S/R = ‘n punt word toegeken indien beide die bewering en rede korrek is

***QUESTION/VRAAG/VRAAG* 1**

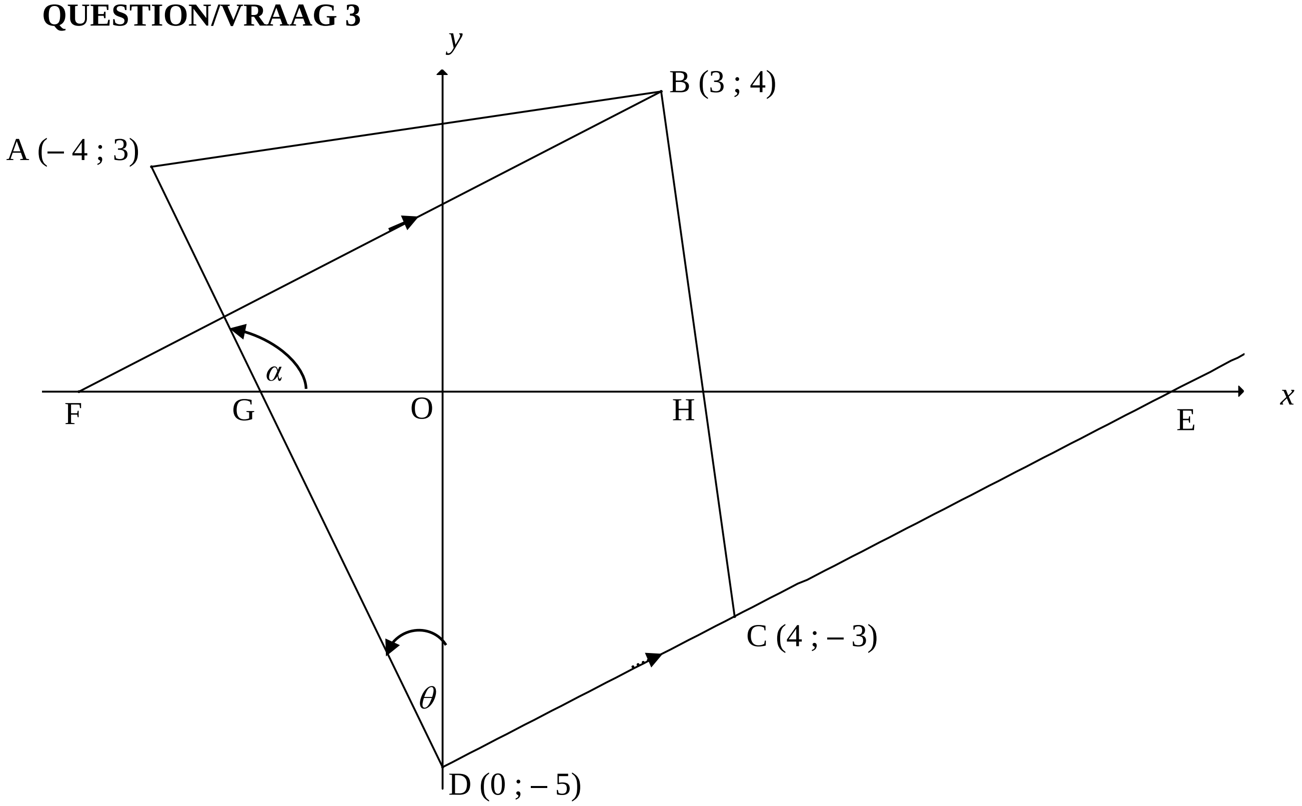
TIME TAKEN (IN HOURS)	5	7	5	8	10	13	15	20	18	25	23
COST (IN THOUSANDS OF RANDS)	10	10	15	12	20	25	28	32	28	40	30

1.1	$a = 4{,}806... = 4{,}81$ $b = 1{,}323... = 1{,}32$ $y = 4{,}81 + 1{,}32x$	✓ $a = 4{,}81$ ✓ $b = 1{,}32$ ✓ equation (3)
1.2	Cost = 25,974… = 25,97 thousand rand (calculator) = R25 970 **OR/*OF*** $y = 4{,}81 + 1{,}32(16)$ $y = 25{,}93$ Cost = R25 930	✓ 25,97 ✓ answer (in Rands) (2) ✓ substitution ✓ answer (in Rands) (2)
1.3	r = 0,949…= 0,95	✓ answer (1)
1.4	$x = 0$ $y = 4{,}81$ **OR** (4,80647) $\therefore$ R4 810 **OR** R4806,47	✓ $x = 0$ ✓ answer (2) **[8]**

QUESTION/VRAAG 2

2.1	modal class: $80 < x \leq 100$	✓ correct class (1)
2.2	<table><tr><th>Commission earned (in thousands of Rands)</th><th>Frequency</th><th>Cumulative Frequency</th></tr><tr><td>$20 < x \leq 40$</td><td>7</td><td>7</td></tr><tr><td>$40 < x \leq 60$</td><td>6</td><td>13</td></tr><tr><td>$60 < x \leq 80$</td><td>8</td><td>21</td></tr><tr><td>$80 < x \leq 100$</td><td>10</td><td>31</td></tr><tr><td>$100 < x \leq 120$</td><td>4</td><td>35</td></tr></table>	✓ 13 ; 21 ✓ 31 ; 35 (2)
2.3		✓ grounded/*geanker* ✓ upper limits/ *boonste limiet* ✓ cum frequency / Kum frekwensie ✓ shape/*vorm* (4)
2.4	No. of salesmen awarded bonuses: 35 – 26 = 9 salesmen	✓ accept (25 – 27) ✓ accept (8 – 10) (2)
2.5	$\text{Estimated mean} = \frac{(30\times 7)+(50\times 6)+(70\times 8)+(90\times 10)+(110\times 4)}{35}$ $= \frac{2410}{35}$ = 68,86 thousand rand or R68 857,14 = R69 000 or 69 thousand rand	✓ top line using midpts & freq ✓ 2410 ✓ answer (nearest) (3) **[12]**

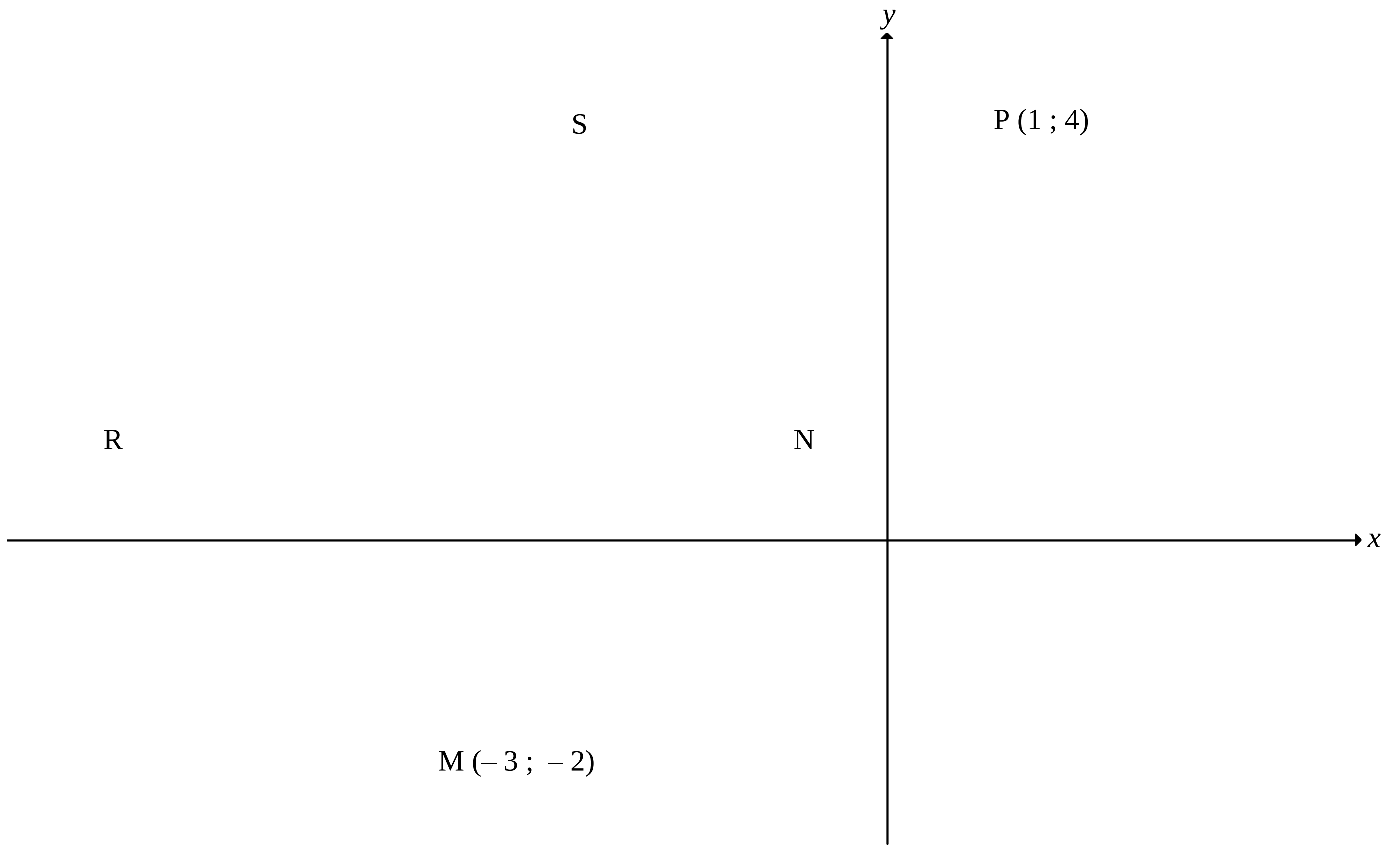
QUESTION/VRAAG 3



3.1	$m_{CD} = \frac{-3-(-5)}{4-0}$ $= \frac{-3+5}{4-0}$ $= \frac{1}{2}$	✓ substitution of C & D ✓answer (2)
3.2	$m_{AD} = \frac{-5-3}{0-(-4)}$ $= -2$ $m_{CD} \times m_{AD} = \frac{1}{2} \times -2$ $= -1$ $\therefore$ AD ⊥ DC	✓ substitution of A & D ✓ $m_{AD} = -2$ ✓ product = –1 (3)
3.3	$AB = \sqrt{(3+4)^2 + (4-3)^2} = \sqrt{50} = 5\sqrt{2}$ $BC = \sqrt{(4-3)^2 + (-3-4)^2} = 5\sqrt{2}$ AB = BC $\therefore$ ΔABC is an iscosceles triangle/'*n gelykbenige driehoek*	✓ correct substitution ✓ length of AB ✓ correct substitution ✓ length of BC (4)

3 .4	$m_{CD} = m_{BF} = \frac{1}{2}$ [BF ∥ DC] $4 = \frac{1}{2}(3) + c$ $c = \frac{5}{2}$ $y = \frac{1}{2}x + \frac{5}{2}$ **OR/*OF*** $y - 4 = \frac{1}{2}(x - 3)$ $y - 4 = \frac{1}{2}x - 1\frac{1}{2}$ $y = \frac{1}{2}x + 2\frac{1}{2}$	✓ $m_{BF} = \frac{1}{2}$ ✓ substitution of B(3 ; 4) ✓equation (3)
3.5	$\tan\alpha = -2$ $\therefore \alpha = 116.57°$ $\alpha = 90° + \theta$ [ext ∠Δ] $\therefore \theta = 26{,}57°$ **OR/*OF*** $\tan\alpha = -2$ OR $m_{AD} = -2$ $\therefore \tan\theta = \frac{1}{2}$ $\therefore \theta = 26{,}57°$ **OR/*OF*** Inclination of DE is β: $\tan\beta = \frac{1}{2}$ $\therefore \beta = 26{,}57°$ $\therefore$ O$\hat{D}$E $= 63{,}43°$ $\therefore \theta = 90° - 63{,}43°$ $= 26{,}57°$	✓ $\tan\alpha = -2$ ✓ $\alpha = 116.57°$ ✓ $\theta = 26{,}57°$ (3) ✓ $\tan\alpha = -2$ ✓ $\tan\theta = \frac{1}{2}$ ✓ $\theta = 26{,}57°$ (3) ✓ $\beta = 26{,}57°$ ✓ O$\hat{D}$E $= 63{,}43°$ ✓ $\theta = 26{,}57°$ (3)
3.6	$x^2 + y^2 = r^2$ $(4)^2 + (-3)^2 = 25$ $x^2 + y^2 = 25$	✓ $r^2 = 25$ ✓ equation (2) **[17]**

QUESTION/VRAAG 4



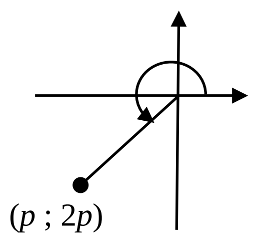
4.1	$N\left(\frac{1+(-3)}{2};\frac{4+(-2)}{2}\right)$ $N(-1;1)$ is the centre of the circle	✓ substitution M & P ✓ x-value of N ✓ y-value of N (3)
4.2	$r=\sqrt{(1-(-1))^2+(4-1)^2}$ $r=\sqrt{13}$ = radius $(x+1)^2+(y-1)^2=13$ **OR/*OR*** $r=\sqrt{(-3-(-1))^2+(-2-1)^2}$ $r=\sqrt{13}$ = radius $(x+1)^2+(y-1)^2=13$	✓ substitution N & P ✓ $r=\sqrt{13}$ ✓ LHS of eq ✓ RHS of eq (4) ✓ substitution N & M ✓ $r=\sqrt{13}$ ✓ LHS of eq ✓ RHS of eq (4)

4.3	$m_{NM} \times m_{MR} = -1$ [radius $\perp$ tangent/*raakklyn*] $m_{NM} = \dfrac{1-(-2)}{-1-(-3)}$ *OR* $m_{PM} = \dfrac{4-(-2)}{1-(-3)}$ $= \dfrac{3}{2}$ $= \dfrac{3}{2}$ $m_{MR} = -\dfrac{2}{3}$ $y - y_1 = -\dfrac{2}{3}(x - x_1)$ **OR/*OF*** $y = -\dfrac{2}{3}x + c$ $y + 2 = -\dfrac{2}{3}(x+3)$ **OR/*OF*** $-2 = -\dfrac{2}{3}(-3) + c$ $y = -\dfrac{2}{3}x - 4$	✓ correct substitution ✓ m_{NM} ✓ m_{MR} ✓substitution of m_{MR} &(–3 ;–2) ✓ equation (5)
4.4	Symmetry of a kite: S(–3 ; 4) **OR/*OF*** $P\hat{S}M = 90°$ [$\angle$ in semi circle] PS $\perp$ SM $\therefore$ S(–3 ; 4) **OR/*OF*** $(NS)^2 = (\text{radius})^2$ $(-3+1)^2 + (y-1)^2 = 13$ $(y-1)^2 = 9$ $y - 1 = \pm 3$ $y = 4$ *OR* $y \neq -2$ $\therefore$ S(–3 ; 4)	✓ *x*-value of S ✓ *y*-value of S (2) ✓ *x*-value of S ✓ *y*-value of S (2) ✓ *x*-value of S ✓ *y*-value of S (2)
4.5	$(SR)^2 = (RM)^2$ ...Tangents from common pt/*rklyne v dies punt* $(x+3)^2 + (y-4)^2 = (x+3)^2 + (y+2)^2$ $y^2 - 8y + 16 = y^2 + 4y + 4$ $-12y = -12$ $y = 1$ $\dfrac{2}{3}x = -4 - 1$ or $1 = -\dfrac{2}{3}x - 4$ $x = -\dfrac{15}{2}$ $x = -7\dfrac{1}{2}$ $\therefore$ $R\left(-7\dfrac{1}{2} ; 1\right)$ **OR/*OF***	✓ equating lengths ✓ simplification ✓ *y*-value of R ✓ *x*-value of R (4)

	$\text{R}(x;1)$ [RN is a horizontal line] $\therefore 1 = -\frac{2}{3}x - 4$ $5 = -\frac{2}{3}x$ $x = -\frac{15}{2}$ $\therefore \text{R}\left(-\frac{15}{2};1\right)$ **OR/*OF*** $m_{\text{NS}} = \frac{1-4}{-1+3} = -\frac{3}{2}$ $\therefore\ m_{\text{RS}} = \frac{2}{3}$ $y - 4 = \frac{2}{3}(x+3)$ $y = \frac{2}{3}x + 6$ $-\frac{2}{3}x - 4 = \frac{2}{3}x + 6$ $x = -7\frac{1}{2}$ $y = \frac{2}{3}(-\frac{15}{2}) + 6 = 1$ $\therefore \text{R}\left(-\frac{15}{2};1\right)$	✓ $y_{\text{R}} = 1$ ✓ horizontal line OR R lies on $y = 1$ ✓ equating ✓ x-value of R ($x < -4,6$) (4) ✓ $y = \frac{2}{3}x + 6$ ✓ equating ✓ x-value of R ($x < -4,6$) ✓ y-value of R (4)
4.6	$\text{RS} = \sqrt{(-3+7,5)^2 + (4-1)^2}$ OR/OF $\text{RM} = \sqrt{(-3+7,5)^2 + (-2-1)^2}$ $\text{RS} = \frac{3\sqrt{13}}{2} = 5,41$ area of RSNM = 2area of ΔRSN $= 2\left(\frac{1}{2}\right)(\sqrt{13})\left(\frac{3\sqrt{13}}{2}\right)$ $= \frac{39}{2}$ **OR/*OF*** 19,5 square units **OR/*OF***	✓ RS **OR** RM ✓ method ✓ $\sqrt{13}$ and $\left(\frac{3\sqrt{13}}{2}\right)$ ✓ answer (4) ✓ method ✓ MS = 6 ✓ RN = 6,5 ✓ answer

	area RSNM $= \frac{1}{2}(\text{MS} \times \text{RN})$ (area of a kite/*opp v vlieër*) $= \frac{1}{2}(6)(6{,}5)$ $= \frac{39}{2}$ OR 19,5 square units **OR/*OF*** $\text{RS} = \sqrt{(-3+7{,}5)^2 + (4-1)^2}$ OR/OF $\text{RM} = \sqrt{(-3+7{,}5)^2 + (-2-1)^2}$ $\text{RS} = \frac{3\sqrt{13}}{2} or 5{,}41$ area of ΔRSN $= \left(\frac{1}{2}\right)(\sqrt{13})\left(\frac{3\sqrt{13}}{2}\right)$ $= \frac{39}{4}$ **OR/*OF*** 9,75 square units area of RSNM = 2area of ΔRSN $= \frac{39}{2}$ **OR/*OF*** 19,5 square units **OR/*OF*** SM = 6 area of RSNM = Area of ΔSMN + Area of ΔRSM $= \frac{1}{2}(6)(1) + \frac{1}{2}(6)(5\frac{1}{2})$ $= 3 + 16\frac{1}{2}$ $= 19\frac{1}{2}$	(4) ✓ RS **OR** RM ✓ $\left(\frac{1}{2}\right)\sqrt{13}\left(\frac{3\sqrt{13}}{2}\right)$ ✓ method ✓ answer (4) ✓ method ✓ MS = 6 ✓ $h = 1$ & $5\frac{1}{2}$ ✓ answer (4)
		[22]

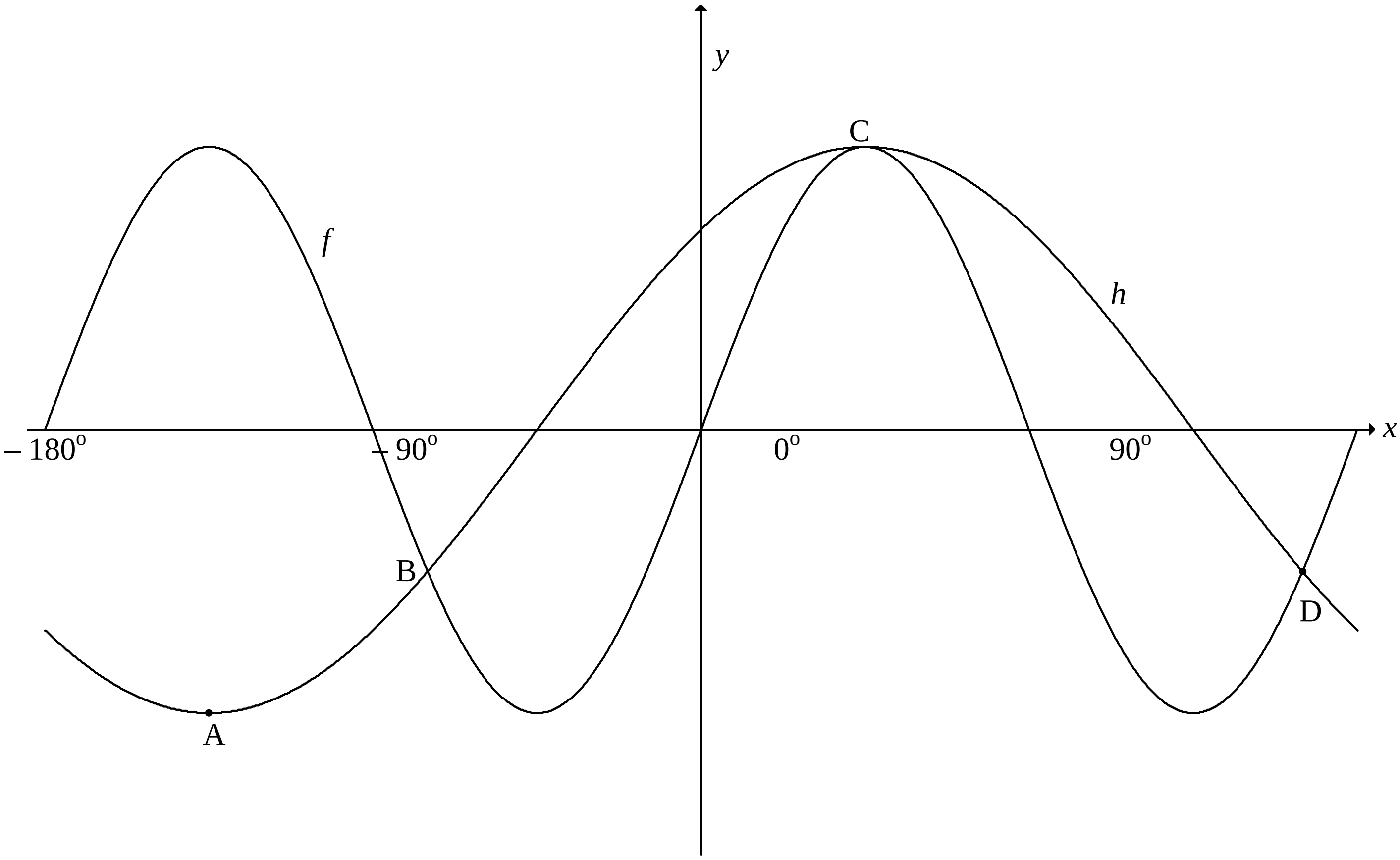
QUESTION/VRAAG 5

5.1.1	$\tan A = \dfrac{\sin\ A}{\cos\ A}$ $= \dfrac{2p}{p}$ $= 2$	✓identity ✓value of tan A (2)
	OR/*OF* $\tan A = \dfrac{2p}{p}$ $= 2$  	✓ $\dfrac{y}{x}$ ✓value of tan A (2)
5.1.2	$\sin^2 A + \cos^2 A = 1$ $(2p)^2 + p^2 = 1$ $4p^2 + p^2 = 1$ $5p^2 = 1$ $p^2 = \dfrac{1}{5}$ $\therefore\ p = -\dfrac{1}{\sqrt{5}}$	✓ $(2p)^2 + p^2 = 1$ ✓ simplification of LHS ✓answer (3)
5.2	$2\sin^2 x - 5\sin x + 2 = 0$ $(2\sin x - 1)(\sin x - 2) = 0$ $\sin x = \dfrac{1}{2}$ or $\sin x = 2$(no solution) ref $\angle = 30°$ $\therefore\ x = 30° + k.360°$ or $x = 150° + k.360°\ ;\ k \in Z$	✓ factors or formula ✓ both equations ✓ no solution/*geen opl* ✓ $30° + k.360°$ ✓ $150° + k.360°$; ✓ $k \in Z$ (6)
5.3.1	$\sin(x + 300°) = \sin x \cos 300° + \cos x \sin 300°$	✓ expansion/*uitbreiding* (1)
5.3.2	$\sin(x + 300°) - \cos(x - 150°)$ $= \sin x \cos 300° + \cos x \sin 300° - (\cos x \cos 150° + \sin x \sin 150°)$ $= \sin x \cos 60° - \cos x \sin 60° - (-\cos x \cos 30° + \sin x \sin 30°)$ $= \sin x \cos 60° - \cos x \sin 60° + \cos x \cos 30° - \sin x \sin 30°$ $= \dfrac{1}{2}\sin x - \dfrac{\sqrt{3}}{2}\cos x + \dfrac{\sqrt{3}}{2}\cos x - \dfrac{1}{2}\sin x$ $= 0$ **OR/*OF***	✓2nd expansion/ *2de uitbreiding* ✓✓ reduction/*reduksie* ✓ special angle values/ *spesiale hoekwaardes* ✓ answer (5)

	$\sin(x+300°)-\cos(x-150°)$ $=\sin x\cos 300°+\cos x\sin 300°-(\cos x\cos 150°+\sin x\sin 150°)$ $=\sin x\cos 60°-\cos x\sin 60°-(-\cos x\cos 30°+\sin x\sin 30°)$ $=\sin x\cos 60°-\cos x\sin 60°+\cos x\cos 30°-\sin x\sin 30°$ $=\sin x\sin 30°-\cos x\sin 60°+\cos x\sin 60°-\sin x\sin 30°$ $=0$	✓2nd expansion/ *2de uitbreiding* ✓✓ reduction/*reduksie* ✓ co-ratios / *ko-verh* ✓ answer (5)
5.4	Consider: $\dfrac{\tan x+1}{\sin x\tan x+\cos x}=\sin x+\cos x$ $\text{LHS}=\dfrac{\left(\dfrac{\sin x}{\cos x}+1\right)}{\left(\sin x.\dfrac{\sin x}{\cos x}+\cos x\right)}=\dfrac{\left(\dfrac{\sin x+\cos x}{\cos x}\right)}{\left(\dfrac{\sin^2 x+\cos^2 x}{\cos x}\right)}$ $=\dfrac{\dfrac{\sin x+\cos x}{\cos x}}{\dfrac{1}{\cos x}}$ $=\dfrac{\sin x+\cos x}{\cos x}\times\dfrac{\cos x}{1}$ $=\sin x+\cos x$ = RHS **OR/*OF*** $\text{LHS}=\dfrac{\left(\dfrac{\sin x}{\cos x}+1\right)}{\left(\sin x.\dfrac{\sin x}{\cos x}+\cos x\right)}=\dfrac{\left(\dfrac{\sin x}{\cos x}+1\right)}{\left(\dfrac{\sin^2 x+\cos^2 x}{\cos x}\right)}$ $=\dfrac{\left(\dfrac{\sin x}{\cos x}+1\right)}{\dfrac{1}{\cos x}}$ $=\left(\dfrac{\sin x}{\cos x}+1\right)\times\dfrac{\cos x}{1}$ $=\sin x+\cos x$ = RHS	✓ identity of tan x ✓ $\dfrac{\sin x+\cos x}{\cos x}$ ✓ $\dfrac{\sin^2 x+\cos^2 x}{\cos x}$ ✓ $\sin^2 x+\cos^2 x=1$ ✓ simplify (5) ✓ identity of tan x ✓ $\dfrac{\sin^2 x+\cos^2 x}{\cos x}$ ✓ $\sin^2 x+\cos^2 x=1$ ✓ simplify ✓ multiplication (5)
5.5.1	$\left(\sqrt{1+k}\right)^2=(\sin x+\cos x)^2$ $1+k=\sin^2 x+2\sin x\cos x+\cos^2 x$ $1+k=1+\sin 2x$ $k=\sin 2x$	✓ square both sides ✓ $\sin^2 x+\cos^2 x=1$ ✓ sin $2x$ (3)

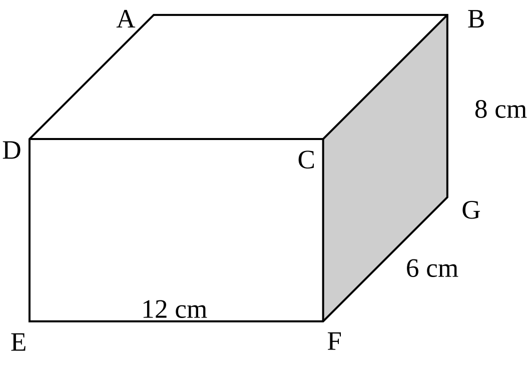
5.5.2	From 5.5.1 $\sin x + \cos x = \sqrt{1+\sin 2x}$ $\therefore$ max value: $\sin x + \cos x = \sqrt{1+1}$ $= \sqrt{2}$	✓ max of sin 2x =1 ✓ answer (2)
	OR/*OF*	
	Maximum value of 1 + sin 2x = 1 + 1 = 2 $\therefore$ maximum value of $\sin x + \cos x = \sqrt{2}$	✓ max of sin 2x =1 ✓ answer (2)
	OR/*OF* $(\sin x + \cos x)^2 = \sin^2 x + 2\sin x \cos x + \cos^2 x$ $= 1 + \sin 2x$ $\therefore$ max value $(\sin x + \cos x)^2 = 1 + 1 = 2$ $\therefore$ max value $\sin x + \cos x = \sqrt{2}$	✓ max of sin 2x =1 ✓ answer (2)
		[27]

QUESTION/VRAAG 6



6.1	Period = 180°	✓ answer (1)
6.2	$-75°$	✓ answer (1)
6.3	$\sin 2x \le \frac{1}{\sqrt{2}}\cos x + \frac{1}{\sqrt{2}}\sin x$ $\sin 2x \le \cos 45°.\cos x + \sin 45°.\sin x$ $\sin 2x \le \cos(x - 45°)$ $x \in [-75°\ ;165°]$	✓ $\cos 45°.\cos x + \sin 45°.\sin x$ ✓ $\cos(x - 45°)$ ✓ ✓ answer (4)
		[6]

QUESTION/VRAAG 7



Figure/*Figuur* (i)

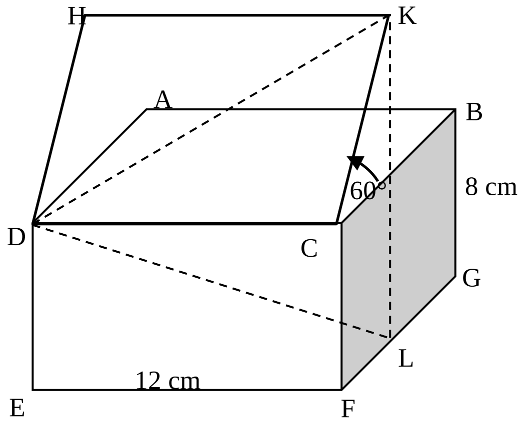
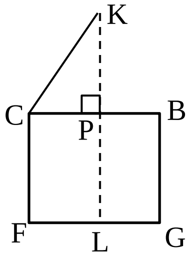
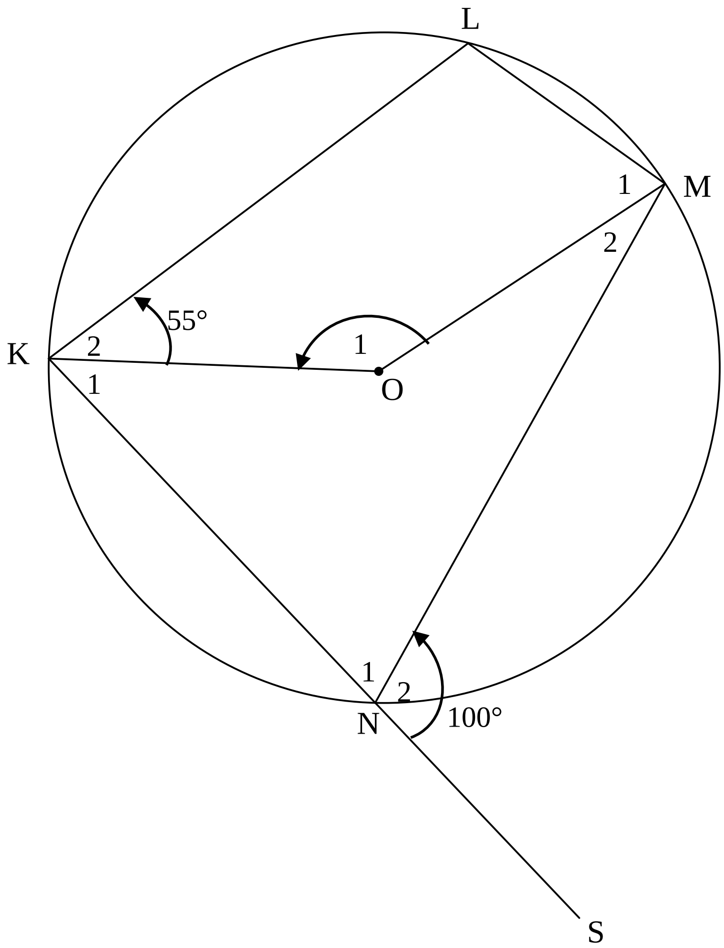


Figure / *Figuur* (ii)

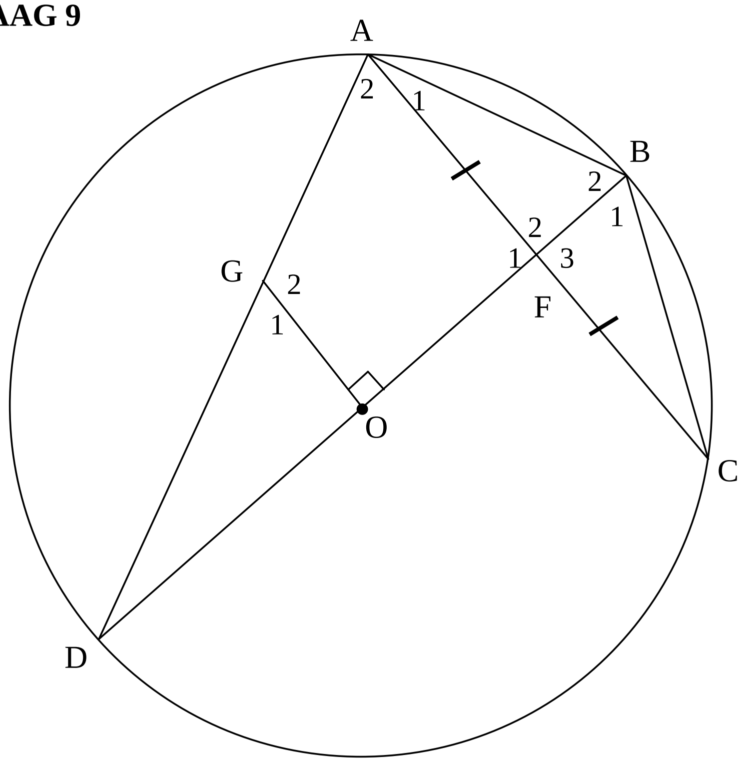
7.1	$KC = 6\text{ cm}$	✓ answer (1)
7.2	Let P be the point of intersection of KL and CB $\frac{KP}{KC} = \sin 60°$ $KP = 6\sin 60°$ $KP = 3\sqrt{3}$ or 5,20 $\therefore KL = 8 + 3\sqrt{3}$ or 13,20 cm 	✓ trig ratio ✓ length of KP ✓ answer (3)
7.3	$DK^2 = 6^2 + 12^2$ $DK = \sqrt{180}$ or $6\sqrt{5}$ or 13,42 cm $\frac{\sin K\hat{D}L}{KL} = \frac{\sin D\hat{L}K}{DK}$ $\frac{\sin K\hat{D}L}{\sin D\hat{L}K} = \frac{KL}{DK}$ $= \frac{8+3\sqrt{3}}{6\sqrt{5}}$ or $\frac{13,20}{13,42}$ or 0,98	✓ $DK = 6\sqrt{5}$ ✓ use of sine rule ✓ $\frac{\sin K\hat{D}L}{\sin D\hat{L}K} = \frac{KL}{DK}$ ✓ answer (4) **[8]**

QUESTION/VRAAG 8



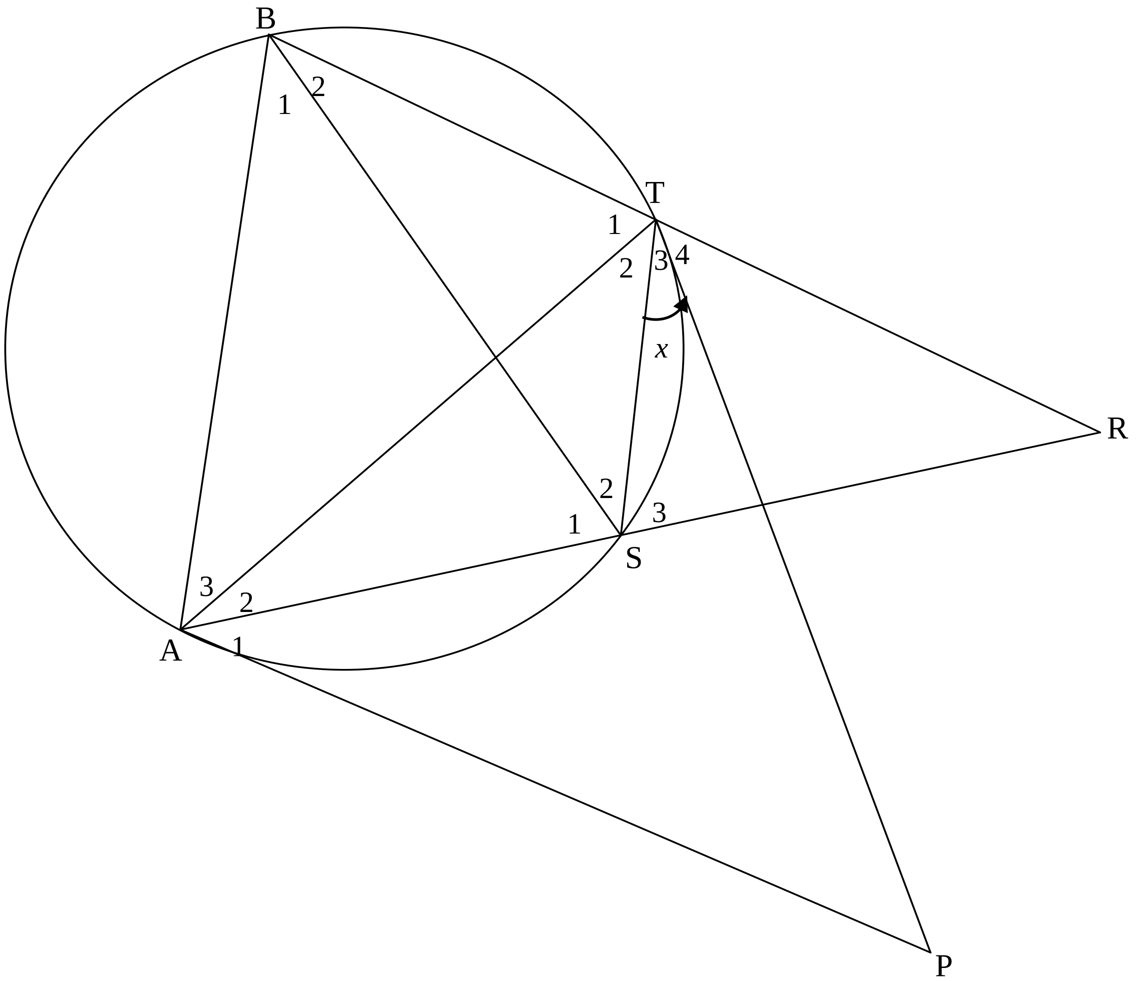
8.1	$\hat{L} = 100°$ [ext $\angle$ cyclic quad = int opp $\angle$ / *buite* $\angle$ *kdvh* = *tos* $\angle$] **OR/*OF*** $\hat{N}_1 = 80°$ [$\angle$s on straight line] $\hat{L} = 100°$ [opp $\angle$s of cyclic quad]	✓S ✓R (2) ✓S ✓R (2)
8.2	$\hat{N}_1 = 80°$ [$\angle$s on straight line/$\angle$e *op reguitlyn*] $\therefore \hat{O}_1 = 160°$ [$\angle$ at centre = 2 × $\angle$ at circumference/midpts $\angle$ = 2 omtreks $\angle$] **OR/*OF*** reflex $K\hat{O}M = 200°$ [$\angle$ at centre=2 × $\angle$ at circumference/*midpts* $\angle$ = *2×omtreks* $\angle$] $\therefore \hat{O}_1 = 160°$ [$\angle$s around a pt/ $\angle$e om ‘n pt]	✓S ✓S ✓R (3) ✓S ✓R ✓S (3)
8.3	$\hat{M}_1 = 360° - (100° + 55° + 160°)$ [sum $\angle$s of quad/*som* $\angle$*e v vierhoek*] $\therefore \hat{M}_1 = 45°$	✓S ✓S (2) **[7]**

QUESTION/VRAAG 9



9.1.1	$\angle$ in semi-circle/$\angle$ *in halfsirkel*	✓answer (1)
9.1.2	**Opp $\angle$s** of quad **= 180°**/*Teenoorst $\angle$e v vierhoek=180°*	✓answer (1)
9.2.1	OF $\perp$ AC [line from centre bisects chord/*lyn v midpt halv kd*] $\therefore$ AC $\parallel$ GO [co-interior/*ko-binne* $\angle$s = 180° **OR/*OF*** corresp/*ooreenkomstige* $\angle$s =]	✓ S ✓ R ✓ R (3)
9.2.2	$\hat{G}_1 = \hat{A}_2$ [corresp/*ooreenk* $\angle$s; AC $\parallel$ GO] $\hat{A}_2 = \hat{B}_1$ [$\angle$s in same segment/$\angle$e *in dies segment*] $\therefore \hat{G}_1 = \hat{B}_1$ **OR/*OF*** $\hat{G}_1 = \hat{B}_2$ [ext $\angle$ cyclic quad/*buite* $\angle$ *koordevh*] but $\Delta ABF \equiv \Delta CBF$ [s, $\angle$, s] $\therefore \hat{B}_2 = \hat{B}_1$ $\therefore \hat{G}_1 = \hat{B}_1$	✓ S ✓ R ✓ S ✓ R (4) ✓ S ✓ R ✓ R ✓ S (4)
9.3	OF : FB = 3 : 2 DB = 2r $\therefore$ DO = 5k and DF = 8k **OR/*OF*** $DF = 2r - \frac{2}{5}r = \frac{8}{5}r$ $\therefore \frac{DG}{DA} = \frac{DO}{DF} = \frac{r}{\frac{8}{5}r}$ [line $\parallel$ side of Δ/*lyn* $\parallel$ *sy v* Δ] $\therefore \frac{DG}{DA} = \frac{5}{8}$	✓ S ✓ R ✓ S (3)
		[12]

QUESTION/VRAAG 10

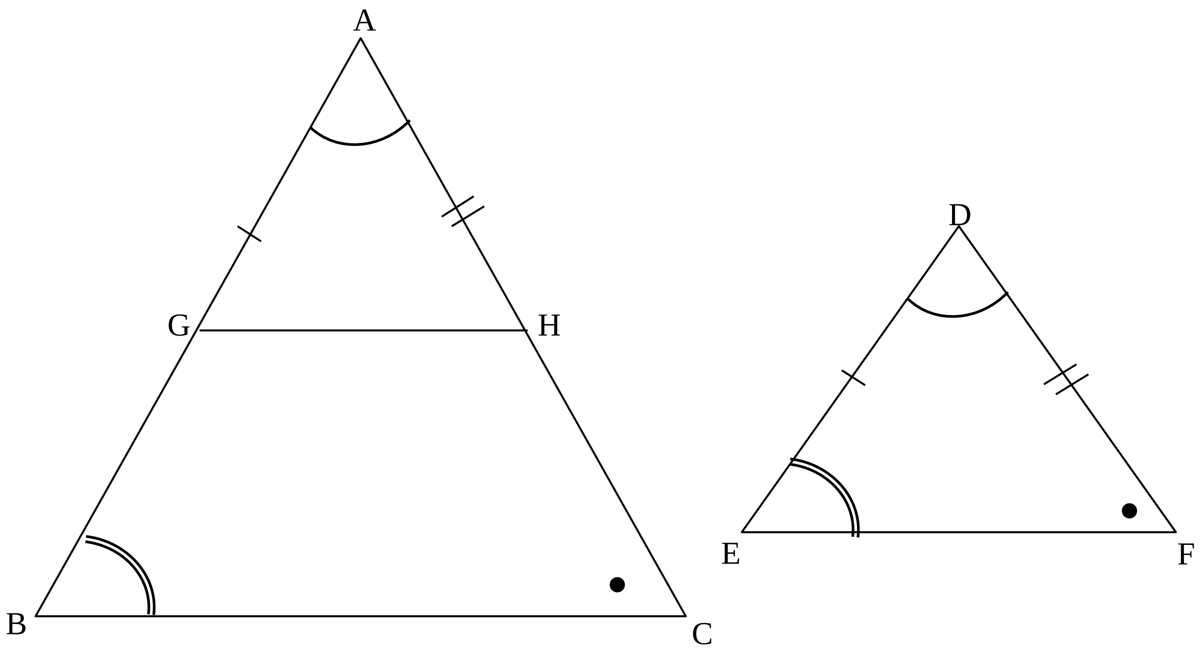


10.1	Tangent-chord theorem	✓ R (1)
10.2.1	$\hat{A}_2 + \hat{A}_3 = \hat{B}_1 + \hat{B}_2$ [$\angle^s$ opp = sides/$\angle$*e teenoor = sye*] $\hat{S}_3 = \hat{B}_1 + \hat{B}_2$ [ext $\angle$ cyclic quad/*buite* $\angle$ *koordevh*] $\therefore \hat{S}_3 = \hat{A}_2 + \hat{A}_3$ $\therefore AB \parallel ST$ [corresp/*ooreenk* $\angle^s$ =] **OR/*OF*** $R\hat{T}S = B\hat{A}S$ [ext $\angle$ cyclic quad/*buite* $\angle$ *koordevh*] $B\hat{A}S = A\hat{B}T$ [$\angle^s$ opp = sides/$\angle$*e teenoor = sye*] $\therefore R\hat{T}S = A\hat{B}T$ $\therefore AB \parallel ST$ [corresp/*ooreenk* $\angle^s$ =]	✓ S ✓ R ✓ S ✓ R ✓ R (5) ✓ S ✓ R ✓ S ✓ R ✓ R (5)

10.2.2	$\hat{B}_2 = x$ [tan chord theorem/*raakl – koordst*] $x + \hat{T}_4 = \hat{B}_1 + \hat{B}_2$ [corresp/*ooreenk* $\angle^s$; AB // ST] $\therefore \hat{T}_4 = \hat{B}_1$ $\hat{B}_1 = \hat{A}_1$ [tan chord theorem/*raakl – koordst*] $\therefore \hat{T}_4 = \hat{A}_1$	✓ S ✓ R ✓ S ✓ R ✓ R (5)
10.2.3	$\hat{T}_4 = \hat{A}_1$ [proven/*bewys in* 10.2.2] $\therefore$ RTAP is a cyclic quadrilateral [line subtends = $\angle^s$] *Is 'n koordevierhoek* [*lyn onderspan* = $\angle$*e*]	✓ S ✓ R (2)
		[13]

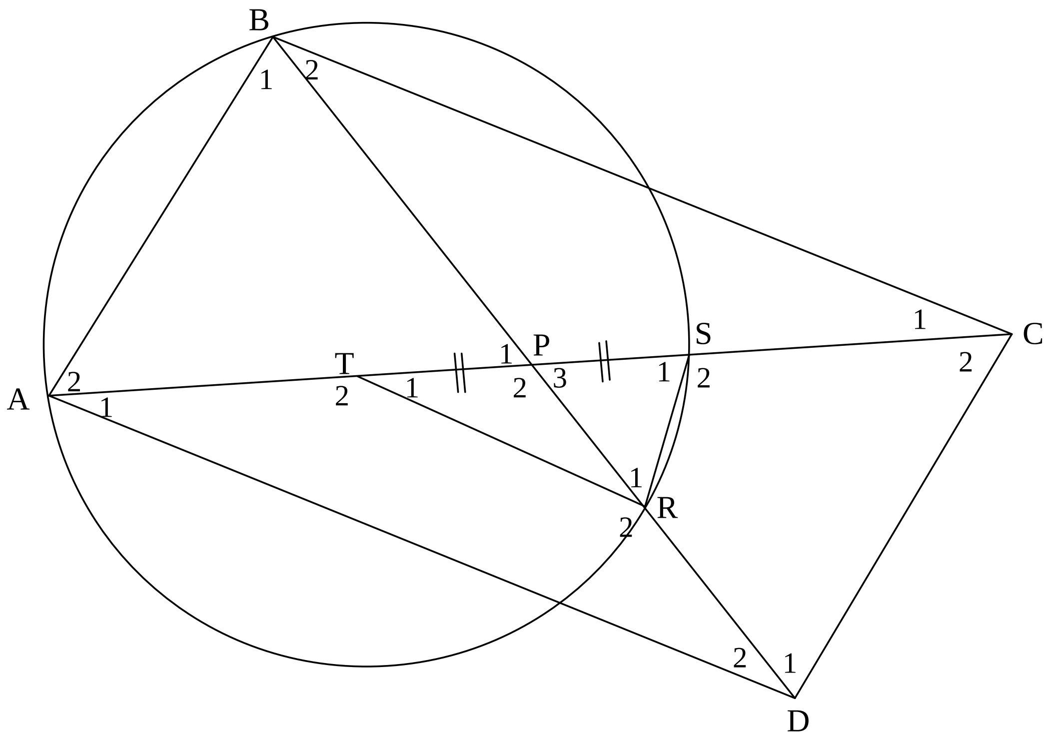
QUESTION/VRAAG 11

11.1



11.1	Constr: On sides AB and AC of ΔABC, mark points G and H respectively such that AG = DE and AH = DF. Draw GH/*Merk punt G en H op sy AB en AC van ΔABC onderskeidelik af sodanig dat AG = DE en AH = DF. Trek GH.* Proof/*Bewys*: $\Delta AGH \equiv \Delta DEF$ [s, ∠, s] $\therefore A\hat{G}H = \hat{E}$ $= \hat{B}$ [$\hat{B} = \hat{E}$, given/*gegee*] $\therefore GH \parallel BC$ [corresp/*ooreenk* $\angle^s$ =] $\therefore \frac{AG}{AB} = \frac{AH}{AC}$ [line ∥ side of Δ / *lyn* ∥ *sye v* Δ] $\therefore \frac{DE}{AB} = \frac{DF}{AC}$ [constr/*konstruksie*]	✓ construction/ *konstruksie* ✓ S/R ✓ S ✓ S /R ✓ S ✓ R (6)

11.2



11.2.1(a)	$AP = PC$ [diag $\|\|^m$ bisect each other/*hoekl* $\|\|^m$ *halveer mekaar*] But $TP = PS$ [given/*gegee*] $AP - TP = PC - PS$ $\therefore AT = SC$	✓S ✓S OR S 2)
11.2.1(b)	In ΔPSR and ΔPBA: $\hat{P}_1 = \hat{P}_3$ [vertically opp $\angle^s$ / *regoorst* $\angle e$] $\hat{B}_1 = \hat{S}_1$ [$\angle^s$ in same segment/$\angle e$ *in dies segment*] $\therefore \Delta PSR \mid\mid\mid \Delta PBA$ [$\angle, \angle, \angle$] **OR/*OF*** In ΔPSR and ΔPBA: $\hat{P}_1 = \hat{P}_3$ [vertically opp $\angle^s$ / *regoorst* $\angle e$] $\hat{B}_1 = \hat{S}_1$ [$\angle^s$ in same segment/$\angle e$ *in dies segment*] $\hat{A}_2 = \hat{R}_1$ [sum $\angle^s$ Δ / *som* $\angle e$ Δ] $\therefore \Delta PSR \mid\mid\mid \Delta PBA$ [$\angle, \angle, \angle$]	✓S ✓R ✓S ✓R ✓R (5) ✓S ✓R ✓S ✓R ✓S (5)

11.2.2(a)	$\frac{PR}{PA}=\frac{PS}{PB}$ [$\|\|\|\Delta$s] $\therefore \frac{PR}{PA}=\frac{TR}{AD}=\frac{PS}{PB}$ [given $\frac{PR}{PA}=\frac{TR}{AD}$] $\therefore \frac{PR}{PA}=\frac{TR}{AD}=\frac{TP}{PD}$ [PS = TP; PB = PD] $\therefore \Delta RPT \|\|\| \Delta APD$ [sides of Δ in prop/*sye v* Δ *in dies verhouding*]	✓ S (all 3 ratios) ✓ S ✓ R (3)
11.2.2(b)	$\hat{T}_1 = \hat{D}_2$ [$\|\|\|\Delta$s] $\therefore$ ATRD is a cyclic quad [converse: ext $\angle$ of cyclic quad/ *Omgekeerde buite* $\angle$ v koordevh]	✓ S ✓ R (2)
		[18]

TOTAL/*TOTAAL*: 150

basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

NATIONAL/*NASIONALE*
SENIOR
CERTIFICATE/*SERTIFIKAAT*

GRADE/*GRAAD* 12

MATHEMATICS P2/*WISKUNDE V2*

FEBRUARY/MARCH/*FEBRUARIE/MAART* 2017

MEMORANDUM

MARKS / PUNTE: 150

This memorandum consists of 21 pages.
Hierdie memorandum bestaan uit 21 bladsye.

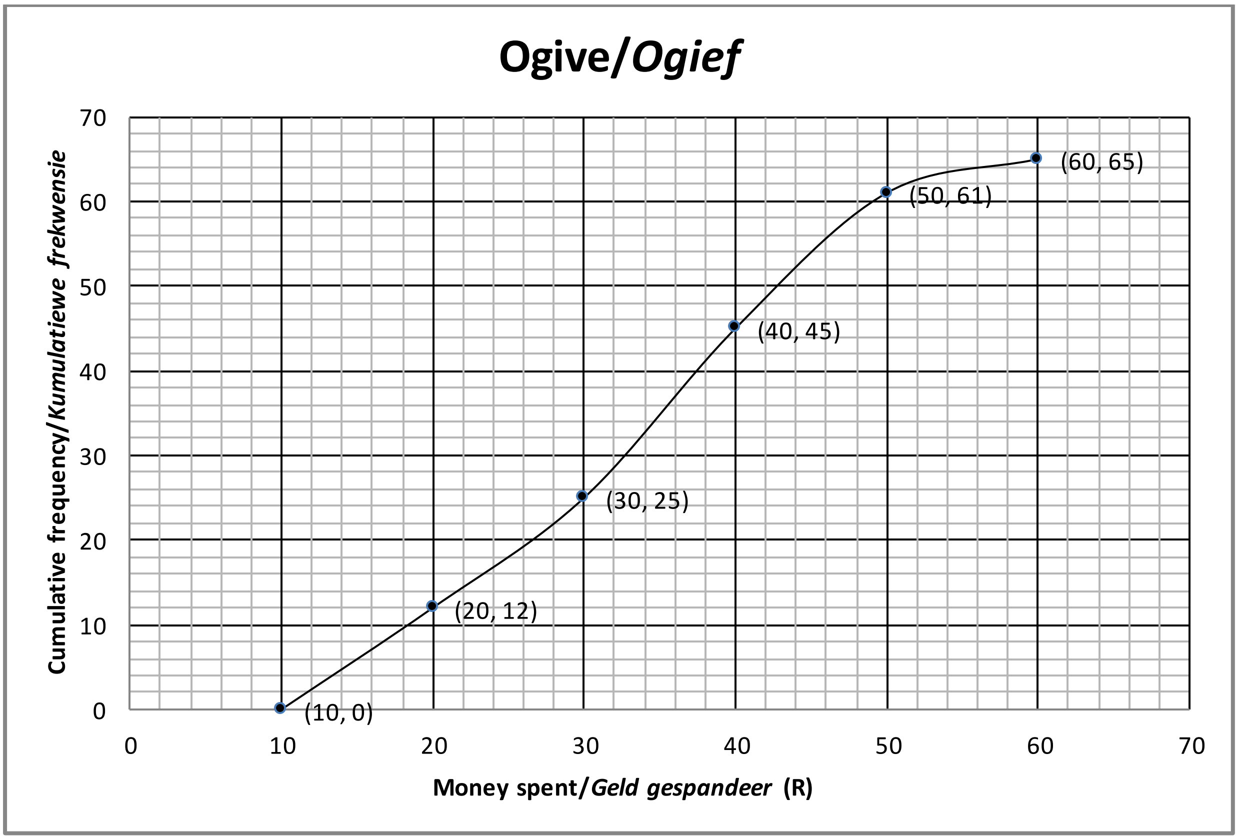
NOTE:

- If a candidate answered a question TWICE, mark only the FIRST attempt.
- If a candidate has crossed out an attempt to answer a question and did not redo it, mark the crossed-out version.
- Consistent accuracy applies in ALL aspects of the marking memorandum. Stop marking at the second calculation error.
- Assuming answers/values in order to solve a problem is NOT acceptable.

LET WEL:

- *Indien 'n kandidaat 'n vraag TWEE keer beantwoord het, sien slegs die EERSTE poging na.*
- *As 'n kandidaat 'n poging om 'n vraag te beantwoord, doodgetrek en nie oorgedoen het nie, sien die doodgetrekte poging na.*
- *Volgehoue akkuraatheid is op ALLE aspekte van die memorandum van toepassing. Staak nasien by die tweede berekeningsfout.*
- *Om antwoorde/waardes om 'n probleem op te los, te veronderstel, word NIE toegelaat NIE.*

QUESTION/*VRAAG* 1

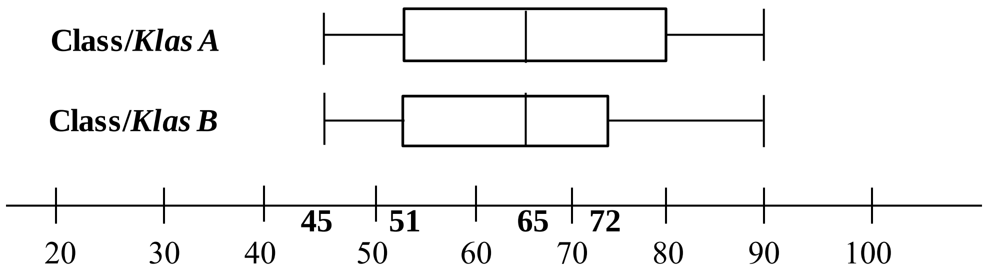


Amount of money/ *Bedrag geld* (in R)	$10 \leq x < 20$	$20 \leq x < 30$	$30 \leq x < 40$	$40 \leq x < 50$	$50 \leq x < 60$
Frequency *Frekwensie*	a	13	20	b	4

1.1	65 learners/*leerders*	✓ answer (1)
1.2	Modal class/*Modale klas*: $30 \leq x < 40$	✓ answer (1)
1.3	$a = 12$ $b = 61 - 45$ $= 16$	✓ answer ✓ answer (2)
1.4	No. of learners/*Aantal leerders* $= 65 - 54$ **OR/*OF*** $65 - 55$ $= 11$ $= 10$ Answer only: full marks	✓ 54 or 55 ✓ 11 or 10 (2) **[6]**

QUESTION/*VRAAG* 2

2.1



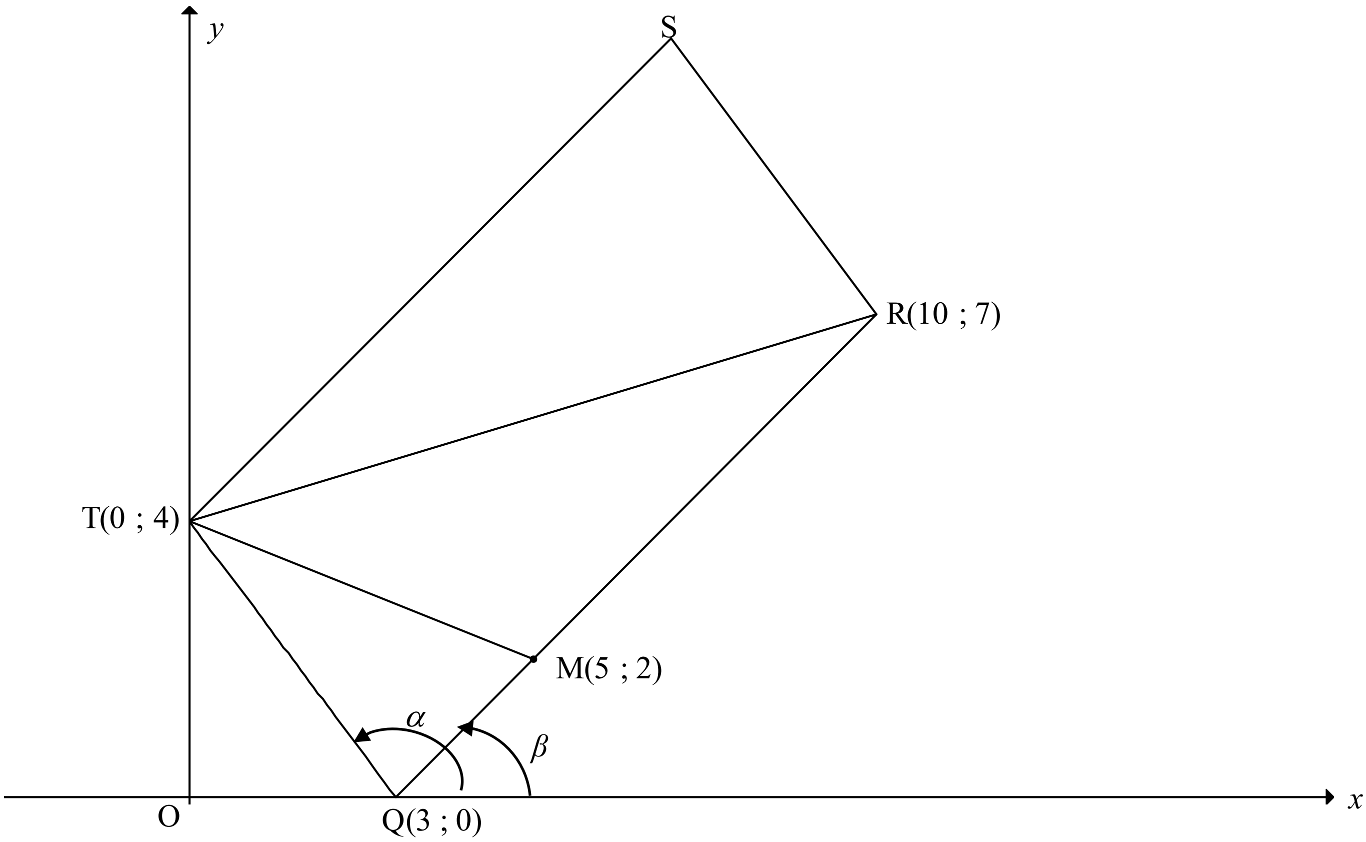
2.1.1	IQR of Class B/*IKV van Klas B* = $Q_3 - Q_1$ $= 72 - 51$ $= 21$ marks/*punte*	✓72 and 51 ✓ 21 only (2)
2.1.2	Although the boxes contain the same number of data points, the marks for Class A are more widely spread./*Alhoewel die monde dieselfde aantal datapunte bevat, is die punte van Klas A meer verspreid.* **OR/OF** Although the boxes contain the same number of data points, the marks for Class B are more clustered./*Alhoewel die monde dieselfde aantal datapunte bevat, is die punte van Klas B nader aan mekaar.*	✓ ✓ Class A is more widely spread (2) ✓ ✓ Class B is more clustered (2)
2.1.3	Medians are the same/*Mediane is dieselfde* Ranges are the same **OR** Maximum and minimum values are the same/*Variasiewydtes is dieselfde **OF** die maksimum en minimum waarde is dieselfde* 75% of both classes obtained 51 and above/*75% van albei klasse behaal 51 en meer.*	✓ ✓ any TWO of the 3 reasons mentioned (2)

2.2

COUPLE/*PAAR*	1	2	3	4	5	6	7	8
JUDGE 1/ *BEOORDELAAR 1*	18	4	6	8	5	12	10	14
JUDGE 2/ *BEOORDELAAR 2*	15	6	3	5	5	14	8	15

2.2.1	$a = -0{,}03$ $b = 0{,}93$ $\hat{y} = -0{,}03 + 0{,}93x$	✓ value a ✓ value b ✓ equation (3)
2.2.2	$\hat{y} = -0{,}03 + 0{,}93(15)$ $= 13{,}92$ **OR/*OF*** $13{,}85$ ≈ 14	✓ substitution ✓ answer (2)
2.2.3	Yes **OR** they are consistent, because $r = 0{,}9$. ($r = 0{,}89567\ldots$)/*Ja OF hulle is konsekwent, want $r = 0{,}9$. ($r = 0{,}89567\ldots$)*	✓ statement ✓ $r = 0{,}9$ (2) **[13]**

QUESTION/*VRAAG* 3



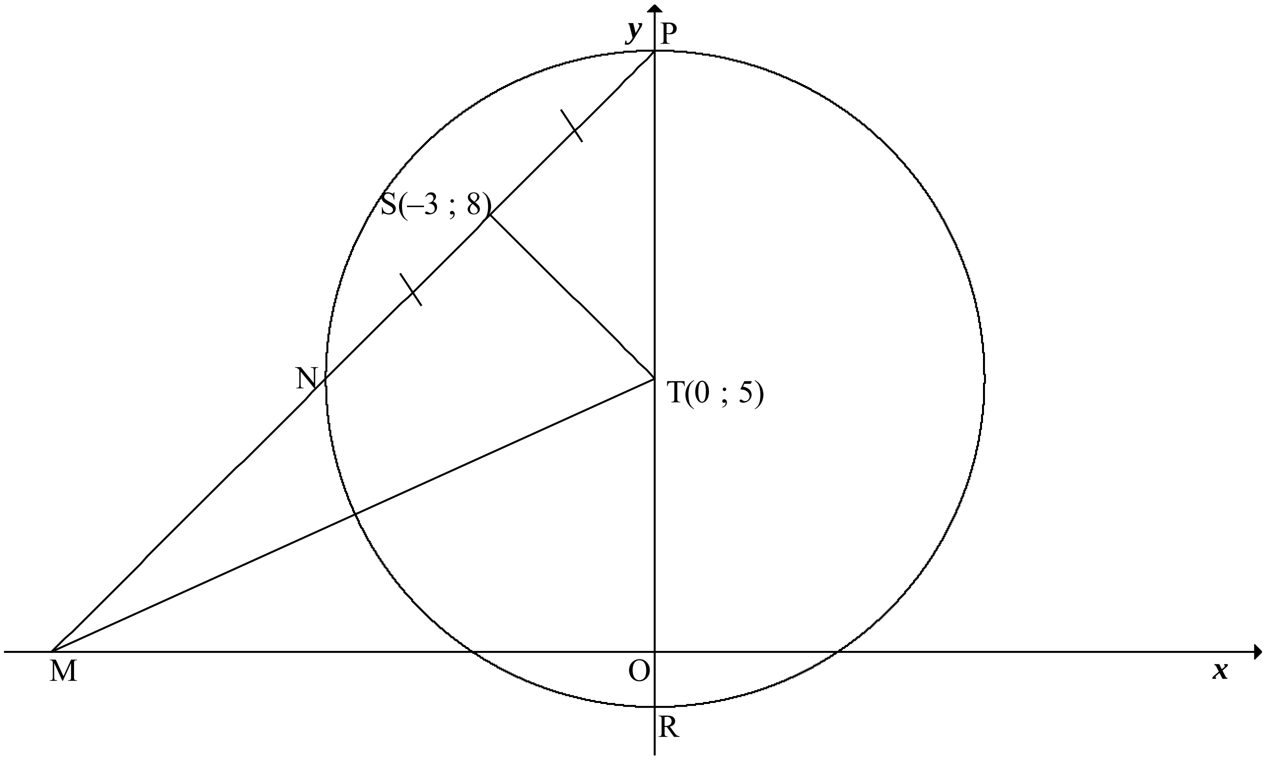
3.1	$m_{TQ} = \dfrac{4-0}{0-3}$ $= -\dfrac{4}{3}$	✓ answer (1)
3.2	$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $RQ = \sqrt{(10-3)^2 + (7-0)^2}$ $RQ = \sqrt{98} = 7\sqrt{2}$	✓substitution/*substitusie* ✓ answer in surd form (2)
3.3	$m_{FQ} = m_{TQ}$ $\dfrac{-8}{k-3} = -\dfrac{4}{3}$ $4k - 12 = 24$ $k = 9$ **OR/*OF*** $m_{FT} = m_{QT}$ $\dfrac{-8-4}{k-0} = -\dfrac{4}{3}$ $-36 = -4k$ $k = 9$ **OR/*OF*** Equation of TQ: $y = -\dfrac{4}{3}x + 4$ $-8 = -\dfrac{4}{3}k + 4$ $k = 9$	✓ equating gradients/*stel gradient gelyk* ✓ $m_{FQ} = \dfrac{-8}{k-3}$ ✓ simplification/*vereenvoudig* ✓ answer (4) ✓ gradient ✓ equation of TQ/*vgl van TQ* ✓ substitution of $(k ; -8)$ /*substitusie van* $(k ; -8)$ ✓ answer (4)

3.4	Using transformation/*Gebruik transformasie*: $\therefore$ S(7 ; 11) **OR/*OF*** Midpoint of TR = midpoint of SQ [diag ||m/*hkle*||*m*] Midpoint of TR = $(5\ ;\ \frac{11}{2})$ $\frac{x_S+3}{2}=5$ and $\frac{y_S+0}{2}=\frac{11}{2}$ $\therefore\ x_S=7$ and $y_S=11$ $\therefore$ S(7 ; 11) **OR/*OF*** Equation of TS: $y=\left(\frac{7-2}{10-5}\right)x+4=x+4$ Equation of RS: $y-7=-\frac{4}{3}(x-10)$ $y=-\frac{4}{3}x+\frac{61}{3}$ $x+4=-\frac{4}{3}x+\frac{61}{3}$ $7x=49$ $x=7$ $\therefore\ y=11$ $\therefore$ S(7 ; 11)	✓ ✓ *x*–value/*waarde* ✓ ✓ *y*–value/*waarde* (4) ✓ *x*–value/*waarde* of/*van* T ✓ *y*–value/*waarde* of/*van* T ✓ *x*–value/*waarde* of/*van* S ✓ *y*–value/*waarde* of/*van* S (4) ✓ equations of TS and RS/*vgls van TS en RS* ✓ equating / *gelykstel* ✓ *x*–value/*waarde* ✓ *y*–value/*waarde* (4)
3.5	$T\hat{S}R=T\hat{Q}R$ [opp ∠s of ||m/*teenoorst ∠e* ||*m*] $T\hat{Q}R=\alpha-\beta$ $tan\,\alpha=m_{TQ}=-\frac{4}{3}$ $\therefore\ \alpha=180°-53{,}13°=126{,}87°$ $tan\,\beta=m_{RQ}=\frac{7}{7}=1$ $\therefore\beta=45°$ $T\hat{Q}R=126{,}87°-45°$ $=81{,}87°$ $T\hat{S}R=81{,}87°$ **OR/*OF***	✓ $T\hat{Q}R=\alpha-\beta$ ✓ $\tan\alpha=m_{TQ}$ ✓ α ✓ $\tan\beta=m_{RQ}$ ✓ β ✓ answer (6)

	TQ = SR = 5 $\text{TR} = \sqrt{100+9} = \sqrt{109}$ $\text{RQ} = \text{TS} = \sqrt{49+49} = \sqrt{98}$ $\cos R\hat{Q}T = \cos T\hat{S}R = \dfrac{TQ^2 + RQ^2 - TR^2}{2.TQ.RQ}$ $= \dfrac{25+98-109}{2(5)(\sqrt{98})}$ $= 0{,}141...$ $R\hat{Q}T = T\hat{S}R = 81{,}87°$	✓ length of TQ **OR** SR ✓ length of TR ✓ length of RQ **OR** TS ✓ correct subst into cosine rule ✓ simplification ✓ answer (6)
3.6.1	$MQ = \sqrt{(5-3)^2 + (2-0)^2}$ $MQ = \sqrt{8}$ $\dfrac{MQ}{RQ} = \dfrac{\sqrt{8}}{\sqrt{98}}$ $= \dfrac{2}{7}$ or 0,29 Answer only: full marks	✓substitution/*substitusie* ✓ $MQ = \sqrt{8} = 2\sqrt{2}$ ✓ answer (3)
3.6.2	$\dfrac{\text{area of } \Delta TQM}{\text{area of } \Delta TQR} = \dfrac{\frac{1}{2}.QM.\perp h}{\frac{1}{2}.QR.\perp h}$ [$\perp$h same/*dieselfde*] $= \dfrac{QM}{QR} = \dfrac{2}{7}$ $\dfrac{\text{area of } \Delta TQM}{\text{area of parm RQTS}} = \dfrac{\text{area of } \Delta TQM}{2 \times \text{area of } \Delta TQR}$ $= \dfrac{1}{2}\left(\dfrac{2}{7}\right) = \dfrac{1}{7}$ **OR/*OF*** $\dfrac{\text{area of } \Delta TQM}{\text{area of } \Delta TQR} = \dfrac{QM}{QR}$ $= \dfrac{2}{7}$ $\dfrac{\text{area of } \Delta TQM}{\text{area of parm RQTS}} = \dfrac{\text{area of } \Delta TQM}{\text{2area of } \Delta TQR}$ $= \dfrac{1}{2}\left(\dfrac{2}{7}\right) = \dfrac{1}{7}$ **OR/*OF***	✓ $\dfrac{\text{area of } \Delta TQM}{\text{area of } \Delta TQR} = \dfrac{2}{7}$ ✓ area parm RQTS = 2area ΔTQR ✓ answer (3) ✓ $\dfrac{\text{area of } \Delta TQM}{\text{area of } \Delta TQR} = \dfrac{2}{7}$ ✓ area parm RQTS = 2area ΔTQR ✓ answer (3)

	$\dfrac{\text{area of }\Delta\text{TQM}}{\text{area of parm RQTS}}=\dfrac{\frac{1}{2}\text{QM}.\perp h}{\text{RQ}.\perp h}$ $=\dfrac{1}{2}\left(\dfrac{2}{7}\right)$ $=\dfrac{1}{7}$ **OR/*OF*** $\dfrac{\text{area of }\Delta\text{TQM}}{\text{area of parm RQTS}}=\dfrac{\frac{1}{2}\text{QT.QM}.\sin(\alpha-\beta)}{2\text{area of }\Delta\text{QTR}}$ $=\dfrac{\frac{1}{2}\text{QT.QM}.\sin(\alpha-\beta)}{2[\frac{1}{2}.\text{QT.QR}.\sin(\alpha-\beta)]}$ $=\dfrac{1}{2}\left(\dfrac{2}{7}\right)$ $=\dfrac{1}{7}$	✓ $\dfrac{\frac{1}{2}\text{QM}.\perp h}{\text{RQ}.\perp h}$ ✓ $\dfrac{1}{2}\left(\dfrac{2}{7}\right)$ ✓ answer (3) ✓ area parm RQTS = 2area ΔTQR ✓ $\dfrac{\frac{1}{2}\text{QT.QM}.\sin(\alpha-\beta)}{2[\frac{1}{2}.\text{QT.QR}.\sin(\alpha-\beta)]}$ ✓ answer (3) **[23]**

QUESTION/*VRAAG* 4



4.1	line from centre to **midpt of chord** / *lyn vanaf midpt na midpt van koord*	✓ answer (1)
4.2	$m_{ST} = \frac{8-5}{-3-0}$ $= -1$ $m_{ST} \times m_{NP} = -1$ [TS ⊥ NP] $\therefore\ m_{NP} = 1$ $\therefore y = x + c$ $8 = -3 + c$ $c = 11$ $\therefore y = x + 11$ **OR/*OF*** $y - y_1 = 1(x - x_1)$ $y - 8 = 1(x + 3)$ $y = x + 11$	✓ subst (–3 ; 8) and (0 ; 5) into gradient formula ✓ m_{ST} ✓ m_{NP} ✓ subst (–3 ; 8) into equation of a line ✓ equation (5)
4.3	P(0 ; 11) [y-intercept of chord NP] $\therefore$ radius is 6 units R(0 ; –1) Equations of the tangents to the circle parallel to the x-axis/ *Vgls van die raaklyne aan die sirkel* || *aan die x-as*: $y = 11$ and $y = -1$	✓ coordinates of P/ koördinate v P ✓ coordinates of R koördinate van R ✓✓ answers (4)
4.4	M(–11 ; 0) [x–intercept of/*x-afsnit van* NP] $MT = \sqrt{(0-11)^2 + (5-0)^2}$ $MT = \sqrt{146} = 12{,}08$	✓✓ coordinates of M ✓ substitution ✓ answer (4)

4.5	MT = diameter/*middellyn* [conv∠ in $\frac{1}{2}$ circle/*omgek ∠ in $\frac{1}{2}$ sirkel*] radius $= \dfrac{\sqrt{146}}{2}$ units Centre of circle/*Middelpunt v sirkel* = Midpoint MT /*Middelpunt MT* $= \left(\dfrac{-11}{2} ; \dfrac{5}{2}\right)$ Equation of circle through S, T and M: $\left(x+\dfrac{11}{2}\right)^2+\left(y-\dfrac{5}{2}\right)^2=\dfrac{146}{4}$ **OR/*OF*** $\left(x+5\dfrac{1}{2}\right)^2+\left(y-2\dfrac{1}{2}\right)^2=\dfrac{73}{2}=6{,}04$	✓ radius of circle ✓ x value of M ✓ y value of M ✓ LHS of equation ✓ RHS of equation (5) **[19]**

QUESTION/*VRAAG* 5

5.1	$a=-1$ $b=2$	✓ answer ✓ answer (2)
5.2	$f(3x) = -\sin 3x$ Period of $f(3x) = \dfrac{360°}{3}$ $= 120°$ Answer only: Full marks	✓ $\dfrac{360°}{3}$ ✓ answer (2)
5.3	$x \in [90° ; 135°) \cup \{180°\}$ **OR/*OF*** $90° \le x < 135°$ or $x = 180°$	✓ 90° and 135° in interval form ✓ 180° as single value ✓ correct brackets (3) ✓ 90° and 135° in interval form ✓ 180° as single value ✓ correct inequalities (3) **[7]**

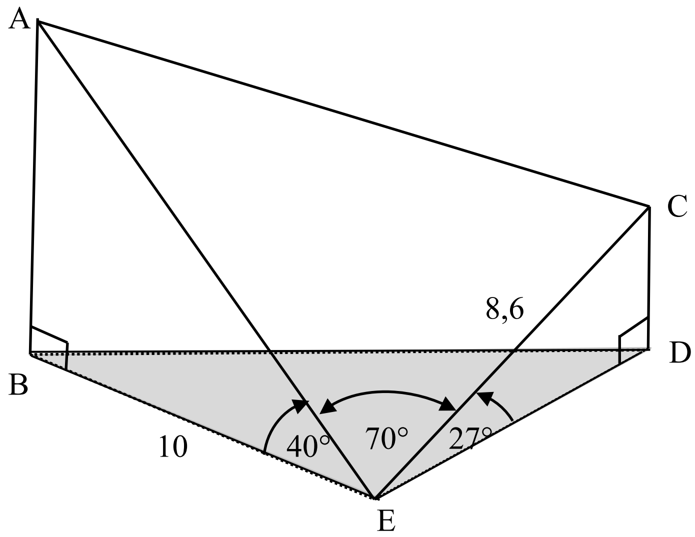
QUESTION/*VRAAG* 6

6.1.1	$\sin(360° - 36°) = -\sin 36°$	✓ answer (1)
6.1.2	$\cos 72° = \cos(2 \times 36°)$ $= 1 - 2\sin^2 36°$ Answer only: Full marks	✓ double angle/*dubbelhoek* ✓answer (2)
6.2	R.T.P.: $1 - \frac{\tan^2\theta}{1+\tan^2\theta} = \cos^2\theta$ $\text{LHS} = \frac{1+\tan^2\theta - \tan^2\theta}{1+\tan^2\theta}$ $= \frac{1}{1+\frac{\sin^2\theta}{\cos^2\theta}}$ $= \frac{1}{\frac{\cos^2\theta+\sin^2\theta}{\cos^2\theta}}$ $= \frac{1}{\frac{1}{\cos^2\theta}}$ $= \cos^2\theta$ = RHS	✓ writing as a single fraction/*skryf as enkelbreuk* ✓ quotient identity/ *kwosiëntidentiteit* ✓ denominator as a single fraction / *Noemer as enkelbreuk* ✓ square identity/*vierkantidentiteit* (4)
	OR/*OF*	
	$\text{LHS} = \frac{1+\tan^2\theta - \tan^2\theta}{1+\tan^2\theta}$ $= \frac{1}{1+\frac{\sin^2\theta}{\cos^2\theta}}$ $= \frac{1}{1+\frac{\sin^2\theta}{\cos^2\theta}} \times \frac{\cos^2\theta}{\cos^2\theta}$ $= \frac{\cos^2\theta}{\cos^2\theta+\sin^2\theta}$ $= \frac{\cos^2\theta}{1}$ $= \cos^2\theta$ = RHS	✓ writing as a single fraction/*skryf as enkelbreuk* ✓ quotient identity / *kwosiëntidentiteit* ✓ $\times \frac{\cos^2\theta}{\cos^2\theta}$ ✓ square identity/*vierkantidentiteit* (4)
	OR/*OF*	✓ quotient identity/

	$\text{LHS} = 1 - \left(\dfrac{\sin^2\theta}{\cos^2\theta} \div \left(1 + \dfrac{\sin^2\theta}{\cos^2\theta} \right) \right)$ $= 1 - \left(\dfrac{\sin^2\theta}{\cos^2\theta} \times \dfrac{\cos^2\theta}{\cos^2\theta + \sin^2\theta} \right)$ $= 1 - \left(\dfrac{\sin^2\theta}{\cos^2\theta} \times \dfrac{\cos^2\theta}{1} \right)$ $= 1 - \sin^2\theta$ $= \cos^2\theta$ $= \text{RHS}$	*kwosiëntidentiteit* ✓ writing as a single fraction/ *skryf as enkelbreuk* ✓ square identity/*vierkantidentiteit* ✓ simplification/*vereenvoudiging* (4)
6.3	$\cos^2 \frac{1}{2}x = \frac{1}{4}$ $\cos \frac{1}{2}x = \frac{1}{2} \text{ or } -\frac{1}{2}$ $\frac{1}{2}x = 60° + k.360°$ or $\frac{1}{2}x = 300° + k.360°$ or $\frac{1}{2}x = 120° + k.360°$ or $\frac{1}{2}x = 240° + k.360°$ $x = 120° + k.720°$ or $x = 600° + k.720°$ or $x = 240° + k.720°$ or $x = 480° + k.720°;\ k \in Z$ **OR/*OF*** $\cos^2 \frac{1}{2}x = \frac{1}{4}$ $\cos \frac{1}{2}x = \frac{1}{2} \text{ or } -\frac{1}{2}$ $\frac{1}{2}x = \pm 60° + k.360°$ or $\frac{1}{2}x = \pm 120° + k.360°$ $x = \pm 120° + k.720°$ or $x = \pm 240° + k.720°;\ k \in Z$	✓✓ $\cos^2 \frac{1}{2}x = \frac{1}{4}$ ✓ 60° and 300° ✓ 120° and 240° ✓ write at least one general solution as $\frac{1}{2}x = \angle + k.360°$ ✓ write at least one general solution as $x = \angle + k.720°;\ k \in Z$ (6) ✓✓ $\cos^2 \frac{1}{2}x = \frac{1}{4}$ ✓ ±60° ✓±120° ✓ write at least one general solution as $\frac{1}{2}x = \angle + k.360°$ ✓ write at least one general solution as $x = \angle + k.720°\ k \in Z$ (6)

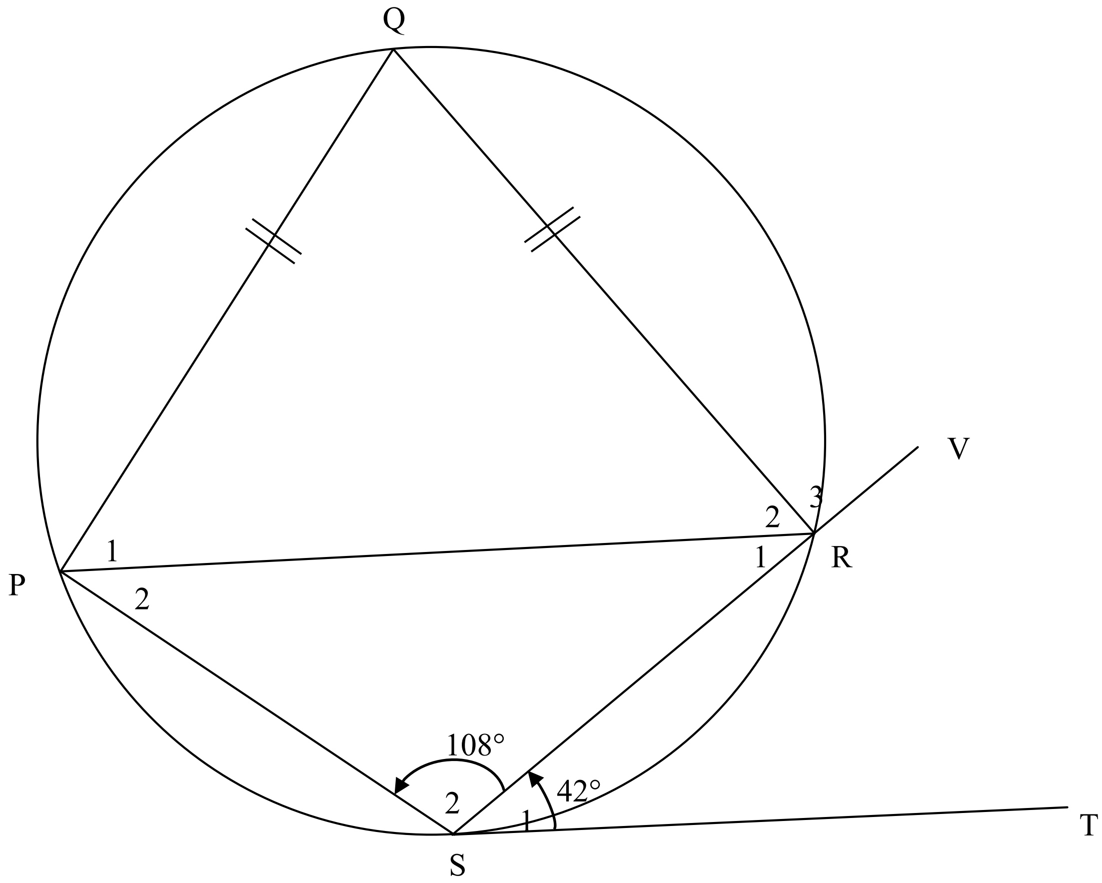
6.4.1	$\sin(A-B)=\cos[90°-(A-B)]$ $=\cos[(90°-A)-(-B)]$ $=\cos(90°-A)\cos(-B)+\sin(90°-A)\sin(-B)$ $=\sin A\cos B+\cos A(-\sin B)$ $=\sin A\cos B-\cos A\sin B$ **OR/*OF*** $\sin(A-B)=\cos[90°-(A-B)]$ $=\cos[(90°+B)-A]$ $=\cos(90°+B)\cos A+\sin(90°+B)\sin A$ $=-\sin B\cos A+\cos B\sin A$ $=\sin A\cos B-\cos A\sin B$	✓ co–ratio/*ko-verhouding* ✓ writing as a difference of A & B/ *skryf as verskil van A & B* ✓ expansion/*uitbreiding* ✓ all reductions/*alle reduksies* (4) ✓ co–ratio/*ko-verhouding* ✓ writing as a difference of A & B/ *skryf as verskil van A & B* ✓ expansion/*uitbreiding* ✓ all reductions/*alle reduksies* (4)
6.4.2	$\sin(x+64°)\cos(x+379°)+\sin(x+19°)\cos(x+244°)$ $=\sin(x+64°)\cos(x+19°)+\sin(x+19°)[-\cos(x+64°)]$ $=\sin(x+64°)\cos(x+19°)-\cos(x+64°)\sin(x+19°)$ $=\sin[x+64°-(x+19°)]$ $=\sin 45°$ $=\frac{1}{\sqrt{2}}$	✓ $\cos(x+379°)=\cos(x+19°)$ ✓✓ $\cos(x+244°)=-\cos(x+64°)$ ✓✓compound formula identity/ *saamgestelde identiteit* ✓ $\sin 45°$ (6) **[23]**

QUESTION/*VRAAG* 7



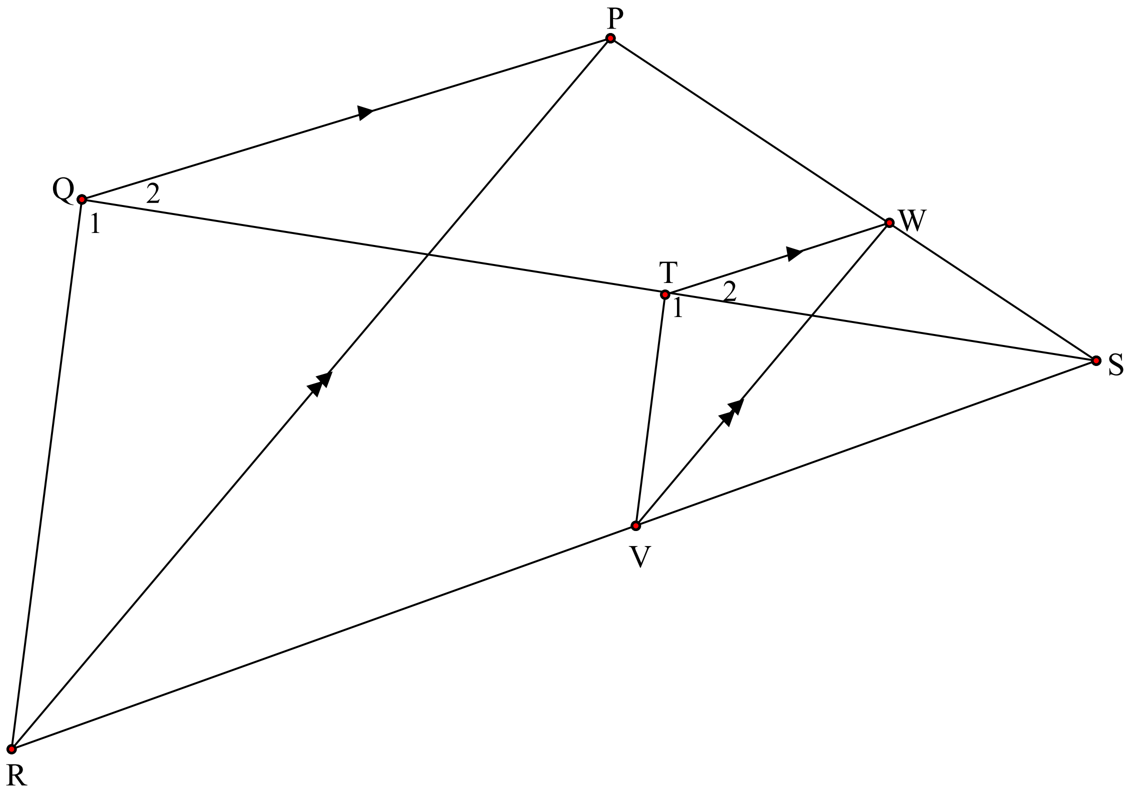
7.1	$\sin 27° = \frac{CD}{8,6}$ $CD = 8,6 \sin 27°$ $CD = 3,90 \text{ m}$	✓ substitution in correct trig ratio / *substitusie in korrekte trig verh* ✓ answer (2)
7.2	$\cos 40° = \frac{10}{AE}$ $AE = \frac{10}{\cos 40°}$ $AE = 13,05 \text{ m}$	✓ substitution in correct trig ratio / *substitusie in korrekte trig verh* ✓ answer (2)
7.3	$AC^2 = CE^2 + AE^2 - 2\,CE.AE(\cos A\hat{E}C)$ $= (8,6)^2 + (13,05)^2 - 2(8,6)(13,05)(\cos 70°)$ $= 167,49$ $AC = 12,94 \text{ m}$	✓ correct use of cosine rule in ΔACE/ *korrekte gebruik van reel in* ΔACE ✓ correct subst into cosine rule ✓ AC^2 ✓ answer (4) **[8]**

QUESTION/*VRAAG* 8



8.1	$\hat{Q} = 72°$ [opp $\angle$s of cyclic quad/*teenoorst $\angle$e koordevh*]	✓ S ✓ R (2)
8.2	$\hat{R}_2 = \hat{P}_1$ [$\angle$s opp equal sides/$\angle$*e teenoor gelyke sye*] $\hat{R}_2 = \dfrac{180° - 72°}{2}$ [sum of $\angle$s in Δ/*som v $\angle$e in Δ*] $= 54°$	✓ S/R ✓ answer (2)
8.3	$\hat{P}_2 = 42°$ [tan chord theorem/*raakl-koordst*]	✓ S ✓ R (2)
8.4	$\hat{R}_3 = \hat{P}_1 + \hat{P}_2$ [ext $\angle$ of cyclic quad/*buite$\angle$ van koordevh*] $= 54° + 42°$ $= 96°$ **OR/*OF*** $\hat{R}_1 = 180° - 108° - 42° = 30°$ [sum of/*som van*$\angle$s/e in Δ] $\hat{R}_3 = 180° - \hat{R}_1 - \hat{R}_2$ [$\angle$s on str line/$\angle$*e op reguitlyn*] $= 180° - 30° - 54°$ [sum of/*som van*$\angle$s/e in Δ] $= 96°$	✓ R ✓ S (2) ✓ $\hat{R}_1 = 30°$ ✓ S (2) **[8]**

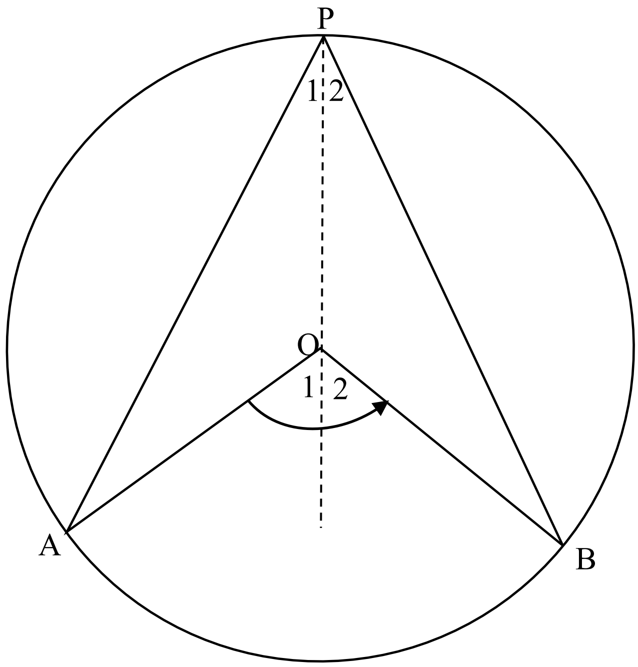
QUESTION/*VRAAG* 9



9.1.1	$\frac{ST}{TQ}=\frac{SW}{WP}$ [prop theorem/*eweredighst*; TW ∥ QP] $=\frac{2}{3}$	✓ S ✓ S (2)
9.1.2	$\frac{SV}{VR}=\frac{SW}{WP}$ [prop theorem/*eweredighst*; VW ∥ RP] $=\frac{2}{3}$	✓ answer (1)
9.2	$\frac{ST}{TQ}=\frac{SV}{VR}$ [both equal/*beide gelyk* $\frac{WS}{PW}$] $\therefore$ TV ∥ QR [line divides 2 sides of Δ in prop/*lyn verdeel 2 sye van Δ in dies verh*] $\therefore \hat{T}_1=\hat{Q}_1$ [corresp/*ooreenkomst* ∠s/e; TV ∥ QR]	✓ S ✓ S ✓ R ✓ R (4)
9.3	ΔVWS ||| <u>ΔRPS</u>	✓ ΔRPS (any order) (1)
9.4	$\frac{WV}{PR}=\frac{SW}{SP}$ [ΔVWS ||| ΔRPS] $=\frac{2}{5}$ **OR/*OF*** $\frac{WV}{PR}=\frac{SV}{SR}$ [ΔVWS ||| Δ RPS] $=\frac{2}{5}$	✓ ratio ✓ answer (2) **[10]**

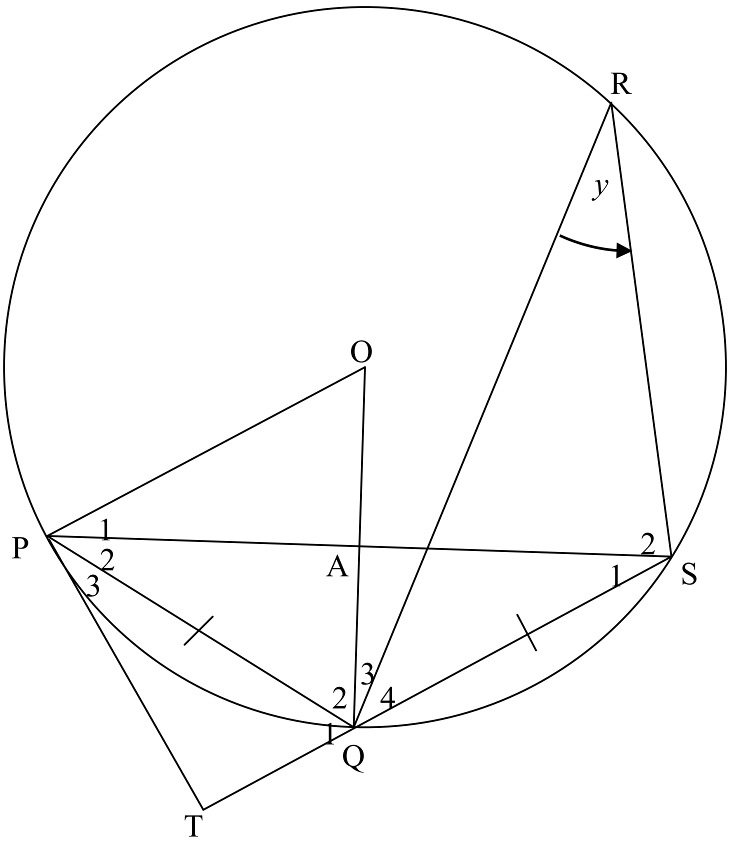
QUESTION/*VRAAG* 10

10.1



	Constr/*Konst* : Draw line PO and extend /*Trek lyn PO en verleng* Proof/*Bewys* : $OP = OA$ [radii] $\therefore \hat{P}_1 = \hat{A}$ [$\angle$s opp/*teenoor* = sides/*sye*] but $\hat{O}_1 = \hat{P}_1 + \hat{A}$ [ext $\angle$ of Δ] $\therefore \hat{O}_1 = 2\hat{P}_1$ Similarly/*Netso*, $\hat{O}_2 = 2\hat{P}_2$ $\therefore \hat{O}_1 + \hat{O}_2 = 2(\hat{P}_1 + \hat{P}_2)$ i.e. $A\hat{O}B = 2A\hat{P}B$	✓ construction ✓ S/R ✓ S/R ✓ S ✓ S (5)

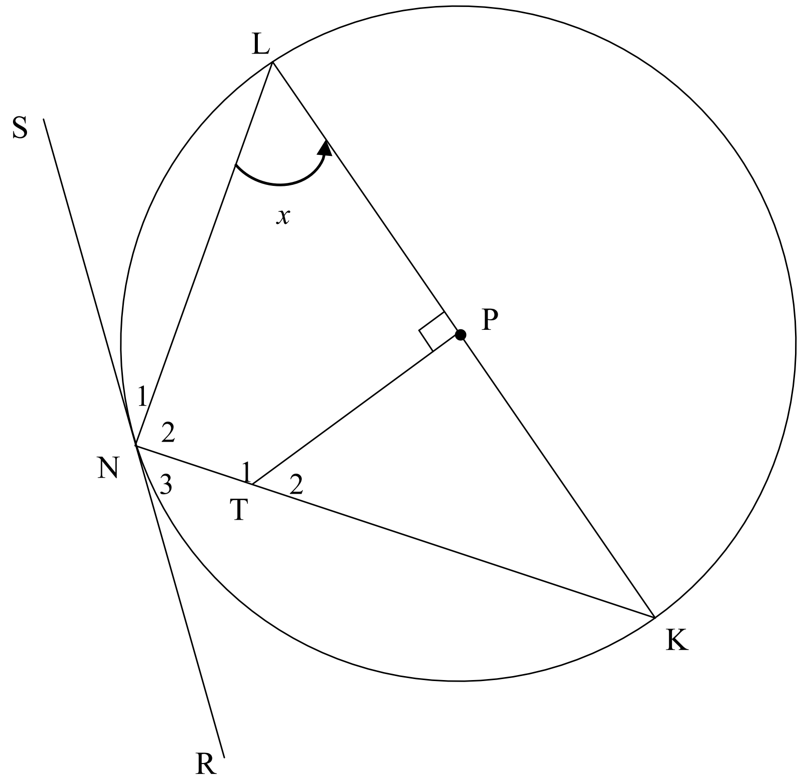
10.2



10.2.1	$\angle$s in the same segment/ $\angle$*e in dieselfde sirkelsegment*	✓ R (1)
10.2.2	$\hat{P}_2 = \hat{S}_1 = y$ [$\angle$s opp equal sides/$\angle$*e teenoor = sye*] $\hat{S}_1 = \hat{P}_3 = y$ [tan chord theorem/*raakl-koordst*] $\therefore\ \hat{P}_2 = \hat{P}_3$ $\therefore$ PQ bisects $T\hat{P}S$	✓ S ✓ R ✓ S ✓ R (4)
10.2.3	$P\hat{O}Q = 2\hat{S}_1 = 2y$ [$\angle$at centre =2×$\angle$at circ/*midpts*$\angle$= *2×omtreks*$\angle$]	✓ S ✓ R (2)
10.2.4	$T\hat{P}A = \hat{P}_2 + \hat{P}_3 = 2y$ [proved/*bewys* in 11.2.2] $\therefore\ T\hat{P}A = P\hat{O}Q$ [proved/*bewys* in 11.2.3] $\therefore$ PT = tangent [converse tan chord theorem/*omgek raakl-koordst*]	✓ $T\hat{P}A = P\hat{O}Q$ ✓ R (2)

10.2.5	$\text{O}\hat{\text{P}}\text{Q} + \text{O}\hat{\text{Q}}\text{P} = 180° - 2y$ [sum of/*sum v* ∠s/*e* in Δ] $\therefore\ \text{O}\hat{\text{Q}}\text{P} = 90° - y$ [∠s opp equal sides/∠*e to = sye;* OP = OQ] In ΔPAQ: $\text{O}\hat{\text{Q}}\text{P} + \hat{\text{P}}_2 + \text{Q}\hat{\text{A}}\text{P} = 180°$ $90° - y + y + \text{Q}\hat{\text{A}}\text{P} = 180°$ [sum of/*sum v* ∠s/*e* in Δ] $\text{Q}\hat{\text{A}}\text{P} = 90°$ $\therefore\ \text{O}\hat{\text{A}}\text{P} = 90°$ [∠s/*e* on straight line/*op reguitlyn*]	✓ S ✓ S ✓ R ✓ S ✓ S (5)
	OR/*OF*	
	$\text{O}\hat{\text{P}}\text{T} = 90°$ [radius ⊥ tangent/*raaklyn*] $\therefore\ \hat{\text{P}}_1 = 90° - 2y$ $\hat{\text{P}}_1 + \hat{\text{O}} + \text{O}\hat{\text{A}}\text{P} = 180°$ [sum of/*sum v* ∠s/*e* in Δ] $(90° - 2y) + 2y + \text{O}\hat{\text{A}}\text{P} = 180°$ $\therefore\ \text{O}\hat{\text{A}}\text{P} = 90°$	✓ S ✓ R ✓ S ✓ S ✓ S (5)
	OR/*OF*	
	POSQ is a kite/'*n vlieër* $\therefore$ OQ ⊥ PS [diag of a kite/*hoeklyne v vlieër*] $\therefore\ \text{O}\hat{\text{A}}\text{P} = 90°$	✓✓✓ S ✓✓ R (5)
	OR/*OF*	
	In ΔOAP and ΔOAS OP = OS (radii) OA is common $\text{P}\hat{\text{O}}\text{A} = 2y$ $= 2\hat{\text{P}}_2$ $= \text{Q}\hat{\text{O}}\text{S}$ ΔOAP ≡ ΔOAS (SAS) $\text{O}\hat{\text{A}}\text{P} = \text{O}\hat{\text{A}}\text{S}$ (≡Δs) $\text{O}\hat{\text{A}}\text{P} = \text{O}\hat{\text{A}}\text{S} = 90°$ (∠s on str line)	 ✓ S ✓ S ✓ S ✓ R ✓ S (5) **[19]**

QUESTION/*VRAAG* 11



11.1	$\hat{N}_2 = 90°$ [$\angle$ in semi-circle/*halfsirkel*] $\therefore$ TPLN is a cyclic quad/ 'n *koordevh* [opp $\angle$s of quad is suppl/ **t***eenoor$\angle$e v vh is suppl*] **OR** $\hat{N}_2 = 90°$ [$\angle$ in semi-circle/*halfsirkel*] $\therefore$ TPLN is a cyclic quad [ext $\angle$ = int opp $\angle$/*buite$\angle$ = to binne $\angle$*]	✓ S ✓ R ✓ R (3) ✓ S ✓ R ✓ R (3)
11.2	$\hat{T}_2 = \text{P}\hat{\text{L}}\text{N} = x$ [ext $\angle$ of cyclic quad/*buite$\angle$ van koordevh*] $\hat{K} = 90° - x$ [sum of/*som v* $\angle$s/e in Δ] $\hat{N}_1 = \hat{K} = 90° - x$ [tan chord theorem/*raakl-koordst*] **OR/*OF*** $\hat{K} = 90° - x$ [sum of/*som v* $\angle$s/e in Δ] $\hat{N}_1 = \hat{K} = 90° - x$ [tan chord theorem/*raakl-koordst*] **OR/*OF*** $\hat{N}_3 = x$ [tan chord theorem/*raakl-koordst*] $\hat{N}_2 = 90°$ [$\angle$in semi circle/ *halfsirkel*] $\hat{N}_1 = 90° - x$ [straight line/*reguitlyn*]	✓ R ✓ S✓ R (3) ✓ R ✓ S✓ R (3) ✓ R ✓ S ✓ S (3)

11.3.1	In ΔKTP and ΔKLN: $P\hat{K}T = L\hat{K}N$ [common/*gemeen*] $K\hat{P}T = K\hat{N}L = 90°$ [given/*gegee*] $\therefore \Delta KTP \mid\mid\mid \Delta KLN$ [$\angle\angle\angle$] **OR/*OF*** In ΔKTP and ΔKLN: $P\hat{K}T = L\hat{K}N$ [common/*gemeen*] $K\hat{P}T = K\hat{N}L = 90°$ [given/*gegee*] $\hat{T}_2 = P\hat{L}N = x$ [proved in 11.2 **OR** sum of $\angle$s in Δ] $\therefore \Delta KTP \mid\mid\mid \Delta KLN$	✓ S ✓ S ✓ R (3) ✓ S ✓ S ✓ S (3)
11.3.2	$\frac{KT}{KL} = \frac{KP}{KN}$ [$\mid\mid\mid$ Δs] $\therefore KT \cdot KN = KP \cdot KL$ But KL = 2KP [radii: PK = LP] $\therefore KT \cdot KN = KP \cdot 2KP$ $= 2KP^2$ $= 2(KT^2 - TP^2)$ [Theorem of Pythagoras] $= 2KT^2 - 2TP^2$	✓ S/R ✓ S ✓ S ✓ S ✓ S (5) **[14]**

TOTAL/*TOTAAL*: 150

basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

NATIONAL/*NASIONALE* SENIOR CERTIFICATE/*SERTIFIKAAT*

GRADE/*GRAAD* 12

MATHEMATICS P2/*WISKUNDE V2*

NOVEMBER 2016

MEMORANDUM

MARKS/*PUNTE*: 150

This memorandum consists of 26 pages.
Hierdie memorandum bestaan uit 26 bladsye.

NOTE:

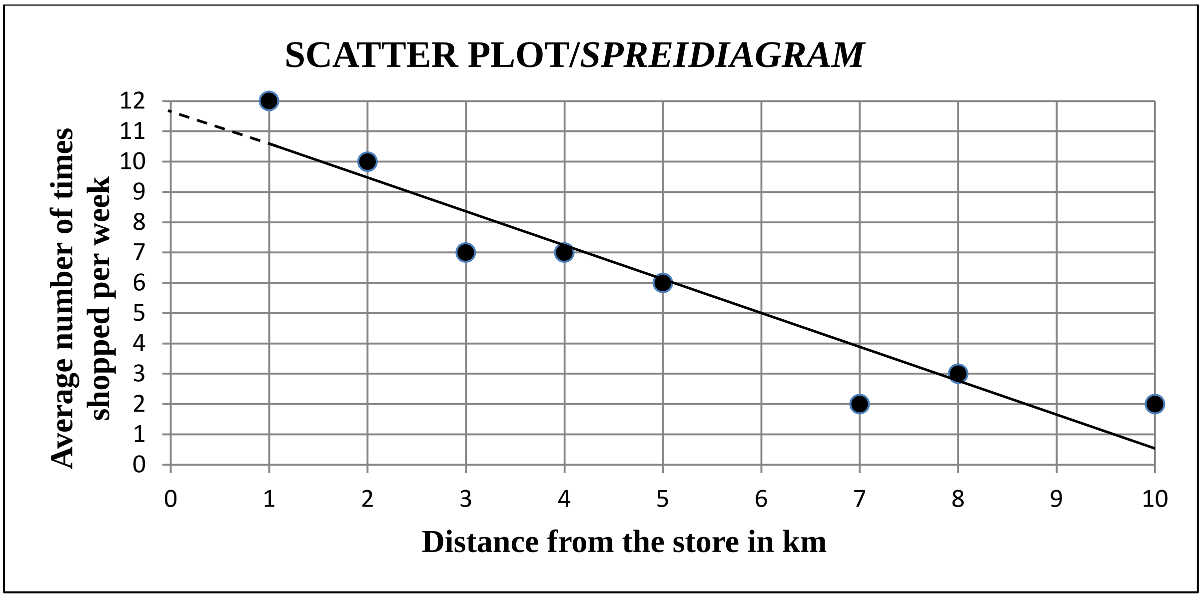
- If a candidate answered a question TWICE, mark only the FIRST attempt.
- If a candidate has crossed out an attempt to answer a question and did not redo it, mark the crossed-out version.
- Consistent accuracy applies in ALL aspects of the marking memorandum. Stop marking at the second calculation error.
- Assuming answers/values in order to solve a problem is NOT acceptable.

LET WEL:

- *Indien 'n kandidaat 'n vraag TWEE keer beantwoord het, sien slegs die EERSTE poging na*.
- *As 'n kandidaat 'n poging om 'n vraag te beantwoord, doodgetrek en nie oorgedoen het nie, sien die doodgetrekte poging na.*
- *Volgehoue akkuraatheid is op ALLE aspekte van die memorandum van toepassing. Staak nasien by die tweede berekeningsfout.*
- *Om antwoorde/waardes om 'n probleem op te los, te veronderstel, word NIE toegelaat NIE.*

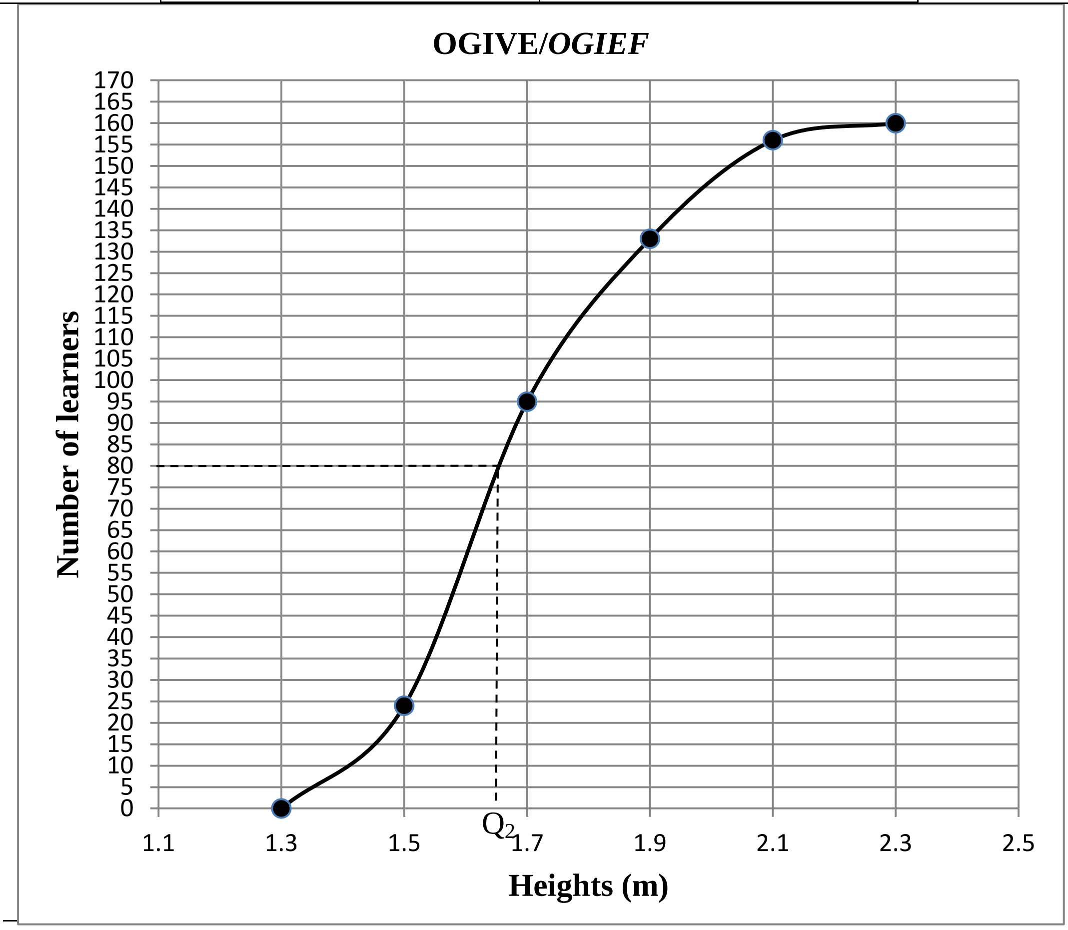
QUESTION/*VRAAG* 1

Distance from the store in km ***Afstand vanaf die winkel in* km**	1	2	3	4	5	7	8	10
Average number of times shopped per week ***Gemiddelde aantal keer wat kopers die winkel per week besoek***	12	10	7	7	6	2	3	2

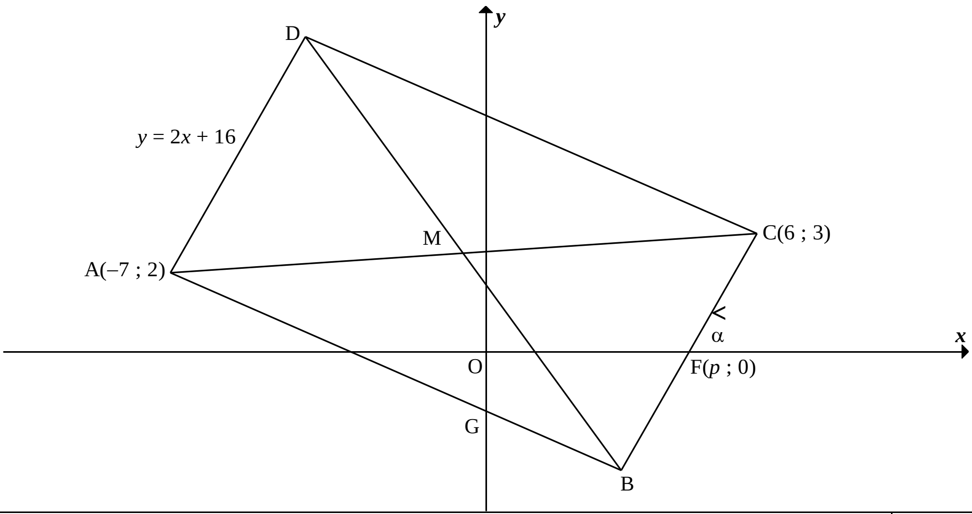


1.1	Strong/*Sterk*	✓ (1)
1.2	–0,95 (–0,9462..)	✓ (1)
1.3	a = 11,71 (11,7132…) b = –1,12 (–1,1176…) $\hat{y} = -1{,}12x + 11{,}71$	✓ value of a ✓ value of b ✓ equation/*vgl* (3)
1.4	$\hat{y} = -1{,}12(6) + 11{,}71$ = 5 times	✓ substitition ✓ answer (2)
1.5	On scatter plot/*Op spreidiagram*	✓✓ A line close to any 2 of the following points: (5 ; 6) or (10 ; $\frac{1}{2}$) or (6 ; 5) or (0 ; 11,7) (2) **[9]**

QUESTION/*VRAAG* 2

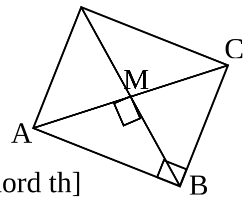
2.1	Positively skewed **OR** skewed to the right/*positief skeef* ***OF*** *skeef na regs*	✓ answer (1)
2.2	Range/*Omvang* = 2,21 – 1,39 = 0,82 m	✓ subtract values ✓answer (2)
2.3	<table><tr><th>Intervals *Klasse*</th><th>Cumulative frequency *Kumulatiewe frekwensie*</th></tr><tr><td>$1{,}3 \le x < 1{,}5$</td><td>24</td></tr><tr><td>$1{,}5 \le x < 1{,}7$</td><td>95</td></tr><tr><td>$1{,}7 \le x < 1{,}9$</td><td>133</td></tr><tr><td>$1{,}9 \le x < 2{,}1$</td><td>156</td></tr><tr><td>$2{,}1 \le x < 2{,}3$</td><td>160</td></tr></table>	✓95 , 133, 156 ✓160 (2)
2.4		✓ upper limits / *boonste limiete* ✓ cum f/ *kum f* ✓ shape/ *vorm* ✓ grounded *geanker* (4)
2.5	method (using 80 to determine the height) 1,65 (accept any value between 1,6 and 1,69)	✓ method ✓ answer (2)
2.6.1	The mean would change by 0,1 *m* *Die gemiddelde sal met 0,1 m verander*	✓ answer (1)
2.6.2	No influence/change as there is no difference in variation of data./*Geen invloed /verandering aangesien daar geen verskil in die variasie van die data is nie.*	✓ answer (1) **[13]**

QUESTION/*VRAAG* 3

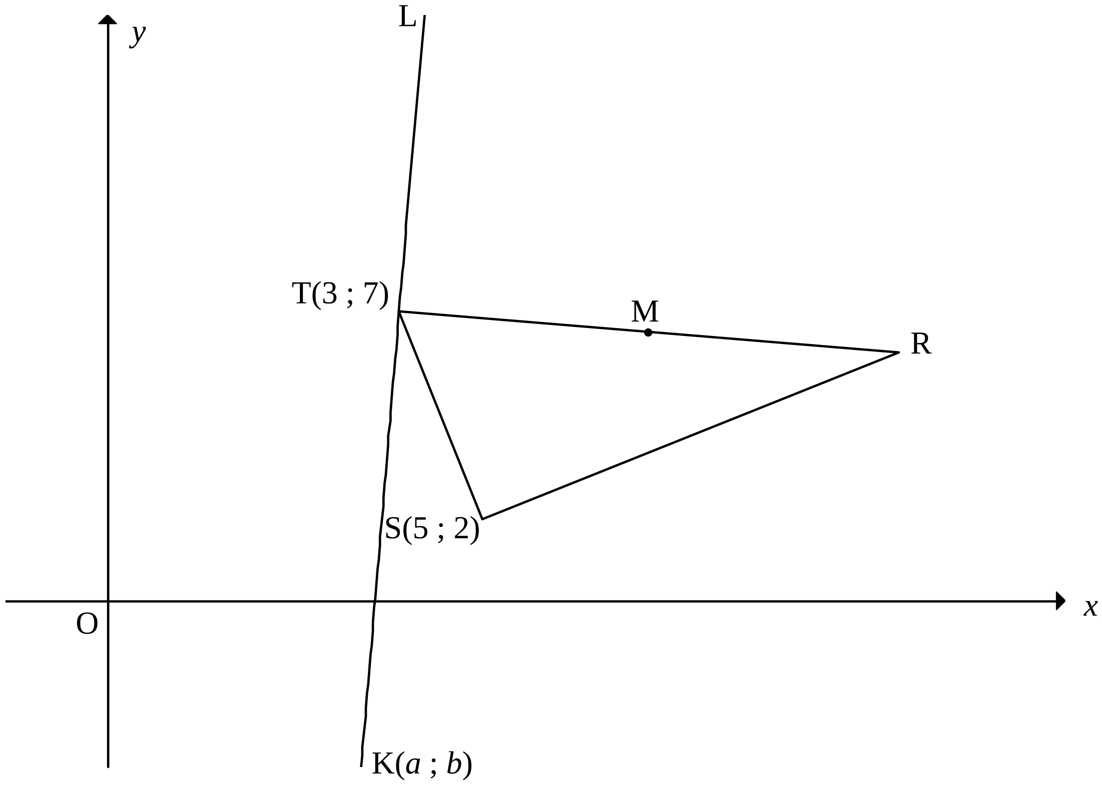


3.1	M = Midpt of AC [diags of rectangle bisect/ *hoekl v reghoek halveer*] $= \text{M}\left(\frac{-7+6}{2}\ ;\ \frac{2+3}{2}\right)$ $= \text{M}\left(-\frac{1}{2}\ ;\ \frac{5}{2}\right)$	✓ x-value of M ✓ y-value of M (2)
3.2	$m_{\text{BC}} = \frac{3-0}{6-p} = \frac{3}{6-p}$ **OR/*OF*** $m_{\text{BC}} = \frac{0-3}{p-6} = \frac{-3}{p-6}$	✓answer (1) ✓answer (1)
3.3	$m_{\text{AD}} = m_{\text{BC}}$ [AD | | BC] $m_{\text{BC}} = 2$ $\frac{3}{6-p} = 2$ $3 = 12 - 2p$ $p = 4\frac{1}{2}$ **OR/OF** $y - y_1 = 2(x - x_1)$ $C(6\,;3)$ $y - 3 = 2(x-6)$ $\therefore y = 2x - 9$ $but\ y = 0$ $\therefore x = 4\frac{1}{2} = p$ **OR/OF**	✓ $m_{\text{BC}} = 2$ ✓ equating ✓answer (3) ✓ $m_{\text{BC}} = 2$ ✓ substituting (6 ; 3) ✓answer (3)

	$y = 2x + c$ $3 = 12 + c$ $-9 = c$ $y = 2x - 9$ $0 = 2x - 9$ $x = \frac{9}{2} \qquad \therefore\ p = \frac{9}{2}$	✓ $m_{BC} = 2$ ✓ substituting ✓answer (3)
3.4	DB =AC [diag of rectangle = / *hoekl v reghoek* =] $AC = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $AC = \sqrt{(6+7)^2 + (3-2)^2}$ $AC = \sqrt{13^2 + 1^2}$ $AC = \sqrt{170}$ $\therefore$ DB = $\sqrt{170}$ or 13,04	✓ substitution ✓ length of AC ✓ AC = BD (3)
3.5	$\tan\alpha = m_{BC} = 2$ $\therefore \alpha = 63{,}43°$	✓ $\tan\alpha = m_{BC}$ ✓ $\alpha = 63{,}43°$ (2)
3.6	In quadrilateral OFBG: $O\hat{F}B = 63{,}43°$ [vert opp $\angle$s/*regoorst* $\angle$*e*] $F\hat{O}G = G\hat{B}F = 90°$ $\therefore\ O\hat{G}B = 360° - [90° + 90° + 63{,}43°]$ [sum $\angle$s quad/*som* $\angle$*e vierh* = 360°] $\therefore\ O\hat{G}B = 116{,}57°$ **OR/*OF*** $m_{AB} = -\frac{1}{2}$ 90° + $O\hat{G}A$ = 153,43° $\therefore\ O\hat{G}A$ = 63,43° $O\hat{G}B$ = 180° – 63,43° = 116,57° **OR/*OF*** $F\hat{O}G = G\hat{B}F = 90°$ $\therefore$GOFB is cyc quad $O\hat{G}B$ = 180° – 63,43° [$\angle$s of cyc quad = 180°] = 116,57° **OR/OF** $O\hat{F}B = 63{,}43°$ $X\hat{O}G = F\hat{B}G = 90°$ $\therefore$ OGBF is a cyclic quad $\therefore O\hat{G}B = 180° - 63{,}43°$ $O\hat{G}B = 116{,}57°$	✓ size of $O\hat{F}B$ ✓ S ✓ answer (3) ✓ $m_{AB} = -\frac{1}{2}$ ✓ S ✓ answer (3) ✓ S ✓ S ✓ answer (3) ✓ S ✓ S ✓ answer (3)

3.7	$M\left(-\frac{1}{2}\,;\,\frac{5}{2}\right)$ is the centre/*is die middelpunt* $r=\frac{\sqrt{170}}{2}$ = radius [BD is diameter/*middellyn*] $\left(x+\frac{1}{2}\right)^2+\left(y-\frac{5}{2}\right)^2=\left(\frac{\sqrt{170}}{2}\right)^2=\frac{85}{2}=42{,}5$	✓ M is centre ✓ $r=\frac{\sqrt{170}}{2}$ ✓ equation (3)
3.8	$\hat{CBM}=\hat{BAM}=45°$ [diag of square bisect ∠s/*hoekl v vierk halv ∠e*] $\therefore$ BC will be a tangent [converse tan chord th/*omgekeerde raakl-koordst*] **OR/*OF***  $\hat{AMB}=90°$ [diag of square bisect ⊥] $\therefore$ AB is diameter BC ⊥ AB $\therefore$ BC is tangent [line ⊥ radius *or* converse tan-chord th]	✓S ✓ R (2) ✓S ✓ R (2) **[19]**

QUESTION/*VRAAG 4*



4.1	$\angle$ in semi circle/ $\angle$ at centre = 2$\angle$ on circle $\angle$ *in halfsirkel* /$\angle$ by middelpt = 2$\angle$ op sirkel	✓ R (1)
4.2	$m_{TS} = \dfrac{7-2}{3-5}$ $= -\dfrac{5}{2}$	✓ substitution ✓ m_{TS} (2)
4.3	$m_{TS} \times m_{RS} = -1$ [TS $\perp$ SR] $\therefore m_{RS} = \dfrac{2}{5}$ $y = \dfrac{2}{5}x + c$ $2 = \dfrac{2}{5}(5) + c$ $c = 0$ $y = \dfrac{2}{5}x$ **OR/*OF***	✓ m_{RS} ✓substitution m and (5 ; 2) ✓ equation (3)

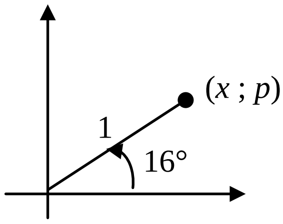
	$m_{TS} \times m_{RS} = -1$ [TS ⊥ SR] $\therefore m_{RS} = \frac{2}{5}$ $y - y_1 = \frac{2}{5}(x - x_1)$ $y - 2 = \frac{2}{5}(x - 5)$ $y = \frac{2}{5}x$	✓ m_{RS} ✓substitution m and (5 ; 2) ✓ equation (3)
4.4.1	$r = \sqrt{36\frac{1}{4}}$ $TR = 2.r = 2\left(\sqrt{36\frac{1}{4}}\right) = \sqrt{145}$	✓ r ✓ answer (2)
	OR/OF $TM = \sqrt{(3-9)^2 + \left(7 - 6\frac{1}{2}\right)^2} = \frac{\sqrt{145}}{2}$ $TR = 2.r = 2\left(\sqrt{36\frac{1}{4}}\right) = \sqrt{145}$	✓ substitution ✓ answer (2)
4.4.2	$M\left(9 ; 6\frac{1}{2}\right)$ $\therefore \frac{x_R + 3}{2} = 9$ and $\frac{y_R + 7}{2} = 6\frac{1}{2}$ $\therefore R(15 ; 6)$ Answer only: full marks Answer only: only 1 coordinate correct (1 mark)	✓ M ✓ x coordinate ✓ y coordinate (3)
	OR/*OF* $M\left(9 ; 6\frac{1}{2}\right)$ $\therefore R\left(9 + 6 ; 6\frac{1}{2} - \frac{1}{2}\right) = R(15 ; 6)$	✓ M ✓ x coordinate ✓ y coordinate (3)
	OR/OF	

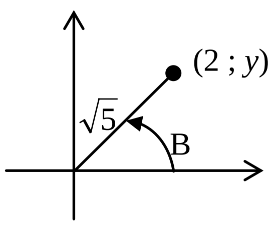
	$m_{TM} = \dfrac{9-3}{6\frac{1}{2}-7} = -\dfrac{1}{12}$ $TM: \quad 7 = -\dfrac{1}{12}(3)+c \quad y = -\dfrac{1}{12}x + \dfrac{29}{4}$ ....(1) $SR: \quad y = \dfrac{2}{5}x$ ....(2) $\dfrac{2}{5}x = -\dfrac{1}{12}x + \dfrac{29}{4}$ $\dfrac{29}{60}x = \dfrac{29}{4}$ $\therefore x = 15$ $\therefore y = \dfrac{2}{5}(15) = 6$	✓ equating ✓ *x* coordinate ✓ *y* coordinate (3)
4.4.3	$\text{ST} = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ $\text{ST} = \sqrt{(5-3)^2+(2-7)^2}$ $\text{ST} = \sqrt{4+25} = \sqrt{29}$ $\sin \text{R} = \dfrac{\text{TS}}{\text{TR}} = \dfrac{\sqrt{29}}{\sqrt{145}}$ *or* $\dfrac{\sqrt{5}}{5}$ *or* $\dfrac{1}{\sqrt{5}}$ *or* $0{,}45$ **OR/*OF*** $TS = \sqrt{29}$ $SR = 2\sqrt{29}$ $\text{area of } \Delta\text{TSR} = \dfrac{1}{2}\left(\sqrt{29}\right)\left(2\sqrt{29}\right) = 29$ $29 = \dfrac{1}{2}(\sqrt{145})(2\sqrt{29})\sin R$ $\sin R = \dfrac{\sqrt{5}}{5}$ *or* $\dfrac{1}{\sqrt{5}}$	✓substitution ✓ answer ✓ ratio (3) ✓area ✓ rule ✓ ratio (3)
4.4.4	$m_{\text{TR}} = \dfrac{7-6\frac{1}{2}}{3-9} = -\dfrac{1}{12}$ **OR/*OF*** $m_{\text{TR}} = \dfrac{7-6}{3-15} = -\dfrac{1}{12}$ $m_{\text{TR}} \times m_{\text{KTL}} = -1$ [$r \perp$ tangent] $m_{\text{KTL}} = 12$ $y - y_1 = 12(x - x_1)$ $y - 7 = 12(x-3)$ $y = 12x - 29$ substitute $\text{K}(a;b)$: $b = 12a - 29$ **OR/*OF***	✓ $m_{\text{TR}} = -\dfrac{1}{12}$ ✓ $m_{\text{KTL}} = 12$ ✓ $y = 12x - 29$ (3)

	$m_{TR} = \dfrac{7-6\frac{1}{2}}{3-9} = -\dfrac{1}{12}$ $m_{TR} \times m_{KTL} = -1$ [$r \perp$ tangent] $\dfrac{b-7}{a-3} = 12$ $b-7 = 12(a-3)$ $b = 12a-29$ **OR/*OF*** $KR^2 = TR^2 + TK^2$ $(a-15)^2 + (b-6)^2 = (15-3)^2 + (6-7)^2 + (a-3)^2 + (b-7)^2$ $-30a + 225 - 12b + 36 = 144 + 1 - 6a + 9 - 14b + 49$ $2b = 24a - 58$ $b = 12a - 29$	✓ $m_{TR} = -\dfrac{1}{12}$ ✓ $m_{KTL} = 12$ ✓substitution (3 ; 7) & (a ; b) (3) ✓ subst into Pyth ✓multiplication ✓simplification (3)
4.4.5	$TK = TR$ $\sqrt{(a-3)^2 + (b-7)^2} = \sqrt{145}$ $(a-3)^2 + (b-7)^2 = 145$ Substitute $b = 12a - 29$ [from 4.4.4] $(a-3)^2 + (12a-29-7)^2 = 145$ $(a-3)^2 + (12a-36)^2 = 145$ $a^2 - 6a + 9 + 144a^2 - 864a + 1296 - 145 = 0$ $145a^2 - 870a + 1160 = 0$ $a = \dfrac{870 \pm \sqrt{(870)^2 - 4(145)(1160)}}{290}$ $a = 2$ or $a = 4$ $\therefore$ $b = 12(2) - 29 = -5$ or $b = 12(4) - 29 = 19$ $\therefore$ K(2 ; –5) **OR/*OF***	✓ substitution into distance formula ✓ substitution of $b = 12a - 29$ ✓standard form ✓ subst into formula or factorise ✓ values of a ✓ value of b (6)

	$\text{TK} = \text{TR}$ $\sqrt{(a-3)^2+(b-7)^2} = \sqrt{145}$ $(a-3)^2+(b-7)^2 = 145$ Substitute $b = 12a - 29$ [from 4.4.4] $(a-3)^2+(12a-29-7)^2 = 145$ $(a-3)^2+(12a-36)^2 = 145$ $(a-3)^2+144(a-3)^2 = 145$ $(a-3)^2 = 1$ $a-3 = \pm 1$ $a = 2$ or 4 $\therefore b = 12(2) - 29$ or $b = 12(4) - 29$ $= -5$ $= 19$ $\therefore$ K(2 ; –5) **OR/*OF*** $KR^2 = TR^2 + TK^2$ $(a-15)^2+(b-6)^2 = 145+145$ $(a-15)^2+(12a-29-6)^2 = 290$ $(a-15)^2+(12a-35)^2 = 290$ $a^2-30a+225+144a^2-840a+1225 = 290$ $145a^2-870a+1160 = 0$ $a^2-6a+8 = 0$ $\therefore (a-2)(a-4) = 0$ $a = 2$ or $a = 4$ $\therefore b = 12(2) - 29$ or $b = 12(4) - 29$ $= -5$ $= 19$ $\text{K}(2\,;-5)$	✓ substitution into distance formula ✓ substitution of $b = 12a - 29$ ✓ $(a-3)^2 = 1$ ✓ ± 1 ✓ values of a ✓ value of b (6) ✓ substitution ✓ substitution of $b = 12a - 29$ ✓standard form ✓ factors ✓ values of a ✓ value of b (6)
		[23]

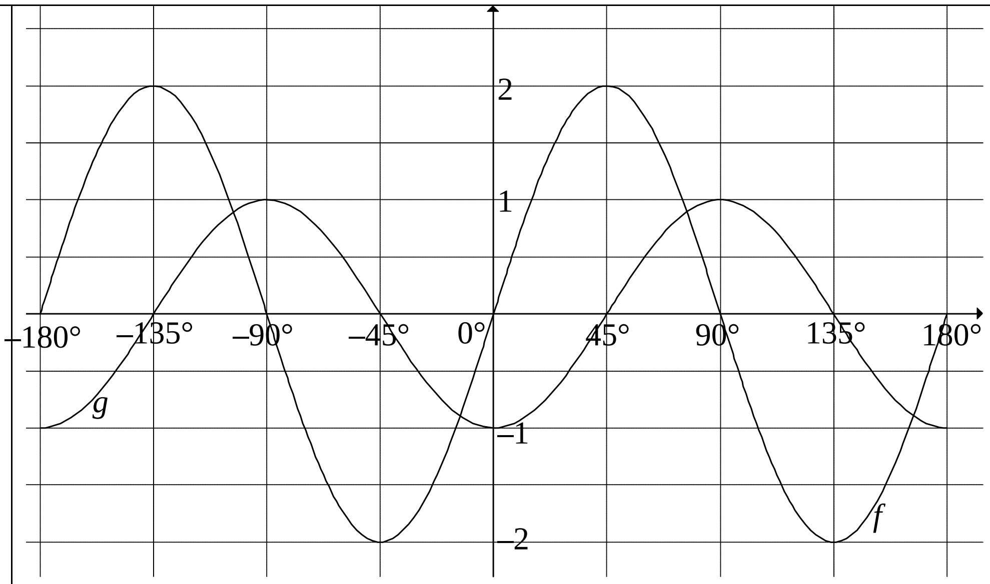
QUESTION/*VRAAG* 5

5.1.1	$\sin 196° = -\sin 16°$ $= -p$	✓reduction ✓answer (2)
5.1.2	$\cos 16° = \sqrt{1-\sin^2 16°}$ $= \sqrt{1-p^2}$ **OR/*OF***  $x^2 + p^2 = 1$ $x = \sqrt{1-p^2}$ $\therefore \cos 16° = \dfrac{\sqrt{1-p^2}}{1} = \sqrt{1-p^2}$	✓statement ✓answer (2) ✓x in terms of p ✓answer (2)
5.2	$\sin(A+B) = \cos[90° - (A+B)]$ $= \cos[(90° - A) - B]$ $= \cos(90° - A)\cos B + \sin(90° - A)\sin B$ $= \sin A \cos B + \cos A \sin B$	✓co-ratio ✓correct form ✓expansion (3)
5.3	$\dfrac{\sqrt{1-\cos^2 2A}}{\cos(-A).\cos(90°+A)}$ $= \dfrac{\sqrt{\sin^2 2A}}{\cos A.(-\sin A)}$ $= \dfrac{\sin 2A}{\cos A.(-\sin A)}$ $= \dfrac{2\sin A \cos A}{\cos A.(-\sin A)}$ $= -2$ **OR/OF** $\dfrac{\sqrt{1-\cos^2 2A}}{\cos(-A)\cos(90°+A)} = \dfrac{\sqrt{1-(2\cos^2 A - 1)^2}}{\cos A. -\sin A}$ $= \dfrac{\sqrt{1-(4\cos^4 A - 4\cos^2 A + 1)}}{\cos A. -\sin A} = \dfrac{\sqrt{4\cos^2 A - 4\cos^4 A}}{\cos A. -\sin A}$ $= \dfrac{\sqrt{4\cos^2 A(1-\cos^2 A)}}{\cos A. -\sin A} = \dfrac{\sqrt{4\cos^2 A \sin^2 A}}{\cos A. -\sin A}$ $= \dfrac{2\cos A.\sin A}{\cos A. -\sin A}$ $= -2$ **OR/OF**	✓ $\sqrt{\sin^2 2A}$ ✓ $\cos A$ ✓ $-\sin A$ ✓ $2\sin A\cos A$ ✓ answer (5) ✓ $2\cos^2 A - 1$ ✓ $\cos A$ ✓ $-\sin A$ ✓identity ✓ answer (5)

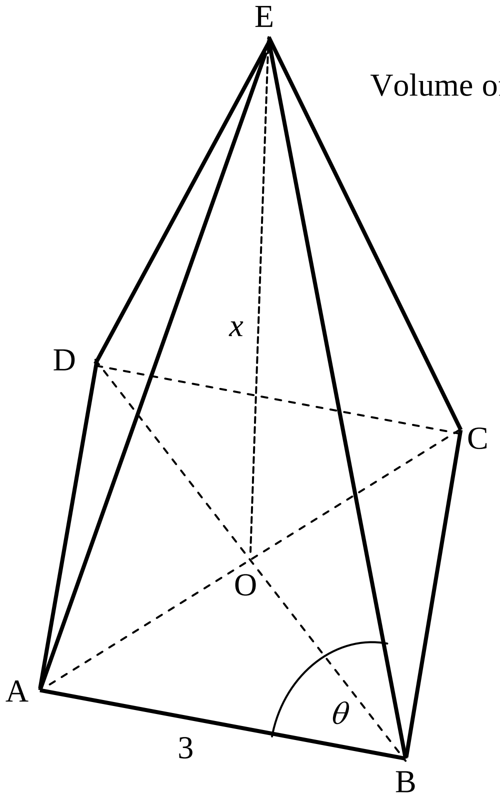
	$\dfrac{\sqrt{1-(1-2\sin^2 A)^2}}{\cos A.-\sin A}$ $=\dfrac{\sqrt{1-(1-4\sin^2 A+4\sin^2 A)}}{\cos A.-\sin A}$ $=\dfrac{\sqrt{4\sin^2 A(1-\sin^2 A)}}{\cos A.-\sin A}$ $=\dfrac{2\sin A\sqrt{\cos^2 A}}{\cos A.-\sin A}$ $=-2$	✓ $1-2\sin^2 A$ ✓ cosA ✓ – sinA ✓identity ✓ answer (5)
5.4.1	$\cos 2B=\dfrac{3}{5}$ $2\cos^2 B-1=\dfrac{3}{5}$ $\cos^2 B=\dfrac{4}{5}$ $\therefore\ \cos B=\sqrt{\dfrac{4}{5}}\ or\ \dfrac{2}{\sqrt{5}}\ or\ \dfrac{2\sqrt{5}}{5}\quad [\,0°\le B\le 90°]$ **OR/*OF*** $\cos B=\dfrac{\sqrt{\cos 2B+1}}{2}$ $=\dfrac{\sqrt{\frac{3}{5}+1}}{2}$ $=\dfrac{2\sqrt{5}}{5}$	✓ identity ✓ value of $\cos^2$B ✓ answer (3) ✓ $=\dfrac{\sqrt{\cos 2B+1}}{2}$ ✓ value of $\cos^2$B ✓ answer (3)
5.4.2	$\sin^2 B=1-\cos^2 B$ $=1-\left(\dfrac{2}{\sqrt{5}}\right)^2$ $=\dfrac{1}{5}\qquad\therefore\ \sin B=\dfrac{1}{\sqrt{5}}\ or\ \dfrac{\sqrt{5}}{5}$ **OR/*OF*** $(2)^2+y^2=(\sqrt{5})^2$ $4+y^2=5$ $y^2=1$ $y=1$ $\therefore\ \sin B=\dfrac{1}{\sqrt{5}}\ or\ \dfrac{\sqrt{5}}{5}$  	✓ $\sin^2 B=\dfrac{1}{5}$ ✓ answer (2) ✓ y = 1 ✓ answer (2)

	OR/*OF* $\cos 2B = \frac{3}{5}$ $1 - 2\sin^2 B = \frac{3}{5}$ $\sin^2 B = \frac{1}{5}$ $\therefore\ \sin B = \frac{1}{\sqrt{5}}\ or\ \frac{\sqrt{5}}{5}$	✓ $\sin^2 B = \frac{1}{5}$ ✓ answer (2)
5.4.3	$\cos(B + 45°) = \cos B.\cos 45° - \sin B.\sin 45°$ $= \left(\frac{2}{\sqrt{5}}\right)\left(\frac{1}{\sqrt{2}}\right) - \left(\frac{1}{\sqrt{5}}\right)\left(\frac{1}{\sqrt{2}}\right)$ $= \frac{2}{\sqrt{10}} - \frac{1}{\sqrt{10}}$ $= \frac{1}{\sqrt{10}}\ or\ \frac{\sqrt{10}}{10}$ **OR/*OF*** $\cos(B + 45°) = \cos B.\cos 45° - \sin B.\sin 45°$ $= \left(\frac{2}{\sqrt{5}}\right)\left(\frac{\sqrt{2}}{2}\right) - \left(\frac{1}{\sqrt{5}}\right)\left(\frac{\sqrt{2}}{2}\right)$ $= \frac{2\sqrt{2}}{2\sqrt{5}} - \frac{\sqrt{2}}{2\sqrt{5}}$ $= \frac{\sqrt{2}}{2\sqrt{5}}\ or\ \frac{\sqrt{10}}{10}$	✓ expansion ✓ $\left(\frac{1}{\sqrt{2}}\right)$ ✓ $\left(\frac{2}{\sqrt{5}}\right)$ & $\left(\frac{1}{\sqrt{5}}\right)$ ✓answer (4) ✓ expansion ✓ $\left(\frac{1}{\sqrt{2}}\right)$ ✓ $\left(\frac{2}{\sqrt{5}}\right)$ & $\left(\frac{1}{\sqrt{5}}\right)$ ✓answer (4)
		[21]

QUESTION/*VRAAG* 6

6.1		✓ *x*- intercepts/ *afsnitte* ✓ *y*- intercept/ *afsnit* ✓ turning pts/ *draaipte* (3)
6.2	$f(x)-3=2\sin 2x-3$ $\therefore$ maximum value = 2 – 3 = –1	✓ ✓ answer (2)
6.3	$2\sin 2x=-\cos 2x$ $\tan 2x=-\frac{1}{2}$ $ref\angle=26{,}57°$ $2x=153{,}43°+k.180°;k\in Z$ $x=76{,}72°+k.90°;k\in Z\ \ or\ \ x=-13.28°+k.90°,k\in Z$ **OR/*OF*** $2\sin 2x=-\cos 2x$ $\tan 2x=-\frac{1}{2}$ $ref\angle=26{,}57°$ $2x=153{,}43°+k.360°\ \ or\ \ 333{,}43°+k.360°,k\in Z$ $x=76{,}72°+k.180°\ \ or\ 166{,}72°+k.180°\ ;k\in Z$	✓ $\tan 2x=-\frac{1}{2}$ ✓ $2x=153{,}43°$ or – 26,56° ✓ 76,72° or – 13,28° ✓ $k.90°;k\in Z$ (4) ✓ $\tan 2x=-\frac{1}{2}$ ✓ $2x=153{,}43°$ & 333,43° ✓ 76,72° & 166,72° ✓ $k.180°;k\in Z$ (4)
6.4	$x\in(-103{,}28°;-13{,}28°)$ **OR/*OF*** $-103{,}28°<x<-13{,}28°$	✓✓ values ✓notation (3) ✓✓ values ✓notation (3) **[12]**

QUESTION/*VRAAG* 7



$$\text{Volume of pyramid} = \frac{1}{3}(\text{area of base}) \times (\perp \text{ height})$$

<table>
<tr><td>7.1</td><td>

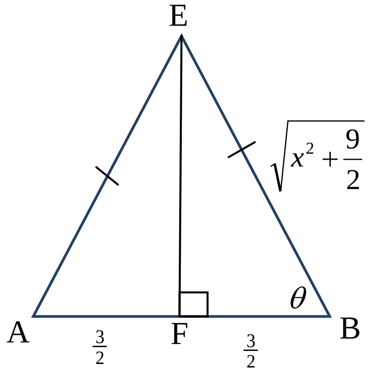
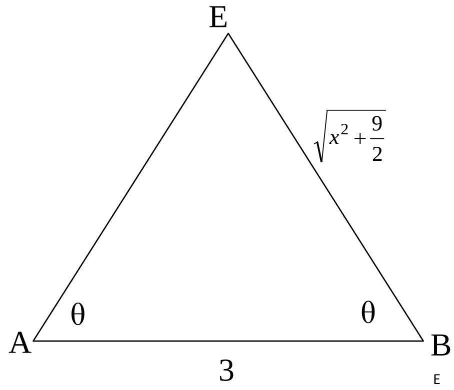
$DB^2 = 3^2 + 3^2$ [Theorem of Pyth]
$= 18$
$DB = \sqrt{18}$
$OB = \frac{1}{2}DB = \frac{\sqrt{18}}{2} \ or \ \frac{3}{\sqrt{2}} \ or \ \frac{3\sqrt{2}}{2} \ or \ 2{,}12$
OR/*OF*
$\sin 45° = \frac{OB}{3}$
$OB = 3\sin 45°$
$OB = \frac{3\sqrt{2}}{2} \ or \ \frac{3}{\sqrt{2}} \ or \ 2{,}12$
OF/OR
$\cos 45° = \frac{OB}{3}$
$\frac{1}{\sqrt{2}} = \frac{OB}{3}$
$OB = \frac{3}{\sqrt{2}} \ or \ \frac{3\sqrt{2}}{2} \ or \ 2{,}12$

</td><td>

✓ substitution into Pyth
✓ value of DB
✓ answer
(3)
✓ correct ratio
✓ OB as subject
✓ answer
(3)
✓ correct ratio
✓ special angle
✓ answer
(3)

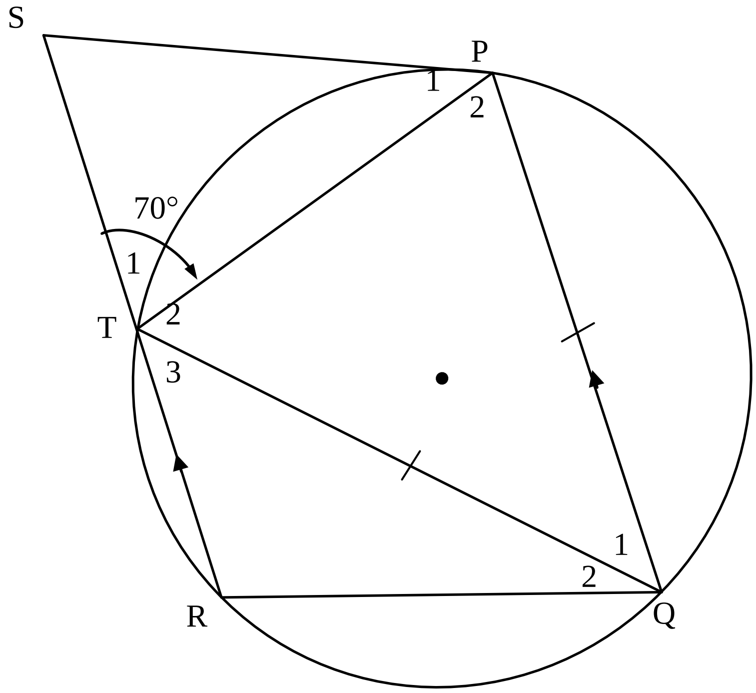
</td></tr>
</table>

	OR/OF $\hat{AOB} = 90°$ (diagonals bisect ⊥) $OB = OA$ $AB^2 = AO^2 + BO^2$ [pyth] $\therefore AB^2 = 2OB^2$ $2OB^2 = 3^2$ $\therefore OB = \frac{3}{\sqrt{2}}$ or $\frac{3\sqrt{2}}{2}$ or 2,12	✓ OB = OA ✓ Pyth ✓ answer (3)
7.2	$BE^2 = EO^2 + OB^2$ (Pyth) $BE^2 = x^2 + \left(\frac{3}{\sqrt{2}}\right)^2$ $BE = \sqrt{x^2 + \frac{9}{2}}$ $AE^2 = AB^2 + EB^2 - 2AB.EB\cos\theta$ $\cos\theta = \frac{AB^2 + EB^2 - AE^2}{2AB.EB} = \frac{AB^2}{2AB.EB}$ [EB = AE] $\cos\theta = \frac{AB}{2EB}$ $\cos\theta = \frac{3}{2\sqrt{x^2 + \frac{9}{2}}}$ **OR/*OF*** $BE^2 = EO^2 + OB^2$ (Pyth) $BE^2 = x^2 + \left(\frac{3}{\sqrt{2}}\right)^2$ $BE = \sqrt{x^2 + \frac{9}{2}}$ $AE^2 = AB^2 + EB^2 - 2AB.EB\cos\theta$ $\left(\sqrt{x^2 + \frac{9}{2}}\right)^2 = 9 + \left(\sqrt{x^2 + \frac{9}{2}}\right)^2 - 2(3)\left(\sqrt{x^2 + \frac{9}{2}}\right).\cos\theta$ $\cos\theta = \frac{9}{6\sqrt{x^2 + \frac{9}{2}}}$ $= \frac{3}{2\sqrt{x^2 + \frac{9}{2}}}$	✓ substitution into Pyth ✓ length of BE ✓correct cosine rule ✓ cos θ as subject ✓ simplification (5) s ✓ substitution into Pyth ✓ length of BE ✓correct cosine rule ✓ substituting ✓ cos θ as subject (5)

	OR/*OF* $\text{BE}^2 = \text{EO}^2 + \text{OB}^2$ (Pyth) $\text{BE}^2 = x^2 + \left(\frac{3}{\sqrt{2}}\right)^2$ $\text{BE} = \sqrt{x^2 + \frac{9}{2}}$ $\cos\theta = \dfrac{\frac{3}{2}}{\sqrt{x^2+\frac{9}{2}}}$ $= \dfrac{3}{2\sqrt{x^2+\frac{9}{2}}}$ 	✓ substitution into Pyth ✓ length of BE ✓ sketch with values ✓ $\frac{3}{2}$ ✓ substitution (5)
	OR/OF $\hat{E} = 180° - 2\theta$ $\sin E = \sin 2\theta$ $\therefore \dfrac{3}{\sin 2\theta} = \dfrac{\sqrt{x^2+\frac{9}{2}}}{\sin\theta}$ $\therefore \dfrac{3}{2\sin\theta\cos\theta} = \dfrac{\sqrt{x^2+\frac{9}{2}}}{\sin\theta}$ $\therefore \dfrac{3}{2\cos\theta} = \sqrt{x^2+\frac{9}{2}}$ $\cos\theta = \dfrac{3}{2\sqrt{x^2+\frac{9}{2}}}$ 	✓ $\hat{E} = 180° - 2\theta$ ✓ $\sin E = \sin 2\theta$ ✓ subst into sine rule ✓ diagram ✓ $2\sin\theta\cos\theta$ (5)
7.3	$\text{Volume} = \frac{1}{3}(\text{area of base}) \times (\perp \text{height})$ $15 = \frac{1}{3}(9) \times x$ $x = 5$ $\cos\theta = \dfrac{3}{2\sqrt{25+\frac{9}{2}}}$ $\therefore\ \theta = 73{,}97°$	✓ substitution ✓ x-value ✓ substitution ✓ answer (4) **[12]**

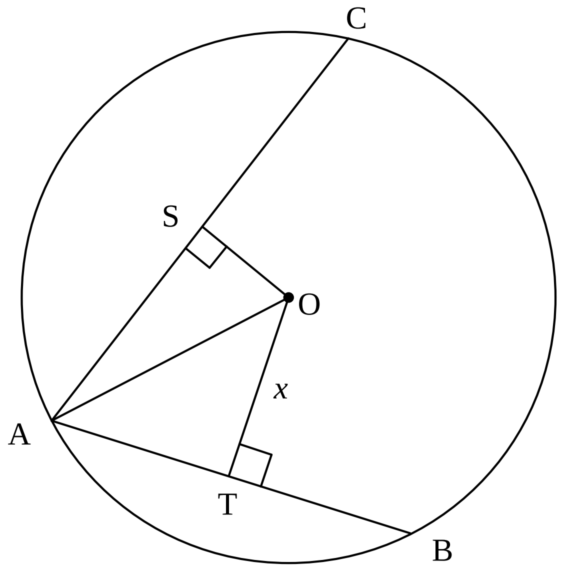
QUESTION/*VRAAG* 8

8.1



8.1.1	Alternate angles / *verwiss hoeke*, PQ ‖ SR	✓ R (1)
8.1.2(a)	$\hat{T}_2 = 70°$ [∠s opp = sides/∠*e teenoor = sye*] $\therefore \hat{Q}_1 = 180° - 2(70°)$ [∠s/*e* Δ = 180°] $= 40°$	✓ S ✓R ✓ answer (3)
8.1.2(b)	$\hat{P}_1 = 40°$ [tangent chord th/*raakl-koordst*]	✓ S ✓R (2)

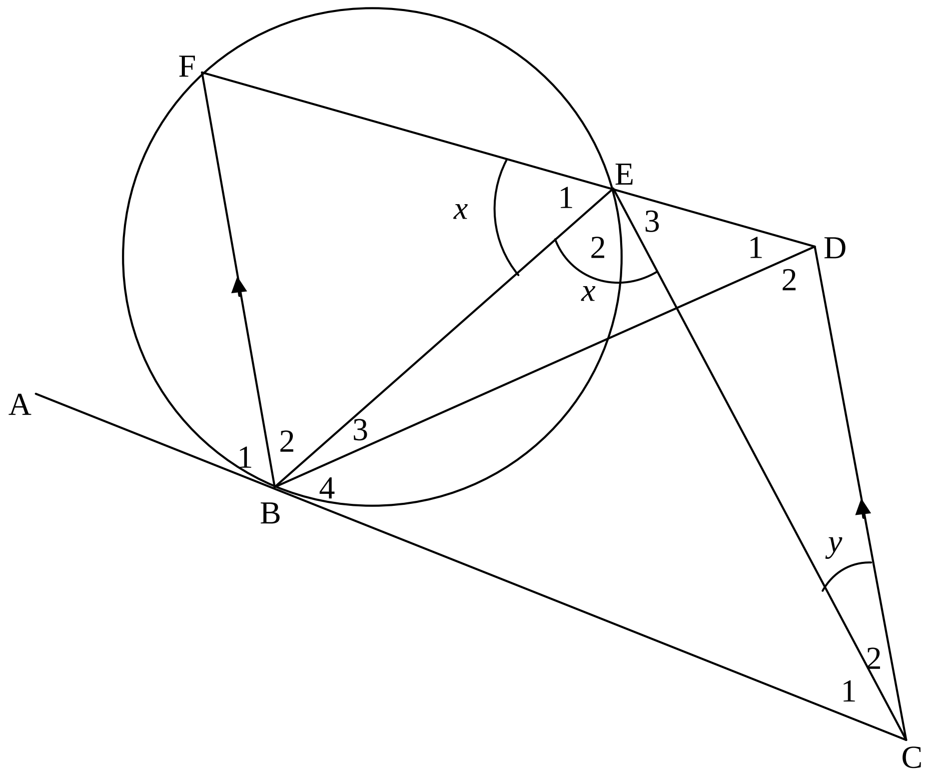
8.2



8.2.1	AT = 20 [line from centre ⊥ to chord/*lyn vanaf midpt ⊥ koord*]	✓S (1)
8.2.2	$AO^2 = OS^2 + AS^2$ [Pyth : ΔAOS] $OT^2 + AT^2 = OS^2 + AS^2$ [Pyth : ΔAOT] But AS = 24 [line from centre ⊥ to chord/*lyn vanaf midpt ⊥ koord*] $OT^2 + 400 = \left(\frac{7}{15}OT\right)^2 + 576$ $176 = \frac{176}{225}OT^2$ $OT^2 = 225$ $OT = 15$ $\therefore AO = \sqrt{225+400}$ $= 25$ **OR/OF** Let OS = 7, then OT = 15 In ΔAOT: $AO^2 = 20^2 + 15^2$ $= 625$ $AO = 25$ In ΔAOS: $AO^2 = 24^2 + 7^2$ $= 625$ $AO = 25$ $\therefore$ OA = 25 **OR/OF**	✓ equating ✓ AS = 24 ✓ substitution $OS = \frac{7}{15}OT$ ✓ OT ✓ radius (5) ✓✓ testing in ΔAOT ✓✓ testing in ΔAOS ✓conclusion (5)

	$AO^2 = OS^2 + AS^2$ [Pyth : ΔAOS] $OT^2 + AT^2 = OS^2 + AS^2$ [Pyth : ΔAOT] Let OT = 15x. Then OS = 7x But AS = 24 [line from centre $\perp$ to chord/*lyn vanaf midpt* $\perp$ *koord*] $(15x)^2 + 400 = (7x)^2 + 576$ $225x^2 + 400 = 49x^2 + 576$ $176x^2 = 176$ $x = 1$ $\therefore$ AO = $\sqrt{225+400}$ = 25 **OR/OF** AS = 24 [line from centre $\perp$ to chord/*lyn vanaf midpt* $\perp$ *koord*] $AO^2 = OS^2 + AS^2$ [Pyth : ΔAOS] $= \left(\frac{7}{15}OT\right)^2 + AS^2$ $AO^2 = \frac{49}{225}(AO^2 - 20^2) + 24^2$ [Pyth : ΔAOT] $\frac{176}{225}AO^2 = \frac{4400}{9}$ $AO^2 = 625$ $AO = 25$	✓ equating ✓ AS = 24 ✓ substitution ✓ $x = 1$ ✓ radius (5) ✓ AS = 24 ✓ substitution OS = $\frac{7}{15}$OT ✓ equating ✓ subst Pyth ✓ radius (5) **[12]**

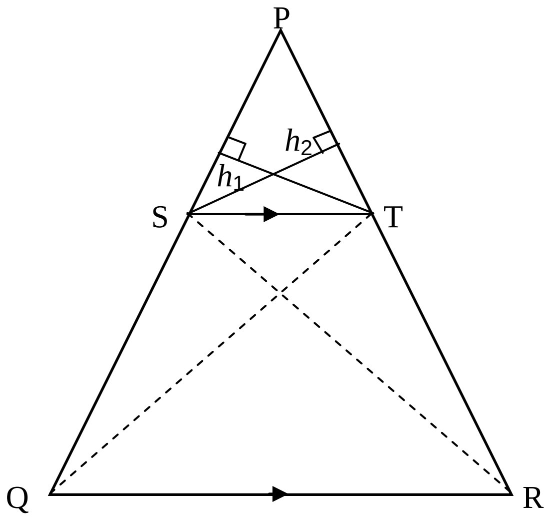
QUESTION/*VRAAG* 9



9.1.1	tangent chord theorem/*raaklyn-koordstelling*	✓ R (1)
9.1.2	corresponding/*ooreenkomstige* ∠s/*e*; FB || DC	✓ R (1)
9.2	$\hat{E}_1 = B\hat{C}D$ $\therefore$ BCDE = cyclic quad [converse ext ∠ cyc quad/*omgek: buite∠kdvh*]	✓ S ✓ R (2)
9.3	$\hat{D}_2 = \hat{E}_2$ [∠*s* in the same segment/∠*e in dies segment*] $\hat{D}_2 = F\hat{B}D$ [alt ∠*s*, BF||CD/*verwiss* ∠*e*,*BF*||*CD*]	✓ S ✓ S (2)
9.4	$\hat{B}_3 = y$ *OR* $\hat{B}_3 = \hat{C}_2$ [∠s in the same segment/∠*e in dies segment*] $\hat{B}_2 = x - y$ *OR* $\hat{B}_3 + \hat{B}_2 = x$ [from 9.3 and 9.4] $\hat{C}_1 = x - y$ [from 9.2 and 9.3] $\therefore \hat{B}_2 = \hat{C}_1$ **OR/*OF*** In ΔBFE and ΔBEC $\hat{E}_1 = \hat{E}_2$ [= *x*] $\hat{F} = \hat{B}_3 + \hat{B}_4$ [tan - chord theorem] $\therefore \Delta BFE /// \Delta CBE$ [∠, ∠, ∠] $\therefore \hat{B}_2 = \hat{C}_1$	✓ S ✓ S ✓ S (3) ✓ identifying Δ's ✓ S ✓ S (3) **[9]**

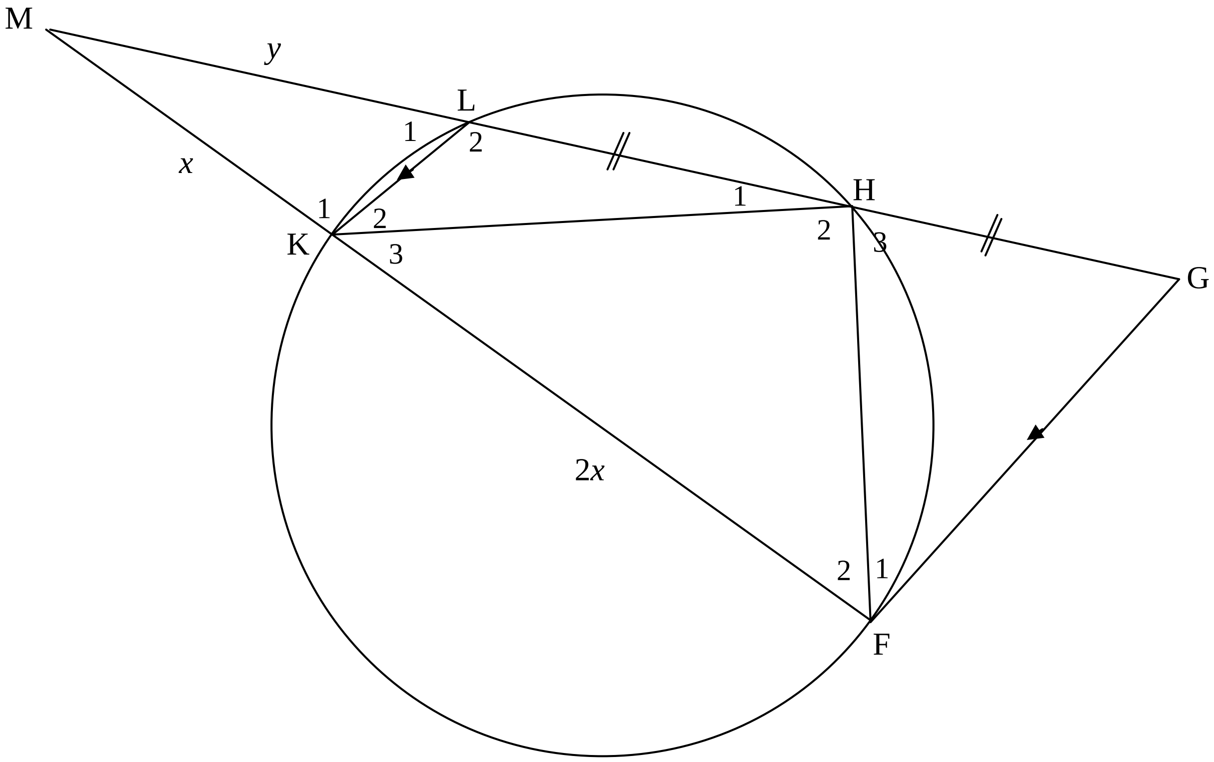
QUESTION/*VRAAG* 10

10.1



10.1	$\boxed{\text{Constr : Join S to R and T to Q and draw } h_1 \text{ from S} \perp \text{PT and } h_2 \text{ from T} \perp \text{PS/ } \textit{Verbind SR en TQ en trek } h_1 \textit{ van S } \perp \textit{ PT en } h_2 \textit{ van T } \perp \textit{ PS}]}$ Proof : $\frac{\text{area }\Delta\text{PST}}{\text{area }\Delta\text{QST}} = \frac{\frac{1}{2}\text{PS}\times h_2}{\frac{1}{2}\text{SQ}\times h_2} = \frac{\text{PS}}{\text{SQ}}$ equal altitudes $\frac{\text{area }\Delta\text{PST}}{\text{area }\Delta\text{STR}} = \frac{\frac{1}{2}\text{PT}\times h_1}{\frac{1}{2}\text{TR}\times h_1} = \frac{\text{PT}}{\text{TR}}$ equal altitudes area ΔPST = area ΔPST [common] But area ΔQST = area ΔSTR [same base, height; ST $\parallel$ QR] $\therefore \frac{\text{area }\Delta\text{PST}}{\text{area }\Delta\text{QST}} = \frac{\text{area }\Delta\text{PST}}{\text{area }\Delta\text{STR}}$ $\therefore \frac{\text{PS}}{\text{SQ}} = \frac{\text{PT}}{\text{TR}}$	✓ constr/*konstruksie* ✓ $\frac{\text{area }\Delta\text{PST}}{\text{area }\Delta\text{QST}} = \frac{\frac{1}{2}\text{PS}\times h_2}{\frac{1}{2}\text{SQ}\times h_2}$ ✓ $\frac{\text{area }\Delta\text{PST}}{\text{area }\Delta\text{STR}} = \frac{\text{PT}}{\text{TR}}$ ✓ S ✓ R ✓ S (6)

10.2



10.2.1	Corresponding/*Ooreenkomstige* ∠s/*e*; GF || LK	✓ R (1)
10.2.2(a)	$\frac{\text{GL}}{\text{LM}} = \frac{\text{FK}}{\text{KM}}$ OR $\frac{\text{GL}}{y} = \frac{2x}{x}$ [prop theorem/*eweredighst*; GF || LK] $\frac{2\text{GH}}{y} = \frac{2x}{x}$ [LH = HG] $\therefore \text{GH} = y$	✓S ✓ R ✓ GL = 2GH (3)

10.2.2(b)	$\overleftarrow{\text{K}}_1 = \text{G}\hat{\text{F}}\text{M}$ [corresponding/*ooreenkomst* $\angle$ s; GF ∥ LK] $\text{L}\hat{\text{K}}\text{M}$ or $\overleftarrow{\text{K}}_1 = \text{M}\hat{\text{H}}\text{F}$ [ext $\angle$ cyclic quad/*buite*$\angle$*koordevh*] $\text{M}\hat{\text{H}}\text{F} = \text{G}\hat{\text{F}}\text{M}$ In ΔMFH and ΔMGF: $\hat{\text{M}} = \hat{\text{M}}$ [common/*gemeen*] $\text{M}\hat{\text{H}}\text{F} = \text{G}\hat{\text{F}}\text{M}$ [proven/*bewys*] $\therefore$ ΔMFH ||| ΔMGF [∠∠∠] **OR/*OR*** $\overleftarrow{\text{K}}_1 = \text{G}\hat{\text{F}}\text{M}$ [corresponding/*ooreenkomst* $\angle$ s; GF ∥ LK] $\text{L}\hat{\text{K}}\text{M}$ or $\overleftarrow{\text{K}}_1 = \text{M}\hat{\text{H}}\text{F}$ [ext $\angle$ cyclic quad/*buite*$\angle$*koordevh*] $\text{M}\hat{\text{H}}\text{F} = \text{G}\hat{\text{F}}\text{M}$ In ΔMFH and ΔMGF: $\hat{\text{M}} = \hat{\text{M}}$ [common/*gemeen*] $\text{M}\hat{\text{H}}\text{F} = \text{G}\hat{\text{F}}\text{M}$ [proven/*bewys*] $\hat{\text{F}}_2 = \hat{\text{G}}$ [∠s of Δ = 180°] $\therefore$ ΔMFH ||| ΔMGF	✓S ✓ R ✓ S ✓ S ✓ R (5) ✓S ✓ R ✓ S ✓ S ✓ S (5)
10.2.2(c)	$\therefore \frac{\text{GF}}{\text{FH}} = \frac{\text{MF}}{\text{MH}}$ [||| Δs] $= \frac{3x}{2y}$	✓S ✓R (2)
10.2.3	$\frac{\text{MF}}{\text{MH}} = \frac{\text{MG}}{\text{MF}}$ [||| Δs] $\frac{3x}{2y} = \frac{3y}{3x}$ [from 10.2.2(*c*)] $\frac{y^2}{x^2} = \frac{9}{6} = \frac{3}{2}$ $\frac{y}{x} = \sqrt{\frac{3}{2}}$	✓ S ✓substitution ✓ simplification (3) **[20]**
	TOTAL MARKS	**150**

basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

SENIOR CERTIFICATE EXAMINATIONS/
SENIORSERTIFIKAAT-EKSAMEN

MATHEMATICS P2/*WISKUNDE V2*

JUNE 2016

MEMORANDUM

MARKS/*PUNTE*: 150

This memorandum consists of 21 pages./
Hierdie memorandum bestaan uit 21 bladsye.

NOTE:

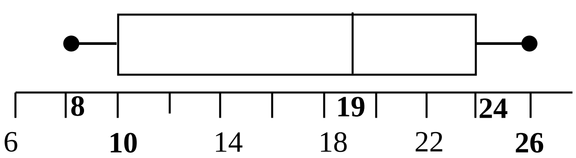
- If a candidate answered a question TWICE, mark only the FIRST attempt.
- If a candidate has crossed out an attempt to answer a question and did not redo it, mark the crossed-out version.
- Consistent accuracy applies in ALL aspects of the marking memorandum. Stop marking at the second calculation error.
- Assuming answers/values in order to solve a problem is NOT acceptable.

LET WEL:

- *Indien 'n kandidaat 'n vraag TWEE keer beantwoord het, sien slegs die EERSTE poging na.*
- *As 'n kandidaat 'n poging om 'n vraag te beantwoord, doodgetrek en nie oorgedoen het nie, sien die doodgetrekte poging na.*
- *Volgehoue akkuraatheid is op ALLE aspekte van die memorandum van toepassing. Staak nasien by die tweede berekeningsfout.*
- *Om antwoorde/waardes om 'n probleem op te los, te veronderstel, word NIE toegelaat NIE.*

QUESTION/*VRAAG* 1

8	8	10	12	16	19	20	21	24	25	26

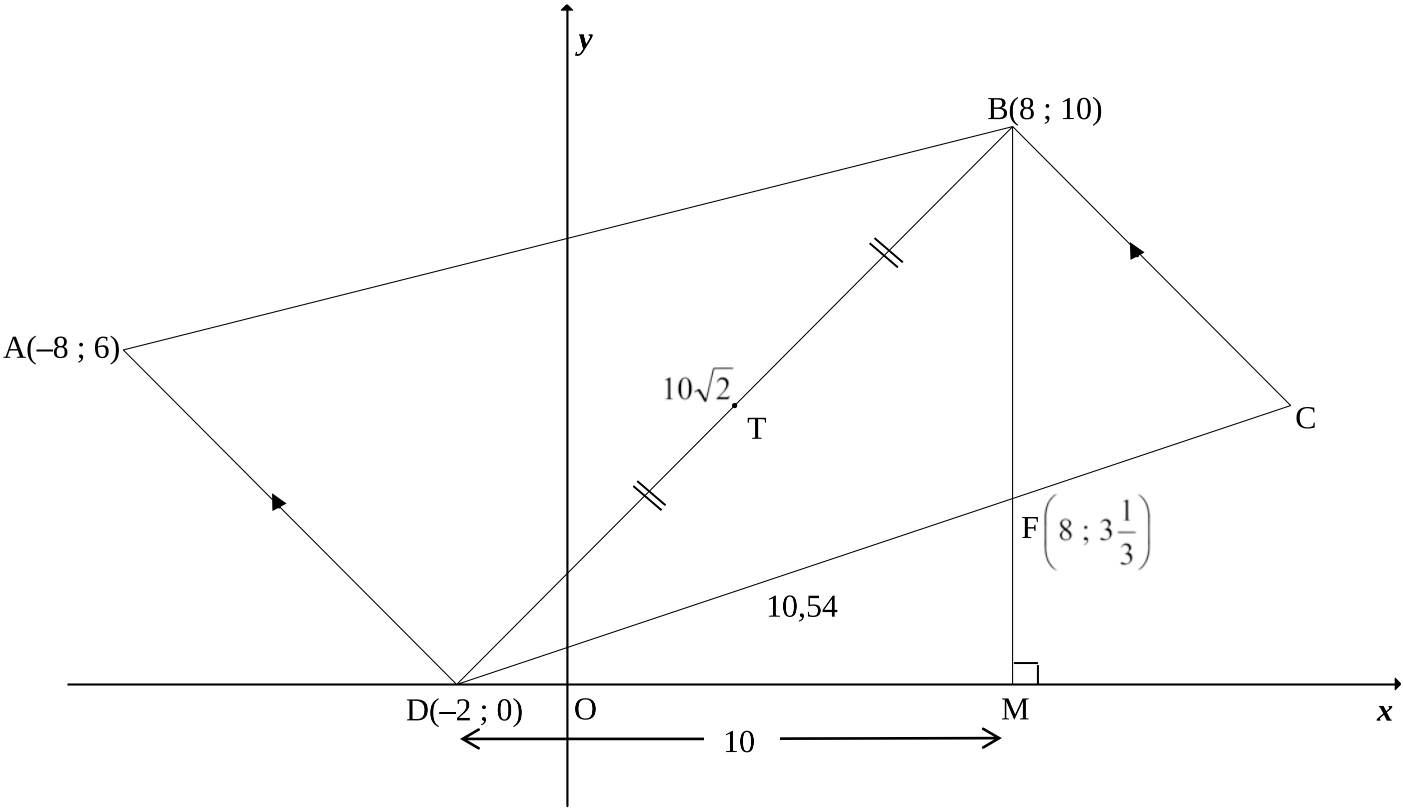
1.1	Mean/*Gemiddelde* $= \frac{189}{11}$ $= 17{,}18$ Answer only: Full marks *Slegs antwoord: Volpunte*	✓189 ✓ answer (2)
1.2	Min = 8, max = 26 Median/*Mediaan* = 19 $Q_1 = 10$, $Q_3 = 24$ $\therefore$ (8 ; 10 ; 19 ; 24 ; 26)	✓ min, max ✓ median ✓ Q_1 & Q_3 (3)
1.3		✓ box/*boks/mond* ✓ whiskers/*snor* (2)
1.4	The data is skewed to the left/*Die data is skeef na links.* **OR/*OF*** Negatively skewed/*Negatief skeef*	✓ answer (1) ✓ answer (1)
1.5	SD/*SA* = 6,46	✓✓ answer (2)
1.6	17,18 + 6,46 = 23,64 $\therefore$ 3 destinations/*bestemmings*	✓ interval ✓ answer (2) **[12]**

QUESTION/*VRAAG* 2

Temperature at midday (in °C) ***Middaguur-temperatuur (in °C)***	18	21	19	26	32	35	36	40	38	30	25
Number of bottles of water (500 mℓ) ***Getal bottels water (500 mℓ)***	12	15	13	31	46	51	57	70	63	53	23

2.1	(30 ; 53)	✓answer (1)
2.2	$a = -38{,}51$ $b = 2{,}68$ $\therefore \hat{y} = 2{,}68x - 38{,}51$	✓ value *a* ✓ value *b* ✓ equation (3)
2.3	$\therefore \hat{y} \approx 36{,}53$ bottles **OR/*OF*** $\hat{y} \approx 2{,}68(28) - 38{,}51$ $\approx 36{,}53$ bottles	✓✓ answer (2) ✓ substitution ✓ answer (2)
2.4	Strong/*Sterk* The majority of the points lie **close to** the regression line./*Die meerderheid punte lê naby die regressielyn.* **OR/*OF*** Strong/*Sterk* $r = 0{,}98$	✓ strong/*sterk* ✓ reason/*rede* (2) ✓ strong/*sterk* ✓ reason/*rede* (2)
2.5	Temperature cannot rise beyond a certain point as this would be life threatening **OR** there is only so much water one can consume before it becomes a risk to your health (hyponatremia)./*Temperatuur kan nie hoër as 'n sekere punt styg nie, anders raak dit lewensgevaarlik.* ***OF*** *'n persoon kan net 'n sekere hoeveelheid water inneem, anders raak dit 'n gesondheidsrisiko*	✓ reason/*rede* (1) **[9]**

QUESTION/*VRAAG* 3



3.1	$m_{AD}=\dfrac{y_2-y_1}{x_2-x_1}$ $=\dfrac{0-6}{-2+8}$ $=\dfrac{-6}{6}=-1$	✓substitution ✓–1 (2)
3.2	$m_{BC}=-1$ [BC||AD] $y=-x+c$ $10=-8+c$ $c=18$ $y=-x+18$ **OR/*OF*** $m_{BC}=-1$ [BC||AD] $y-y_1=m(x-x_1)$ $y-10=-(x-8)$ $y=-x+18$	✓ gradient ✓substitute *m* and (8 ; 10) ✓ equation (3) ✓ gradient ✓substitute *m* and (8 ; 10) ✓ equation (3)

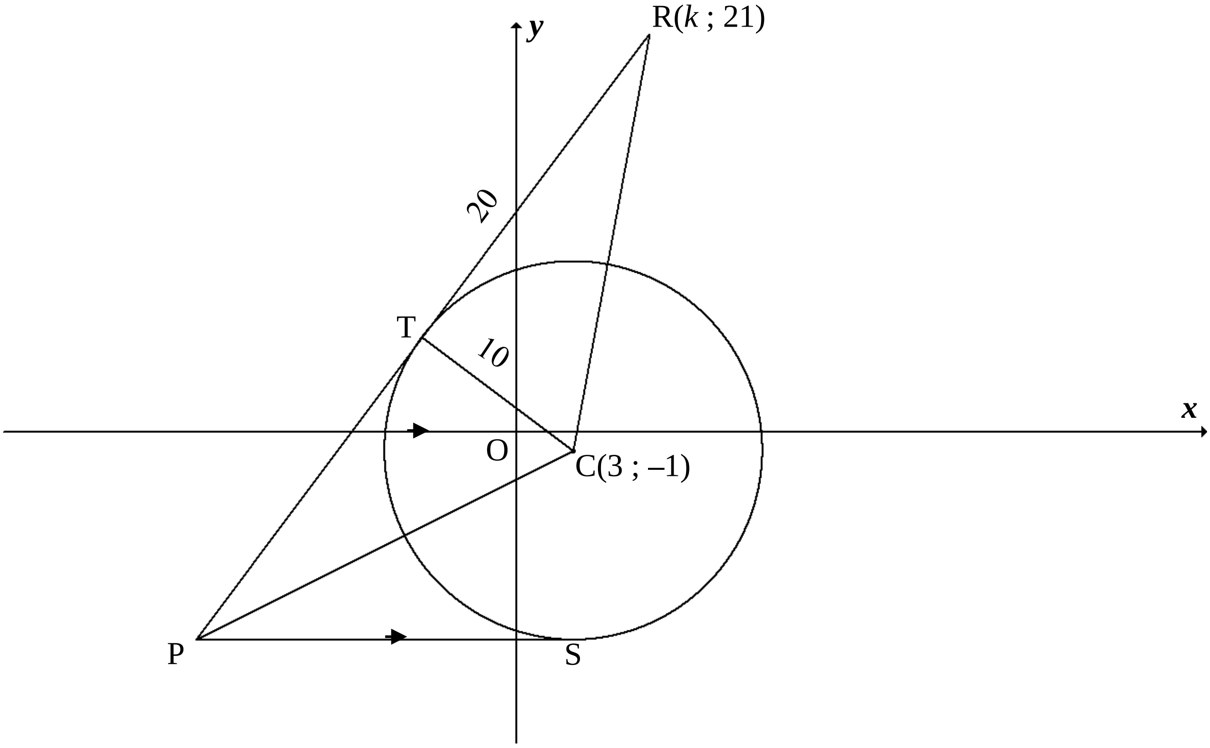
3.3	$m_{\text{BD}} = \dfrac{y_2 - y_1}{x_2 - x_1}$ $= \dfrac{10-0}{8+2} = 1$ $m_{\text{BD}} \times m_{\text{AD}} = 1 \times -1 = -1$ $\therefore \text{DB} \perp \text{AD}$	✓substitution ✓ answer ✓ $m_{BD} \times m_{AD} = -1$ (3)
	OR $\text{AD}^2 = 72$ *or* $\text{AD} = \sqrt{72}$ *or* $6\sqrt{2}$ $\text{AB}^2 = 272$ *or* $\text{AB} = \sqrt{272}$ *or* $4\sqrt{17}$ $\text{BD}^2 = 200$ *or* $\text{BD} = \sqrt{200}$ *or* $10\sqrt{2}$ $\therefore \text{AB}^2 = \text{AD}^2 + \text{BD}^2$ $\therefore \text{A}\hat{\text{D}}\text{B} = 90°$ [converse Pyth th/ *omgekeerde Pyth st*]	✓✓ calculating all 3 sides ✓ $\text{AB}^2 = \text{AD}^2 + \text{BD}^2$ (3)
3.4	$\tan \text{B}\hat{\text{D}}\text{M} = m_{\text{BD}} = 1$ $\therefore \text{B}\hat{\text{D}}\text{M} = 45°$	✓ $\tan \text{B}\hat{\text{D}}\text{M} = m_{\text{BD}}$ ✓ answer (2)
	OR $\sin \text{B}\hat{\text{D}}\text{M} = \dfrac{\text{BM}}{\text{BD}} = \dfrac{10}{10\sqrt{2}} = \dfrac{1}{\sqrt{2}}$ $\therefore \text{B}\hat{\text{D}}\text{M} = 45°$	✓ $\sin \text{B}\hat{\text{D}}\text{M} = \dfrac{1}{\sqrt{2}}$ ✓ answer (2)
3.5	$\text{T}(x\,;y) = \left(\dfrac{x_1+x_2}{2}; \dfrac{y_1+y_2}{2}\right)$ $= \left(\dfrac{-2+8}{2}; \dfrac{0+10}{2}\right)$ $= (3\ ;\ 5)$ T symmetrical about BM/*T is simmetries om BM* $\therefore$ distance of T to BM = 5 units = distance from BM to C $\therefore$ C(13 ; 5) **OR/*OF***	✓T(3 ; 5) ✓value of *x* ✓ value of *y* (3)

	$m_{DF} = \dfrac{3\frac{1}{3} - 0}{8-(-2)} = \dfrac{1}{3}$ Equation of DF: $y - y_1 = m(x - x_1)$ $y - 0 = \dfrac{1}{3}(x+2)$ $y = \dfrac{1}{3}x + \dfrac{2}{3}$ Equation of BC: $y = -x + 18$ $\dfrac{1}{3}x + \dfrac{2}{3} = -x + 18$ $4x = 52$ $x = 13$ $\therefore$ $y = -13 + 18 = 5$ $\therefore$ C(13 ; 5)	✓eq of DF ✓value of x ✓ value of y (3)
3.6	area/*opp* ΔBDF $=$ area/*opp* ΔBDM $-$ area/*opp* ΔDFM $= \dfrac{1}{2}(10)(10) - \dfrac{1}{2}(10)\left(\dfrac{10}{3}\right)$ $= \dfrac{100}{3}$ or $33\dfrac{1}{3}$ or 33,3 square units/*vk eenh* **OR/*OF*** area/*opp* ΔBDF $= \dfrac{1}{2}$.BF.DM $= \dfrac{1}{2}\left(\dfrac{20}{3}\right)(10)$ $= \dfrac{100}{3}$ or $33\dfrac{1}{3}$ or 33,3 square units/*vk eenh* **OR/*OF***	✓ formula/method ✓ 10 (DM) ✓ 10 (BM) ✓ $\dfrac{10}{3}$ or $3\dfrac{1}{3}$ ($\perp$h) ✓ answer (5) ✓ formula/method ✓ BF ✓✓ DM ✓ answer (5)

	$\tan F\hat{D}M = m_{DC} = \frac{5-0}{13+2} = \frac{1}{3}$ or $\tan F\hat{D}M = \frac{FM}{DM} = \frac{\frac{10}{3}}{10} = \frac{1}{3}$	✓ gradient/ratio
	$F\hat{D}M = 18{,}43°$ $\therefore B\hat{F}D = 108{,}43°$ [ext ∠ Δ]	✓ $B\hat{F}D$
	$BF = \frac{20}{3}$ or $6\frac{2}{3}$ $DF^2 = (10)^2 + \left(3\frac{1}{3}\right)^2$ [Pyth ΔDFM]	✓ DF
	$DF = 10{,}54$ or $\frac{\sqrt{1000}}{3}$ or $\frac{10\sqrt{10}}{3}$ $\boxed{\begin{aligned} BD &= \sqrt{(10-0)^2+(8+2)^2} \\ &= \sqrt{200} \text{ or } 10\sqrt{2}\end{aligned}}$ $\therefore$ area/*opp* $\Delta BDF = \frac{1}{2}.BF.FD.\sin B\hat{F}D$ $= \frac{1}{2}\left(\frac{20}{3}\right)\left(\frac{10\sqrt{10}}{3}\right)(\sin 108{,}43)$ $= \frac{100}{3}$ or $33\frac{1}{3}$ or 33,33 square units/*vk eenh*	✓correct substitution into area rule ✓ answer (5)
	OR/*OF* $BF = \frac{20}{3}$ or $6\frac{2}{3}$	✓ BF
	$BD = \sqrt{(10-0)^2+(8+2)^2}$ $= \sqrt{200}$ *or* $10\sqrt{2}$ area/*opp* $\Delta BDF = \frac{1}{2}.BF.BD.\sin D\hat{B}F$	✓ BD ✓ formula/method
	$= \frac{1}{2}\left(\frac{20}{3}\right)\left\langle\sqrt{200}\right\rangle(\sin 45°)$ $= \frac{100}{3}$ or $33\frac{1}{3}$ or 33,33 square units/*vk eenh*	✓correct substitution into area rule ✓ answer (5)
	OR/*OF* area/*opp* ΔBDF = area/*opp* ΔBCD – area/*opp* ΔBCF $= \frac{1}{2}\left(10\sqrt{2}\right)\left(5\sqrt{2}\right) - \frac{1}{2}\left(\frac{20}{3}\right)(5)$ $= \frac{100}{3}$ or $33\frac{1}{3}$ or 33,33 square units/*vk eenh*	✓ formula/method ✓ BD = $10\sqrt{2}$ ✓ BC = $5\sqrt{2}$ ✓ BF = $\frac{20}{3}$ ✓ answer (5)
	OR/*OF*	

	$\tan \hat{FDM} = m_{DC} = \frac{5-0}{13+2} = \frac{1}{3}$ or $\tan \hat{FDM} = \frac{FM}{DM} = \frac{\frac{10}{3}}{10} = \frac{1}{3}$ $\hat{FDM} = 18,43°$ $\hat{BDF} = 26,56°$ area / *opp* ΔBDF $= \frac{1}{2}.BD.DF.\sin \hat{BDF}$ $= \frac{1}{2}.(10\sqrt{2}).\left(\frac{10\sqrt{10}}{3}\right).\sin 26,56°$ $= \frac{100}{3}$ or $33\frac{1}{3}$ or $33,33$ square units/*vk eenh*	✓ gradient/ratio ✓ $\hat{BDF}$ ✓ DF **OR/*OF*** BD ✓correct substitution into area rule ✓ answer (5) **[18]**

QUESTION/*VRAAG* 4



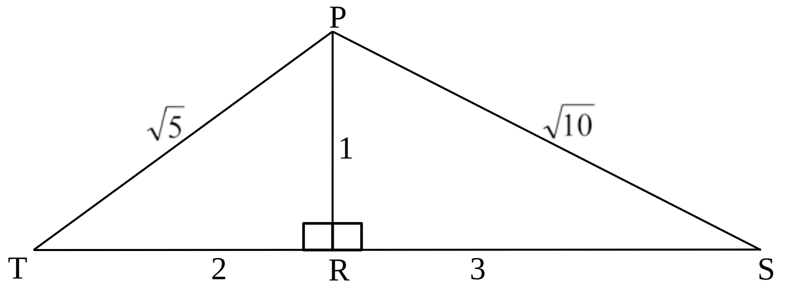
4.1	radius $\perp$ tangent /*raaklyn*	✓ R (1)
4.2	$CR^2 = TR^2 + CT^2$ (Pyth) $CR^2 = 20^2 + 10^2 = 500$ $CR = \sqrt{500}$ or $10\sqrt{5}$	✓ substitution ✓ answer (2)
4.3	$CR^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$ $500 = (k-3)^2 + (21+1)^2$ $k^2 - 6k + 9 + 484 = 500$ $k^2 - 6k - 7 = 0$ $(k-7)(k+1) = 0$ $k = 7$ or $k \neq -1$ **OR/*OF*** $CR^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$ $500 = (k-3)^2 + (21+1)^2$ $(k-3)^2 = 16$ $k - 3 = 4$ or $k - 3 = -4$ $k = 7$ or $k \neq -1$	✓ substitution ✓ standard form ✓ factors ✓ $k = 7$ (4) ✓ substitution ✓ square form ✓ square root ✓ $k = 7$ (4)

4.4	$(x-3)^2+(y+1)^2=100$	✓✓ answer (2)
4.5	CS = 10 and CS $\perp$ PS $\therefore S(3;-11)$ $\therefore y=-11$	✓ $S(3;-11)$ ✓ answer (2)
4.6.1	$S(3;-11)$ $\therefore 3(-11)-4x=35$ $x=-17$ $\therefore P(-17;-11)$ **OR/*OF*** $\frac{4}{3}x+\frac{35}{3}=-11$ $\frac{4}{3}x=\frac{-68}{3}$ $x=-17$ $P(-17;-11)$	✓ substituting ✓ answer (2) ✓ equating ✓ answer (2)
4.6.2	PT = PS [tangents from common point/*rklyne vanaf dies pt*] = 17 + 3 = 20 units **OR** $PC=\sqrt{(-17-3)^2+(-11+1)^2}$ $=\sqrt{500}$ *or* $10\sqrt{5}$ $PT^2=PC^2-TC^2$ [Pyth th] = 500 – 100 = 400 $\therefore$ PT = 20 **OR** $PC=\sqrt{(-17-3)^2+(-11+1)^2}$ $=\sqrt{500}$ *or* $10\sqrt{5}$ $\Delta PTC\equiv\Delta RTC$ [90°HS] $\therefore$ PT = TR $\therefore$ PT = 20	✓ S ✓R ✓ answer (3) ✓ value of PC ✓ using Pyth ✓ answer (3) ✓ value of PC ✓ S/R or proved ✓ answer (3)
4.7.1	$M(3;-16)$	✓answer (1)

4.7.2	Radius = 4	✓ answer (1)
4.7.3	$r_1 + r_2 = 10 + 4 = 14$ distance $\text{CM} = \sqrt{(3-3)^2 + (-1+16)^2}$ $= \sqrt{225}$ $= 15$ CM > $r_1 + r_2$ Therefore the two circles do not intersect or touch./*Daarom sny of raak die twee sirkels nie.*	✓ $r_1 + r_2$ ✓ 15 ✓explanation (3) **[21]**

QUESTION/*VRAAG* 5

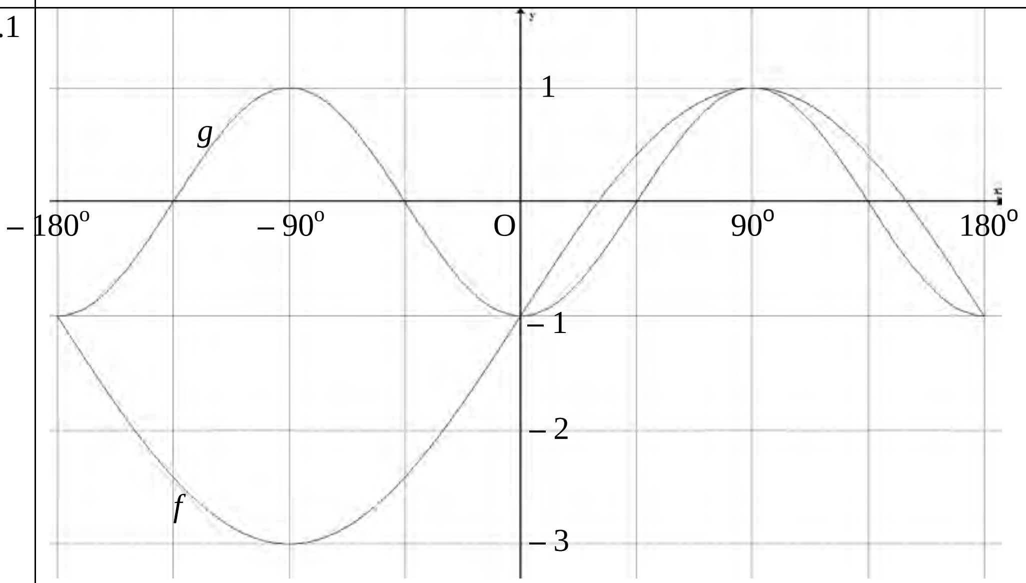
5.1



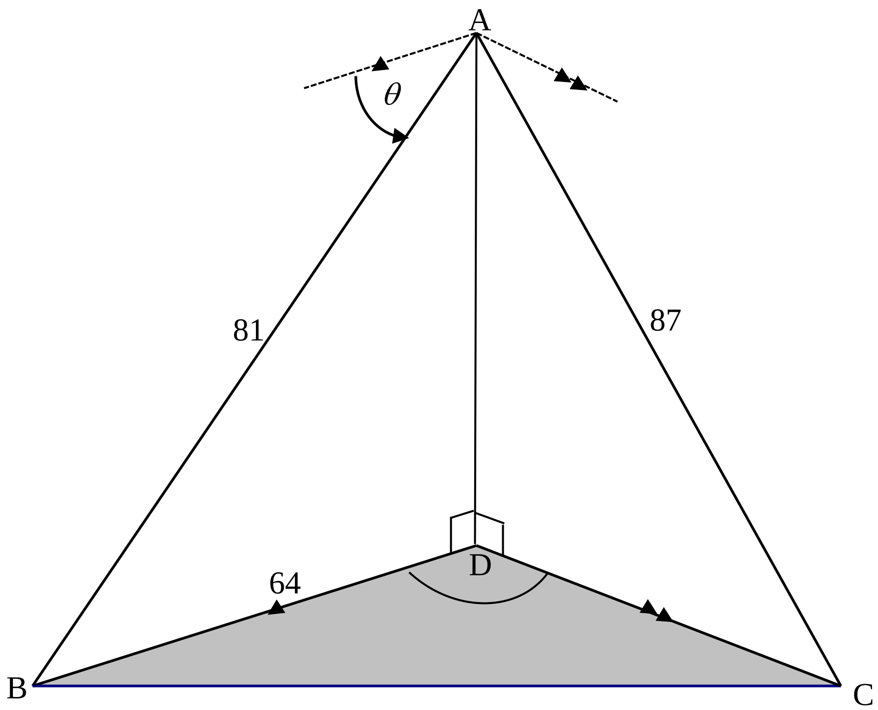
5.1.1(a)	$\sin T = \frac{1}{\sqrt{5}} = \frac{\sqrt{5}}{5} = 0{,}45$	✓value (1)
5.1.1(b)	$\cos S = \frac{3}{\sqrt{10}} = \frac{3\sqrt{10}}{10} = 0{,}95$	✓value (1)
5.1.2	$\cos(T+S) = \cos T \cos S - \sin T \sin S$ $= \left(\frac{2}{\sqrt{5}}\right)\left(\frac{3}{\sqrt{10}}\right) - \left(\frac{1}{\sqrt{5}}\right)\left(\frac{1}{\sqrt{10}}\right)$ $= \frac{6}{\sqrt{50}} - \frac{1}{\sqrt{50}}$ $= \frac{5}{\sqrt{50}}$ *or* $\frac{1}{\sqrt{2}}$ *or* $\frac{\sqrt{2}}{2}$	✓expansion ✓ $\frac{2}{\sqrt{5}}$ ✓ $\frac{1}{\sqrt{10}}$ ✓ simplification ✓ answer (5)
5.2	$\frac{1}{\cos(360° - \theta)\sin(90° - \theta)} - \tan^2(180° + \theta)$ $= \frac{1}{(\cos\theta)(\cos\theta)} - \tan^2\theta$ $= \frac{1}{\cos^2\theta} - \left(\frac{\sin^2\theta}{\cos^2\theta}\right)$ $= \frac{1-\sin^2\theta}{\cos^2\theta}$ $= \frac{\cos^2\theta}{\cos^2\theta}$ *OR* $\frac{1-\sin^2\theta}{1-\sin^2\theta}$ $= 1$	✓ $\cos\theta$ ✓ $\cos\theta$ ✓ $\tan^2\theta$ ✓ $\frac{\sin^2\theta}{\cos^2\theta}$ ✓identity ✓ answer (6)

5.3	$(\sin x - \cos x)^2 = \left(\frac{3}{4}\right)^2$ $\sin^2 x - 2\sin x\cos x + \cos^2 x = \frac{9}{16}$ $1 - 2\sin x\cos x = \frac{9}{16}$ $2\sin x\cos x = \frac{7}{16}$ $\therefore \sin 2x = \frac{7}{16}$	✓ squaring both sides ✓expanding LHS ✓ using identity ✓ simplifying ✓answer (5) **[18]**

QUESTION/*VRAAG* 6

6.1	$4\sin x + 2\cos 2x = 2$ $2\sin x + \cos 2x - 1 = 0$ $2\sin x + (1 - 2\sin^2 x) - 1 = 0$ $2\sin^2 x - 2\sin x = 0$ $2\sin x(\sin x - 1) = 0$ $2\sin x = 0$ or $\sin x - 1 = 0$ $\sin x = 0$ $\sin x = 1$ $x = k.180°$ or $x = 90° + k.360\,, k \in Z$	✓using identity ✓ standard form ✓ factors ✓ $\sin x = 0$ or $\sin x = 1$ ✓ $k.180°$ ✓ $90° + k.360, k \in Z$ (6)
6.2.1		✓ turning point (–90° ; –3) ✓ turning point (90° ; 1) ✓ (–180° ; –1) & (0° ; –1) (3)
6.2.2	$(-90°; 0°)$ **OR/*OF*** $-90° < x < 0°$	✓ ✓ answer (2) ✓ ✓ answer (2)
6.2.3	$f(x) = g(x)$ $\therefore -180°; 0°; 90°; 180°$ $f(x + 30°) = g(x + 30°)$ $\therefore x = -30°; 60°; 150°$	✓ any ONE correct ✓ other 2 correct (2) **[13]**

QUESTION/*VRAAG* 7

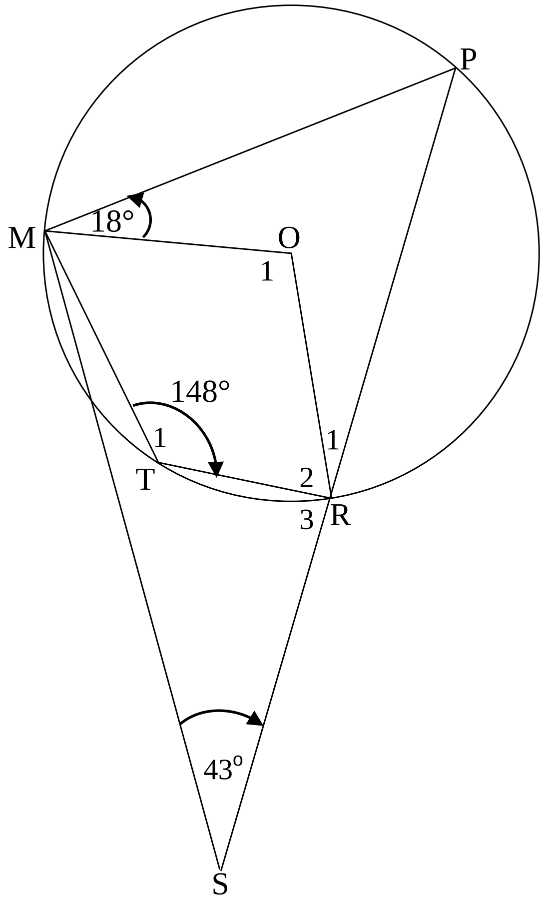


7.1	$\hat{ABD} = \theta$ [alternate $\angle$ s; $\parallel$ lines] $\cos\theta = \frac{BD}{AB} = \frac{64}{81}$ $\theta = 38°$ OR/***OF*** $\sin B\hat{A}D = \frac{64}{81}$ $B\hat{A}D = 52{,}18°$ $\theta = 38°$	✓correct trig ratio ✓substitution into correct ratio ✓ answer (to the nearest degree) (3) ✓correct trig ratio ✓substitution into correct ratio ✓ answer (to the nearest degree) (3)
7.2	$BC^2 = AB^2 + AC^2 - 2(AB)(AC)\cos B\hat{A}C$ $= 81^2 + 87^2 - 2(81)(87)\cos 82{,}6°$ $= 12314{,}754...$ $BC = 110{,}97\text{ m}$	✓use cosine rule ✓correct substitution into cosine rule ✓answer (3)

7.3	$\frac{\sin D\hat{C}B}{BD} = \frac{\sin B\hat{D}C}{BC}$ $\sin D\hat{C}B = \frac{BD.\sin B\hat{D}C}{BC}$ $\sin D\hat{C}B = \frac{64.\sin 110°}{110{,}97}$ $\therefore D\hat{C}B = 32{,}82°$	✓ use sine rule ✓ substitution ✓ answer (3) **[9]**

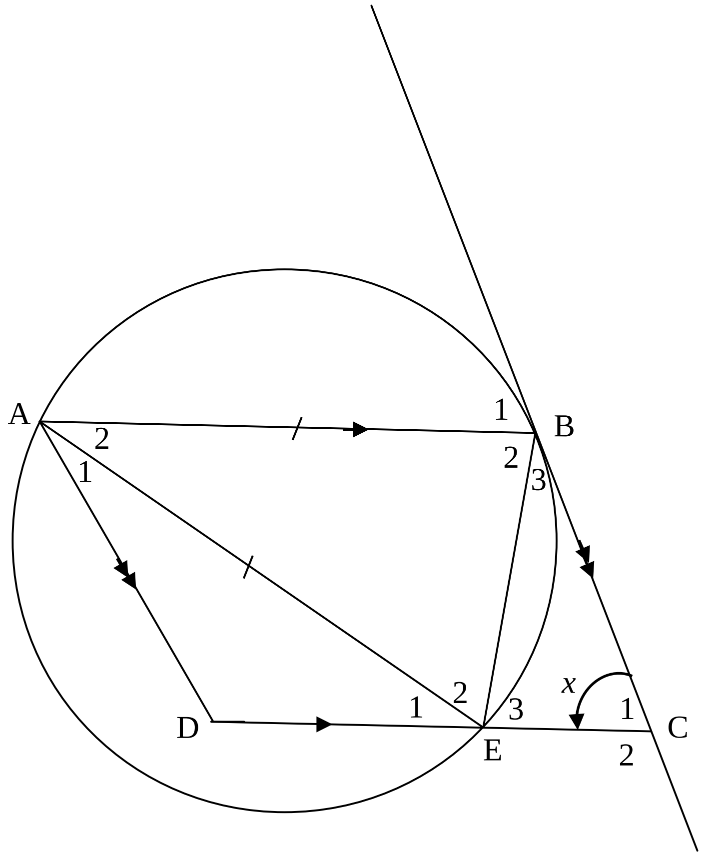
QUESTION/*VRAAG* 8

8.1



8.1.1	$\hat{P} = 32°$ [opp $\angle$s of cyclic quad/*teenoorst$\angle$e v koordevh*]	✓ S ✓ R (2)
8.1.2	$\hat{O}_1 = 2(32°) = 64°$ [$\angle$ centre = 2 $\angle$ at circum/*midpts$\angle$ = 2 omtreks$\angle$*] **OR/*OF*** reflex $\hat{O} = 296°$ [$\angle$ centre = 2 $\angle$ at circum/*midpts$\angle$ = 2 omtreks$\angle$*] $\hat{O}_1 = 64°$ [$\angle$s around a point/*$\angle$e om 'n punt*]	✓ S ✓R (2) ✓ S and R ✓ S (2)
8.1.3	$O\hat{M}S = 180° - (32° + 18° + 43°)$ [sum $\angle$s Δ / *som $\angle$e Δ*] $= 87°$	✓ S ✓ S (2)
8.1.4	$\hat{R}_3 = T\hat{M}P$ [ext $\angle$ cyclic quad/*buite $\angle$ koordevh*] $= 87° + 18° - 6°$ $= 99°$	✓ R ✓ S (2)

8.2

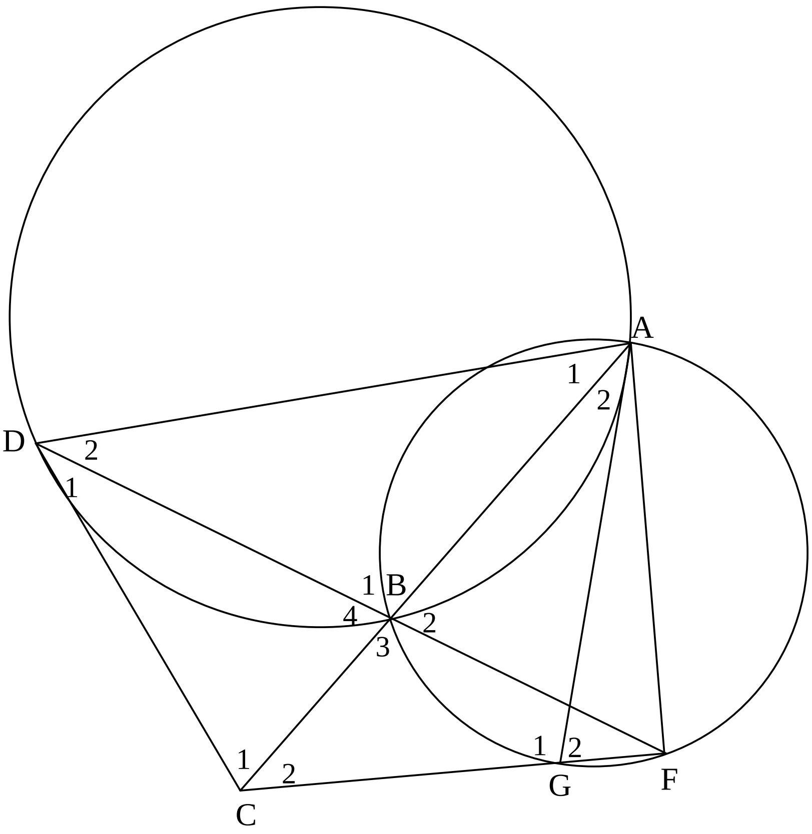


8.2.1	corres $\angle$s/*ooreenk* $\angle e$; AB | | DC	✓ R (1)
8.2.2	$\hat{E}_2 = x$ [tan - chord theorem/*raakl – koordst*] $\hat{B}_2 = x$ [$\angle$s opp = sides/$\angle e$ *teenoor* = *sye*] $\hat{E}_3 = x$ [alt $\angle$s/*verwiss* $\angle e$; AB || DC] $D\hat{A}B = x$ [opp $\angle$s $\|^m$ /*teenoor* $\angle e$ $\|^m$ **OR/*OF*** alternate/*verwiss* $\angle$s/*e*; BC ||AD] Any 3 $\angle$s correct	✓S ✓R ✓ S ✓ R ✓ S ✓ R (6)
8.2.3	$\hat{D} = 180° - x$ [co - int $\angle$s suppl/*ko – binne*$\angle e$ suppl; AD | | BC] $\therefore \hat{B}_2 + \hat{D} = 180°$ $\therefore$ ABED a cyc quad/*kdvh* [converse opp $\angle$s of cyclic quad/ *omgek teenoorst* $\angle e$ *koordevh*] **OR/*OF*** $D\hat{A}B = x$ [opp $\angle$s/*teenoor* $\angle e$ | |^m] OR/*OF* [alt$\angle$s/*verwiss* $\angle e$;BC | | AD] $\hat{E}_3 = D\hat{A}B = x$ $\therefore$ ABED a cyc quad/*kdvh* [converse ext $\angle$ of cyc quad/*omgek buite*$\angle$ *v koordevh*]	✓S ✓ R ✓ R (3) ✓S ✓ R ✓ R (3) **[18]**

QUESTION/*VRAAG* 9

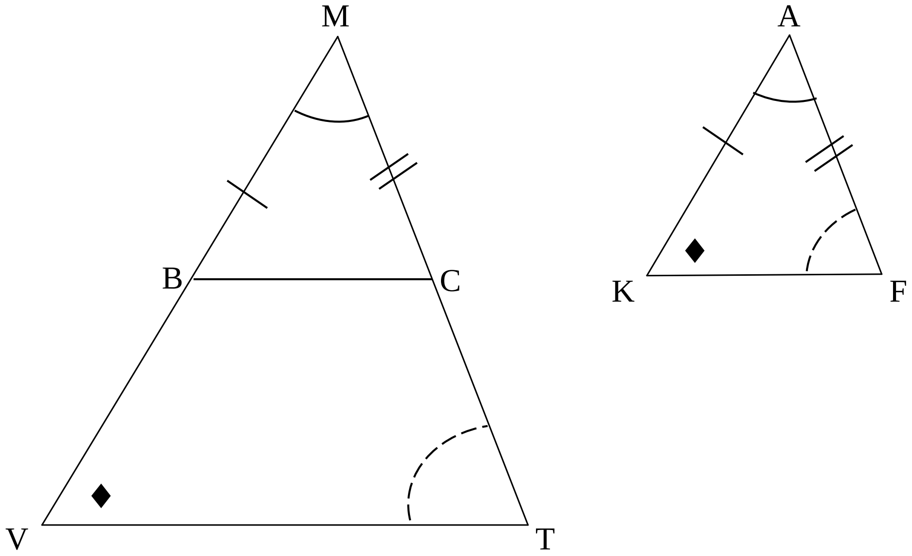
9.1	… in the alternate segment/…*in die(teen)oorstaande segment*	✓ answer (1)

9.2



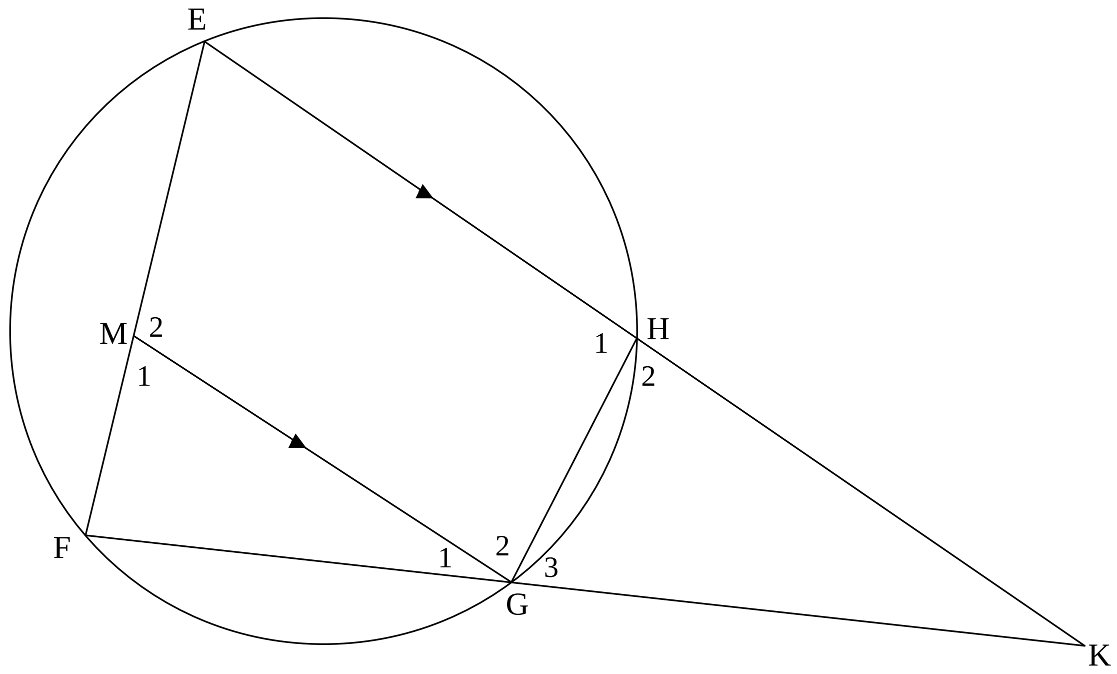
9.2.1	$\hat{A}_1 = \hat{D}_1$ [tan chord theorem/*raakl – koordst*] $\hat{B}_4 = \hat{A}_1 + \hat{D}_2$ [ext $\angle\ \Delta$/*buite*$\angle\ \Delta$] $= \hat{D}_1 + \hat{D}_2$	✓S ✓ R ✓ S ✓ R (4)
9.2.2	$\hat{B}_4 = \hat{B}_2$ [vert opp $\angle$s/*regoorst* $\angle$*e*] $\hat{D}_1 + \hat{D}_2 = \hat{B}_2$ [proven/*bewys*] $= \hat{G}_2$ [$\angle$s in same segment/$\angle$e *in dies segment*] $\therefore$ AGCD is cyc quad/*kvh* [converse ext$\angle$ cyc quad/*omgek buite*$\angle$ *kvh*]	✓ S ✓ S ✓ R ✓ R (4)
9.2.3	$\hat{D}_1 = \hat{A}_2$ [$\angle$s in same segment/$\angle$*e in dies segment*] $\hat{A}_2 = \hat{F}$ [$\angle$s in same segment/$\angle$*e in dies segment*] $\therefore \hat{D}_1 = \hat{F}$ $\therefore$ DC = CF [sides opp = $\angle$s/*sye teenoor* = $\angle$*e*]	✓ S ✓ R ✓ S ✓ R (4) **[13]**

QUESTION/*VRAAG* 10



10.1	Constr/*Konstr* : Draw line BC such that MB = AK and MC = AF *Trek lyn BC sodat MB = AK en MC = AF* Proof/*Bewys* : In ΔBMC and/*en* ΔKAF MB = AK [constr/*konstr*] $\hat{M} = \hat{A}$ [given/*gegee*] MC = AF [constr/*konstr*] ΔBMC ≡ ΔKAF [s ∠ s] $\therefore M\hat{B}C = A\hat{K}F$ or $M\hat{C}B = A\hat{F}K$ [≡ Δ] but /*maar* $\hat{V} = \hat{K}$ or $\hat{T} = \hat{F}$ [given/*gegee*] $\therefore M\hat{B}C = \hat{V}$ or $M\hat{C}B = \hat{T}$ But these are corresponding ∠s/*maar hulle is ooreenk ∠e* ∴ BC || VT [corr ∠s = /*ooreenk∠e* =] $\therefore \frac{MV}{MB} = \frac{MT}{MC}$ [prop theorem/*eweredighst*; BC | | VT] but /*maar* MB = AK and MC = AF [constr/*konstr*] $\therefore \frac{MV}{AK} = \frac{MT}{AF}$	✓ constr/*konstr* ✓ S / R ✓ S ✓ S ✓ S / R ✓S ✓R (7)

10.2



10.2.1(a)	In ΔKGH and ΔKEF $\hat{K}$ is common/*gemeen* $\hat{H}_2 = \hat{F}$ [ext $\angle$ cyclic quad/*buite* $\angle$ *koordevh*] $\hat{G}_3 = \hat{E}$ [sum$\angle$s Δ OR ext $\angle$ cyclic quad/*som*$\angle$*e*Δ *OR buite*$\angle$*koordevh*] $\therefore \Delta KGH \parallel\mid \Delta KEF$ [$\angle\angle\angle$]	✓ S ✓S ✓R ✓ naming third angle OR $\angle\angle\angle$ (4)
10.2.1(b)	$\frac{EF}{GH} = \frac{KE}{KG}$ [$\parallel\mid \Delta$s] $\therefore \frac{EF}{GH} = \frac{KE}{EF}$ [KG = EF] $\therefore EF^2 = KE.GH$	✓ S ✓ S (2)
10.2.1(c)	$\frac{KG}{KF} = \frac{EM}{EF}$ [prop theorem/*eweredighst*; MG | | EK] but EF = KG [given/*gegee*] $\frac{KG}{KF} = \frac{EM}{KG}$ $KG^2 = EM.KF$	✓ S ✓ R ✓ S (3)
10.2.2	KE.GH = EM.KF $EM = \frac{20 \times 4}{16}$ $= 5$ units	✓ KE.GH = EM.KF ✓ substitution ✓ answer (3) **[19]**

TOTAL/*TOTAAL*: 150

basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE/*GRAAD* 12

MATHEMATICS P2/*WISKUNDE V2*

FEBRUARY/MARCH/*FEBRUARIE/MAART* 2016

MEMORANDUM

MARKS: 150
PUNTE: 150

This memorandum consists of 21 pages./
Hierdie memorandum bestaan uit 21 bladsye.

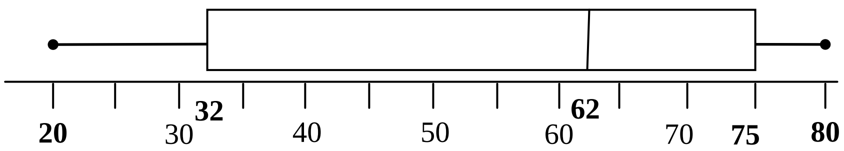
NOTE:

- If a candidate answers a question TWICE, mark only the FIRST attempt.
- If a candidate crossed out an attempt of a question and did not redo the question, mark the crossed-out version.
- Consistent accuracy applies in ALL aspects of the marking memorandum. Stop marking at the second calculation error.
- Assuming answers/values in order to solve a problem is NOT acceptable.

LET WEL:

- Indien 'n kandidaat 'n vraag TWEE KEER beantwoord, sien slegs die EERSTE poging na.
- Indien 'n kandidaat 'n antwoord doodgetrek en nie oorgedoen het nie, sien die doodgetrekte poging na.
- Volgehoue akkuraatheid word in ALLE aspekte van die memorandum toegepas. Hou op nasien by die tweede berekeningsfout.
- Om antwoorde/waardes om 'n probleem op te los, te veronderstel, word NIE toegelaat NIE.

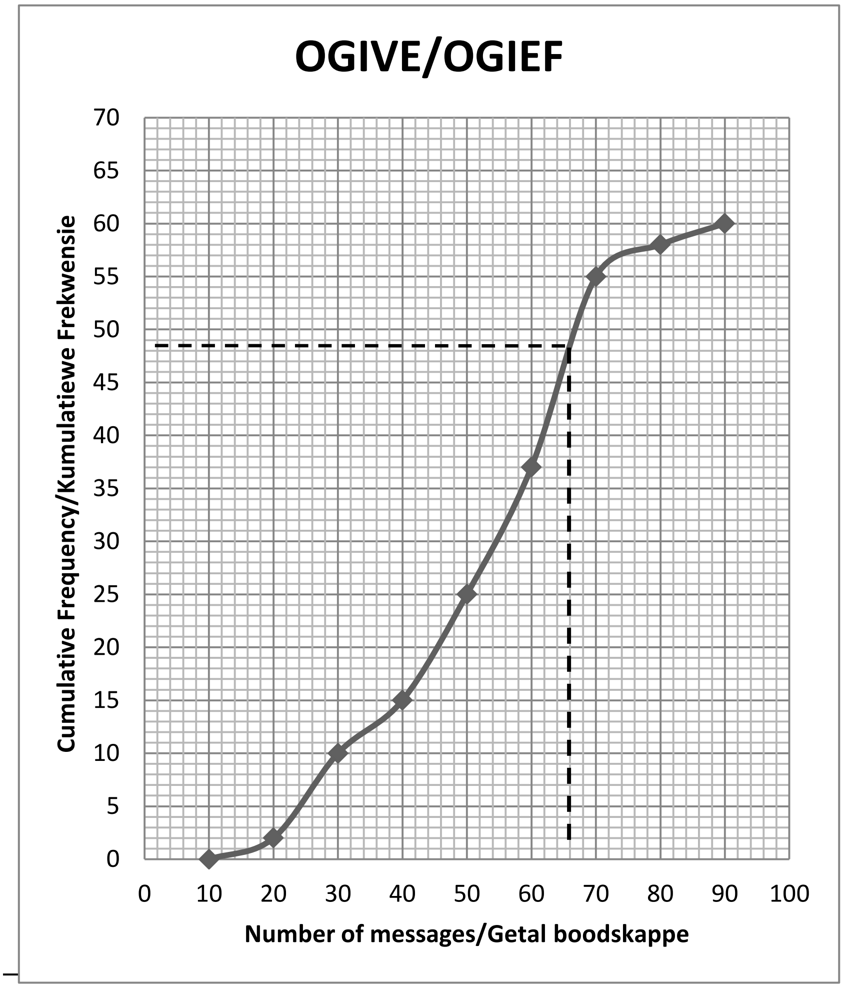
QUESTION/*VRAAG* 1



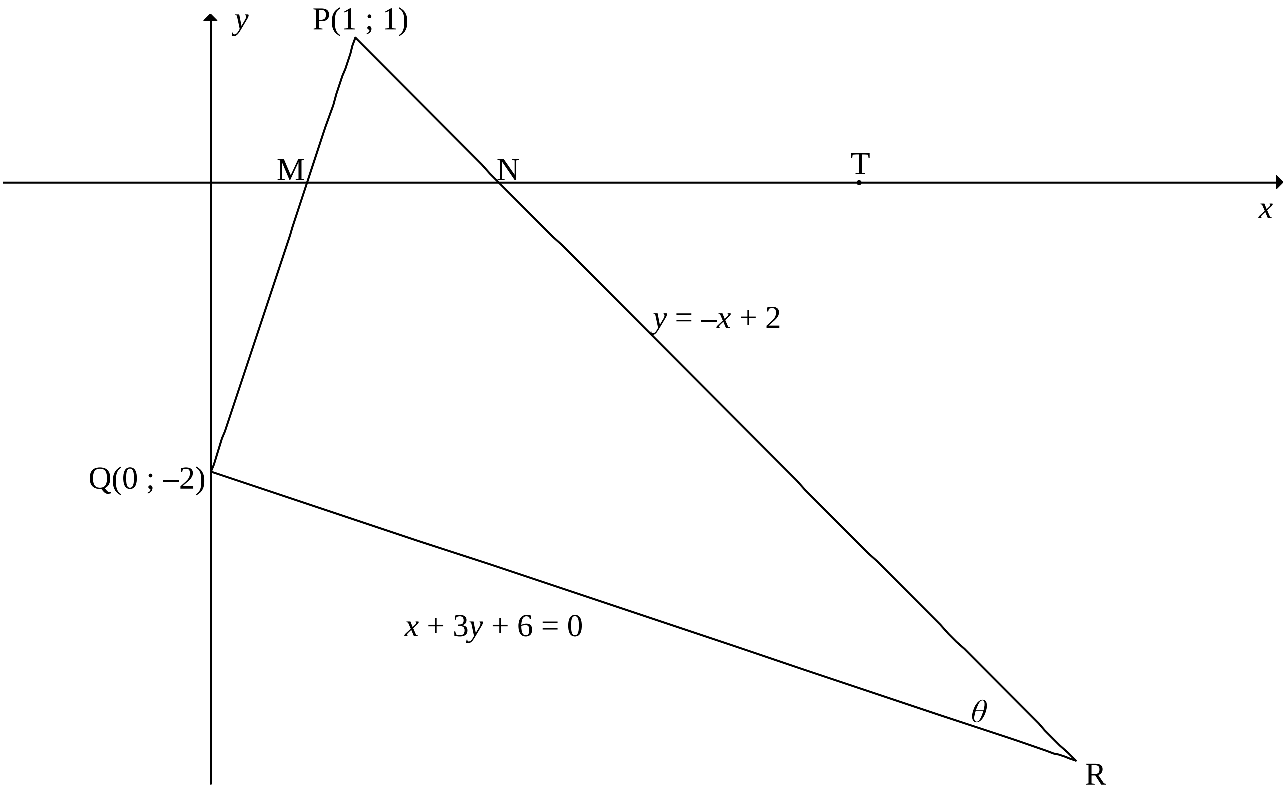
<table>
<tr><td>1.1</td><td>The data is skewed to the left/Die data is skeef na links.
OR/OF
The data is negatively skewed/Die data is negatief skeef.</td><td>✓answ/antw

✓answ/antw
(1)</td></tr>
<tr><td>1.2</td><td>Range/Omvang = 80 – 20
= 60</td><td>✓ max. – min.
✓answ/antw
(2)</td></tr>
<tr><td>1.3</td><td>25% of the learners failed/van die leerders het gedruip</td><td>✓ ✓answ/antw
(2)</td></tr>
<tr><td>1.4</td><td>$54 = \frac{445 + T_4}{9}$
$T_4 = 41$
<table><tr><td>20</td><td>28</td><td>36</td><td>41</td><td>62</td><td>69</td><td>75</td><td>75</td><td>80</td></tr></table></td><td>✓ 20
✓✓ 41
✓ 62
✓ 75
✓ 80
(6)
[11]</td></tr>
</table>

QUESTION/*VRAAG* 2

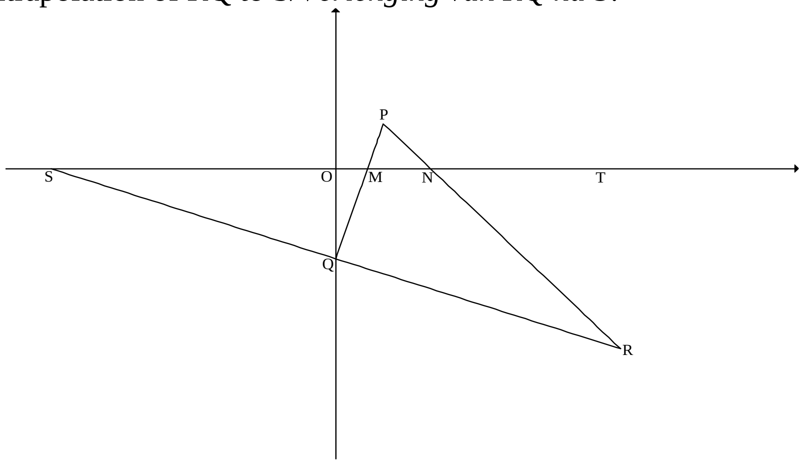
2.1	Mean/*Gemiddelde* = $\frac{2(15)+8(25)+...2(85)}{60} = \frac{3080}{60}$ = 51,33 messages per day/*boodskappe per dag*	✓ 3 080 ✓ $\frac{3080}{60}$ ✓answ/*antw* (3)
2.2		✓grounding at (10 ; 0) ✓plotting at upper limits ✓plotting. cumulative *f* ✓smooth shape of curve ✓*geanker by* (10 ; 0) ✓*stip by boonste limiete* ✓*plot kumulatiewe f* ✓*gladde vorm van kurwe* (4)
2.3	Number of days/*Getal dae* = 60 – 46 (see on graph above/*sien op grafiek hierbo*) = 14 days/*dae* **OR/*OF*** Number of days/*Getal dae* = 2 + 3 + $\frac{1}{2}\times 18$ = 14 days/*dae*	✓46 (accept 45 – 49) ✓answ/*antw* (accept 11 – 15) (2) ✓add correct values/*tel korrekte waardes by* ✓answ/*antw* (2) **[9]**

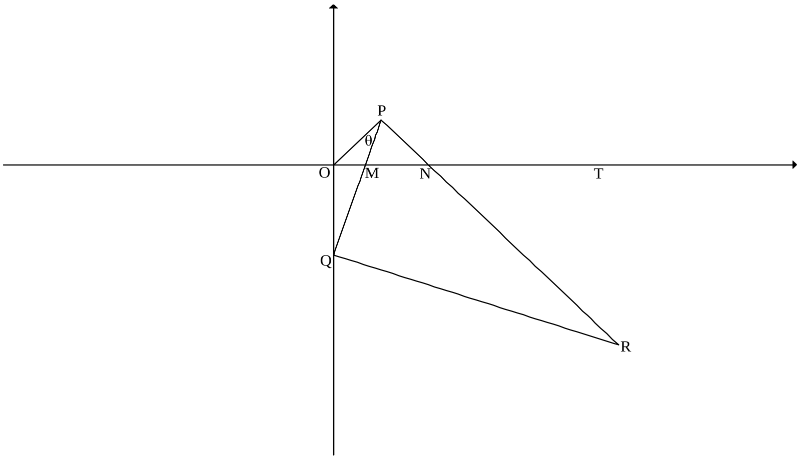
QUESTION/*VRAAG* 3



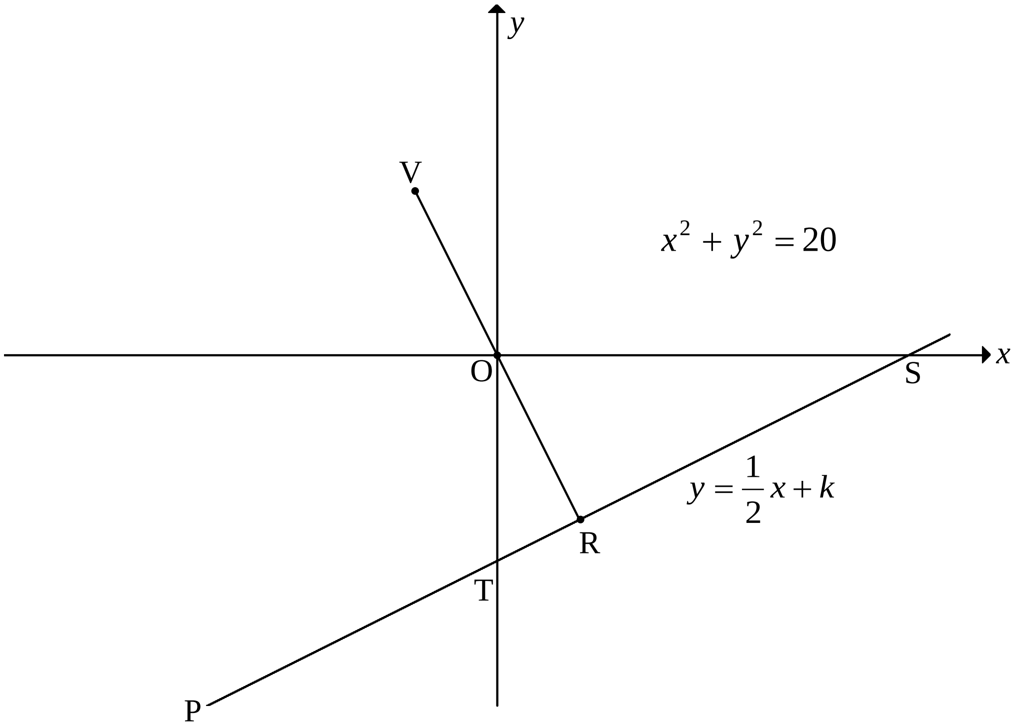
3.1	$m_{PQ} = \frac{1-(-2)}{1-0}$ $= 3$	✓subst (1 ; 1) & (0 ; –2) ✓answ/*antw* (2)
3.2	QR: $y = -\frac{1}{3}x - 2$ $\therefore m_{QR} = -\frac{1}{3}$ $m_{PQ} \times m_{QR} = 3 \times -\frac{1}{3}$ $= -1$ $\therefore PQ \perp QR \quad \therefore P\hat{Q}R = 90°$	✓ $m_{QR} = -\frac{1}{3}$ ✓ $m_{PQ} \times m_{QR} = -1$ (2)

3.3	$-\frac{1}{3}x-2=-x+2$ $\frac{2}{3}x=4$ $x=6$ $y=-4$ $\therefore$ R(6 ; –4)	✓equating/*gelyk stel* ✓x-value/*waarde* ✓y-value/*waarde* (3)
3.4	$\text{PR}=\sqrt{(1-6)^2+(1-(-4))^2}$ $=\sqrt{50}=5\sqrt{2}$ **OR/*OF*** $\text{PR}^2=(1-6)^2+(1-(-4))^2$ $=50$ $\therefore \text{PR}=\sqrt{50}=5\sqrt{2}$	✓subst into/*in* distance formula/ *afstandsformule* ✓answ/*antw* in surd form/ *wortelvorm* (2) ✓subst into/*in* distance formula/ *afstandsformule* ✓answ/*antw* in surd form/ *wortelvorm* (2)
3.5	PR is a diameter/*'n middellyn* [chord subtends/*kd onderspan* 90°] Centre of circle/*Midpt v sirkel*: $\left(\frac{1+6}{2};\frac{1-4}{2}\right)$ $=\left(3\frac{1}{2};-1\frac{1}{2}\right)$ $r=\frac{\sqrt{50}}{2}$ OR $\frac{5\sqrt{2}}{2}$ OR 3,54 $\therefore \left(x-\frac{7}{2}\right)^2+\left(y+\frac{3}{2}\right)^2=\frac{50}{4}$ OR $\frac{25}{2}$ OR 12,5	✓✓S ✓✓ $\left(3\frac{1}{2};-1\frac{1}{2}\right)$ ✓r-value/*waarde* ✓answ/*antw* (6)
3.6	m of/*van* radius = –1 $\therefore m$ of/*van* tangent/*raaklyn* = 1 Equation of tangent/*Vgl van raaklyn*: $y-y_1=(x-x_1)$ $y-1=x-1$ $\therefore y=x$ **OR/*OF*** $y=x+c$ $1=1+c$ $y=x$	✓m of tang/*rkl* ✓subst m & P(1 ; 1) into/*in* eq of line/*vgl v lyn* ✓answ/*antw* (3)
3.7	$\tan \text{P}\hat{\text{N}}\text{T}=m_{\text{PR}}=-1$ $\therefore \text{P}\hat{\text{N}}\text{T}=135°$ $\tan \text{P}\hat{\text{M}}\text{T}=m_{\text{PQ}}=3$ $\therefore \text{P}\hat{\text{M}}\text{T}=71{,}57°$ $\hat{\text{P}}=63{,}43°$ [ext ∠ of Δ/*buite* ∠ *v* Δ] $\therefore \theta=26{,}57°$ [sum of ∠s in Δ/*som v* ∠*e in* Δ] **OR/*OF***	✓ $\tan \text{P}\hat{\text{N}}\text{T}=-1$ ✓ $\text{P}\hat{\text{N}}\text{T}=135°$ ✓ $\text{P}\hat{\text{M}}\text{T}=71{,}57°$ ✓ $\hat{\text{P}}=63{,}43°$ ✓answ/*antw* (5)

	Extrapolation of RQ to S/*Verlenging van RQ na S*:	
		
	$\tan P\hat{N}T = m_{PR} = -1$ $\therefore S\hat{N}R = 135°$ $\tan N\hat{S}R = m_{RS} = -\frac{1}{3}$ $\therefore N\hat{S}R = 18{,}43°$ $\theta = 180° - (135° + 18{,}43°)$ [sum of ∠s in Δ/*som v ∠e in* Δ] $= 26{,}57°$	✓ $\tan P\hat{N}T = -1$ ✓ $S\hat{N}R = 135°$ ✓ $\tan N\hat{S}R = -\frac{1}{3}$ ✓ $N\hat{S}R = 18{,}43°$ ✓answ/*antw* (5)
	OR/*OF*	
	$PQ^2 = 1^2 + 3^2 = 10$ $PQ = \sqrt{10}$ $\therefore \sin\theta = \frac{PQ}{PR} = \frac{\sqrt{10}}{\sqrt{50}} = \frac{1}{\sqrt{5}}$ $\therefore \theta = 26{,}57°$	✓subst into/*in* distance formula/ *afstandsformule* ✓distance/*afst* PQ ✓correct trig ratio/ *korrekte trig vh* ✓correct trig eq/ *korrekte trig vgl* ✓answ/*antw* (5)
	OR/*OF*	
	$QR^2 = 6^2 + 2^2 = 40$ $QR = 2\sqrt{10}$ $\therefore \cos\theta = \frac{2\sqrt{10}}{\sqrt{50}} = \frac{2}{\sqrt{5}}$ $\therefore \theta = 26{,}57°$	✓subst into/*in* distance formula/ *afstandsformule* ✓distance/*afst* PQ ✓correct trig ratio/ *korrekte trig vh* ✓correct trig eq/ *korrekte trig vgl* ✓answ/*antw* (5)
	OR/*OF*	

	$\tan\theta = \dfrac{m_{\text{RQ}} - m_{\text{PR}}}{1 + m_{\text{RQ}}.m_{\text{PR}}}$ $= \dfrac{-\frac{1}{3} - (-1)}{1 + (-\frac{1}{3})(-1)}$ $= \dfrac{1}{2}$ $\therefore\ \theta = 26{,}57°$	✓correct formula/ *korrekte formule* ✓ $m_{\text{RQ}} = -\frac{1}{3}$ ✓correct subst/ *subst korrek* ✓tan $\theta = \dfrac{1}{2}$ ✓ $\theta = 26{,}57°$ (5)
	 tangent OP goes through the origin/*raakl OP gaan deur oorsprong* $\hat{\text{POM}} = 45°$ $\text{O}\hat{\text{P}}\text{M} = \theta = \hat{\text{P}}$ [tan-chord theorem/*raakl-kdst*] $\tan \text{P}\hat{\text{M}}\text{T} = m_{\text{PQ}} = 3$ $\therefore \text{P}\hat{\text{M}}\text{T} = 71{,}57°$ $\therefore\ \theta + 45° = 71{,}57°$ [ext $\angle$ of Δ/*buite-*$\angle$ *v* Δ] $\therefore\ \theta = 26{,}57°$	✓ $\text{P}\hat{\text{O}}\text{M} = 45°$ ✓R ✓ $\text{P}\hat{\text{M}}\text{T} = 71{,}57°$ ✓S ✓ $\theta = 26{,}57°$ (5) **[23]**

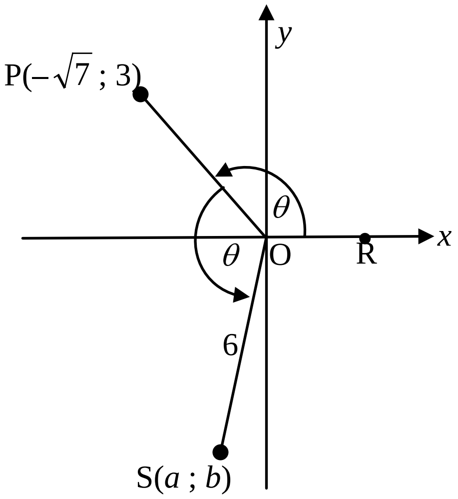
QUESTION/*VRAAG* 4



4.1	OR ⊥ TR [radius ⊥ tangent/*raakl*] $\therefore\ m_{TR} \times m_{OR} = -1$ $\therefore\ m_{OR} = -2$ $\therefore\ y = -2x$	✓S/R ✓*m* of/*van* OR ✓equation/*vgl* (3)
4.2	$x^2 + (-2x)^2 = 20$ $x^2 + 4x^2 = 20$ $5x^2 - 20 = 0$ $x^2 - 4 = 0$ $(x+2)(x-2) = 0$ $\therefore\ x = 2$ $y = -2(2) = -4$ $\therefore$ R(2 ; –4)	✓subst eq of OR into circle eq/ *subst vgl OR in sirkelvgl* ✓st. form/*st. vorm* ✓*x*-value/*waarde* ✓*y*-value/*waarde* (4)

4.3	Subst R(2 ; –4) into the equation of/*in vgl van* PRS: $-4 = \frac{1}{2}(2) + k$ $k = -5$ $\therefore$ OT = 5 $0 = \frac{1}{2}x - 5$ $x = 10$ $\therefore$ OS = 10 Area/*Oppervlakte* = $\frac{1}{2}$OS . OT $= \frac{1}{2}(10)(5)$ = 25 sq units/*vk eenh*	✓correct subst/ *korrekte subst* ✓value of k ✓$y = 0$ ✓x-intercept/*afsnit* ✓correct subst into area form/ *subst korrek in opp-formule* ✓answ/*antw* (6)
4.4	$0 = \frac{x_V + 2}{2}$ and/*en* $0 = \frac{y_V - 4}{2}$ $\therefore$ V(–2 ; 4) T(0 ; –5) .... from/*van* 4.3 $VT = \sqrt{(-2-0)^2 + (4-(-5))^2}$ $= \sqrt{4+81}$ $= \sqrt{85}$	✓x-value/*waarde*V ✓y-value/*waarde*V ✓subst of points V and T into distance formula/ *subst punte V en T in afst-form* ✓answ/*antw* (4) **[17]**

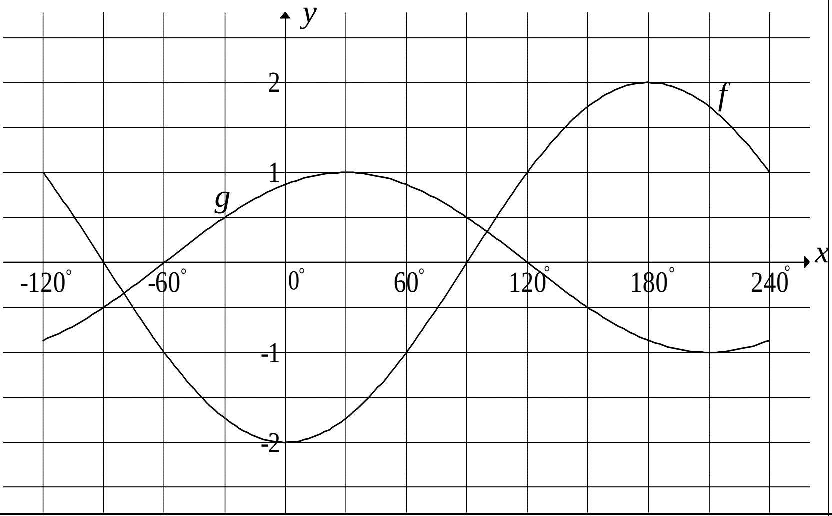
QUESTION/*VRAAG* 5



5.1.1	$\tan\theta = -\frac{3}{\sqrt{7}}$	✓answ/*antw* (1)
5.1.2	$\sin(-\theta) = -\sin\theta$ $OP^2 = \left(-\sqrt{7}\right)^2 + 3^2$ $OP^2 = 16$ $OP = 4$ $\sin(-\theta) = -\frac{3}{4}$	✓reduction/ *reduksie* ✓ OP = 4 ✓answ/*antw* (3)
5.1.3	$\frac{a}{6} = \cos 2\theta$ $a = 6(1-2\sin^2\theta)$ $= 6 - 12\left(\frac{3}{4}\right)^2$ $= \frac{24}{4} - \frac{27}{4}$ $= -\frac{3}{4}$ **OR/*OF*** $\frac{a}{6} = \cos 2\theta$ $a = 6(2\cos^2\theta - 1)$ $= 12\left(\frac{-\sqrt{7}}{4}\right)^2 - 6$ $= \frac{21}{4} - \frac{24}{4}$ $= -\frac{3}{4}$ **OR/*OF***	✓trig ratio/*verh* ✓expansion/ *uitbreiding* ✓ $\sin\theta = \frac{3}{4}$ ✓answ/*antw* (4) ✓trig ratio/*verh* ✓expansion/ *uitbreiding* ✓ $\cos\theta = \frac{-\sqrt{7}}{4}$ ✓answ/*antw* (4)

	$\frac{a}{6} = \cos 2\theta$ $a = 6(\cos^2\theta - \sin^2\theta)$ $= 6\left[\left(\frac{-\sqrt{7}}{4}\right)^2 - \left(\frac{3}{4}\right)^2\right]$ $= 6\left(-\frac{2}{16}\right)$ $= -\frac{3}{4}$	✓ trig ratio/*verh* ✓expansion/ *uitbreiding* ✓ $\cos\theta = \frac{-\sqrt{7}}{4}$ & $\sin\theta = \frac{3}{4}$ ✓answ/*antw* (4)
5.2.1	$\frac{4\sin x.\cos x}{2\sin^2 x - 1} = \frac{2(2\sin x.\cos x)}{-(1-2\sin^2 x)}$ $= \frac{2\sin 2x}{-\cos 2x}$ $= -2\tan 2x$	✓2sin 2*x* ✓–cos 2*x* ✓answ/*antw* (3)
5.2.2	$\frac{4\sin 15°\cos 15°}{2\sin^2 15° - 1} = -2\tan 2(15°)$ $= -2\tan 30°$ $= -2\left(\frac{1}{\sqrt{3}}\right)$ $= -\frac{2}{\sqrt{3}}$ **OR/*OF*** $-\frac{2\sqrt{3}}{3}$	✓ $-2\tan 2(15°)$ ✓answ/*antw* (2) **[13]**

QUESTION/*VRAAG* 6

6.1	sin (x + 60°) + 2cos x = 0 sin x cos 60° + cos x sin 60° + 2cos x = 0 $\frac{1}{2}\sin x + \frac{\sqrt{3}}{2}\cos x + 2\cos x = 0$ $\frac{1}{2}\sin x = -2\cos x - \frac{\sqrt{3}}{2}\cos x$ $\sin x = -4\cos x - \sqrt{3}\cos x$ $\sin x = \cos x(-4-\sqrt{3})$ $\frac{\sin x}{\cos x} = \frac{\cos x(-4-\sqrt{3})}{\cos x}$ $\therefore \tan x = -4-\sqrt{3}$	✓expansion/*uitbreiding* ✓special angle values/ *spesiale ∠-waardes* ✓simpl/*vereenv* ✓ $\sin x = \cos x(-4-\sqrt{3})$ (4)
6.2	$\tan x = -4-\sqrt{3}$ $\tan x = -(4+\sqrt{3})$ *ref* ∠ = 80,10° x = –80,1° or/*of* 99,9°	✓80,10° ✓99,90° ✓–80,1° (3)
6.3.1		✓(30° ; 1) ✓(–60° ; 0) ✓shape/*vorm* (3)
6.3.2	∴ sin (x + 60°) > –2cos x x ∈ (–80,10° ; 99,90°) **OR/*OF*** $-80{,}10° < x < 99{,}90°$	✓✓critical values/ *kritiese waardes* ✓notation/*notasie* (3) **[13]**

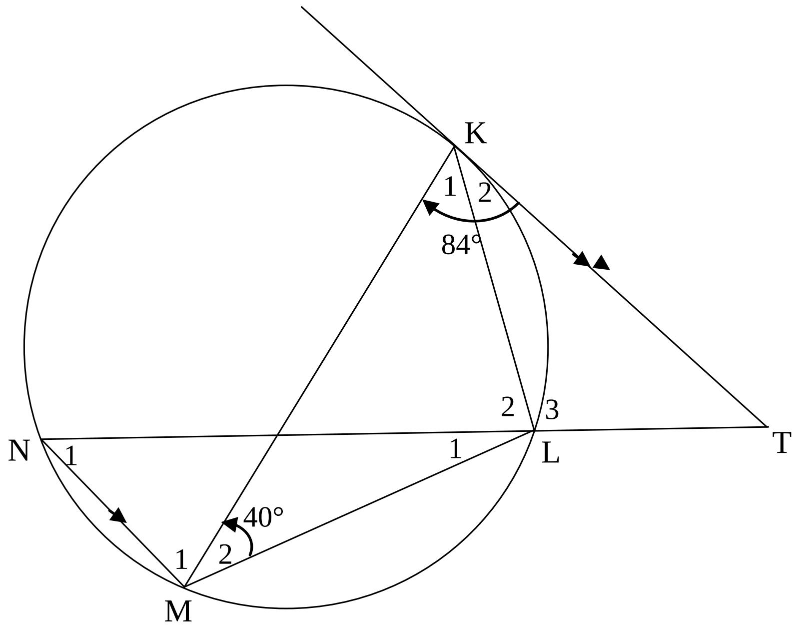
QUESTION/*VRAAG* 7

7.1.1	Area of/*Oppervlakte van* ΔPQR $= \frac{1}{2}$PQ.QR.$\sin\hat{Q}$ $= \frac{1}{2}x(20-4x)(\sin 60°)$ $= 10x - 2x^2\left(\frac{\sqrt{3}}{2}\right)$ $= 5\sqrt{3}x - \sqrt{3}x^2$	✓subst into area rule/ *subst in opp-reël* ✓subst & simpl/ *subst en vereenv* (2)
7.1.2	For maximum area/*Vir maksimum opp:* (Area ΔPQR)$^/$ = 0 $5\sqrt{3} - 2\sqrt{3}x = 0$ $2\sqrt{3}x = 5\sqrt{3}$ $\therefore\ x_{\max} = \frac{5}{2}$ or $2\frac{1}{2}$ or/*of* 2,5	✓(Area ΔPQR)$^/$ = 0 ✓$5\sqrt{3} - 2\sqrt{3}x$ ✓answ/*antw* (3)
	OR/*OF* $x_{\max} = -\frac{b}{2a}$ $= -\frac{5\sqrt{3}}{2(-\sqrt{3})} = \frac{5}{2}$ or $2\frac{1}{2}$ or 2,5	✓formula/*e* ✓ subst ✓answ/*antw* (3)
	OR/*OF* $5\sqrt{3}x - \sqrt{3}x^2 = 0$ $\sqrt{3}x(5-x) = 0$ $\therefore$ x = 0 or 5 $\therefore\ x_{\max} = \frac{0+5}{2} = \frac{5}{2}$ or/*of* 2,5	✓x-intercepts/ *x-afsnitte* ✓subst ✓answ/*antw* (3)
7.1.3	$RP^2 = QP^2 + QR^2 - 2.QP.QR.\cos Q$ $= 10^2 + 2{,}5^2 - 2(10)(2{,}5)\cos 60°$ $= 81{,}25$ $\therefore$ RP = 9,01	✓subst into cosine rule/*in cos-reël* ✓simpl/*vereenv* ✓answ/*antw* (3)

7.2	In ΔABC: $\sin\beta = \dfrac{h}{AB}$ $\therefore\ AB = \dfrac{h}{\sin\beta}$ In ΔABD: AB = BD and/*en* $A\hat{D}B = 90° - \beta$ [∠s of/*v* Δ = 180°] $\dfrac{\sin 2\beta}{AD} = \dfrac{\sin(90° - \beta)}{AB}$ $AD = \dfrac{AB.\sin 2\beta}{\sin(90° - \beta)}$ $= \dfrac{h}{\sin\beta} \times \dfrac{2\sin\beta.\cos\beta}{\cos\beta}$ $= 2h$	✓AB ito h and/*en* β ✓ $A\hat{D}B = 90° - \beta$ ✓correct subst into cosine rule/*subst korrek in cos-reël* ✓AD as subject/ *onderwerp* ✓expansion/*uitbrei* ✓sin (90°– β) = cos β ✓answer ito h (7)
	OR/*OF* In ΔABC: $\sin\beta = \dfrac{h}{AB}$ $\therefore\ AB = \dfrac{h}{\sin\beta}$ In ΔABD: AB = BD $AD^2 = AB^2 + AB^2 - 2AB.AB.\cos 2\beta$ $= \left(\dfrac{h}{\sin\beta}\right)^2 + \left(\dfrac{h}{\sin\beta}\right)^2 - 2\left(\dfrac{h}{\sin\beta}\right)^2.\cos 2\beta$ $= \left(\dfrac{h}{\sin\beta}\right)^2 + \left(\dfrac{h}{\sin\beta}\right)^2 - 2\left(\dfrac{h}{\sin\beta}\right)^2 (1 - 2\sin^2\beta)$ $= \left(\dfrac{h}{\sin\beta}\right)^2 + \left(\dfrac{h}{\sin\beta}\right)^2 - 2\left(\dfrac{h}{\sin\beta}\right)^2 + 4h^2$ $= 4h^2$ $\therefore$ AD = 2h	✓AB ito h and/*en* β ✓correct subst into cosine rule/*subst korrek in cos-reël* ✓expansion/*uitbrei* ✓multiplication/ *vermenigv* ✓simpl/*vereenv* ✓answer ito h (7)
	OR/*OF* Split isosceles triangle ABQ into two congruent triangles AEB and DEB. Then ΔABC ≡ ΔBAE (AB = AC, $A\hat{B}E = B\hat{A}C = \beta$, h) $\therefore$ AE = ED = BC = h $\therefore$ AD = 2h	(7)
		[15]

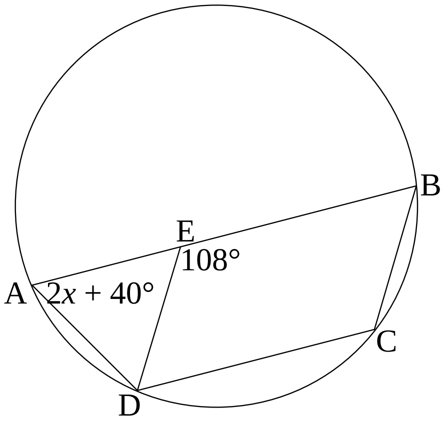
QUESTION/*VRAAG* 8

8.1



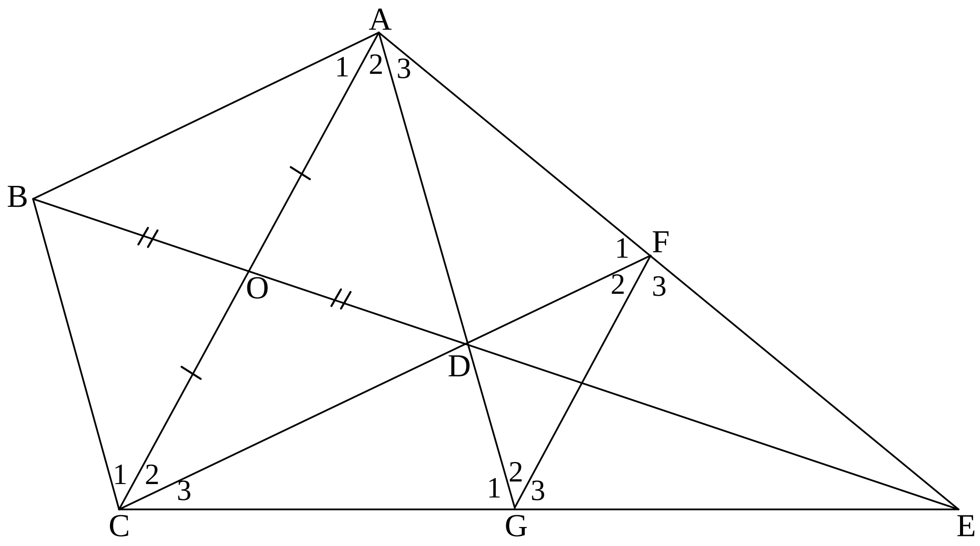
8.1.1	$\hat{K}_2 = \hat{M}_2 = 40°$ [tan chord theorem/*raakl-kdst*]	✓S ✓R (2)
8.1.2	$\hat{N}_1 = \hat{K}_1$ [∠s in the same seg/∠*e in dies segm*] $\hat{K}_1 = 84° - 40° = 44°$ $\therefore\ \hat{N}_1 = 44°$	✓S ✓R ✓S (3)
8.1.3	$\hat{T} = \hat{N}_1 = 44°$ [alt/*verw* ∠s/*e*; KT | | NM]	✓S ✓R (2)
8.1.4	$\hat{L}_2 = \hat{K}_2 + \hat{T}$ [ext ∠ of Δ/*buite* ∠ *v Δ*] $= 40° + 44°$ $= 84°$	✓R ✓S (2)
8.1.5	In ΔKLM: $44° + 84° + 40° + \hat{L}_1 = 180°$ [∠s sum in Δ/∠*e som in* Δ] $\therefore \hat{L}_1 = 12°$	 ✓S (1)

8.2



8.2	$\hat{C} = 108°$ [opp∠s of ∥m/*tos* ∠*e v* ∥*m*] $2x + 40° + 108° = 180°$ [opp∠s of cyc quad/*tos*∠*e v kdvh*] $2x = 32°$ $x = 16°$ **OR/*OF*** $\hat{C} = 180° - (2x + 40°)$ [opp∠s of cyc quad/*tos*∠*e v kdvh*] $180° - (2x + 40°) = 108°$ [opp∠s of ∥m/*tos* ∠*e v* ∥*m*] $2x = 32°$ $x = 16°$	✓S ✓R ✓S ✓R ✓answ/*antw* (5) ✓S ✓R ✓S ✓R ✓answ/*antw* (5) **[15]**

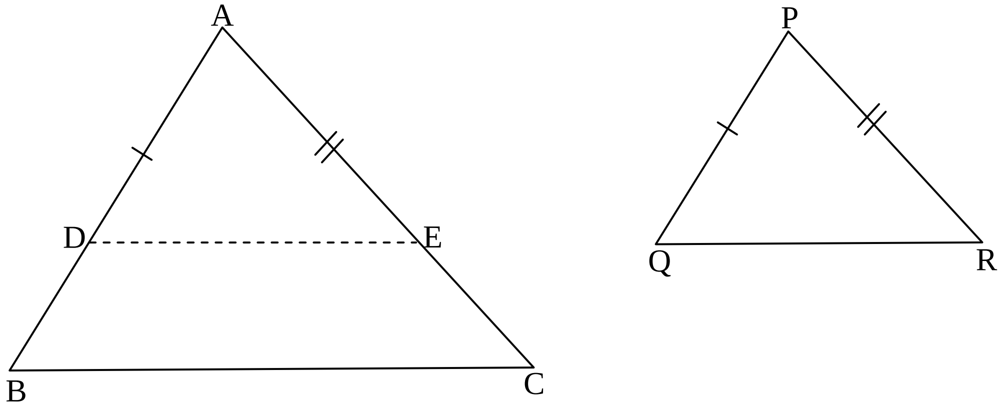
QUESTION/*VRAAG* 9



9.1	ABCD is a ∥m [diags of quad bisect each other/ *hoekl v vh halveer mekaar*]	✓ R (1)
9.2	$\frac{ED}{DB}=\frac{FE}{AF}$ [Prop Th/*Eweredigh st*; DF ∥ BA] $\frac{ED}{DB}=\frac{GE}{CG}$ [Prop Th/*Eweredigh st*; DG ∥ BC]	✓S ✓ R ✓S ✓ R (4)
9.3	$\frac{FE}{AF}=\frac{GE}{CG}$ [proved/*bewys*] $\therefore$ AC ∥ FG [line divides two sides of Δ in prop/ *lyn verdeel 2 sye van Δ eweredig*] $\hat{C}_2=\hat{F}_2$ [alt/*verw* ∠s/*e*; AC ∥ FG] $\hat{A}_1=\hat{C}_2$ [alt/*verw* ∠s/*e*; AB ∥ CD] $\therefore\ \hat{A}_1=\hat{F}_2$	✓S ✓ S ✓R ✓S ✓ S (5)
9.4	$\hat{A}_1=\hat{A}_2$ [diags of rhombus/*hoekl v ruit*] $\hat{A}_2=\hat{F}_2$ [$\hat{A}_1=\hat{F}_2$] $\therefore$ ACGF = cyc quad/*kdvh* [∠s in the same seg =/ *∠e in dies segm* =] **OR/*OF*** $\hat{C}_2=\hat{A}_2$ [∠s opp equal sides of rhombus/ *∠e to gelyke sye v ruit*] $\hat{A}_2=\hat{G}_2$ [alt/*verw*-∠s/*e*; AC ∥ FG] $\therefore\ \hat{C}_2=\hat{G}_2$ $\therefore$ ACGF is a cyc quad/*kdvh* [∠s in the same seg =/ *∠e in dies segm* =]	✓S ✓ S ✓R (3) ✓S ✓ S ✓R (3) **[13]**

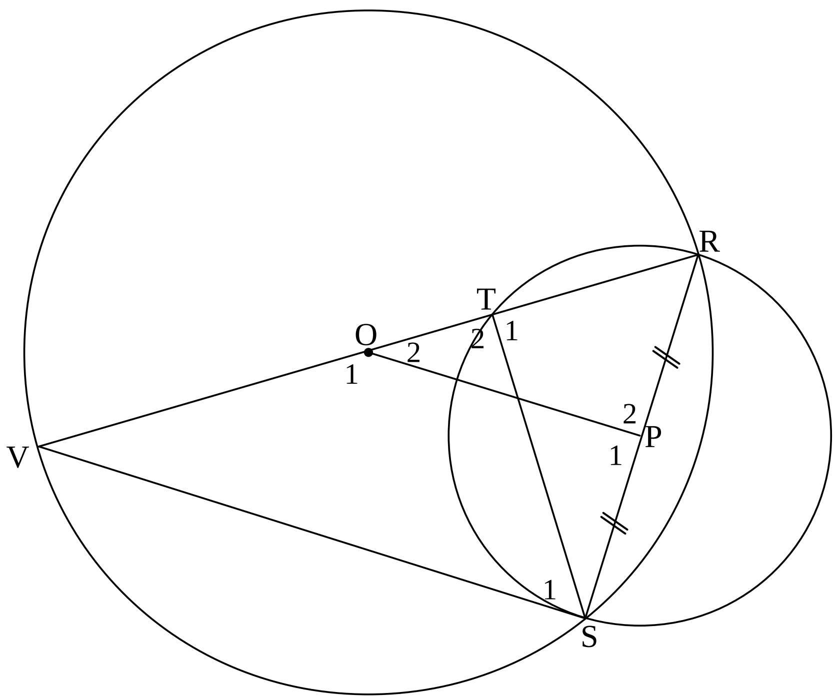
QUESTION/*VRAAG* 10

10.1



10.1.1	In ΔADE and/*en* ΔPQR: AD = PQ [construction/*konstr*] $\hat{A} = \hat{P}$ [given/*gegee*] AE = PR [construction/*konstr*] $\therefore \Delta ADE \equiv \Delta PQR$ [S$\angle$S]	✓all/*al* 3 S's/*e* ✓reason/*rede* (2)
10.1.2	$A\hat{D}E = \hat{Q}$ [Δs $\equiv$ $\therefore$ corres/*ooreenk* $\angle$s/*e* =] But $\hat{B} = \hat{Q}$ [given/*gegee*] $\therefore A\hat{D}E = \hat{B}$ $\therefore$ DE | | BC [corres/*ooreenk* $\angle$s/*e* =]	✓ $A\hat{D}E = \hat{Q}$ ✓ $A\hat{D}E = \hat{B}$ ✓reason/*rede* (3)
10.1.3	$\frac{AB}{AD} = \frac{AC}{AE}$ [Prop Th/*Eweredigh st*; DE | | BC] But/*Maar* AD = PQ and/*en* AE = PR [construction/*konstr*] $\therefore \frac{AB}{PQ} = \frac{AC}{PR}$	✓S/R ✓S (2)

10.2



10.2.1	line from centre to midpt of chord/*lyn van midpt na midpt van koord*	✓answ/*antw* (1)
10.2.2	OP || VS [Midpt Theorem/*Midpt-stelling*] In ΔROP and/*en* ΔRVS: $\hat{R} = \hat{R}$ [common/*gemeen*] $\hat{O}_2 = \hat{V}$ [corresp/*ooreenk* ∠s/*e*; OP || VS] $\therefore \Delta ROP \ \|\|\| \ \Delta RVS$ [∠,∠,∠] **OR/*OF*** In ΔROP and/*en* ΔRVS: $\hat{P}_2 = V\hat{S}R$ [corresponding ∠s/ *ooreenkomstige* ∠'e] $\hat{R} = \hat{R}$ [common/*gemeen*] $\therefore \Delta ROP \ \|\|\| \ \Delta RVS$ [∠,∠,∠]	✓S ✓ R ✓S ✓S & ∠;∠;∠ **OR/*OF*** 3 angles/*hoeke* (4) ✓S ✓ R ✓S ✓S & ∠;∠;∠ **OR/*OF*** 3 angles/*hoeke* (4)

10.2.3	In ΔRVS and/*en* ΔRST: $V\hat{S}R = S\hat{T}R = 90°$ [$\angle$ in semi-circle/$\angle$ *in halfsirkel*] $\hat{R}$ is common/*gemeen* $\hat{V} = T\hat{S}R$ $\therefore \Delta RVS \mid\mid\mid \Delta RST$ [$\angle,\angle,\angle$]	✓S ✓ R ✓S & $\angle;\angle;\angle$ **OR/*OF*** 3 angles/*hoeke* (3)
10.2.4	In ΔRTS and/*en* ΔSTV: $R\hat{T}S = V\hat{T}S = 90°$ [$\angle$ s on straight line/$\angle$*e op rt lyn*] $\hat{R} = 90° - T\hat{S}R$ $= T\hat{S}V$ $T\hat{S}R = \hat{V}$ $\therefore \Delta RTS \mid\mid\mid \Delta STV$ [$\angle,\angle,\angle$] $\therefore \frac{RT}{ST} = \frac{TS}{VT}$ $\therefore ST^2 = VT.TR$	✓ΔRTS & ΔSTV ✓S ✓S ✓S (with justification/*met motivering*) ✓$\Delta RTS \mid\mid\mid \Delta STV$ ✓ratio/*verh* (6)
		[21]

TOTAL/*TOTAAL*: 150

basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE/ *NASIONALE SENIOR SERTIFIKAAT*

GRADE/*GRAAD* 12

MATHEMATICS P2/*WISKUNDE V2*

NOVEMBER 2015

MEMORANDUM

MARKS: 150
PUNTE: 150

This memorandum consists of 27 pages./
Hierdie memorandum bestaan uit 27 bladsye.

NOTE:

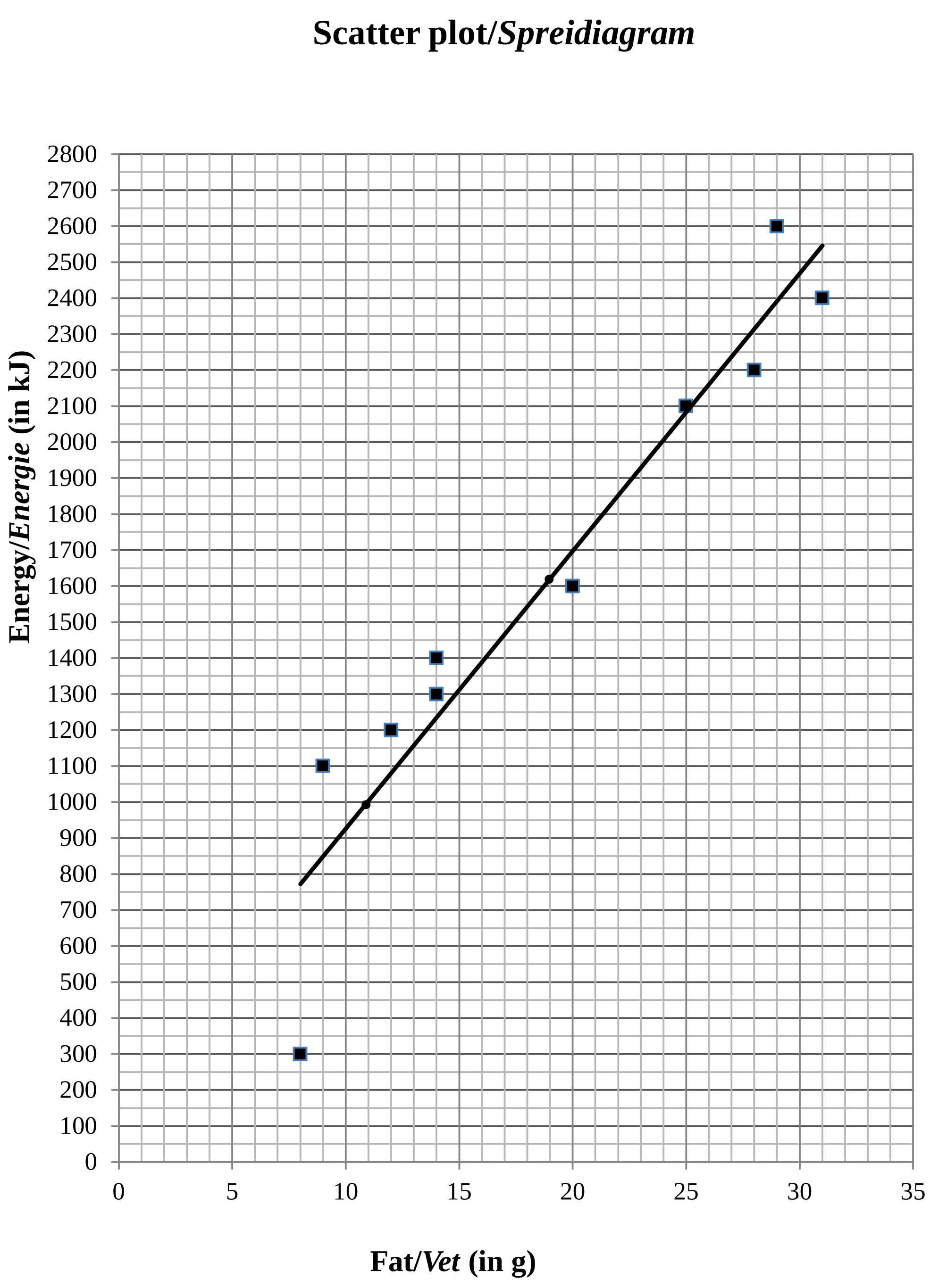
- If a candidate answers a question TWICE, mark only the FIRST attempt.
- If a candidate crossed out an attempt of a question and did not redo the question, mark the crossed-out version.
- Consistent accuracy applies in ALL aspects of the marking memorandum. Stop marking at the second calculation error.
- Assuming answers/values in order to solve a problem is NOT acceptable.
- Penalty of only 1 mark for incorrect rounding throughout the paper (Q1.2.1)

LET WEL:

- Indien 'n kandidaat 'n vraag TWEE KEER beantwoord, sien slegs die EERSTE poging na.
- Indien 'n kandidaat 'n antwoord doodgetrek het en nie oorgedoen het nie, sien die doodgetrekte poging na.
- Volgehoue akkuraatheid word in ALLE aspekte van die memorandum toegepas. Hou op nasien by die tweede berekeningsfout.
- Om antwoorde/waardes om 'n probleem op te los, te veronderstel, word NIE toegelaat NIE.

QUESTION/*VRAAG* 1

Fat/*Vet* (in g)	9	14	25	8	12	31	28	14	29	20
Energy/*Energie* (in kJ)	1 100	1 300	2 100	300	1 200	2 400	2 200	1 400	2 600	1 600

1.1 1.2.2	 	<u>1.1</u> no marks: 0 – 2 points correctly ✓plotting 3 – 5 points correctly ✓✓plotting 6 – 9points correctly ✓✓✓plotting all 10 points correctly *geen punte: 0 – 2 punte korrek* ✓ *stip 3 – 5 pte korrek* ✓✓*stip 6 –9 pte korrek* ✓✓✓ *stip al 10 pte korrek* (3) <u>1.2.2</u> ✓ *y* – int close to (0 ; 150) ✓ one pt close to (25 ; 2100) or (20 ; 1700) (2)

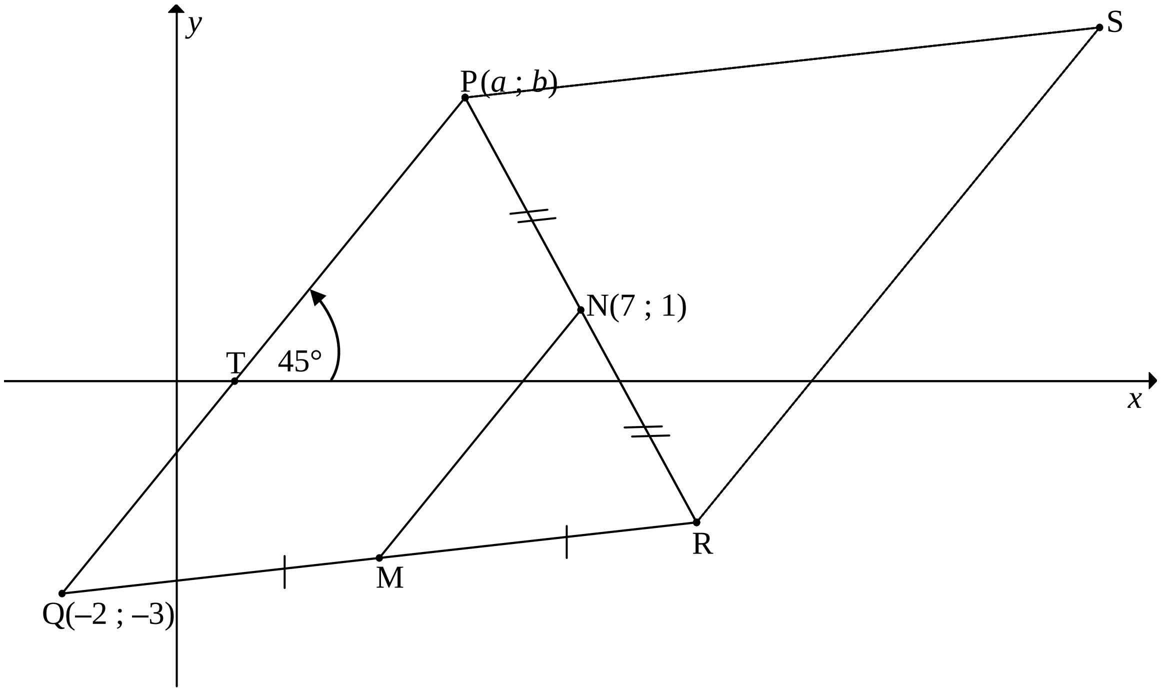
1.2.1	$\hat{y} = 154{,}60 + 77{,}13(18)$ $= 1\ 542{,}94 \approx 1\ 500$ kJ	✓ subst ✓ answ rounded off correctly/ *antw korrek afgerond* (2)
1.3	(8 ; 300)	✓ answ/*antw* (1)
1.4	$r = 0{,}9520... \approx 0{,}95$	✓✓ answ/*antw* (2)
1.5	very strong positive relationship/ *baie sterk positiewe verband*	✓ strong/ *sterk* (1) **[11]**

QUESTION/*VRAAG* 2

Sum of the values on uppermost faces/ ***Som van die waardes op boonste vlakke***	**Frequency/** ***Frekwensie***
2	0
3	3
4	2
5	4
6	4
7	8
8	3
9	2
10	2
11	1
12	1

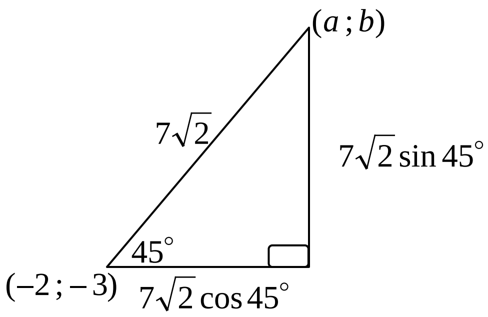
2.1	mean/*gemiddelde* $= \dfrac{2(0)+3(3)+4(2)+\ldots 12(1)}{30} = \dfrac{202}{30}$ $= 6{,}73$	✓202 ✓ answ/*antw* (2)
2.2	median/*mediaan* $= \dfrac{T_{15}+T_{16}}{2} = \dfrac{7+7}{2} = 7$	✓✓ answ/*antw* (2)
2.3	SD/*SA* $= 2{,}264... \approx 2{,}26$	✓✓ answ/*antw* (2)
2.4	$(6{,}73 - 2{,}26\ ;\ 6{,}73 + 2{,}26)$ $= (4{,}47\ ;\ 8{,}99)$ $\therefore 4 + 4 + 8 + 3 = 19$ times/*keer*	✓lower boundary ✓upper boundary ✓ answ/*antw* (3) **[9]**

QUESTION/*VRAAG* 3

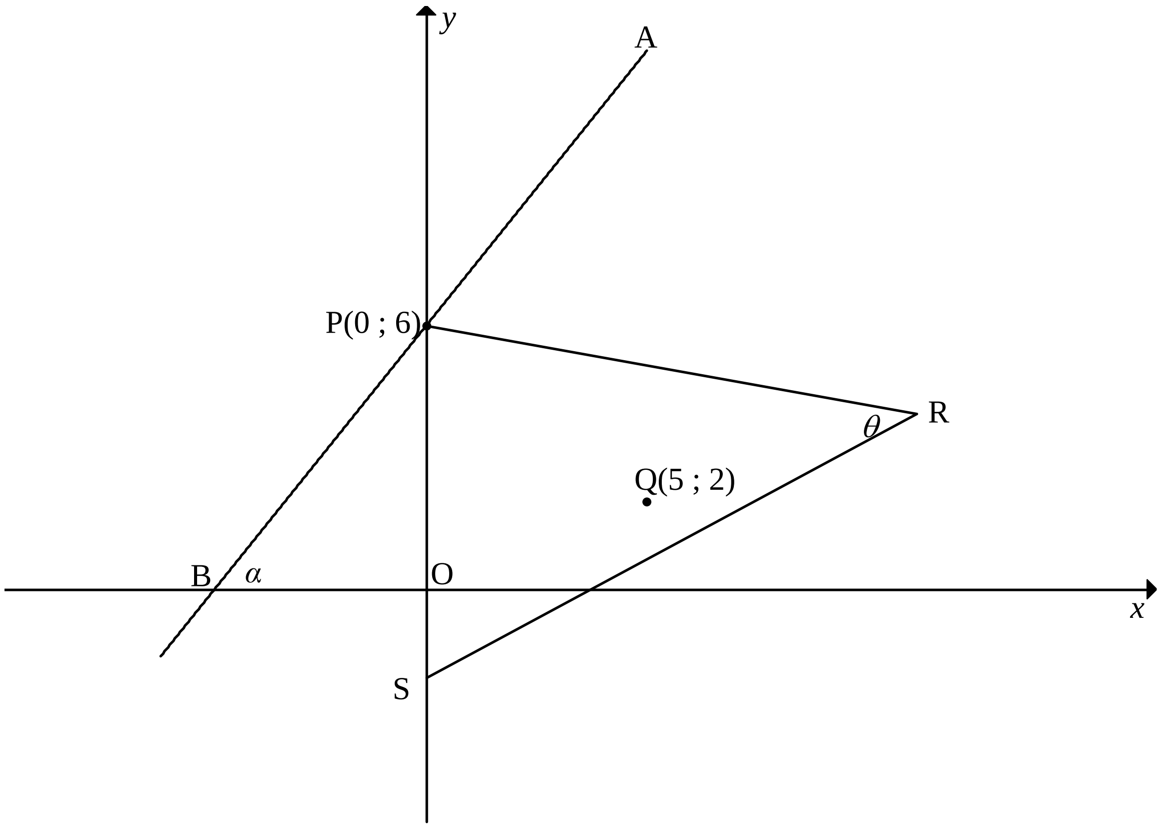


3.1	$m_{PQ} = \tan 45°$ $= 1$	✓ $m = \tan 45°$ ✓ answ/*antw* (2)
3.2	MN | | QP [midpt theorem/*midpt-stelling*] $\therefore m_{MN} = 1$ $\therefore y - y_1 = m(x - x_1)$ $\therefore y - 1 = 1(x - 7)$ $\therefore \quad y = x - 6$ **OR/*OF*** MN | | PQ [midpt theorem/*midpt-stelling*] $\therefore m_{MN} = 1$ $\therefore y = mx + c$ $\therefore 1 = 1(7) + c$ $-6 = c$ $\therefore \quad y = x - 6$	✓ S **OR** R ✓ m_{MN} ✓ subst m and/*en* N(7 ; 1) ✓ equation/*vgl* (4) ✓ S **OR** R ✓ m_{MN} ✓ subst m and/*en* N(7 ; 1) ✓ equation/*vgl* (4)
3.3	MN = $\frac{1}{2}$PQ [midpoint theorem/midp stelling] $\therefore$ MN = $\frac{7\sqrt{2}}{2} \approx 4{,}95$	✓ S ✓ answ/antw (2)

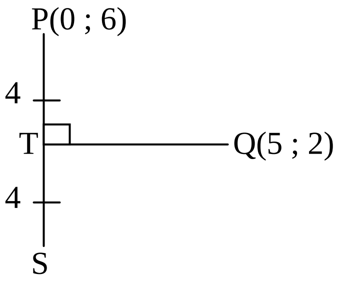
3.5	QN = NS [diag of ||m/hoekl van ||m] $\frac{-2+x_S}{2}=7$ and/en $\frac{-3+y_S}{2}=1$ $\therefore\ x_S = 16$ $\therefore\ y_S = 5$ **OR/*OF*** QN = NS [diag of ||m/hoekl van ||m] $\therefore$ by inspection/deur inspeksie: S(16 ; 5)	✓ method/metode ✓ x-value/waarde ✓ y-value/waarde (3) ✓ method/metode ✓ x-value/waarde ✓ y-value/waarde (3)
3.6	Equation of/*Vgl van PQ*: $y = x + c$ $-3 = -2 + c$ $y = x - 1$ $\therefore a = b + 1$ .....(1) From distance formula/*Van afstandsformule*: $PQ=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ $7\sqrt{2}=\sqrt{(a-(-2))^2+(b-(-3))^2}$ $\therefore 98=(a+2)^2+(b+3)^2$ .....(2) Subst (1) into (2): $98=(b+1+2)^2+(b+3)^2$ $98=b^2+6b+9+b^2+6b+9$ $0=2b^2+12b-80$ $0=b^2+6b-40$ $\therefore 0 = (b + 10)(b - 4)$ $\therefore\ b = 4$ (since $b > 0$) Subst $b = 4$ into (1): $\therefore a = 4 + 1 = 5$ $\therefore$ P(5 ; 4) **OR/*OF*** Equation of/*Vgl van PQ*: $y = x + c$ $-3 = -2 + c$ $y = x - 1$ $\therefore a = b + 1$ .....(1) From distance formula/*Van afstandsformule*: $7\sqrt{2}=\sqrt{(a-(-2))^2+(b-(-3))^2}$ $\therefore 98=(a+2)^2+(b+3)^2$ .....(2) Subst (1) into (2): $98=(b+1+2)^2+(b+3)^2$ $98=2(b+3)^2$ $49=(b+3)^2$ $\pm 7=b+3$ $\pm 7-3=b$ $\therefore\ b = 4$ (since $b > 0$) Subst $b = 4$ into (1): $\therefore a = 4 + 1 = 5$ $\therefore$ P(5 ; 4)	✓ eq of/*vgl van* PQ ✓ subst Q & $7\sqrt{2}$ into/*in* distance formula/ *afstandsformule* ✓ subst eq of/*vgl v.* PQ ✓ st form/*st vorm* ✓ value of/*waarde van b* ✓ value of/*waarde van a* (6) ✓ eq of/*vgl van* PQ ✓ subst Q & $7\sqrt{2}$ into/*in* distance formula/ *afstandsformule* ✓ subst eq of/*vgl v.* PQ ✓ simplification/ *vereenvoudig* ✓ value of/*waarde van b* ✓ value of/*waarde van a* (6)

	OR/*OF* Equation of/*Vgl van PQ*: $y = x + c$ $-3 = -2 + c$ $y = x - 1 \qquad \therefore a = b + 1$ .....(1) From distance formula/*Van afstandsformule*: $7\sqrt{2} = \sqrt{(a-(-2))^2 + (b-(-3))^2}$ $98 = (a+2)^2 + (a-1+3)^2$ $= 2(a+2)^2$ $\therefore$ $a + 2 = 7$ (since/*aangesien a* > 0) $\therefore$ $a = 5$ Subst $a = 4$ into (1): $\therefore b = 5 - 1 = 4$ $\therefore$ P(5 ; 4)	✓ eq of/*vgl van* PQ ✓ subst Q & $7\sqrt{2}$ into/*in* distance formula/ *afstandsformule* ✓ subst eq of/*vgl v.* PQ ✓ simplification/ *vereenvoudig* ✓ value of/*waarde van a* ✓ value of/*waarde van b* (6)
	OR/*OF*  $a = -2 + 7\sqrt{2}\cos 45^\circ = 5$ $b = -3 + 7\sqrt{2}\sin 45^\circ = 4$	✓✓✓✓ ✓ ✓ (6) **[17]**

QUESTION/*VRAAG* 4

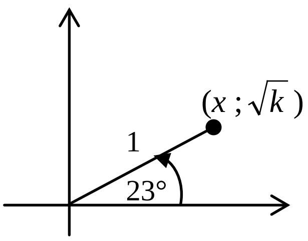


4.1	$(x-5)^2+(y-2)^2=r^2$ $(0-5)^2+(6-2)^2=r^2$ $25+16=r^2$ $41=r^2$ $\therefore\ (x-5)^2+(y-2)^2=41$ **OR/*OF*** $PQ=\sqrt{(0-5)^2+(6-2)^2}$ $=\sqrt{25+16}$ $r=\sqrt{41}$ $\therefore\ (x-5)^2+(y-2)^2=41$	✓ subst (5 ; 2) into circle eq/*in sirkelvgl* ✓ value of/*waarde van* r^2 ✓ equation/*vgl* (3) ✓ subst (5 ; 2) & (0 ; 6) into dist. form/*in afst. form* ✓ value of/*waarde van r* ✓ equation/*vgl* (3)
4.2	$(0-5)^2+(y-2)^2=41$ $25+(y-2)^2=41$ $25+y^2-4y+4=41$ $y^2-4y-12=0$ $(y-6)(y+2)=0$ $y\neq 6\quad or/of\quad y=-2$ $\therefore$ S(0 ; –2) or $y=-2$	✓ $x=0$ ✓ st form/*st. vorm* ✓ answ/*antw* (neg value) (3)

	OR/*OF* $(0-5)^2+(y-2)^2 = 41$ $25+(y-2)^2 = 41$ $(y-2)^2 = 16$ $y-2 = \pm 4$ $y = 2 \pm 4$ $y \neq 6$ or/*of* $y = -2$ $\therefore$ S(0 ; –2)	✓ $x = 0$ ✓ square form/ *kwadraatvorm* ✓ answ/*antw* (neg value)
	OR/*OF* Draw/*Trek* QT ⊥ PS PT = TS [line from centre ⊥ to chord/ *lyn van midpt ⊥ koord*] PT = $y_P - y_Q = 6-2 = 4$ $y_Q - y_S = 4$ $y_S = 2-4 = -2$ $\therefore$ S(0 ; –2) 	(3) ✓ $x = 0$ ✓✓ $y = -2$ (3)
4.3	$m_{PQ} = \dfrac{6-2}{0-5}$ $= -\dfrac{4}{5}$ $m_{PQ} \times m_{APB} = -1$ [tan/*raakl* ⊥ radius] $\therefore m_{APB} = \dfrac{5}{4}$ $\therefore y = \dfrac{5}{4}x + 6$	✓ subst (0 ; 6) & (5 ; 2) into grad form/*in grad. formule* ✓ m_{PQ} ✓ m_{APB} ✓ equation/*vgl* (4)
4.4	tan $\alpha = \dfrac{5}{4}$ $\therefore \alpha = 51{,}34°$	✓ tan $\alpha = m_{APB}$ ✓ answ/*antw* (2)
	OR/*OF* B(4,8 ; 0) $\therefore$ tan $\alpha = \dfrac{6}{4{,}8}$ $\therefore \alpha = 51{,}34°$	✓ tan $\alpha = \dfrac{6}{4{,}8}$ ✓ answ/*antw* (2)

4.5	$\theta = B\hat{P}S$ [tan-chord th/*raakl-koordst.*] $= 90° - \alpha$ [$\angle$ sum in Δ/$\angle$ *som van* Δ] $= 90° - 51{,}34°$ $= 38{,}66°$ **OR/*OF*** PS = 8 $PQ = SQ = \sqrt{41}$ $PS^2 = PQ^2 + SQ^2 - 2.PQ.SQ.\cos P\hat{Q}S$ $64 = 41 + 41 - 2.41.\cos P\hat{Q}S$ $\cos P\hat{Q}S = \frac{18}{82}$ $P\hat{Q}S = 77{,}32°$ $\theta = \frac{1}{2}P\hat{Q}S$ [$\angle$ at centre = 2 × $\angle$ circumf] $= 38{,}66°$	✓ S ✓ R ✓ $90° - \alpha$ ✓ answ/*antw* (4) ✓ correct subst into cosine rule ✓ $P\hat{Q}S = 77{,}32°$ ✓ R ✓ answ/*antw* (4)
4.6	Area $\Delta PQS = \frac{1}{2}$ PS ×height/*hoogte* $= \frac{1}{2}(8)(5)$ = 20 sq units/*vk eenh* **OR/*OF*** $P\hat{Q}S = 2 \times 38{,}66°$ [$\angle$at centre = 2×$\angle$ at circum/ *midpts$\angle$=2omtreks$\angle$*] $= 77{,}32°$ Area $\Delta PQS = \frac{1}{2}PQ.QS.\sin P\hat{Q}S$ $= \frac{1}{2}.\sqrt{41}.\sqrt{41}.\sin 77{,}32°$ = 20 sq units/*vk eenh*	✓ area formula/*e*: ΔPQS ✓ PS = 8 ✓ $\perp h = 5$ ✓ answ/*antw* (4) ✓ size of/*grootte v* $P\hat{Q}S$ ✓ area rule/*reël*: ΔPQS ✓ subst correctly/ *subst korrek* ✓ answ/*antw* (4) **[20]**

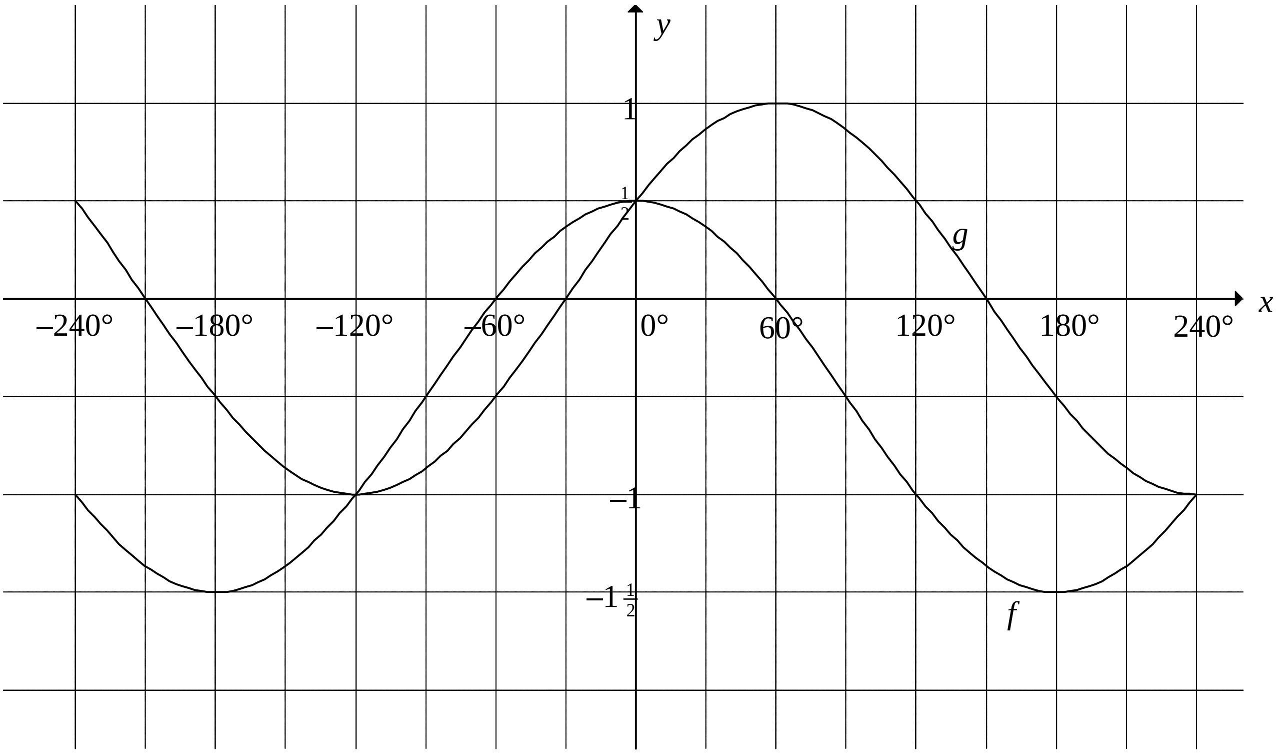
QUESTION/*VRAAG* 5

5.1.1	$\sin 203°$ $= -\sin 23°$ $= -\sqrt{k}$	✓ reduction/ *reduksie* ✓ answ ito/*antw itv k* (2)
5.1.2	$\cos^2 23° = 1 - \sin^2 23°$ $= 1 - k$ $\cos 23° = \sqrt{1-k}$ **OR/*OF*** $x^2 + (\sqrt{k})^2 = 1$ $x^2 = 1 - k$ $x = \sqrt{1-k}$ $\cos 23° = \dfrac{\sqrt{1-k}}{1} = \sqrt{1-k}$  	✓identity/*identiteit* ✓ $\cos^2 23°$ ito/*itv k* ✓ answ/*antw* (3) ✓ $x^2 = 1 - k$ ✓ x ito/*itv k* ✓ answ/*antw* (3)
5.1.3	$\tan(-23°) = -\tan 23°$ $= -\dfrac{\sin 23°}{\cos 23°}$ $= -\dfrac{\sqrt{k}}{\sqrt{1-k}} = -\sqrt{\dfrac{k}{1-k}}$ **OR/*OF*** $\tan(-23°) = -\tan 23°$ $= -\dfrac{\sqrt{k}}{\sqrt{1-k}} = -\sqrt{\dfrac{k}{1-k}}$	✓ reduction/ *reduksie* ✓ answ ito/*antw itv k* (2) ✓ reduction/ *reduksie* ✓ answ ito/*antw itv k* (2)
5.2	$\dfrac{4\cos x.(-\sin x)}{\sin(30° - x + x)}$ $= \dfrac{-4\sin x.\cos x}{\sin 30°}$ $= \dfrac{-4\sin x.\cos x}{\frac{1}{2}}$ $= -8\sin x.\cos x$ $= -4(2\sin x.\cos x)$ $= -4\sin 2x$	✓ $\cos x$ ✓$-\sin x$ ✓ $\sin(\alpha + \beta)$ ✓ $\dfrac{1}{2}$ ✓ double sine form */ dubbel sin form* ✓ answ/*antw* (6)

	OR/OF $\dfrac{4\cos x.(-\sin x)}{(\sin 30°\cos x-\cos 30°\sin x)\cos x+(\cos 30°\cos x+\sin 30°\sin x)\sin x}$ $=\dfrac{-4\sin x.\cos x}{(\frac{1}{2}\cos x-\frac{\sqrt{3}}{2}\sin x)\cos x+(\frac{\sqrt{3}}{2}\cos x+\frac{1}{2}\sin x)\sin x}$ $=\dfrac{-2(2\sin x.\cos x)}{\frac{1}{2}\cos^2 x+\frac{1}{2}\sin^2 x}$ $=\dfrac{-2(2\sin x.\cos x)}{\frac{1}{2}(\cos^2 x+\sin^2 x)}$ $=\dfrac{-2(2\sin x.\cos x)}{\frac{1}{2}(1)}$ $=-8\cos x\sin x$ $=-4(2\sin x\cos x)$ $=-4\sin 2x$	✓ $\cos x$ ✓ $-\sin x$ ✓ $\frac{1}{2}\cos^2 x+\frac{1}{2}\sin^2 x$ ✓ $\frac{1}{2}$ ✓ double sine form / *dubbel sin form* ✓ answ/*antw* (6)

5.3	$\cos 2x - 7\cos x - 3 = 0$ $2\cos^2 x - 1 - 7\cos x - 3 = 0$ $2\cos^2 x - 7\cos x - 4 = 0$ $(2\cos x + 1)(\cos x - 4) = 0$ $\therefore \cos x = -\frac{1}{2}$ or/*of* $\cos x = 4$ (no solution) $\therefore x = 120° + n.360°$ or/*of* $x = 240° + n.360°$; $n \in Z$ **OR/OF** $\therefore x = \pm 120° + n.360°$; $n \in Z$	✓ expansion/ *uitbreiding* ✓ $2\cos^2 x - 7\cos x - 4 = 0$ ✓ factors/*faktore* ✓ $\cos x = -\frac{1}{2}$ ✓ 120° & 240° ✓ + n.360° **OR/OF** ✓ ±120° ✓ + n.360° (6)
5.4	$\sin 3\theta = \sin(2\theta + \theta)$ $= \sin 2\theta \cos\theta + \cos 2\theta \sin\theta$ $= 2\sin\theta\cos\theta\cos\theta + (1 - 2\sin^2\theta)\sin\theta$ $= 2\sin\theta(1 - \sin^2\theta) + \sin\theta - 2\sin^3\theta$ $= 3\sin\theta - 4\sin^3\theta$ $= 3(\frac{1}{3}) - 4(\frac{1}{3})^3$ $= 1 - \frac{4}{27}$ $= \frac{23}{27}$	✓ expansion of/ *uitbreiding van* $\sin(2\theta + \theta)$ ✓ expansions of $\sin 2\theta$ AND $\cos 2\theta$ ✓ $1 - \sin^2\theta$ ✓ subst ✓ answ/*antw* (5) **[24]**

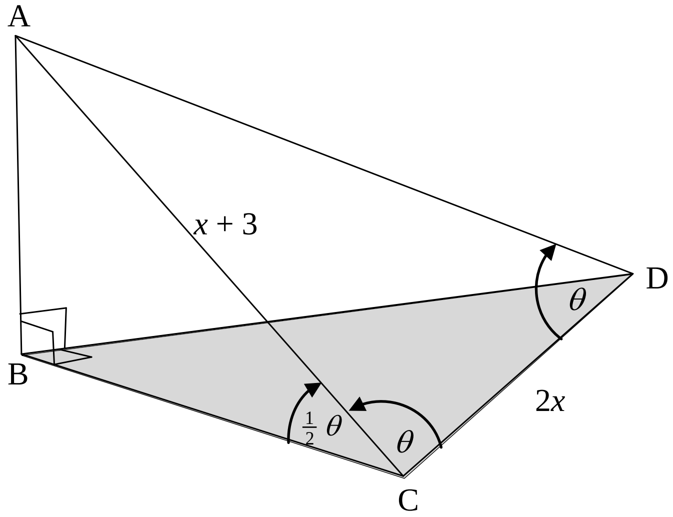
QUESTION/*VRAAG* 6

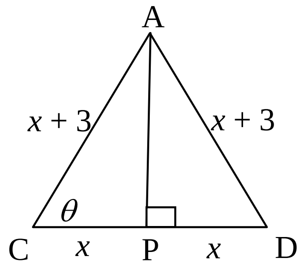


6.1	$f(x) = \cos x - \frac{1}{2}$ and/*en* $g(x) = \sin(x + 30°)$ $\therefore\ p = 30°$ and/*en* $q = -\frac{1}{2}$ **OR/*OF*** $\sin(60° + p) = 1$ and/*en* $\cos 0° + q = \frac{1}{2}$ $\therefore\ p = 30°$ $\therefore\ q = -\frac{1}{2}$	✓ $f(x) = \cos x - \frac{1}{2}$ ✓ $g(x) = \sin(x + 30°)$ ✓ value of/*waarde v p* ✓ value of/*waarde v q* (4) ✓ $\sin(60° + p) = 1$ ✓ $\cos 0° + q = \frac{1}{2}$ ✓ value of/*waarde v p* ✓ value of/*waarde v q* (4)
6.2	$x \in (-120° ; 0°)$ **OR/*OF*** $-120° < x < 0°$	✓ critical values/ *kritiese waardes* ✓ correct interval/ *korrekte interval* (2)

6.3	The graph of *g* has to shift 60° to the left and then be reflected about the *x*-axis./*Die grafiek van g moet 60° na links skuif en dan om die x-as gereflekteer word.* **OR/*OF*** The graph of *g* must be reflected about the *x*-axis and then be shifted 60° to the left./*Die grafiek van g moet om die x-as gereflekteer word en dan met 60° na links geskuif word.* **OR/*OF*** The graph of *g* has to shift 120° to the right./*Die grafiek van g moet 120° na regs geskuif word.* **OR/*OF*** The graph of *g* has to shift 240° to the left./*Die grafiek van g moet met 240° na links geskuif word*	✓ 60° left/*links* ✓ reflection about *x*-axis/*refleksie om x-as* (2) ✓ reflection about *x*-axis/*refleksie om x-as* ✓ 60° left/*links* (2) ✓ ✓ 120° right/*regs* (2) ✓ ✓ 240° left/*links* (2) **[8]**

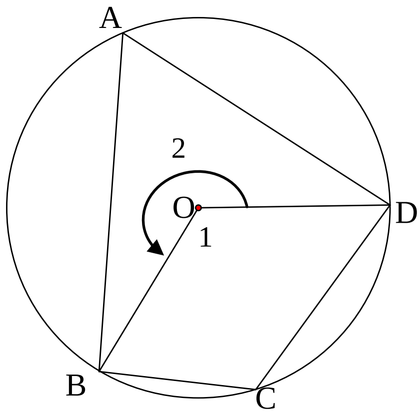
QUESTION/*VRAAG* 7



7.1	$\mathrm{C\hat{A}D} = 180° - 2\theta$ [$\angle$s sum of Δ/$\angle$*e som van* Δ]	✓ answ/*antw* (1)
7.2	$\frac{\sin\theta}{x+3} = \frac{\sin(180° - 2\theta)}{2x}$ $\frac{\sin\theta}{x+3} = \frac{\sin 2\theta}{2x}$ $\frac{\sin\theta}{x+3} = \frac{2\sin\theta.\cos\theta}{2x}$ $\cos\theta = \frac{2x\sin\theta}{2(x+3)\sin\theta}$ $\cos\theta = \frac{x}{x+3}$	✓ correct subst into sine rule/*korrekte subst in sin-reël* ✓ sin 2θ ✓ 2 sinθ. cos θ ✓ cos θ as subject/ *as onderwerp* (4)
	OR/*OF* AD = x + 3 [sides opp = $\angle$s/*sye to* = $\angle$*e*] $AC^2 = AD^2 + CD^2 - 2AD.CD.\cos\theta$ $(x+3)^2 = (x+3)^2 + (2x)^2 - 2(2x)(x+3).\cos\theta$ $0 = 4x^2 - 4x(x+3)\cos\theta$ $\cos\theta = \frac{4x^2}{4x(x+3)}$ $= \frac{x}{x+3}$	✓ AD = x + 3 ✓ correct subst into cosine rule/*korrekte subst in cos-reël* ✓ simplification/ *vereenvoudiging* ✓ cos θ as subject/ *as onderwerp* (4)
	OR/*OF* Draw/*Trek* AP $\perp$ CD cos $\theta = \frac{x}{x+3}$ 	✓ ✓ constr/*konstr* ✓ ✓ sketch shown/ *toon skets* (4)

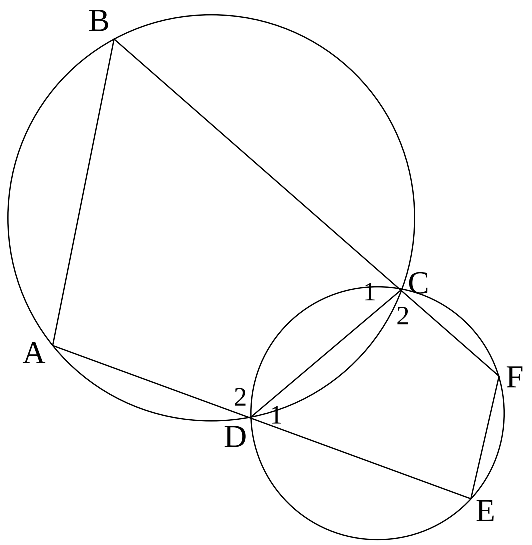
7.3	$\cos\theta = \frac{2}{5}$ $\therefore\ \theta = 66{,}42°$ In ΔABC: $\sin\ \frac{1}{2}\theta = \frac{AB}{AC}$ $\sin 33{,}21° = \frac{AB}{5}$ $\therefore\ AB = 5\sin 33{,}21°$ $= 2{,}74$ **OR/*OF*** $\sin\ \frac{\theta}{2} = \frac{AB}{5}$ $\therefore\ AB = 5\sin\ \frac{\theta}{2}$ but/*maar*: $\cos\theta = \frac{2}{5}$ $1 - 2\sin^2\frac{\theta}{2} = \frac{2}{5}$ $\sin^2\frac{\theta}{2} = \frac{3}{10}$ $\sin\frac{\theta}{2} = \sqrt{\frac{3}{10}}$ $\therefore\ AB = 5\sqrt{\frac{3}{10}} = \sqrt{\frac{15}{2}} = 2{,}74$	✓ $\cos\theta = \frac{2}{5}$ ✓ size of/*grootte v* θ ✓ correct ratio/ *korrekte verh* ✓ subst correctly/ *korrek* ✓ answ/*antw* (5) ✓ AB = 5 sin $\frac{\theta}{2}$ ✓ equation/*vgl* ✓ simplification/ *vereenvoudiging* ✓ value of/*waarde v* sin $\frac{\theta}{2}$ ✓ answ/*antw* (5) **[10]**

QUESTION/*VRAAG* 8



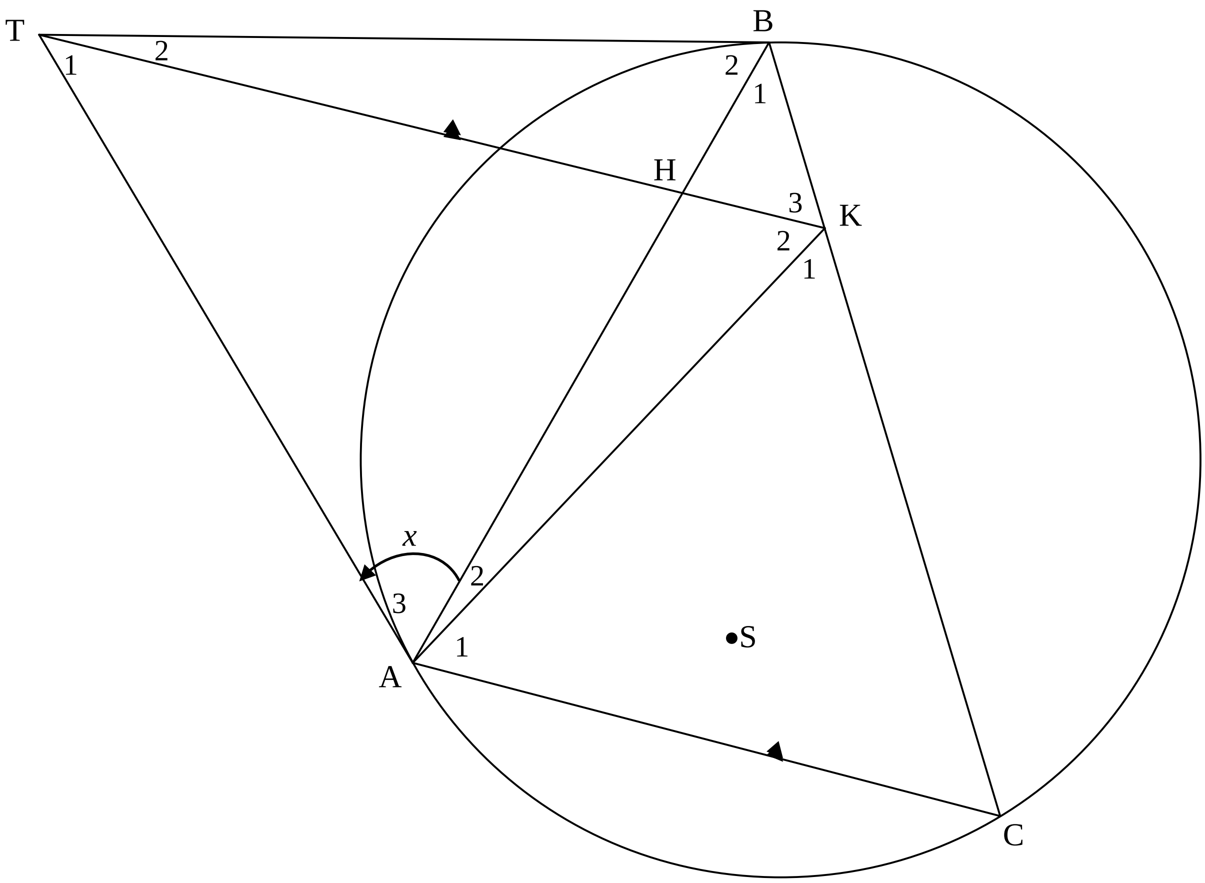
8.1.1	twice or double /*twee keer of dubbel*	✓ R (1)
8.1.2	$\hat{O}_1 = 2\hat{A}$ [∠ at centre = 2×∠ at circ/*midpts∠ =2×omtreks∠*] $\hat{O}_2 = 2\hat{C}$ [∠ at centre = 2×∠ at circ/*midpts∠ =2×omtreks∠*] $\hat{O}_1 + \hat{O}_2 = 360°$ [∠s in a rev/*∠e in omw of om 'n pt*] $2\hat{A} + 2\hat{C} = 360°$ $\therefore \hat{A} + \hat{C} = 180°$ **OR/OF** Let/*Gestel* $\hat{O}_1 = 2x$ $\hat{A} = x$ [∠ at centre = 2×∠ at circ/*midpts∠ =2×omtreks∠*] $\hat{O}_2 = 360° - 2x$ [∠s in a rev/*∠e in omw of om 'n pt*] $\hat{C} = 180° - x$ [∠ at centre = 2×∠ at circ/*midpts∠ =2×omtreks∠*] $\therefore \hat{A} + \hat{C} = 180°$	✓ S ✓ S ✓ S (3) ✓ S ✓ S ✓ S (3)

8.2



8.2	$\hat{A} = \hat{C}_2$ [ext $\angle$ of cyclic quad/*buite$\angle$ v kdvh*] $\hat{E} = 180° - \hat{C}_2$ [opp $\angle$s of cyclic quad/*tos$\angle$e v kdvh*] $\therefore\ \hat{E} = 180° - \hat{A}$ $\therefore$ EF || AB [co-interior $\angle$s 180°/*ko-binne$\angle$e 180°*] **OR/*OF*** $\hat{B} = \hat{D}_1$ [ext $\angle$ of cyclic quad/*buite$\angle$ v kdvh*] $\hat{F} = 180° - \hat{D}_1$ [opp $\angle$s of cyclic quad/*tos$\angle$e v kdvh*] $\therefore\ \hat{F} = 180° - \hat{B}$ $\therefore$ EF || AB [co-interior $\angle$s 180°/*ko-binne$\angle$e 180°*]	✓ S ✓ R ✓ S ✓ R ✓ R (5) ✓ S ✓ R ✓ S ✓ R ✓ R (5) **[9]**

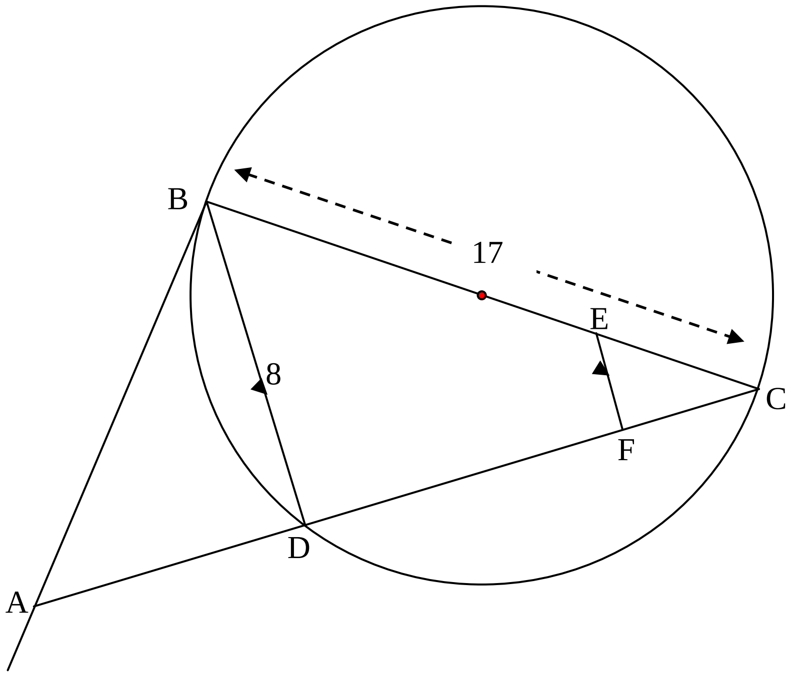
QUESTION/*VRAAG* 9



9.1	$\hat{K}_3 = \hat{C}$ $= \hat{A}_3$ $= x$	[corresp ∠s/*ooreenk ∠e* ; CA| |KT] [tan-chord th/*raakl-koordst*]	✓ S ✓ R ✓ S ✓ R (4)
9.2	$\hat{K}_3 = x = \hat{A}_3$ ∴ AKBT is cyc quad	[proved/*bewys* in 9.1] [line (BT) subtends equal ∠s/ *lyn (BT) onderspan gelyke ∠e*] **OR/OF** [converse ∠s in same segment/ *omgek ∠e in dies segment*]	✓ S ✓ R (2)
9.3	$\hat{K}_3 = \hat{C}$ $= \hat{B}_2$ $= \hat{K}_2$ ∴ TK bisects/*halveer* $A\hat{K}B$ **OR/*OF*** $\hat{K}_2 = \hat{B}_2$ $= \hat{A}_3$	[proven in 9.1] [tan-chord th/*raakl-koordst*] [∠s in the same segm/*∠e in dies segm*] [∠s in the same seg/*∠e in dies segm*] [tans from same pt; ∠s opp equal sides/ *rkle v dies pt; ∠e to gelyke sye*]	 ✓ S ✓ R ✓ S ✓ R (4) ✓ S ✓ R ✓ S ✓ R

	$\therefore \quad = \hat{K}_3$ [proven in 9.1] $\therefore$ TK bisects/*halveer* $A\hat{K}B$	(4)
9.4	$\hat{A}_3 = \hat{K}_2 = x$ [proven/*bewys*] $\therefore$ TA tangent [converse tan chord theorem OR $\angle$ between line and chord/ *omgekeerde raakl-kdst* ***OF*** $\angle$ *tussen lyn en koord*]	✓ S ✓ R (2)
9.5	$B\hat{S}A = B\hat{K}A = 2x$ [A,S,K & B concyclic/*konsiklies*] $A\hat{T}B = 180° - 2x$ [A,T,B & K concyclic/*konsiklies*] $\therefore$ points A, S, B and T are also concyclic/*punte A, S, B en T is ook konsiklies* [opp $\angle$s of quad = 180°/*tos* $\angle$*e van vierhoek=180°*] **OR/*OF*** A, S K and B are concyclic. A, K, B and T are concyclic. $\therefore$ A, S, B and T are concyclic. **OR/*OF*** The circle passing through points A, K and B contains the point S on the circumference (A, ,S, K and B concyclic)./*Die sirkel deur punt A, K en B bevat die punt S op die omtrek (A, S, K en B konsiklies).* The circle passing through A, K and B contains the point T on the circumference (proven in 9.2)./*Die sirkel deur punt A, K en B bevat die punt T op die omtrek (bewys in 9.2).* $\therefore$ points A, S, B and T are also concyclic/*punte A, S, B en T is konsiklies*	✓ S (both/*beide* statements/ *bewerings*) ✓ R (2) ✓ S ✓ S (2) ✓ S ✓ S (2) **[14]**

QUESTION/*VRAAG* 10



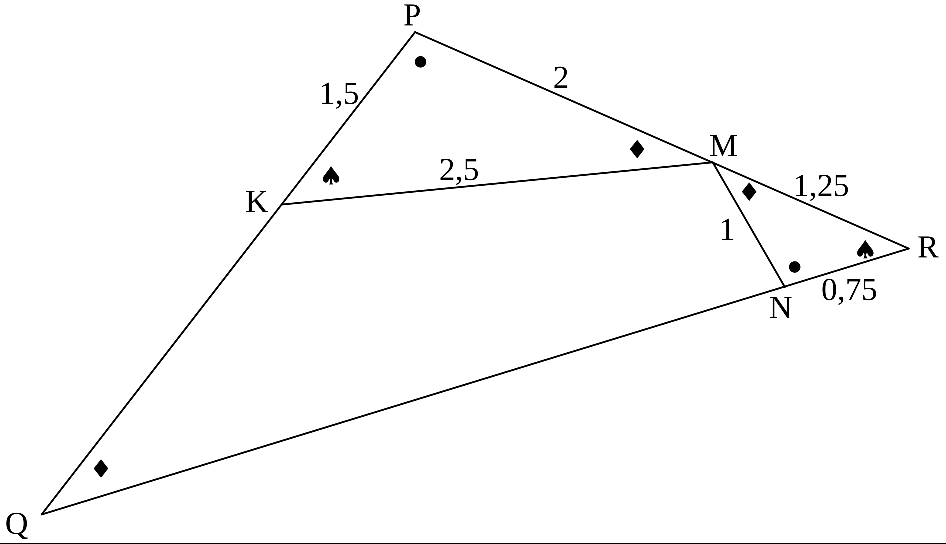
10.1	$B\hat{D}C = 90°$ [$\angle$ in semi circle/$\angle$ *in halfsirkel*] $DC^2 = 17^2 - 8^2$ [Th of/*stelling v* Pythagoras] $= 225$ $\therefore$ DC = 15	✓ S ✓ using/*gebruik* Pyth *korrek/* correctly ✓ answ/*antw* (3)
10.2.1	$\frac{CF}{CD} = \frac{CE}{CB}$ [line | | one side of Δ/*lyn* | | *een sy van* Δ] **OR/OF** $\Delta CEF \mid\mid\mid \Delta CBD$ $\therefore \frac{CF}{15} = \frac{1}{4}$ $\therefore$ CF = 3,75	✓ S/R ✓ subst correctly/ *korrek* ✓ answ/*antw* (3)
10.2.2	$B\hat{D}C = 90°$ [$\angle$ in semi circle/$\angle$ *in halfsirkel*] $E\hat{F}C = B\hat{D}C$ [corresp $\angle$s/*ooreenk* $\angle e$; EF| |BD] $A\hat{B}C = 90°$ [tan $\perp$ diameter/*raakl* $\perp$ *middellyn*] In ΔBAC and/*en* Δ FEC: $A\hat{B}C = E\hat{F}C$ [proven/bewys] $\hat{C} = \hat{C}$ [common/*gemeen*] $\therefore \Delta BAC \mid\mid\mid \Delta FEC$ [$\angle\angle\angle$] **OR/*OF*** $B\hat{D}C = 90°$ [$\angle$ in semi circle/$\angle$ *in halfsirkel*] $E\hat{F}C = B\hat{D}C$ [corresp $\angle$s/*ooreenk* $\angle e$; EF| |BD] $A\hat{B}C = 90°$ [tan $\perp$ diameter/*raakl* $\perp$ *middellyn*] In ΔBAC and/*en* Δ FEC: $A\hat{B}C = E\hat{F}C$ [proven/bewys] $\hat{C} = \hat{C}$ [common/*gemeen*]	✓ S/R ✓ S ✓ R ✓ S ✓ R (5) ✓ S/R ✓ S ✓ R ✓ S

	$\hat{BAC} = \hat{FEC}$ [$\angle$ sum in Δ/$\angle$ *som van* Δ] $\therefore \Delta BAC \mid\mid\mid \Delta FEC$	✓ S (5)
10.2.3	$EC = \frac{1}{4} \times 17 = 4{,}25$ $\frac{AC}{EC} = \frac{BC}{FC}$ [$\Delta BAC \mid\mid\mid \Delta FEC$] $\frac{AC}{4{,}25} = \frac{17}{3{,}75}$ $\therefore AC = 19{,}27$ or/*of* $19\frac{4}{15}$ **OR/*OF*** $\cos\ \hat{C} = \frac{CF}{CE} = \frac{BC}{AC}$ $\therefore\ \frac{3{,}75}{4{,}25} = \frac{17}{AC}$ $\therefore\ AC = 19{,}27$ or/*of* $19\frac{4}{15}$ **OR/*OF*** $\Delta BCA \mid\mid\mid \Delta DBC$ $CB^2 = CD\ .\ AC$ $AC = \frac{BC^2}{DC}$ $= \frac{17^2}{15}$ $= 19{,}27$ or/*of* $19\frac{4}{15}$ **OR/*OF*** $\hat{C} = A\hat{B}D$ [tan-chord theorem/*rkl-kdstelling*] $\frac{AD}{8} = \tan A\hat{B}D$ $= \tan\hat{C}$ $= \frac{8}{15}$ $\therefore\ AD = \frac{64}{15}$ $\therefore\ AC = 19{,}27$ or/*of* $19\frac{4}{15}$	✓ length of/*lengte v* EC ✓ S ✓ subst correctly/ *korrek* ✓ answ/*antw* (4) ✓✓ correct ratios/ *korrekte verh's* ✓ subst correctly/ *korrek* ✓ answ/*antw* (4) ✓ S **OR** Pyth th ✓ correct ratio ✓ subst ✓ answ/*antw* (4) ✓ S ✓ correct ratio ✓ subst ✓ answ/*antw* (4)

10.2.4	AC is diameter of the circle passing through A, B and C [chord subtends 90° **OR** converse $\angle$ in semi circle] *AC is middellyn van die sirkel wat deur die punte A, B en C gaan* [*koord onderspan 90°* ***OF*** *omgek* $\angle$ *in halfsirkel*] $\therefore$ radius = $\frac{1}{2}\times 19{,}27 = 9{,}63$ or/*of* $9\frac{19}{30}$ or/*of* $\frac{1}{2}\text{AC}$	✓ S/R ✓ answ/*antw* (2) **[17]**

QUESTION/*VRAAG* 11

11.1	equiangular or similar/*gelykhoekig of gelykvormig*	✓ answ/*antw* (1)



11.2.1	$\frac{KP}{RN}=\frac{1,5}{0,75}=2$; $\frac{PM}{NM}=\frac{2}{1}=2$; $\frac{KM}{RM}=\frac{2,5}{1.25}=2$ $\therefore\ \frac{KP}{RN}=\frac{PM}{NM}=\frac{KM}{RM}$ $\therefore\ \Delta KPM\ \|\|\|\ \Delta RNM$ [Sides of Δ in prop/*sye v Δ eweredig*] **OR/*OF*** $\frac{RN}{KP}=\frac{0,75}{1,5}=\frac{1}{2}$; $\frac{NM}{PM}=\frac{1}{2}$; $\frac{RM}{KM}=\frac{1,25}{2,5}=\frac{1}{2}$ $\therefore\ \frac{RN}{KP}=\frac{NM}{PM}=\frac{RM}{KM}$ $\therefore\ \Delta KPM\ \|\|\|\ \Delta RNM$ [Sides of Δ in prop/*sye v Δ eweredig*] **OR/*OF*** In ΔMNR: $1,25^2 = 1^2 + 0,75^2 = 1,5625$ $\therefore\ M\hat{N}R = 90°$ [converse Pyth theorem] In ΔPKM: $2,5^2 = 1,5^2 + 2^2 = 6,25$ $\therefore\ \hat{P} = 90°$ [converse Pyth theorem] $\cos\ P\hat{K}M=\frac{1,5}{2,5}=\frac{3}{5}$ and $\cos\ \hat{R}=\frac{0,75}{1,25}=\frac{3}{5}$ $\therefore\ P\hat{K}M=\hat{R}$ In ΔKPM and ΔRNM $P\hat{K}M=\hat{R}$ [proved] $\hat{P}=M\hat{N}R$ [proved] $\therefore \Delta KPM\|\|\|\Delta RNM$ [∠;∠;∠ OR 3rd ∠]	✓✓✓ all 3 statements/ *al 3 bewerings* (3) ✓✓✓ all 3 statements/ *al 3 bewerings* (3) ✓ $\hat{P}=M\hat{N}R$ ✓ $P\hat{K}M=\hat{R}$ ✓[∠;∠;∠ OR 3rd ∠] (3)

11.2.2	$P\hat{K}M = \hat{R}$ [$\Delta KPM \mid\mid\mid \Delta RNM$] $\therefore$ $\hat{P}$ is common/*gemeen* $\therefore \Delta RPQ \mid\mid\mid \Delta KPM$ [$\angle\angle\angle$] $\frac{RP}{KP} = \frac{RQ}{KM}$ [$\Delta RPQ \mid\mid\mid \Delta KPM$] $\therefore \frac{3{,}25}{1{,}5} = \frac{RQ}{2{,}5}$ $\therefore RQ = \frac{2{,}5 \times 3{,}25}{1{,}5} = 5{,}42$ or $5\frac{5}{12}$ $\therefore NQ = 5{,}42 - 0{,}75 = 4{,}67$ or $4\frac{2}{3}$	✓ S ✓ $\Delta RPQ \mid\mid\mid \Delta KPM$ ✓ S ✓ subst correctly/ *korrek* ✓ $RQ = 5\frac{5}{12}$ ✓ NQ = answ/*antw* (6)
	OR/OF	
	$R\hat{N}M = \hat{P}$ [$\Delta KPM \mid\mid\mid \Delta RNM$] $\therefore$ $\hat{R}$ is common/*gemeen* $\therefore \Delta RNM \mid\mid\mid \Delta RPQ$ [$\angle\angle\angle$] $\frac{RP}{RN} = \frac{RQ}{RM}$ [$\Delta RNM \mid\mid\mid \Delta RPQ$] $\therefore \frac{3{,}25}{0{,}75} = \frac{RQ}{1{,}25}$ $\therefore RQ = 5{,}42$ or $5\frac{5}{12}$ $\therefore NQ = 5{,}42 - 0{,}75 = 4{,}67$ or $4\frac{2}{3}$	✓ S ✓ $\Delta RNM \mid\mid\mid \Delta RPQ$ ✓ S ✓ subst correctly/ *korrek* ✓ $RQ = 5\frac{5}{12}$ ✓ NQ = answ/*antw* (6)
	OR/*OF* In ΔMNR: $1{,}25^2 = 1^2 + 0{,}75^2 = 1{,}5625$ $\therefore M\hat{N}R = 90°$ [converse Pyth theorem] In ΔPKM: $2{,}5^2 = 1{,}5^2 + 2^2 = 6{,}25$ $\therefore \hat{P} = 90°$ [converse Pyth theorem] In ΔMNR and ΔQPR $\angle R$ is common $M\hat{N}R = \hat{P} = 90°$ $\therefore \Delta MNR \mid\mid\mid \Delta QPR$ [$\angle\angle\angle$] $\frac{RP}{RN} = \frac{RQ}{RM}$ [$\Delta RNM \mid\mid\mid \Delta RPQ$] $\therefore \frac{3{,}25}{0{,}75} = \frac{RQ}{1{,}25}$ $\therefore RQ = 5{,}42$ or $5\frac{5}{12}$ $\therefore NQ = 5{,}42 - 0{,}75 = 4{,}67$ or $4\frac{2}{3}$	✓ S ✓ $\Delta MNR \mid\mid\mid \Delta QPR$ ✓ S ✓ subst correctly/ *korrek* ✓ $RQ = 5\frac{5}{12}$ ✓ NQ = answ/*antw* (6) **[10]**

TOTAL/*TOTAAL*: 149

basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE/GRAAD 12

MATHEMATICS P2/*WISKUNDE V2*

FEBRUARY/MARCH/*FEBRUARIE/MAART* 2015

MEMORANDUM

MARKS/*PUNTE*: 150

This memorandum consists of 24 pages.
Hierdie memorandum bestaan uit 24 bladsye.

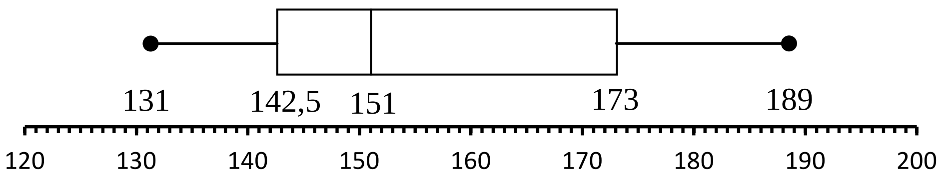
NOTE:

- If a candidate answers a question TWICE, only mark the FIRST attempt.
- If a candidate has crossed out an attempt of a question and not redone the question, mark the crossed out version.
- Consistent accuracy applies in ALL aspects of the marking memorandum.
- Assuming answers/values in order to solve a problem is NOT acceptable.

NOTA:

- *As 'n kandidaat 'n vraag TWEEKEER beantwoord, merk slegs die EERSTE poging.*
- *As 'n kandidaat 'n poging om die vraag te beantwoord, doodgetrek het en nie dit oorgedoen het nie, merk die doodgetrekte poging.*
- *Volgehoue akkuraatheid word in ALLE aspekte van die nasienmemorandum toegepas.*
- *Aanvaarding van antwoorde/waardes om 'n probleem op te los, is ONaanvaarbaar.*

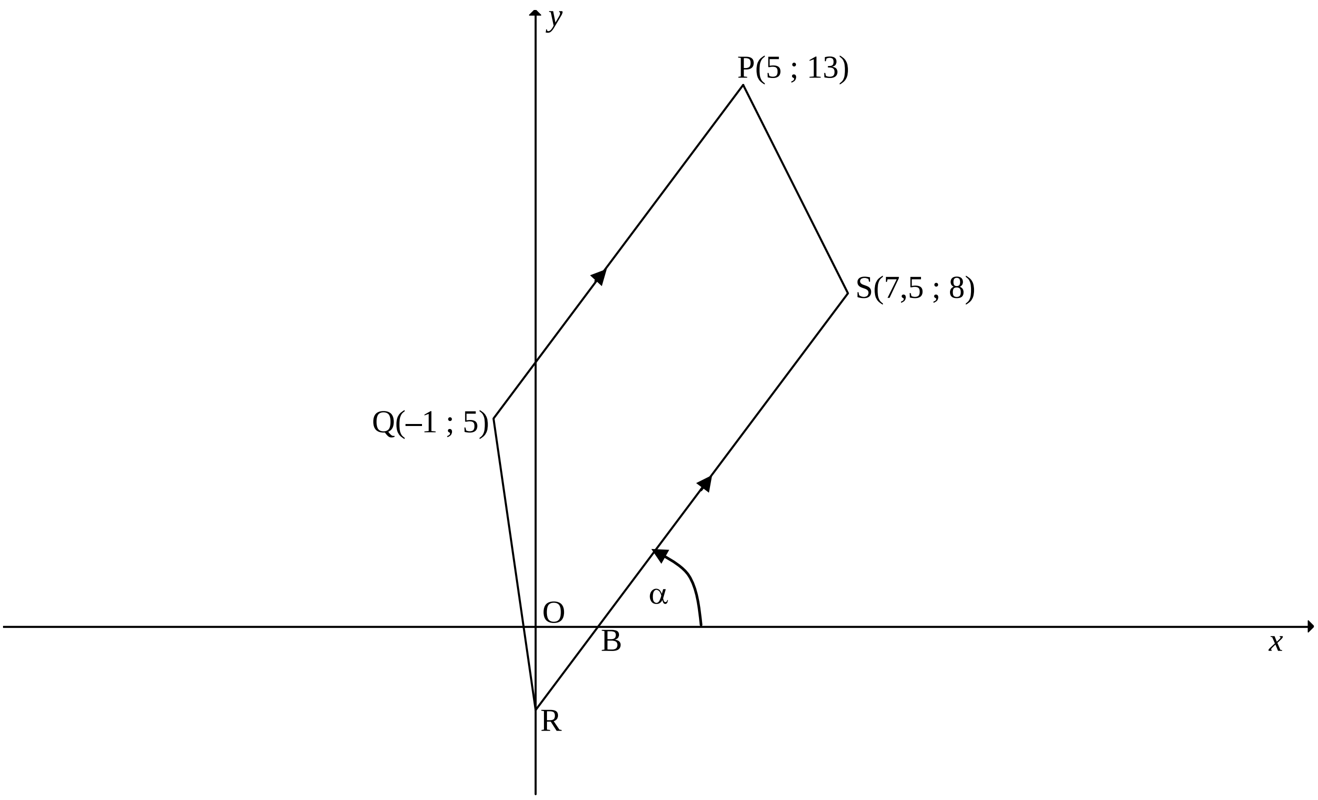
QUESTION/*VRAAG* 1

1.1	$\bar{x} = \frac{3310}{21}$ $= 157,62$ Answer only: Full marks *slegs antw: volpunte*	✓ $\frac{3310}{21}$ ✓157,62 (2)
1.2	(131 ; 142,5 ; 151 ; 173 ; 189)	✓ 131 and/ *en* 189 ✓ 142,5 ✓ 173 ✓ 151 (4)
1.3		✓box/*mond* ✓ whiskers/ *snor* (2)
1.4	positively skewed/*positief skeef* **OR/*OF*** skewed to the right/*skeef na regs*	✓ answer/ *antwoord* (1)
1.5	σ = 17,27	✓✓answer/ *antwoord* (2)
1.6.1	$\bar{x} = 157,62 + p$	✓ answer (1)
1.6.2	σ = 17,27	✓ answer/ *antwoord* (1) **[13]**

QUESTION/*VRAAG* 2

2.1	As the temperature increases, the sales of ice-creams increase/*Soos die temperatuur styg, neem die verkope toe.* **OR/*OF*** As the temperature decreases, the sales of ice-creams decrease/*Soos die temperatuur daal, neem die verkope af.*	✓ reason/*rede* (1) ✓ reason/*rede* (1)
2.2	The liveable temperature cannot keep on increasing/*Die leefbare temperatuur kan nie aanhou styg nie*.	✓ reason/*rede* (1)
2.3	$a = -460{,}35$ $b = 30{,}09$ $\hat{y} = 30{,}09x - 460{,}35$ **OR/*OF*** $\hat{y} = -460{,}35 + 30{,}09x$ Answer only: Full marks *slegs antw: volpunte*	✓✓ –460,35 ✓ 30,09 ✓ equation/*vgl* (4)
2.4	$r = 0{,}96$	✓ 0,96 (1)
2.5	There is a <u>very strong</u> positive relationship (correlation)/*Daar is 'n <u>baie sterk</u> positiewe verband (korrelasie).*	✓ very strong/*baie sterk* (1) **[8]**

QUESTION/VRAAG 3

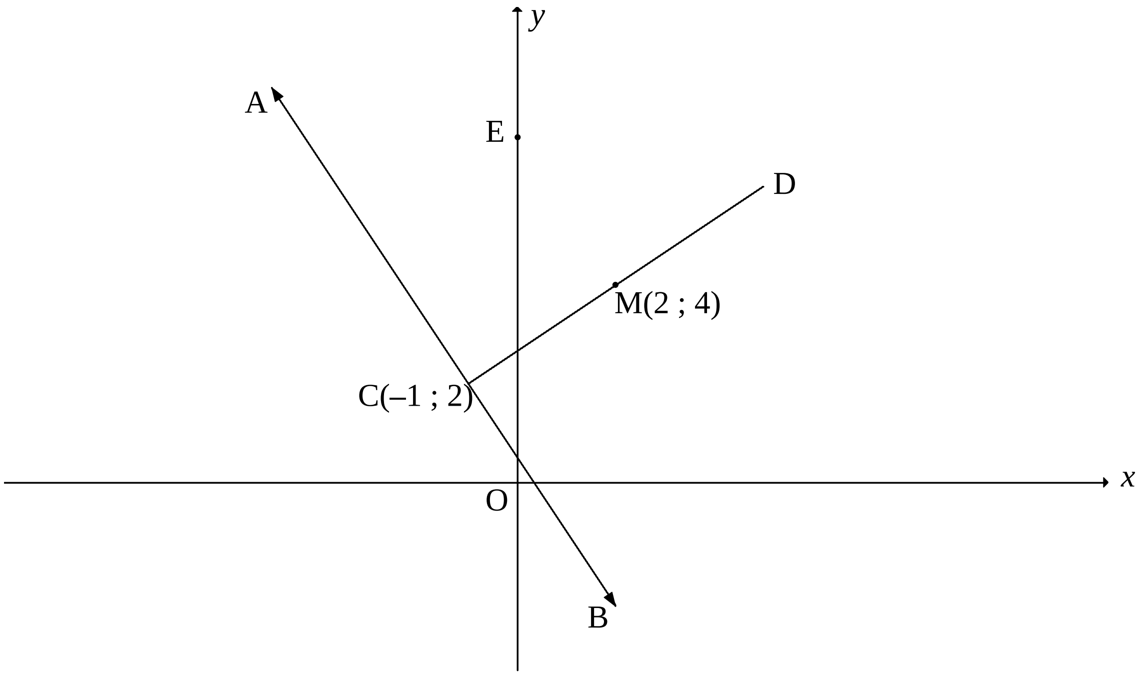


3.1	$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $= \sqrt{(5+1)^2 + (13-5)^2}$ $= 10$	✓ use of distance formula/*gebruik afstandformule* ✓ correct subst into form/*korrekte subst in formule* ✓ 10 (3)
3.2	$m_{PQ} = \frac{y_2 - y_1}{x_2 - x_1}$ $= \frac{13-5}{5-(-1)}$ $= \frac{8}{6} = \frac{4}{3}$ Answer only: Full marks *slegs antw: volpunte*	✓ correct subst into gradient formula/ *korrekte subst in gradiëntformule* ✓ gradient/*gradiënt* (2)

3.3	Equation of line RS/*Vgl van lyn RS*: $m_{RS} = m_{PQ} = \frac{4}{3}$ (= gradients, | | lines/=*gradiënte*, | | *lyne*) $y = mx + c$ $8 = \frac{4}{3}\left(\frac{15}{2}\right) + c$ $c = -2$ $y = \frac{4}{3}x - 2$ $\therefore 4x - 3y - 6 = 0$ **OR/*OF*** $y - y_1 = m(x - x_1)$ $y - 8 = \frac{4}{3}\left(x - \frac{15}{2}\right)$ $y = \frac{4}{3}x - 2$ $\therefore 4x - 3y - 6 = 0$	✓ $m_{RS} = \frac{4}{3}$ ✓ subst of S(7,5 ; 8) and *m* into eq /*subst van S(7,5 ; 8) en m in vgl* ✓ value of *c* /*waarde van c* or/of st form/*st vorm* ✓ equation/*vgl* (4)
3.4	B is the *x*-intercept of/*is die x-afsnit van* $y = \frac{4}{3}x - 2$ $0 = \frac{4}{3}x - 2$ $4x - 6 = 0$ $x = \frac{3}{2}$ **OR/*OF*** $4x - 3(0) - 6 = 0$ $4x - 6 = 0$ $x = \frac{3}{2}$	✓ $y = 0$ ✓ $x = \frac{3}{2}$ (2)
3.5	$\tan\ \alpha = \frac{4}{3}$ $\alpha = 53{,}13° = \mathrm{O}\hat{\mathrm{B}}\mathrm{R}$ (vert opp ∠s/*regoorst ∠e*) $\mathrm{O}\hat{\mathrm{R}}\mathrm{B} = 180° - (90° + 53{,}13°)$ (∠s of Δ/*∠e van Δ*) $= 36{,}87°$	✓ $\tan\ \alpha = \frac{4}{3}$ ✓ 53,13° ✓ 36,87° (3)
3.6	$\mathrm{BS} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $= \sqrt{\left(\frac{15}{2} - \frac{3}{2}\right)^2 + (8 - 0)^2}$ $= 10$ PQ || BS and/*en* PQ = BS PQBS = parallelogram (1 pair opp sides = and ||/*1 pr tos sye =en* ||) **OR/*OF*** midpoint of/*midpt van* QS: $\left(\frac{-1 + 7.5}{2}; \frac{5 + 8}{2}\right) = \left(\frac{13}{4}; \frac{13}{2}\right)$ midpoint of/*midpt van* PB: $\left(\frac{5 + 1.5}{2}; \frac{13 + 0}{2}\right) = \left(\frac{13}{4}; \frac{13}{2}\right)$ PQBS = parallelogram (diags bisect each other/*hoekl halv mekaar)* **OR/*OF***	✓ correct subst into form/*korrekte subst in formule* ✓ BS = 10 ✓ BS = PQ ✓ reason/*rede* (4) ✓ $\left(\frac{-1 + 7.5}{2}; \frac{5 + 8}{2}\right)$ ✓ $\left(\frac{5 + 1.5}{2}; \frac{13 + 0}{2}\right)$ ✓ $\left(\frac{13}{4}; \frac{13}{2}\right)$ ✓ reason/*rede* (4)

	$m_{QB} = \frac{5-0}{-1-1,5} = \frac{5}{-2,5} = -2$ $m_{PS} = \frac{13-8}{5-7,5} = \frac{5}{-2,5} = -2$ $m_{QB} = m_{PS}$ $\therefore$ QB || PS PQ || BS PQBS = parallelogram (both pairs opp sides ||/*beide pr tos sye* ||) **OR/*OF*** $BS = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ $= \sqrt{\left(\frac{15}{2}-\frac{3}{2}\right)^2+(8-0)^2}$ $\therefore$ PQ = BS $=10$ $QB = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ $= \sqrt{(-1-1,5)^2+(5-0)^2} = \sqrt{(2,5)^2+(5)^2} = \frac{5\sqrt{5}}{2}$ or 5,59 $PS = \sqrt{(5-7,5)^2+(13-8)^2} = \sqrt{(2,5)^2+(5)^2} = \frac{\sqrt{125}}{2}$ or 5,59 QB = PS PQBS = parallelogram (both pairs opp sides =/ *beide pr tos sye* =)	✓ m_{QB} ✓ m_{PS} ✓ QB || PS ✓ reason/*rede* (4) ✓ correct subst into form/*korrekte subst in formule* ✓ PQ = 10 ✓ QB = PS ✓ reason/*rede* (4) **[18]**

QUESTION/VRAAG 4



4.1.1	Radius = $\sqrt{(2+1)^2+(4-2)^2}$ $r = \sqrt{13}$ Equation of circle/*vgl van sirkel*: $(x-2)^2+(y-4)^2=13$ **OR/*OF*** $(x-2)^2+(y-4)^2=r^2$ $(-1-2)^2+(2-4)^2=r^2$ $r^2=13$ $\therefore\ (x-2)^2+(y-4)^2=13$	✓ $\sqrt{(2+1)^2+(4-2)^2}$ or/*of* $\sqrt{13}$ ✓ $(x-2)^2+(y-4)^2$ ✓13 (3) ✓ $(x-2)^2+(y-4)^2$ ✓ $(-1-2)^2+(2-4)^2$ ✓13 (3)
4.1.2	At/*by* D: $\frac{-1+x_D}{2}=2$ $\frac{2+y_D}{2}=4$ $-1+x_D=4$ and/*en* $2+y_D=8$ $x_D=5$ $y_D=6$ D(5 ; 6) **OR/*OF*** By inspection/*deur inspeksie*: D(5 ; 6)	✓ x – value/*waarde* ✓ y - value/*waarde* (2) ✓ x - value/*waarde* ✓ y - value/*waarde* (2)

4.1.3	$m_{MC} = \frac{4-2}{2+1} = \frac{2}{3}$ $m_{AB} \times m_{MC} = -1$ (Tangent ⊥ radius/*raaklyn ⊥ radius*) $m_{AB} = -\frac{3}{2}$ $y - y_1 = m(x - x_1)$ ***OR/OF*** $y = mx + c$ $y - 2 = -\frac{3}{2}(x+1)$ $2 = -\frac{3}{2}(-1) + c$ $y = -\frac{3}{2}x + \frac{1}{2}$ $y = -\frac{3}{2}x + \frac{1}{2}$	✓ $m_{MC} = \frac{4-2}{2+1} = \frac{2}{3}$ ✓ $m_{AB} \times m_{MC} = -1$ ✓ $m_{AB} = -\frac{3}{2}$ ✓subst *m* and (–1 ; 2) into eq /*subst m en (–1 ; 2) in vgl* ✓ eq in standard form/ *vgl in st vorm* (5)
4.1.4	At/*by* E: $(0-2)^2 + (y-4)^2 = 13$ $(y-4)^2 = 9$ $y - 4 = \pm 3$ $y = 7$ or $y = 1$ E(0 ; 7) **OR/*OF*** At/*by* E: $(0-2)^2 + (y-4)^2 = 13$ $4 + y^2 - 8y + 16 = 13$ $y^2 - 8y + 7 = 0$ $(y-7)(y-1) = 0$ $y = 7$ or $y = 1$ E(0 ; 7)	✓ $x = 0$ ✓ simplification/ *vereenvoudiging* ✓ *y* - values/*waardes* ✓ E(0 ; 7) (4) ✓ $x = 0$ ✓ simplification/ *vereenvoudiging* ✓ *y* - values/*waardes* ✓ E(0 ; 7) (4)
4.1.5	$m_{EM} = \frac{y_2 - y_1}{x_2 - x_1}$ $= \frac{4-7}{2-0}$ $= -\frac{3}{2}$ $m_{AB} = -\frac{3}{2}$ $\therefore$ EM | | AB ($m_{EM} = m_{AB}$)	✓ $m_{EM} = -\frac{3}{2}$ ✓ reason/*rede* (2)

4.2	The centres of the circles are / *Die middelpunte van die sirkels is* P(–2 ; 4) and / *en* Q(5 ; –1) $QP^2 = (-2-5)^2 + (4-(-1))^2$ $QP = \sqrt{74} \approx 8{,}60$ units $r_M + r_P = 5+3$ $= 8$ $\therefore\ r_M + r_P < QP$ $\therefore$ The two circles do not intersect/*Die twee sirkels sny nie*	✓ both centres/*albei Midpte* ✓ QP ✓ correct subst into form/*korrekte subst in formule* ✓ distance between 2 centres/*afstand tussen 2 midpte* ✓✓ $r_M + r_P < QP$ (6) **[22]**

QUESTION/VRAAG 5

5.1	$x^2 + y^2$ $=(3\sin\theta)^2 + (3\cos\theta)^2$ $=9\sin^2\theta + 9\cos^2\theta$ $=9(\sin^2\theta + \cos^2\theta)$ $=9(1)$ $=9$	✓ simpl/*vereenv* ✓ CF/*GF* = 9 ✓ answer/*antw* (3)
5.2	$\sin(540° - x).\sin(-x) - \cos(180° - x).\sin(90° + x)$ $\sin(180° - x).\sin(-x) - \cos(180° - x).\sin(90° + x)$ $= (\sin x)(-\sin x) - (-\cos x)(\cos x)$ $= -\sin^2 x + \cos^2 x$ $= \cos 2x$	✓ $\sin(540° - x) = \sin x$ ✓ $\sin(-x) = -\sin x$ ✓ $\cos(180° - x) = -\cos x$ ✓ $\sin(90° + x) = \cos x$ ✓ $-\sin^2 x + \cos^2 x$ ✓ $\cos 2x$ (6)
5.3.1	$OT = \sqrt{x^2 + p^2}$ $\sin\alpha = \frac{y_T}{OT}$ $= \frac{p}{\sqrt{x^2 + p^2}}$ $\frac{p}{\sqrt{x^2 + p^2}} = \frac{p}{\sqrt{1 + p^2}}$ $x^2 = 1$ $x = -1$ **OR/*OF*** (P lies in 3rd quadrant) $x^2 + y^2 = r^2$ $x^2 + p^2 = \left(\sqrt{1 + p^2}\right)^2$ $x^2 + p^2 = 1 + p^2$ $x^2 = 1$ $x = -1$ (P lies in 3rd quadrant)	✓ $OT = \sqrt{x^2 + p^2}$ ✓ $\sin\alpha = \frac{y_T}{OT}$ ✓ $x^2 = 1$ (3) ✓ $x^2 + y^2 = r^2$ ✓ subst ✓ $x^2 = 1$ (3)
5.3.2	$\cos(180° + \alpha)$ $= -\cos\alpha$ $= -\left(\frac{-1}{\sqrt{1 + p^2}}\right)$ $= \frac{1}{\sqrt{1 + p^2}}$	✓ $-\cos\alpha$ ✓ answer/*antw* (2)

5.3.3	$\cos 2\alpha$ $= \cos^2\alpha - \sin^2\alpha$ $= \left(\dfrac{-1}{\sqrt{1+p^2}}\right)^2 - \left(\dfrac{p}{\sqrt{1+p^2}}\right)^2$ $= \dfrac{1}{1+p^2} - \dfrac{p^2}{1+p^2}$ $= \dfrac{1-p^2}{1+p^2}$	✓ expansion/ *uitbreiding* ✓✓ squaring each term/*kwadreer elke term* (3)
	OR/*OF*	
	$\cos 2\alpha$ $= 1 - 2\sin^2\alpha$ $= 1 - 2\left(\dfrac{p}{\sqrt{1+p^2}}\right)^2$ $= 1 - 2\left(\dfrac{p^2}{1+p^2}\right)$ $= 1 - \dfrac{2p^2}{1+p^2}$ $= \dfrac{1+p^2-2p^2}{1+p^2}$ $= \dfrac{1-p^2}{1+p^2}$	✓ expansion/ *uitbreiding* ✓squaring/*kwadrering* ✓ writing as single fraction/*skryf as enkelterm* (3)
	OR/*OF*	
	$\cos 2\alpha$ $= 2\cos^2\alpha - 1$ $= 2\left(\dfrac{-1}{\sqrt{1+p^2}}\right)^2 - 1$ $= 2\left(\dfrac{1}{1+p^2}\right) - 1$ $= \dfrac{2}{1+p^2} - 1$ $= \dfrac{2-1-p^2}{1+p^2}$ $= \dfrac{1-p^2}{1+p^2}$	✓ expansion/ *uitbreiding* ✓squaring/*kwadrering* ✓ writing as single fraction/*skryf as enkelterm* (3)

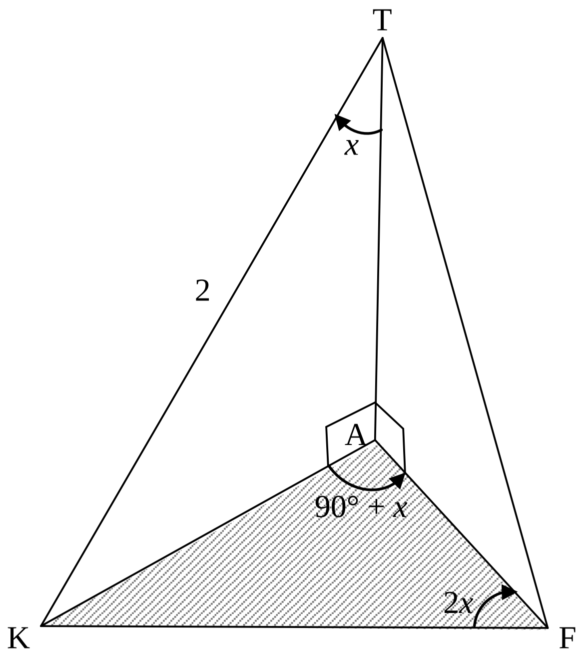
<table>
<tr><td>5.4.1</td><td>The identity is undefined for/die identiteit is ongedefinieerd as:
$2\sin^2 x = 0$
$\therefore$ <u>$\sin x = 0$:</u> $x = 0°$; $180°$
or/of
<u>$\tan x = \infty$:</u> $x = 90°$
$\therefore$ $x = 0°$; $90°$; $180°$</td><td>✓ $x = 0°$
✓ $x = 90°$
✓ $x = 180°$

(3)</td></tr>
<tr><td>5.4.2</td><td>LHS/LK $= \dfrac{2\tan x - \sin 2x}{2\sin^2 x}$
$= \dfrac{2\left(\dfrac{\sin x}{\cos x}\right) - 2\sin x\cos x}{2\sin^2 x}$
$= \left(\dfrac{2\sin x - 2\sin x\cos^2 x}{\cos x}\right) \times \dfrac{1}{2\sin^2 x}$
$= \dfrac{2\sin x(1-\cos^2 x)}{\cos x} \times \dfrac{1}{2\sin^2 x}$
$= \dfrac{2\sin x(\sin^2 x)}{\cos x} \times \dfrac{1}{2\sin^2 x}$
$= \dfrac{\sin x}{\cos x}$
$= \tan x$
$=$ RHS/RK
OR/OF
LHS/LK $= \dfrac{2\tan x - \sin 2x}{2\sin^2 x}$
$= \dfrac{2\left(\dfrac{\sin x}{\cos x}\right) - 2\sin x\cos x}{2\sin^2 x} \times \dfrac{\cos x}{\cos x}$
$= \dfrac{2\sin x - 2\sin x\cos^2 x}{2\sin^2 x\cos x}$
$= \dfrac{2\sin x(1-\cos^2 x)}{2\sin^2 x\cos x}$
$= \dfrac{2\sin x.\sin^2 x}{2\sin^2 x\cos x}$
$= \dfrac{\sin x}{\cos x}$
$= \tan x$
$=$ RHS/RK</td><td>✓ $\dfrac{\sin x}{\cos x}$
✓ $2\sin x.\cos x$
✓ simplify numerator/ vereenv teller
✓ factorising/fakt
✓ $1 - \cos^2 x = \sin^2 x$
✓ simplify to/vereenv na $\dfrac{\sin x}{\cos x}$
(6)

✓ $\dfrac{\sin x}{\cos x}$
✓ $2\sin x.\cos x$
✓ simpl/vereenv
✓ factorising/fakt
✓ $1 - \cos^2 x = \sin^2 x$
✓ simplify to /vereenv na $\dfrac{\sin x}{\cos x}$
(6)
[26]</td></tr>
</table>

QUESTION/VRAAG 6

6.1



6.1.1	In ΔTAK: $\frac{\text{AK}}{\text{KT}} = \sin \text{K}\hat{\text{T}}\text{A}$ $\text{AK} = \text{KT}.\sin x$ $= 2\sin x$ **OR/*OF*** $\frac{\sin \text{K}\hat{\text{T}}\text{A}}{\text{AK}} = \frac{\sin \text{K}\hat{\text{A}}\text{T}}{\text{KT}}$ $\frac{\sin 90^\circ}{2} = \frac{\sin x}{\text{AK}}$ $\text{AK} = 2\sin x$	✓ correct trig ratio/ *korrekte trigverh.* ✓answer/*antw* (2) ✓ correct subst into sine rule/*korrekte subst in sin-reël* ✓answer/*antw* (2)

<table>
<tr>
<td>6.1.2</td>
<td>In ΔAKF:
$\frac{\text{KF}}{\sin \text{K}\hat{\text{A}}\text{F}} = \frac{\text{AK}}{\sin \text{A}\hat{\text{F}}\text{K}}$
$\frac{\text{KF}}{\sin(90° + x)} = \frac{\text{AK}}{\sin 2x}$
$\text{KF} = \frac{\text{AK}.\sin(90° + x)}{\sin 2x}$
$= \frac{2\sin x.\cos x}{2\sin x.\cos x}$
$= 1$

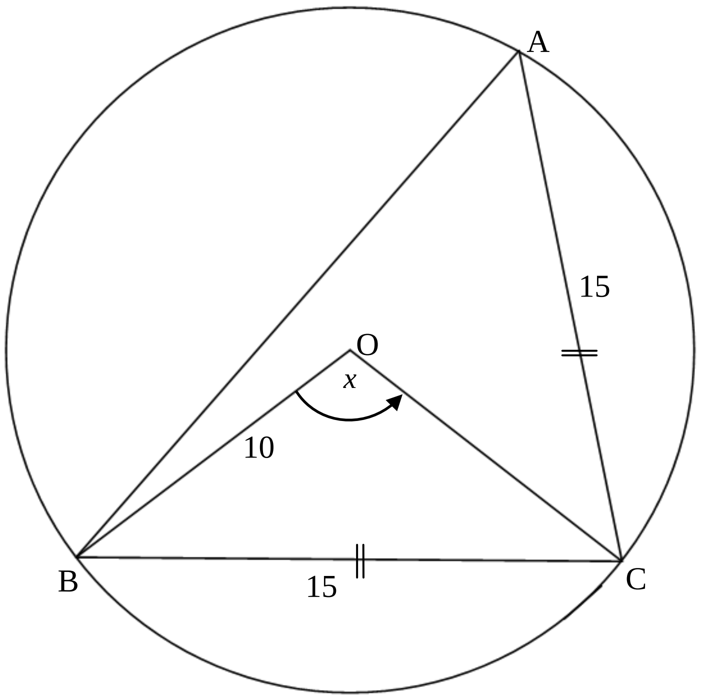
OR/OF

In ΔAKF:
$\frac{\text{KF}}{\sin \text{K}\hat{\text{A}}\text{F}} = \frac{\text{AK}}{\sin \text{A}\hat{\text{F}}\text{K}}$
$\frac{\text{KF}}{\sin(90° + x)} = \frac{\text{AK}}{\sin 2x}$
$\text{KF} = \frac{\text{AK}.\sin(90° + x)}{\sin 2x}$
$= \frac{\text{AT}.\tan x.\cos x}{2\sin x.\cos x}$
$= \frac{2\cos x.\frac{\sin x}{\cos x}.\cos x}{2\sin x.\cos x}$
$= 1$

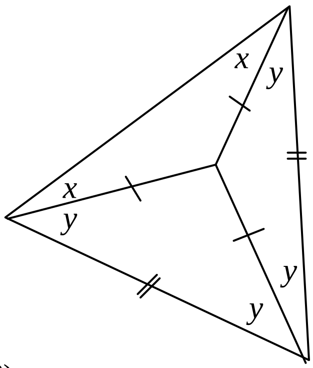
$\boxed{\cos x = \frac{\text{AT}}{2} \quad \therefore \text{AT} = 2\cos x}$</td>
<td>✓ using sine rule/ gebruik sin-reël
✓ correct subst into sine rule/korrekte subst in sin-reël
✓ sin(90° +x) = cos x
✓ 2sin$x$.cos$x$
✓ 1
(5)

✓ using sine rule/ gebruik sin-reël
✓ correct subst into sine rule/korrekte subst in sin-reël
✓ sin(90° +x) = cos x
✓ 2sin$x$.cos$x$
✓ 1
(5)</td>
</tr>
</table>

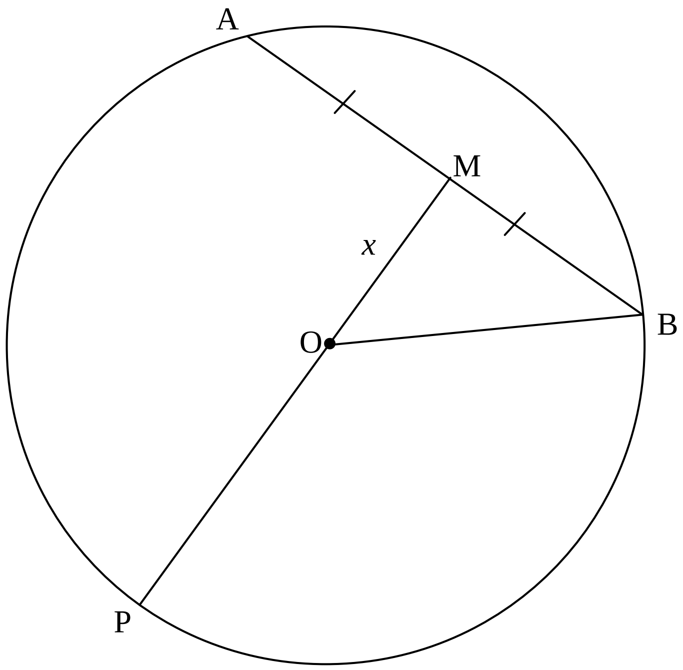
6.2



6.2.1	In ΔBOC: $BC^2 = BO^2 + CO^2 - 2.BO.\ CO\ .\cos x$ $15^2 = 10^2 + 10^2 - 2(10)(10).\cos x$ $200 \cos x = -25$ $\cos x = -0{,}125$ $x = 180° - 82{,}82°$ $= 97{,}18°$ **OR/*OF*** Draw a line OD$\perp$ BC: BD = DC (line from centre $\perp$ on chord) $\Delta OBD \equiv \Delta OCD$ (90°; h; s) $\sin \frac{x}{2} = \frac{7{,}5}{10}$ $\frac{x}{2} = 48{,}59°$ $\therefore\ x = 97{,}18°$	✓ using cosine rule/ *gebruik cos-reël* ✓correct subst/ *korrekte subst* ✓ $\cos x = -0{,}125$ ✓97,18° (4) ✓ S/R ✓correct ratio/ *korrekte verh* ✓ value of/*waarde van* $\frac{x}{2}$ ✓97,18° (4)

6.2.2	$\text{B}\hat{\text{A}}\text{C} = 48{,}59°$ ($\angle$ at centre=2×$\angle$at circ/$\angle$ *by midpt=2×*$\angle$*omt*) $\text{A}\hat{\text{B}}\text{C} = \text{B}\hat{\text{A}}\text{C} = 48{,}59°$ ($\angle$'s opp equal sides/$\angle$*e teenoor = sye*) $\therefore \text{A}\hat{\text{C}}\text{B} = 82{,}82°$ (sum of $\angle$s of Δ/*som van* $\angle$*e van Δ*)	✓ S ✓ S ✓82,82° (3)
	OR/*OF* $\text{A}\hat{\text{C}}\text{B} = \frac{1}{2}\text{A}\hat{\text{O}}\text{B}$ ($\angle$at centre = 2× $\angle$at circle) ($\angle$ *by midpt=2×*$\angle$*omt*) $= \frac{1}{2}[360° - 2(97{,}18°)]$ $= 82{,}82°$	✓ S ✓ S ✓82,82° (3)
	OR/*OF* $\text{O}\hat{\text{C}}\text{B} = \frac{1}{2}(180° - 97{,}18°)$ ($\angle$'s opp equal sides; sum of $\angle$s of Δ) ($\angle$*e teenoor = sye; som van* $\angle$*e van Δ*) $= 41{,}41°$  $\text{A}\hat{\text{C}}\text{B} = 2(41{,}41°)$ $= 82{,}82°$	✓ S ✓ S ✓82,82° (3)
6.2.3	Area/*Oppervlakte* ΔABC $= \frac{1}{2}(\text{BC})(\text{AC})\sin \text{A}\hat{\text{C}}\text{B}$ $= \frac{1}{2}(15)(15)(\sin 82{,}82°)$ $= 111{,}62 \text{ cm}^2$	✓correct subst into area rule/*korrekte subst in opp-reël* ✓ $111{,}62 \text{ cm}^2$ (2) **[16]**

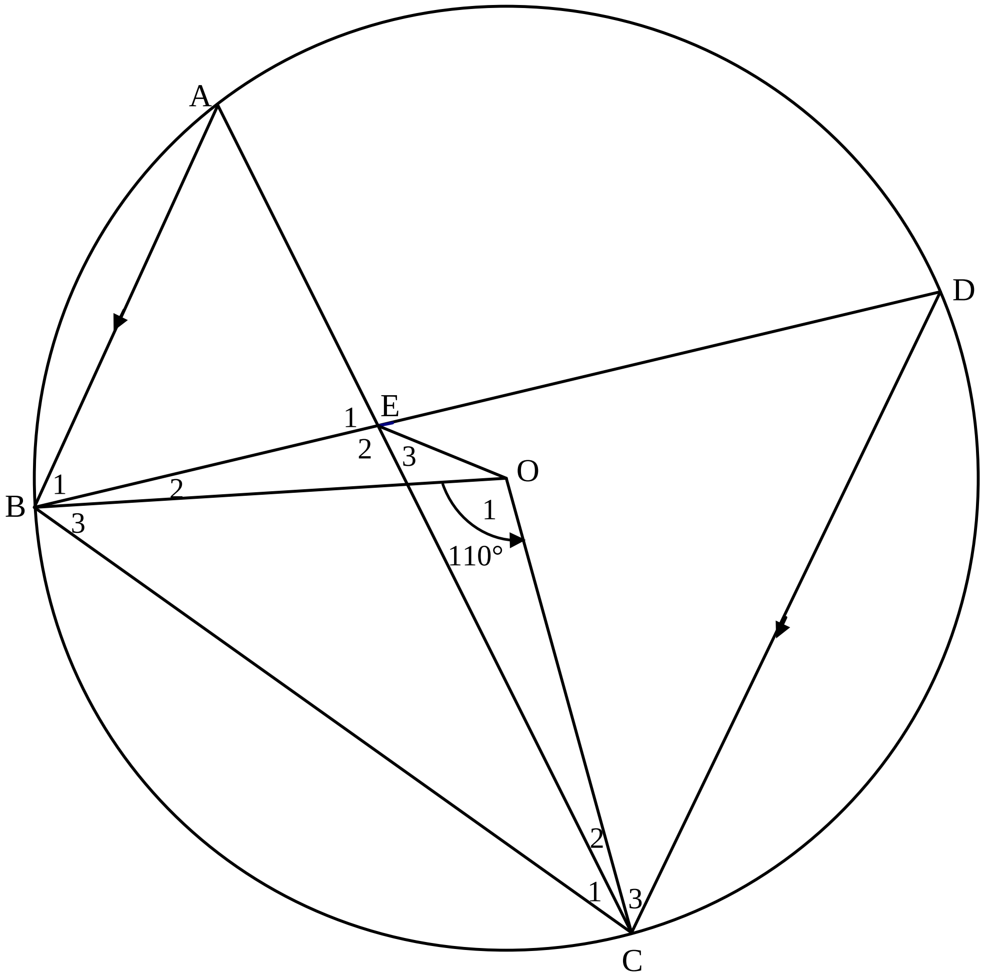
QUESTION/VRAAG 7



7.1	MB = 10 cm	✓ answer/*antw* (1)
7.2	line from centre to midpoint of chord is perpendicular to chord/*lyn vanaf midpt na midpt van koord is loodreg op koord* **OR/*OF*** line from centre bisects chord/*lyn vanaf midpt halveer koord*	✓ answer/*antw* (1) ✓ answer/*antw* (1)
7.3	$\frac{\text{MP}}{\text{OM}}=\frac{5}{2}$ $\frac{x+\text{OP}}{x}=\frac{5}{2}$ $2x+2\text{OP}=5x$ $\text{OP}=\frac{3x}{2}$ **OR/*OF*** $\frac{\text{OP}}{\text{OM}}=\frac{3}{2}$ $\text{OP}=\frac{3x}{2}$	✓ $\frac{x+\text{OP}}{x}=\frac{5}{2}$ ✓ $\text{OP}=\frac{3x}{2}$ (2) ✓ $\frac{\text{OP}}{\text{OM}}=\frac{3}{2}$ ✓ $\text{OP}=\frac{3x}{2}$ (2)

7.4	OM² + MB² = OB² $x^2+10^2=\left(\frac{3x}{2}\right)^2$ $4x^2+400=9x^2$ $5x^2=400$ $x^2=80$ $x=8{,}94$ or $4\sqrt{5}$ or $\sqrt{80}$	✓ subst into/*subst* Pythagoras ✓ $4x^2+400=9x^2$ ✓ answer/*antw* (3) **[7]**

QUESTION/VRAAG 8

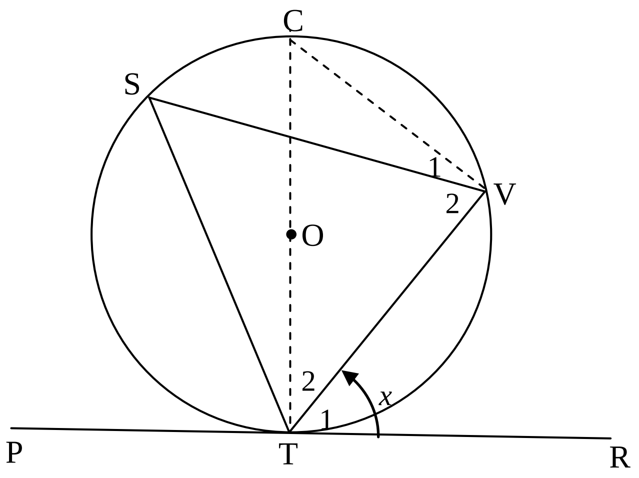


8.1.1	$\hat{D} = \frac{1}{2}\hat{O}_1 = 55°$ ($\angle$ at centre=2×$\angle$at circ/$\angle$ *by midpt=2×$\angle$by omt*)	✓S ✓R (2)
8.1.2	$\hat{A} = \frac{1}{2}\hat{O}_1 = 55°$ ($\angle$ at centre=2×$\angle$at circ/$\angle$ *by midpt=2×$\angle$by omt*) **OR/*OF*** $\hat{A} = \hat{D} = 55°$ ($\angle$s in same segment/$\angle$*e in dieselfde segment*)	✓S ✓R (2) ✓S ✓R (2)
8.1.3	$\hat{B}_1 = \hat{D} = 55°$ (alternate $\angle$s/*verwiss$\angle$e*; AB | | DC) $\hat{E}_2 = \hat{B}_1 + \hat{A}$ (ext$\angle$ ofΔ = sum of opp$\angle^s$/*buite$\angle$ vΔ=som v tos$\angle$e*) $= 55° + 55°$ $\hat{E}_2 = 110°$	✓S ✓R ✓R ✓ answer/*antw* (4)
8.2	$\hat{E}_2 = \hat{O}_1 = 110°$ (proven in/*bewys in* 8.1.3) BEOC is a cyclic quadrilateral (equal $\angle$s subtended by line/ *gelyke $\angle$e onderspan deur lyn*)	✓S ✓R (2) **[10]**

QUESTION/VRAAG 9

9.1	the interior opposite angle/*die teenoorstaande binnehoek.*	✓ answer/*antw* (1)

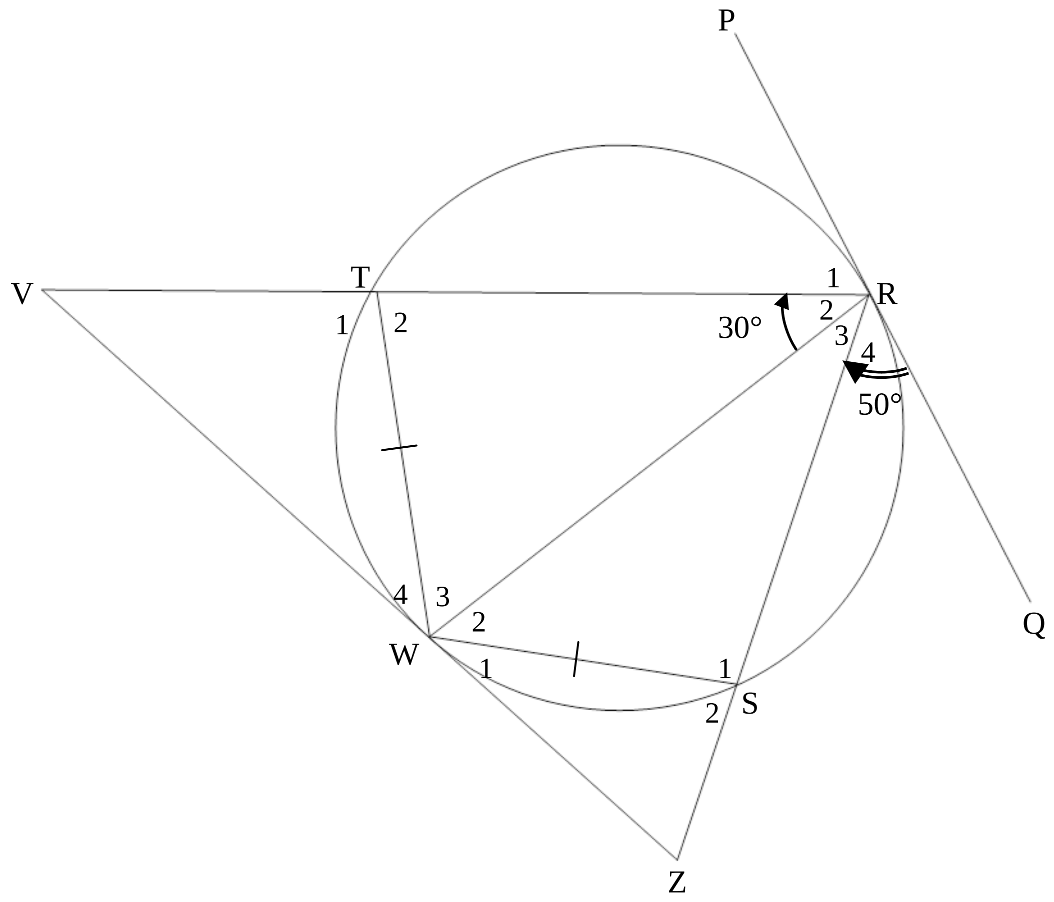
9.2



Construction: Draw diameter CT and join CV.
Konstruksie: Trek middellyn CT en verbind CV.

$\hat{V}_1 + \hat{V}_2 = \mathbf{90°}$	**∠ in semi-circle/*∠ in halfsirkel***	✓ S ✓ R
$\hat{T}_2 = 90° - x$	**Tangent ⊥ diameter/radius/*raaklyn ⊥ middellyn/radius***	✓ R
$\therefore \hat{C} = \boldsymbol{x}$	Sum of the angles of triangle/*Som van die hoeke van 'n driehoek*	✓ S
$\therefore \hat{S} = x$	**∠'s same segment/*∠e in dieselfde segment***	✓ R
$\therefore V\hat{T}R = \hat{S}$		(5)

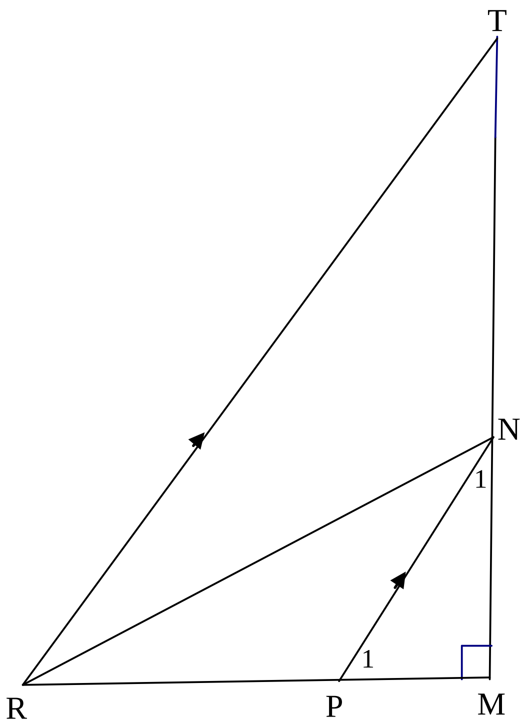
9.3



9.3.1	Equal chords subtend equal ∠s/*Gelyke koorde onderspan gelyke ∠e*	✓ R (1)
9.3.2	$\hat{W}_4 = 30°$ (tan chord theorem/*rkl-koordst*) $\hat{W}_1 = 30°$	✓ answer/*antw* ✓ reason/*rede* ✓ answer/*antw* (3)
9.3.3(a)	$\hat{R}_4 = \hat{W}_2 = 50°$ (tan chord theorem/*rkl-koordst*) $\hat{S}_2 = \hat{R}_3 + \hat{W}_2$ (ext∠ of Δ/*buite ∠ v Δ*) $\therefore \hat{S}_2 = 80°$ **OR/*OF*** $\hat{R}_2 = \hat{R}_3 = 30°$ (= chords subtend =∠s /= *kde onderspan=∠e*) $\hat{R}_4 = \hat{W}_2 = 50°$ (tan chord theorem/*rkl-koordst*) $\therefore \hat{S}_2 = 80°$	✓ S ✓R ✓ S (3) ✓ S ✓R ✓ S (3)

9.3.3(b)	$\hat{T}_2 = \hat{S}_2 = 80°$ (ext $\angle$ of cyclic quad/*buite$\angle$ van koordevh*) $V + \hat{W}_4 = \hat{T}_2$ (ext$\angle$ of Δ/*buite$\angle$ van Δ*) $\therefore\ \hat{V} = 50°$	✓ S ✓ R ✓ S ✓ S (4)
9.3.4	In ΔRVW and/*en* ΔRWS: $\hat{R}_2 = \hat{R}_3 = 30°$ (proven/*bewys* in 9.3.1) $\hat{V} = \hat{W}_2 = 50°$ (proven/*bewys* in 9.3.3) $V\hat{W}R = \hat{S}_1$ (3rd $\angle$ in Δ) $\therefore \Delta RVW \mid\mid\mid \Delta RWS$ ($\angle\angle\angle$) $\therefore \frac{WR}{RV} = \frac{RS}{WR}$ ($\Delta RVW \mid\mid\mid \Delta RWS$) $\therefore WR^2 = RV.RS$	✓ using the correct Δs/ *gebruik korrekte Δe* ✓ S ✓ S ✓ R (3rd $\angle$ in Δ) or ($\angle\angle\angle$) ✓ S (5) **[22]**

QUESTION/VRAAG 10



10.1.1	corresponding ∠s/*ooreenkomstige∠e*; PN | | RT	✓ answer/*antw* (1)
10.1.2	∠; ∠; ∠ **OR/*OF*** ∠; ∠	✓ answer/*antw* (1)
10.2	$\frac{PM}{RM}=\frac{PN}{RT}$ $(\Delta PNM \mid\mid\mid \Delta RTM)$ $=\frac{PN}{3PN}$ $=\frac{1}{3}$	✓ S ✓ S (2)
10.3	$\frac{PM}{RM}=\frac{1}{3}$ $\therefore \frac{RP}{RM}=\frac{2}{3}$ $RN^2 - PN^2 = (RM^2 + NM^2) - (PM^2 + NM^2)$ (Pyth) $= RM^2 - PM^2$ $=\left(\frac{3}{2}RP\right)^2-\left(\frac{1}{2}RP\right)^2$ $=\frac{9}{4}RP^2-\frac{1}{4}RP^2$ $= 2RP^2$ **OR/*OF***	✓ Use of Pyth. for RN^2 and PN^2 ✓ $RM = \frac{3}{2}RP$ ✓ $PM=\frac{1}{2}RP$ ✓ $\frac{9}{4}RP^2$ & $\frac{1}{4}RP^2$ (4)

	$RN^2 - PN^2 = (RM^2 + NM^2) - (PM^2 + NM^2)$ (Pyth) $= RM^2 - PM^2$ $= (3PM)^2 - PM^2$ $= 8PM^2$ $= 2(2PM)^2$ $= 2RP^2$	✓ Use of Pyth. for RN^2 and PN^2 ✓RM = RP + PM ✓ $(3PM)^2 - PM^2$ ✓ RP = 2PM (4)
	OR/*OF*	
	$RN^2 - PN^2 = (RM^2 + NM^2) - (PM^2 + NM^2)$ (Pyth) $= RM^2 - PM^2$ $= (RP + PM)^2 - PM^2$ $= RP^2 + 2RP.PM + PM^2 - PM^2$ $= RP^2 + 2RP.\frac{1}{2}RP$ $= 2RP^2$	✓ Use of Pyth. for RN^2 and PN^2 ✓RM = RP + PM ✓ expansion/ *uitbreiding* ✓ $PM = \frac{1}{2}RP$ (4) **[8]**

TOTAL/*TOTAAL*: 150

basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE/*GRAAD 12*

MATHEMATICS P2/*WISKUNDE V2*

NOVEMBER 2014

MEMORANDUM

MARKS/*PUNTE*: 150

This memorandum consists of 23 pages.
Hierdie memorandum bestaan uit 23 bladsye.

NOTE:

- If a candidate answers a question TWICE, only mark the FIRST attempt.
- If a candidate has crossed out an attempt of a question and not redone the question, mark the crossed out version.
- Consistent accuracy applies in ALL aspects of the marking memorandum.
- Assuming answers/values in order to solve a problem is NOT acceptable.

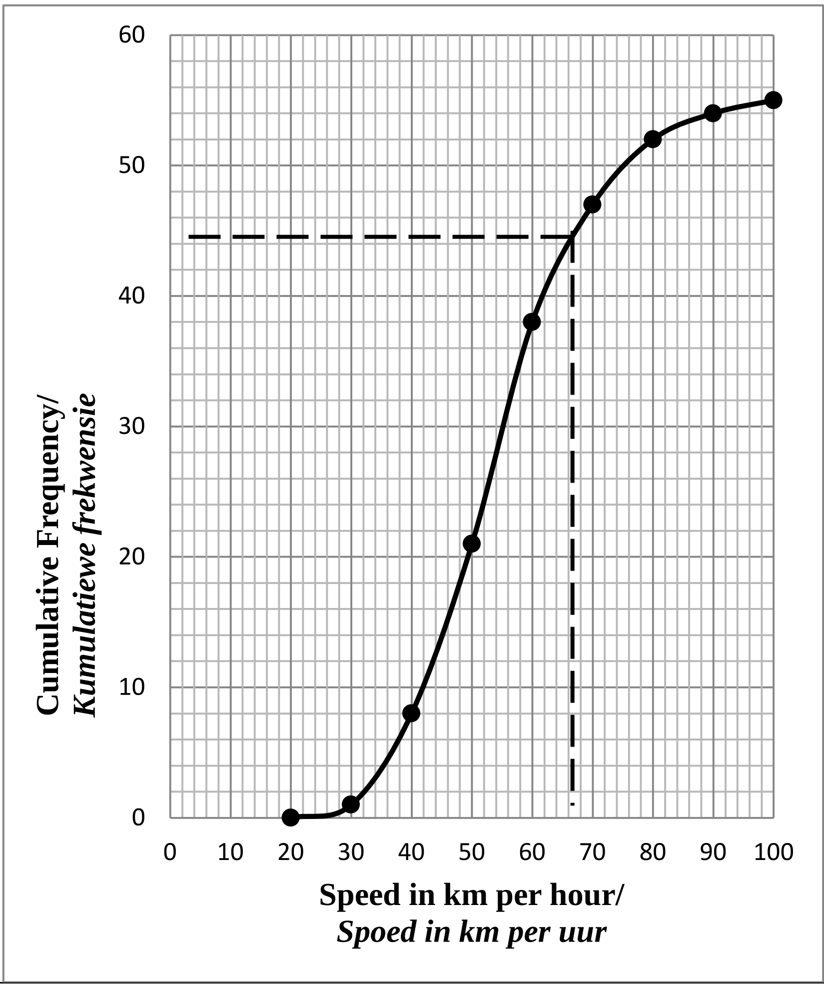
NOTA:

- *As 'n kandidaat 'n vraag TWEEKEER beantwoord, merk slegs die EERSTE poging.*
- *As 'n kandidaat 'n poging om die vraag te beantwoord, doodgetrek het en nie dit oorgedoen het nie, merk die doodgetrekte poging.*
- *Volgehoue akkuraatheid word in ALLE aspekte van die nasienmemorandum toegepas.*
- *Aanvaarding van antwoorde/waardes om 'n probleem op te los, is ONaanvaarbaar.*

QUESTION/*VRAAG* 1

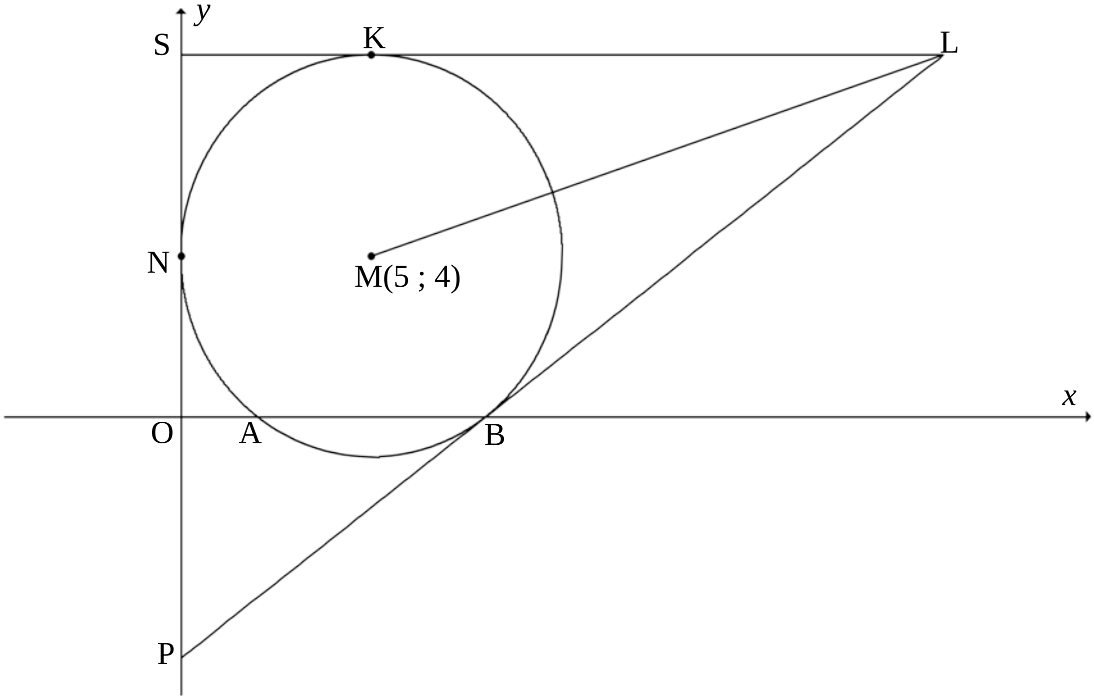
1.1	$\bar{x} = \frac{816}{12} = 68$	✓ $\frac{816}{12}$ ✓ 68 (2)
1.2	σ = 18,42	✓ answer/*antw* (1)
1.3	(68 – 18,42 ; 68 + 18,42) = (49,58 ; 86,42) $\therefore$ 6 candidates had a mark within one standard deviation of the mean/*6 kandidate het 'n punt binne een standaardafwyking vanaf die gemiddelde*.	✓✓ interval ✓ answer/*antw* (3)
1.4	a = 22,828... = 22,83 b = 0,66429... = 0,66 $\therefore\ \hat{y} = 0{,}66x + 22{,}83$ **OR/*OF*** $\hat{y} = 22{,}83 + 0{,}66x$	✓ value of *a/ waarde van a* ✓ value of *b/ waarde van b* ✓ equation/*vgl* (3)
1.5	$\hat{y} = 0{,}66x + 22{,}83$ y = 0,66(60) + 22,83 62,43...% ≈ 62% **OR/*OF*** 62,69% ≈ 63%	✓ subs of 60 into equation ✓ answer/*antw* (2) ✓✓ answer/*antw* (2)
1.6	(82 ; 62)	✓ answer/*antw* (1) **[12]**

QUESTION/*VRAAG* 2

<table>
<tr><td>2.1</td><td>$50 < x \leq 60$ OR/OF $50 \leq x < 60$ OR/OF
between 50 and 60/tussen 50 en 60</td><td>✓ answer/antw
(1)</td></tr>
<tr><td>2.2.1</td><td>
<table>
<tr><th>Class
Klas</th><th>Frequency
Frekwensie</th><th>Cumulative frequency
Kumulatiewe frekwensie</th></tr>
<tr><td>$20 < x \leq 30$</td><td>1</td><td>1</td></tr>
<tr><td>$30 < x \leq 40$</td><td>7</td><td>8</td></tr>
<tr><td>$40 < x \leq 50$</td><td>13</td><td>21</td></tr>
<tr><td>$50 < x \leq 60$</td><td>17</td><td>38</td></tr>
<tr><td>$60 < x \leq 70$</td><td>9</td><td>47</td></tr>
<tr><td>$70 < x \leq 80$</td><td>5</td><td>52</td></tr>
<tr><td>$80 < x \leq 90$</td><td>2</td><td>54</td></tr>
<tr><td>$90 < x \leq 100$</td><td>1</td><td>55</td></tr>
</table>
</td><td>✓ 8
✓ 55
(2)</td></tr>
<tr><td>2.2.2</td><td>


</td><td>✓ grounding at (20 ; 0)/ anker by (20 ; 0)
✓ plotting at upper limits/ plot by boonste limiete
✓ smooth shape of curve/gladde kurwe
(3)</td></tr>
<tr><td>2.3</td><td>55 – 44 (accept/aanvaar 43 – 45)
≈ 11 motorists/motoriste
(accept/aanvaar 10 – 12 motorists/motoriste)</td><td>✓ 44
✓ 11
(2)
[8]</td></tr>
</table>

QUESTION/*VRAAG* 3

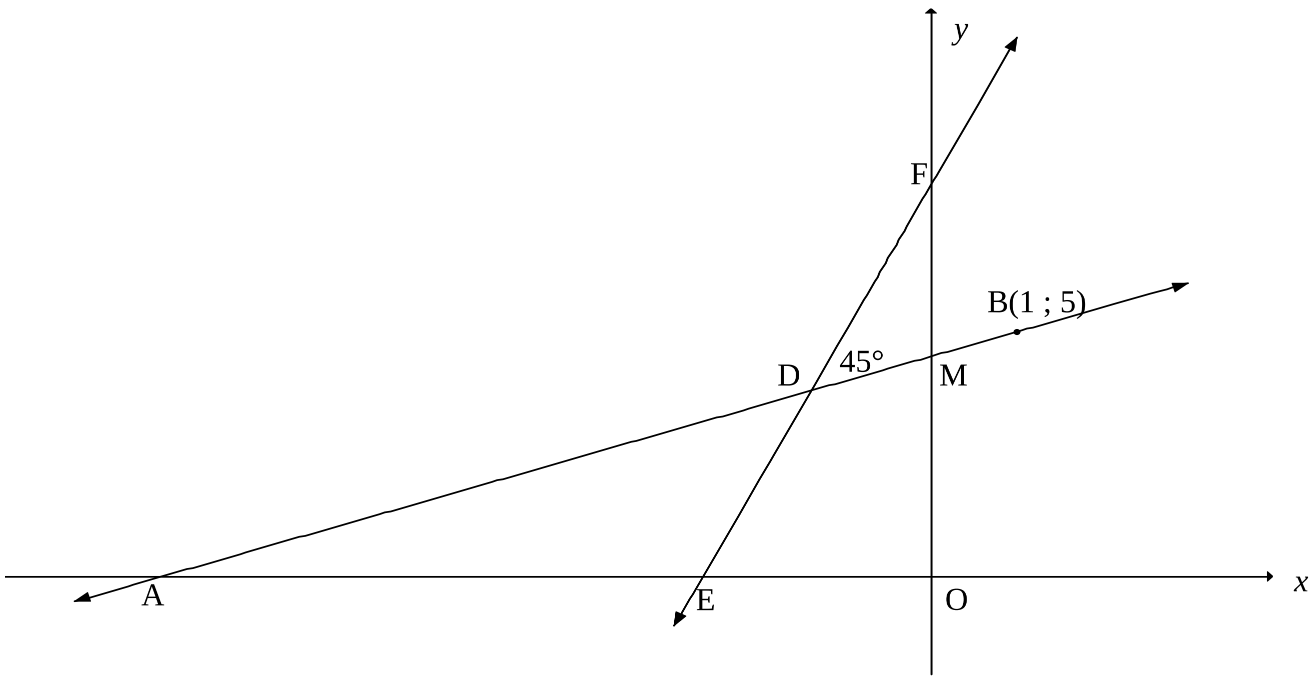


3.1	r = MN = 5	✓ answer/*antw* (1)
3.2	$(x-5)^2+(y-4)^2=25$	✓equation/*vgl* (1)
3.3	A(x ; 0) $(x-5)^2+(0-4)^2=25$ $x^2-10x+25+16=25$ $x^2-10x+16=0$ $(x-8)(x-2)=0$ $\therefore x=8$ or/*of* $x=2$ $\therefore$ A(2 ; 0) **OR/*OF*** $(x-5)^2+(0-4)^2=25$ $(x-5)^2+16=25$ $(x-5)^2=9$ $(x-5)=\pm 3$ $\therefore x=8$ or/*of* $x=2$ $\therefore$ A(2 ; 0)	✓ substitute into eq/ *vervang in vgl* $y=0$ ✓ standard form/ *standaardvorm* or perfect square form/*kwadr vorm* ✓ answer/*antw* (3)
3.4.1	$m_{\text{MB}}=\dfrac{4-0}{5-8}$ $=-\dfrac{4}{3}$	✓ subst M and B into form/*vervang M and B in form* ✓ $m_{\text{MB}}=-\dfrac{4}{3}$ (2)

3.4.2	$m_{MB} \times m_{PB} = -1$ (tangent ⊥ radius/ *rkl* ⊥ *radius*) $m_{PB} = \frac{3}{4}$ $y = \frac{3}{4}x + c$ **OR/*OF*** $y - y_1 = \frac{3}{4}(x - x_1)$ $0 = \frac{3}{4}(8) + c$ $y - 0 = \frac{3}{4}(x - 8)$ $y = \frac{3}{4}x - 6$ $y = \frac{3}{4}x - 6$	✓ $m_{MB} \times m_{PB} = -1$ ✓ $m_{PB} = \frac{3}{4}$ ✓ equation/*vgl* (3)
3.5	$y_K = y_M + r = 4 + 5$ $y = 9$	✓ 9 ✓ equation/*vgl* (2)
3.6	At/*By* L: $\frac{3}{4}x - 6 = 9$ $3x - 24 = 36$ $3x = 60$ $x = 20$ $\therefore$ L(20 ; 9)	✓ equating simultaneously ✓ simplification (2)
3.7	L(20 ; 9) $\text{ML} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ **OR/OF** $\text{ML} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $= \sqrt{(20-5)^2 + (9-4)^2}$ $= \sqrt{(15)^2 + (5)^2}$ $= \sqrt{225 + 25}$ $= \sqrt{(5)^2(9+1)}$ $= \sqrt{250}$ *or / of* $5\sqrt{10}$ $= \sqrt{250}$ *or / of* $5\sqrt{10}$	✓ correct subst into distance formula/ *korrekte subst in afstand-formule* ✓ answer in surd form/*antw in wortelvorm* (2)
3.8	**MK ⊥ KL OR/OF** $\text{M}\hat{\text{K}}\text{L} = 90°$ (radius ⊥ tangent/*radius* ⊥ *rkl*) $\therefore$ ML is a diameter as it subtends a right angle/*ML is middellyn* $r = \frac{\text{ML}}{2} = \frac{\sqrt{250}}{2} = \sqrt{\frac{125}{2}}$ or 7,91 Centre of circle = midpoint of ML/*Midpt van sirkel = midpt v ML* $x = \frac{5+20}{2} = \frac{25}{2} = 12{,}5$ $y = \frac{4+9}{2} = \frac{13}{2} = 6{,}5$ Centre/*midpt*: (12,5 ; 6,5) Equation of the circle KLM /*Vgl van sirkel KLM*: $\therefore \quad (x - 12{,}5)^2 + (y - 6{,}5)^2 = \frac{250}{4} = \frac{125}{2} = 62{,}5$ **OR/*OF***	✓ S ✓ value of/*waarde van r* ✓ $x = 12{,}5$ ✓ $y = 6{,}5$ ✓ answer in correct form/ *antw in korrekte vorm* (5)

	MK ⊥ KL OR/OF $\mathbf{M\hat{K}L} = 90°$ (radius ⊥ tangent/*radius ⊥ rkl*) $\therefore$ ML is a diameter as it subtends a right angle/*ML is middellyn* Centre of circle = midpoint of ML/*Midpt van sirkel = midpt v ML* $x = \frac{5+20}{2} = \frac{25}{2} = 12{,}5$ $y = \frac{4+9}{2} = \frac{13}{2} = 6{,}5$ Centre/*midpt*: (12,5 ; 6,5) Equation of the circle KLM /*Vgl van sirkel KLM*: $(x-12{,}5)^2 + (y-6{,}5)^2 = r^2$ subst (5 ; 4): $(5-12{,}5)^2 + (4-6{,}5)^2 = r^2$ $62{,}5 = r^2$ $\therefore \quad (x-12{,}5)^2 + (y-6{,}5)^2 = \frac{250}{4} = \frac{125}{2} = 62{,}5$ **OR/*OF***	✓ S ✓ $x = 12{,}5$ ✓ $y = 6{,}5$ ✓ value of/*waarde van* r^2 ✓ answer in correct form/*antw in korrekte vorm* (5)
	By symmetry about LM/*deur simmetrie om LM*: **MK ⊥ KL OR/OF** $\mathbf{M\hat{K}L} = 90°$ (radius ⊥ tangent/*radius ⊥ rkl*) $\therefore$ ML is a diameter as it subtends a right angle/*ML is middellyn* ML is a diameter /*ML is 'n middellyn* $r = \frac{\text{ML}}{2} = \frac{\sqrt{250}}{2} = \sqrt{\frac{125}{2}}$ or /of 7,91 Centre of circle = midpoint of ML/*Midpt van sirkel = midpt v ML* $x = \frac{5+20}{2} = \frac{25}{2} = 12{,}5$ $y = \frac{4+9}{2} = \frac{13}{2} = 6{,}5$ Centre/*midpt*: (12,5 ; 6,5) Equation of the circle KLM /*Vgl van sirkel KLM*: $\therefore \quad (x-12{,}5)^2 + (y-6{,}5)^2 = \frac{250}{4} = \frac{125}{2} = 62{,}5$	✓ S ✓ value of/*waarde van r* ✓ $x = 12{,}5$ ✓ $y = 6{,}5$ ✓ answer in correct form/*antw in korrekte vorm* (5) **[21]**

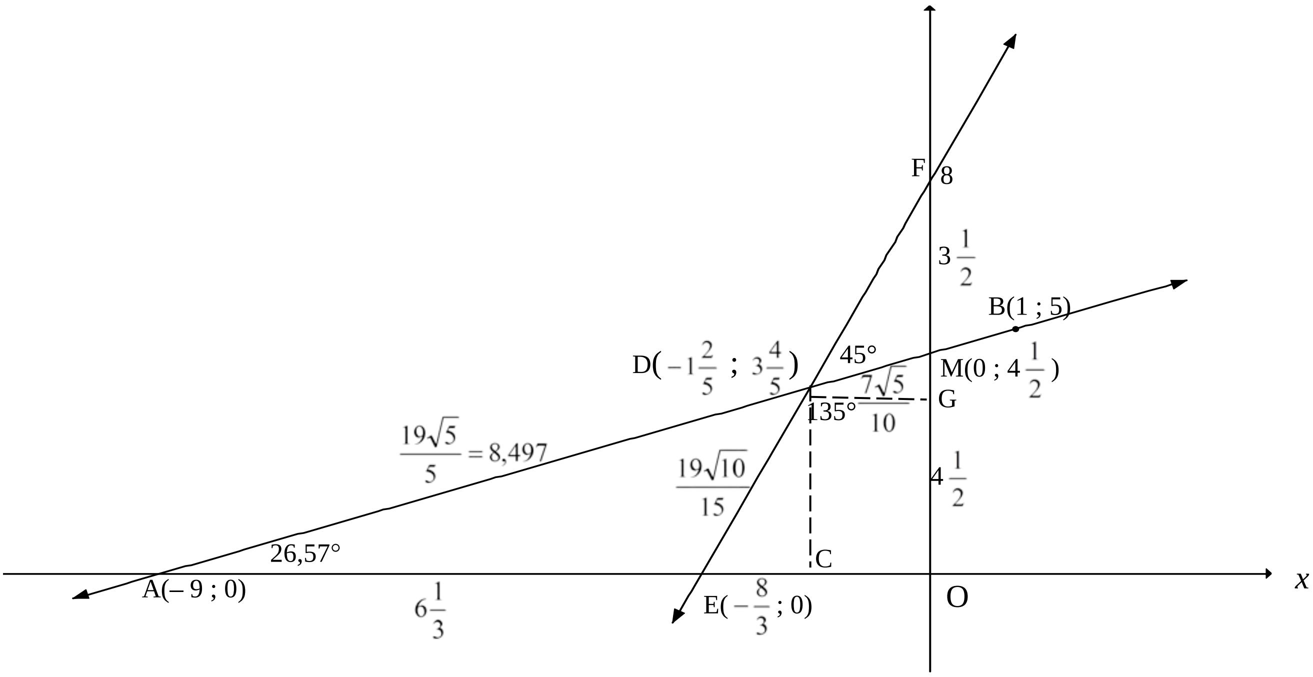
QUESTION/VRAAG 4



4.1	$y = 0$: $3x + 8 = 0$ $x = -\frac{8}{3}$ $\therefore$ E$\left(-2\frac{2}{3}; 0\right)$ **OR/*OF*** E$\left(-\frac{8}{3}; 0\right)$	✓ *y*-value/*waarde* ✓ *x*-value/*waarde* (2)
4.2	$\tan D\hat{E}O = m_{DE} = 3$ $\therefore D\hat{E}O = 71{,}565... = 71{,}57°$ $D\hat{A}E = 71{,}565...° - 45°$ $= 26{,}57°$	✓ $\tan D\hat{E}O = 3$ ✓ 71,565…° ✓ 26,57° (3)
4.3	$m_{AB} = \tan 26{,}57°$ $= \frac{1}{2}$ $y = \frac{1}{2}x + c$ **OR/*OF*** $y - y_1 = \frac{1}{2}(x - x_1)$ $5 = \frac{1}{2}(1) + c$ $y - 5 = \frac{1}{2}(x - 1)$ $y = \frac{1}{2}x + 4\frac{1}{2}$ $y = \frac{1}{2}x + \frac{9}{2}$	✓ $m_{AB} = \tan 26{,}57°$ ✓ $m_{AB} = \frac{1}{2}$ ✓ subst of *m* and (1 ; 5)into formula/ *subst m en (1 ; 5) in formule* ✓ equation/*vgl* (4)

4.4	Solve $x - 2y + 9 = 0$ and $y = 3x + 8$ simultaneously: $x - 2(3x+8) + 9 = 0$ $x - 6x - 16 + 9 = 0$ $-5x = 7$ $x = -1\frac{2}{5}$ $\therefore y = 3(-1\frac{2}{5}) + 8$ **OR/*OF*** $-1\frac{2}{5} - 2y + 9 = 0$ $y = 3\frac{4}{5}$ $y = 3\frac{4}{5}$ $\therefore D(-1\frac{2}{5}\ ;\ 3\frac{4}{5})$	✓ subst/*vervang* ✓ x-value/*waarde* ✓ subst/*vervang* ✓ y-value/*waarde* (4)
	OR/OF $x = 2y - 9$ $y = 3(2y - 9) + 8$ $y = 6y - 27 + 8$ $\therefore y = 3\frac{4}{5}$ $x = 2(3\frac{4}{5}) - 9$ **OR/OF** $3\frac{4}{5} = 3x + 8$ $x = -1\frac{2}{5}$ $x = -1\frac{2}{5}$ $\therefore D(-1\frac{2}{5}\ ;\ 3\frac{4}{5})$	✓ subst/*vervang* ✓ y value/*waarde* ✓ subst/*vervang* ✓ x-value/*waarde* (4)
	OR/*OF* $3x + 8 = \frac{1}{2}x + 4\frac{1}{2}$ $6x + 16 = x + 9$ $5x = -7$ $\therefore x = -1\frac{2}{5}$ $\therefore y = 3(-1\frac{2}{5}) + 8$ **OR/*OF*** $y = \frac{1}{2}(-1\frac{2}{5}) + 4\frac{1}{2}$ $y = 3\frac{4}{5}$ $y = 3\frac{4}{5}$ $\therefore D(-1\frac{2}{5}\ ;\ 3\frac{4}{5})$	✓ equating/*gelyk stel* ✓ x value/*waarde* ✓ subst/*vervang* ✓ y-value/*waarde* (4)
	OR/*OF*	

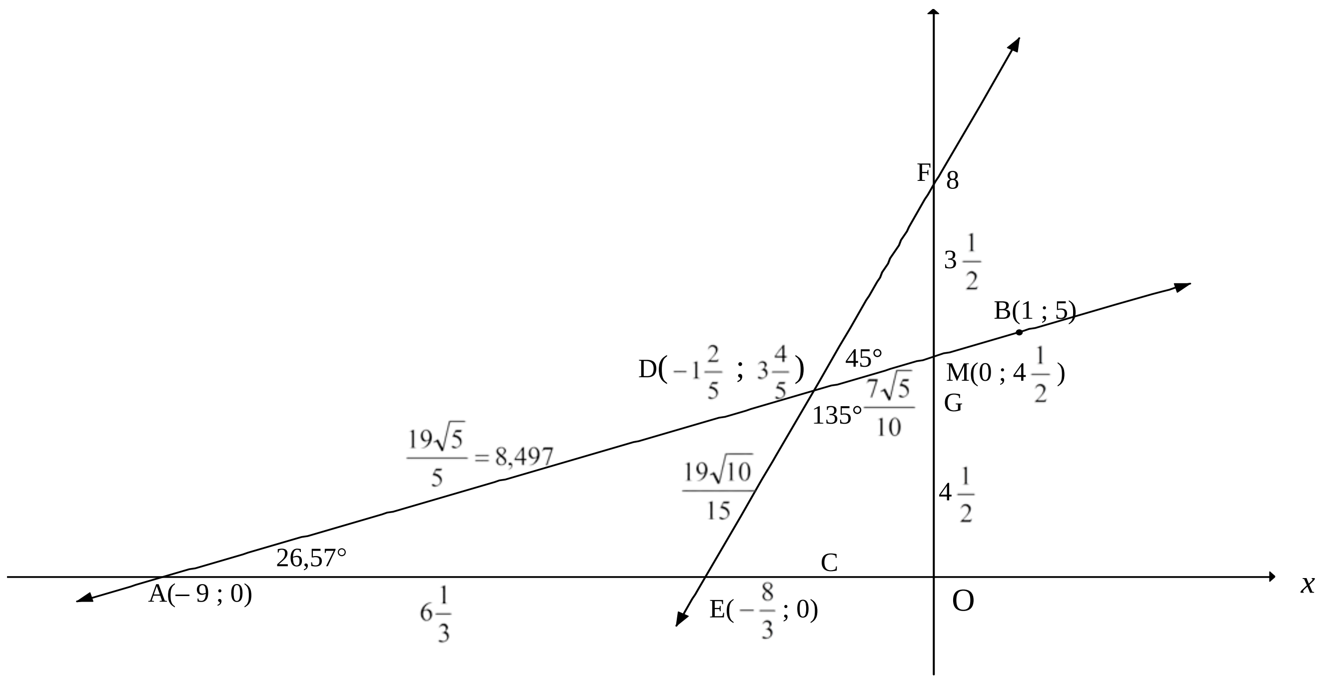
	$x - 2y = -9$ ……(1) $-6x + 2y = 16$ …...(2) (1) + (2): $-5x = 7$ $\therefore x = -1\frac{2}{5}$ $\therefore -1\frac{2}{5} - 2y = -9$ **OR/*OF*** $y = 3(-1\frac{2}{5}) + 8$ $y = 3\frac{4}{5}$ $y = 3\frac{4}{5}$ $\therefore$ D$(-1\frac{2}{5}\ ;\ 3\frac{4}{5})$ **OR/*OF*** $y = 3x + 8$ ……….(1) $6y = 3x + 27$ …….(2) (1) – (2): $-5y = -19$ $\therefore y = 3\frac{4}{5}$ $3\frac{4}{5} = 3x + 8$ **OR/*OF*** $x = 2(3\frac{4}{5}) - 9$ $x = -1\frac{2}{5}$ $x = -1\frac{2}{5}$ $\therefore$ D$(-1\frac{2}{5}\ ;\ 3\frac{4}{5})$	✓ adding/*optelling* ✓ x-value/*waarde* ✓ subst/*vervang* ✓ y-value/*waarde* (4) ✓ subtracting/*aftrekking* ✓ y-value/*waarde* ✓ subst/*vervang* ✓ x-value/*waarde* (4)



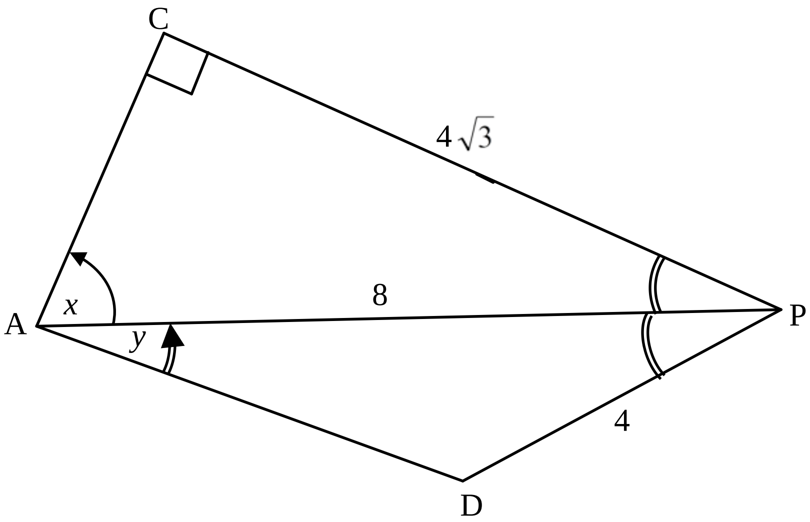
4.5	area DMOE = area ΔAMO – area ΔADE $x_A = 2(0) - 9 \quad \therefore$ A(–9 ; 0) area ΔAMO $= \frac{1}{2}$. AO. OM $= \frac{1}{2}(9)(4\frac{1}{2})$ = 20,25 area ΔADE $= \frac{1}{2}$. AE . y_D $= \frac{1}{2}$. (AO – EO). y_D $= \frac{1}{2}\left(9 - 2\frac{2}{3}\right)\left(3\frac{4}{5}\right)$ = 12,03 **OR/OF** area ΔADE $= \frac{1}{2}$ AD.AE.sin $D\hat{A}E$ $= \frac{1}{2}\ (\frac{19\sqrt{5}}{5}).6\frac{1}{3}$.sin 26,57° = 12,03 $\therefore$ area DMOE = 8,22 square units/*vk eenh* **OR/*OF***	✓ correct method/ *korrekte metode* ✓ $x_A = -9$ ✓ $\frac{1}{2}(9)(4\frac{1}{2})$ ✓ AE = $9 - 2\frac{2}{3} = 6\frac{1}{3}$ ✓ $y_D = 3\frac{4}{5}$ **OR/OF** ✓ AD = $\frac{19\sqrt{5}}{5}$ ✓ AE = $6\frac{1}{3}$ ✓ answer/*antw* (6)

	area DMOE = area rectangle DCOG + area ΔDMG + area ΔDEC $= (1\frac{2}{5} \times 3\frac{4}{5}) + \frac{1}{2}\left(1\frac{2}{5}\right)(\frac{7}{10}) + \frac{1}{2}(3\frac{4}{5})(\frac{19}{15})$ = 8,22 square units/*vk eenh*	✓ correct method/ *korrekte metode* ✓ $3\frac{4}{5}$ ✓ $1\frac{2}{5}$ ✓ 0,7 ✓ $\frac{19}{15}$ ✓ answer (6)
	OR/*OF*	
	area DMOE = area ΔEDO + area ΔODM $= \frac{1}{2}(\text{EO} \times y_{\text{D}}) + \frac{1}{2}(\text{OM} \times -x_D)$ $= \frac{1}{2}\left[\left(\frac{8}{3} \times \frac{19}{5}\right) + \left(\frac{9}{2} \times \frac{7}{5}\right)\right]$ $= \frac{1}{2}\left(\frac{304+189}{30}\right)$ $= \frac{493}{60}$ or/*of* $8\frac{13}{60}$ or/*of* 8,22 square units/*vk eenh*	✓ correct method/ *korrekte metode* ✓ $y_{\text{D}} = \frac{19}{5}$ or $3\frac{4}{5}$ ✓ EO = $\frac{8}{3}$ ✓ $-x_{\text{D}} = \frac{7}{5}$ ✓ OM = $\frac{9}{2}$ or $4\frac{1}{2}$ ✓ answer/*antw* (6)
	OR/*OF*	
	area DMOE = area ΔEOF – area ΔDMF $= \frac{1}{2}(\text{EO} \times \text{OF}) - \frac{1}{2}(\text{OF} - \text{OM})(-x_D)$ $= \frac{1}{2}\left[\left(\frac{8}{3} \times 8\right) + \left(\frac{7}{2} \times \frac{7}{5}\right)\right]$ $= \frac{1}{2}\left(\frac{640-147}{30}\right)$ $= \frac{493}{60}$ or $8\frac{13}{60}$ or 8,22 square units/*vk eenh*	✓ correct method/ *korrekte metode* ✓ $y_{\text{F}} = 8$ ✓ EO = $\frac{8}{3}$ ✓ $-x_{\text{D}} = \frac{7}{5}$ ✓ FM = $3\frac{1}{2}$ ✓ answer/*antw* (6)
	OR/*OF*	

	area ΔEOM $= \frac{1}{2}(\text{EO} \times \text{OM})$ $= \frac{1}{2}\left(\frac{8}{3} \times \frac{9}{2}\right)$ $=$ 6 sq units/*vk eenh*	✓ area ΔEOM
	ED $= \sqrt{\left(-\frac{7}{5}+\frac{8}{3}\right)^2 + \left(\frac{19}{5}\right)^2}$ and DM $= \sqrt{\left(\frac{7}{5}\right)^2 + \left(\frac{9}{2}-\frac{19}{5}\right)^2}$ $= \frac{19\sqrt{10}}{15}$ or 4,005... $= \frac{7\sqrt{5}}{10}$ or 1,565..	✓ ED $= \frac{19\sqrt{10}}{15}$ ✓ DM $= \frac{7\sqrt{5}}{10}$
	area ΔEDM $= \frac{1}{2}(\text{ED} \times \text{DM} \times \sin \text{E}\hat{\text{D}}\text{M})$ $= \frac{1}{2}\left(\frac{19\sqrt{10}}{15}\right)\left(\frac{7\sqrt{5}}{10}\right)\sin 135°$ $= \frac{133}{60}$ or 2,216...	✓ area ΔEDM
	$\therefore$ area DMOE = area ΔEOM + area ΔEDM $=$ 6 + 2,216... $= \frac{493}{60}$ or/*of* $8\frac{13}{60}$ or/*of* 8,22 square units/*eenh*^{2}	✓ correct method/ *korrekte metode* ✓ answer/*antw*
		(6) **[19]**



QUESTION/VRAAG 5



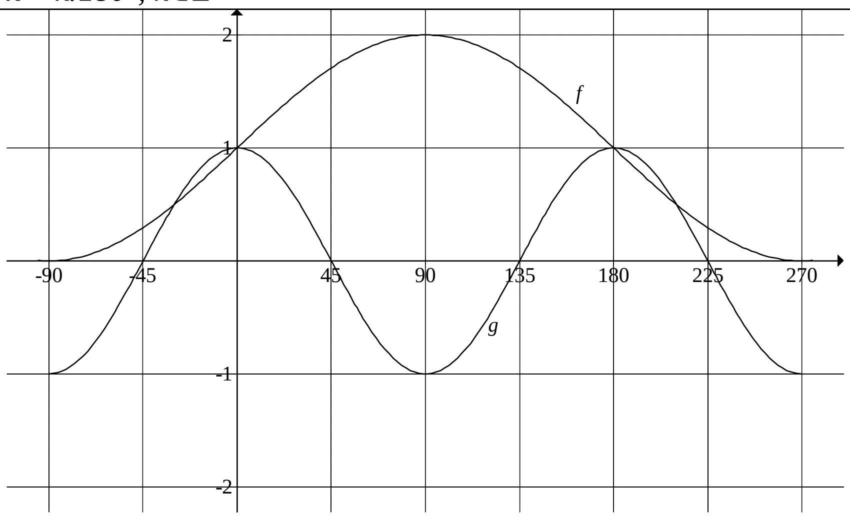
5.1	$\sin \hat{CAP} = \frac{CP}{AP}$ $\sin x = \frac{4\sqrt{3}}{8} = \frac{\sqrt{3}}{2}$ $x = 60°$ OR/OF $\frac{\sin 90°}{8} = \frac{\sin x}{4\sqrt{3}}$ $\sin x = \frac{4\sqrt{3}}{8} = \frac{\sqrt{3}}{2}$ $x = 60°$	✓ correct sine ratio/ *korrekte sin-verh* ✓ $\frac{\sqrt{3}}{2}$ (2) ✓ correct sine ratio/ *korrekte sin-verh* ✓ $\frac{\sqrt{3}}{2}$ (2)
5.2	$\hat{CPA} = \hat{DPA} = 30°$ (AP bisects $\hat{DPC}$) $AD^2 = AP^2 + DP^2 - 2.AP.DP.\cos \hat{APD}$ $= 8^2 + 4^2 - 2(8)(4)\cos 30°$ $= 8^2 + 4^2 - 2(8)(4)(\frac{\sqrt{3}}{2})$ $= 24,57...$ $AD = 4,96$	✓ $\hat{DPA} = 30°$ ✓ correct subst into cosine rule/ *korrekte subst in cos-reël* ✓ 24,57... ✓ 4,96 (4)

5.3	$\frac{\sin \hat{DAP}}{DP} = \frac{\sin \hat{APD}}{AD}$ $\frac{\sin y}{4} = \frac{\sin 30°}{4,96}$ $\sin y = \frac{4\sin 30°}{4,96}$ $= 0,403...$ $y = 23,78°$ **OR/*OF*** $AD^2 = AP^2 + DP^2 - 2.AP.DP.\cos \hat{DAP}$ $4^2 = 8^2 + (4,96)^2 - 2(8)(4,96).\cos y$ $\cos y = \frac{8^2 + (4,96)^2 - 4^2}{2(8)(4,96)}$ $\cos y = 0,9148...$ $y = 23,82°$	✓ correct subst into sine rule/ *korrekte subst in sin-reël* ✓ sin *y* subject ✓ 23,78° (3) ✓ correct subst into cosine rule/ *korrekte subst in cos-reël* ✓ cos *y* subject ✓ 23,82° (3) **[9]**

QUESTION/VRAAG 6

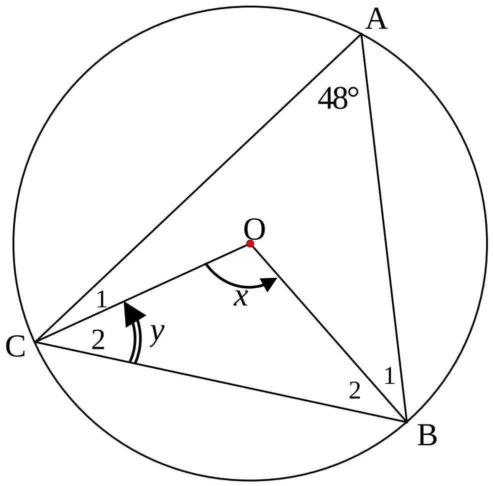
6.1	$\cos^2(180°+x)+\tan(x-180°)\sin(720°-x)\cos x$ $= (-\cos x)^2 + [-(-\tan x)](-\sin x)(\cos x)$ $= \cos^2 x + \left(\dfrac{\sin x}{\cos x}\right)(-\sin x)(\cos x)$ $= \cos^2 x - \sin^2 x$ $= \cos 2x$	✓ $(-\cos x)^2$ or $\cos^2 x$ ✓ $\tan x$ or $-(-\tan x)$ ✓ $-\sin x$ ✓ $\tan x = \dfrac{\sin x}{\cos x}$ ✓ $\cos^2 x - \sin^2 x$ (5)
6.2	$\sin(\alpha-\beta)$ $= \cos[90° - (\alpha-\beta)]$ $= \cos[(90°-\alpha)+\beta]$ $= \cos(90°-\alpha)\cos\beta - \sin(90°-\alpha)\sin\beta$ $= \sin\alpha\cos\beta - \cos\alpha\sin\beta$ **OR/*OF*** $\sin(\alpha-\beta)$ $= \cos[90° - (\alpha-\beta)]$ $= \cos[(90°+\beta)+(-\alpha)]$ $= \cos(90°+\beta)\cos(-\alpha) - \sin(90°+\beta)\sin(-\alpha)$ $= (-\sin\beta)\cos\alpha - \cos\beta(-\sin\alpha)$ $= \sin\alpha\cos\beta - \cos\alpha\sin\beta$	✓rewrite as/*herskryf* $\cos[(90°-\alpha)+\beta]$ ✓ expansion/ *uitbreiding* ✓simpl/*vereenv* (3) ✓ rewrite as/*herskryf* $\cos[(90°+\beta)+(-\alpha)]$ ✓ expansion/ *uitbreiding* ✓simpl/*vereenv* (3)
6.3	$x^2 - y^2$ $= \sin^2 76° - \cos^2 76°$ $= -(\cos^2 76° - \sin^2 76°)$ $= -\cos 2(76°)$ $= -\cos 152°$ $= -(-\cos 28°)$ **OR/*OF*** $= -\cos(90° + 62°)$ $= \cos 28°$ $= -(-\sin 62°)$ $= \cos(90° - 62°)$ $= \sin 62°$ $= \sin 62°$ **OR/*OF*** $x^2 - y^2$ $= \sin^2 76° - \cos^2 76°$ $= \sin 76° \sin 76° - \cos 76° \cos 76°$ $= \sin 76° \cos 14° - \cos 76° \sin 14°$ $= \sin(76° - 14°)$ $= \sin 62°$ **OR/*OF*** $x^2 - y^2$ $= \sin^2 76° - \cos^2 76°$ $= \cos^2 14° - \sin^2 14°$ $= \cos 2(14°)$ $= \cos 28°$ $= \sin 62°$	✓$-(\cos^2 76° - \sin^2 76°)$ ✓ recognition of cos double angle ✓ $-\cos 152°$ ✓ $\cos 28°$ (4) ✓ $\cos 14°$ ✓ $\sin 14°$ ✓ recognition of sine compound angle ✓ $\sin(76° - 14°)$ (4) ✓ $\cos^2 14°$ ✓ $\sin^2 14°$ ✓ recognition of cos double angle ✓ $\cos 28°$ (4) **[12]**

QUESTION/VRAAG 7

7.1	$0 \leq y \leq 2$ or $y \in [0 ; 2]$	✓ critical values/ *kritieke waardes* ✓ notation/*notasie* (2)
7.2	$\sin x + 1 = \cos 2x$ $\sin x + 1 = 1 - 2\sin^2 x$ $2\sin^2 x + \sin x = 0$ $\sin x(2\sin x + 1) = 0$	✓ $1 - 2\sin^2 x$ ✓ st form/*st vorm* (2)
7.3	$\sin x(2\sin x + 1) = 0$ $\sin x = 0$ *or* $\sin x = -\frac{1}{2}$ $x = 0° + k.360°$ or $x = 210° + k.360°$ or $x = 180° + k.360°$ $x = 330° + k.360°, k \in Z$ **OR/*OF*** $x = k.180°, k \in Z$	✓ $\sin x = 0$ *or* $\sin x = -\frac{1}{2}$ ✓ 0° ; 180° **OR/*OF*** $x = k.180°$ ✓ 210° ; 330° ✓ $k.360°$, $k \in Z$ (4)
7.4		✓ *y*-intercept/*afsnit* ✓ *x*-intercepts/*afsnitte* ✓ min/max points/ *min/maks punte* (3)
7.5	$f(x) = g(x)$ at/*by*: $x = -30° ; 0° ; 180° ; 210°$ $\therefore f(x + 30°) = g(x + 30°)$ at/*by*: $x = -60° ; -30° ; 150° ; 180°$	✓ –30° ; 0° ; 180° ; 210° ✓✓ –60° ; –30° ; 150° ; 180° (3)
7.6	Series will converge if/*Reeks sal konvergeer as:* $-1 < r < 1$ $-1 < 2\cos 2x < 1$ $-\frac{1}{2} < \cos 2x < \frac{1}{2}$ $\therefore$ $30° < x < 60°$ or $x \in (30° ; 60°)$	✓ $-1 < r < 1$ ✓ $r = 2\cos 2x$ ✓ $-\frac{1}{2} < \cos 2x < \frac{1}{2}$ ✓✓ $30° < x < 60°$ (5) **[19]**

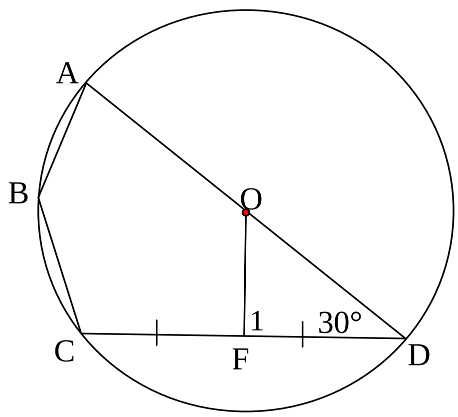
QUESTION/VRAAG 8

8.1



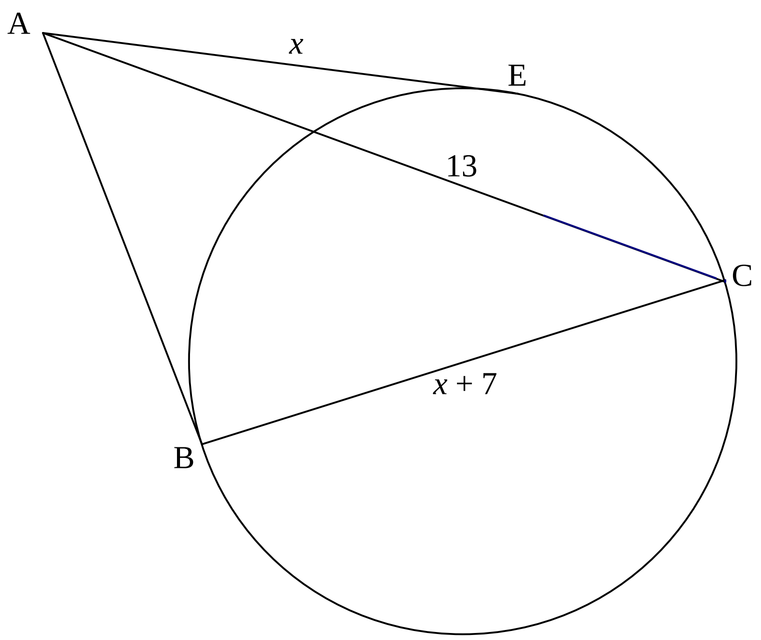
8.1.1	$x = 96°$ (∠ at centre = 2∠ at circumference/ ∠ *by midpt = 2∠ by omtrek*)	✓ S ✓ R (2)
8.1.2	$\hat{C}_2 + \hat{B}_2 = 180° - 96° = 84°$ (sum of ∠s in Δ/ *som v∠e in* Δ) $y = \hat{B}_2 = 42°$ (∠s opp = sides/∠*e teenoor = sye*)	✓ S ✓ S (2)

8.2



8.2.1	$\hat{F}_1 = 90°$ (line from centre to midpt chord/ *lyn vanaf midpt na midpt kd)*	✓ S ✓ R (2)
8.2.2	$A\hat{B}C = 150°$ (opposite ∠s of cyclic quad/ *tos* ∠*e v koordevh*)	✓ S ✓ R (2)

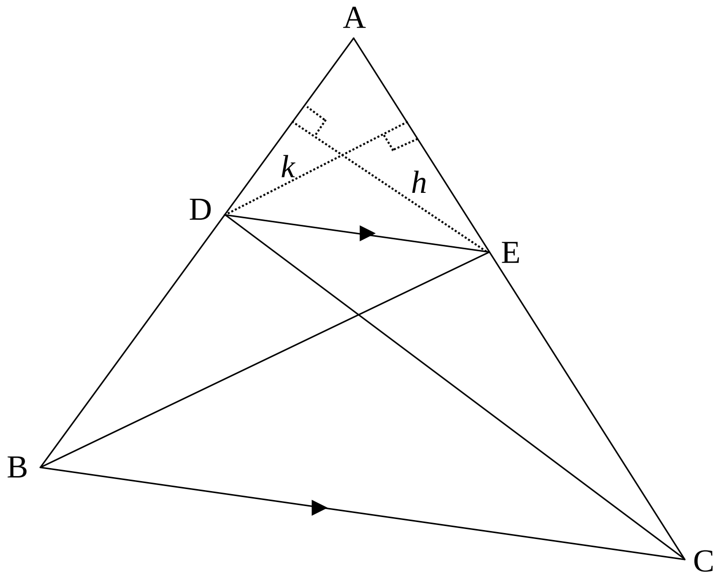
8.3



8.3.1 (a)	tangent ⊥ radius/diameter / *raaklyn ⊥ radius/middellyn*	✓ R (1)
8.3.1 (b)	tangents from common pt **OR** tangents from same pt / *raaklyne v gemeensk pt* ***OF*** *raaklyne vanaf dies pt*	✓ R (1)
8.3.2	AB² + BC² = AC² $x^2 + (x+7)^2 = 13^2$ (Theorem of/*Stelling van*Pythagoras) $x^2 + x^2 + 14x + 49 = 169$ $2x^2 + 14x - 120 = 0$ $x^2 + 7x - 60 = 0$ $(x-5)(x+12) = 0$ $x = 5 \quad (x \neq -12)$	✓ AB² + BC² = AC² ✓ $x^2 + (x+7)^2 = 13^2$ ✓ standard form ✓ answer (4) **[14]**

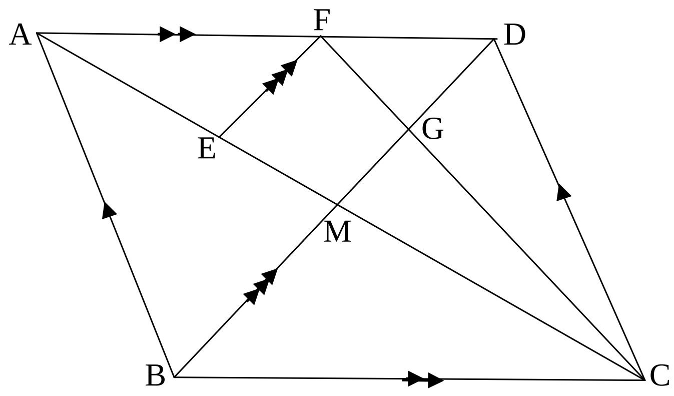
QUESTION/VRAAG 9

9.1



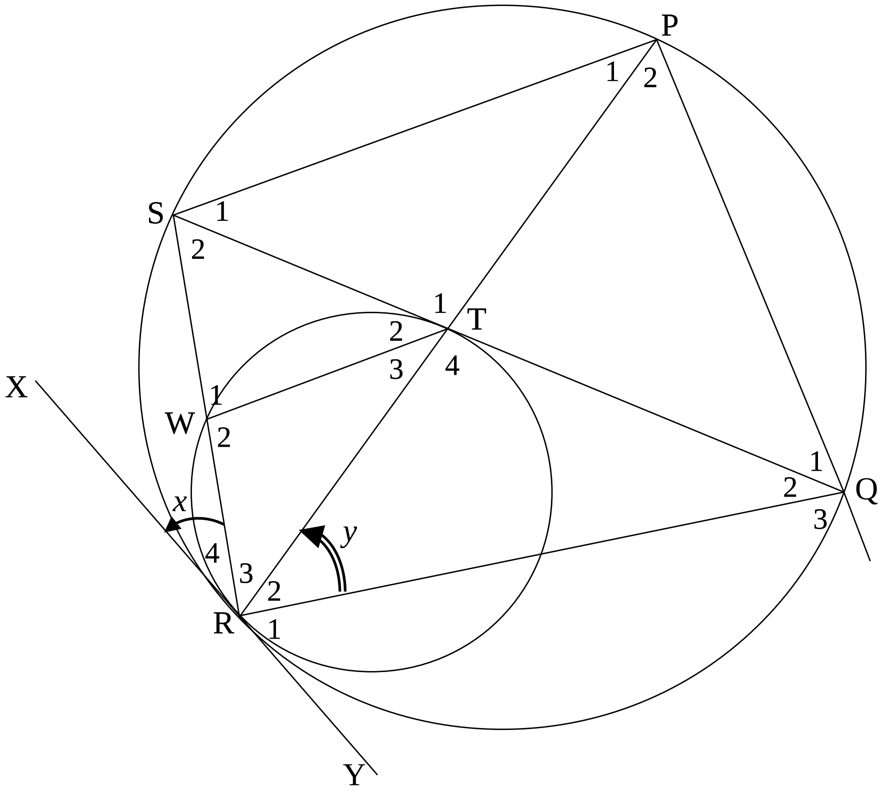
9.1.1	Same base (DE) and same height (between parallel lines) *Dieselfde basis (DE) en dieselfde hoogte (tussen ewewydige lyne)*	✓ same base/*dies basis* between || lines/ *tussen || lyne* (1)
9.1.2	$\dfrac{\text{AD}}{\text{DB}} = \dfrac{\frac{1}{2}\text{AE}\times k}{\frac{1}{2}\text{EC}\times k}$ But/*Maar* area ΔDEB = area ΔDEC (Same base and same height/*dieselfde basis en dieselfde hoogte*) $\therefore \dfrac{\text{area }\Delta\text{ADE}}{\text{area }\Delta\text{DEB}} = \dfrac{\text{area }\Delta\text{ADE}}{\text{area }\Delta\text{DEC}}$ $\therefore \dfrac{\text{AD}}{\text{DB}} = \dfrac{\text{AE}}{\text{EC}}$	✓ S ✓ S ✓ S ✓ R ✓ S (5)

9.2

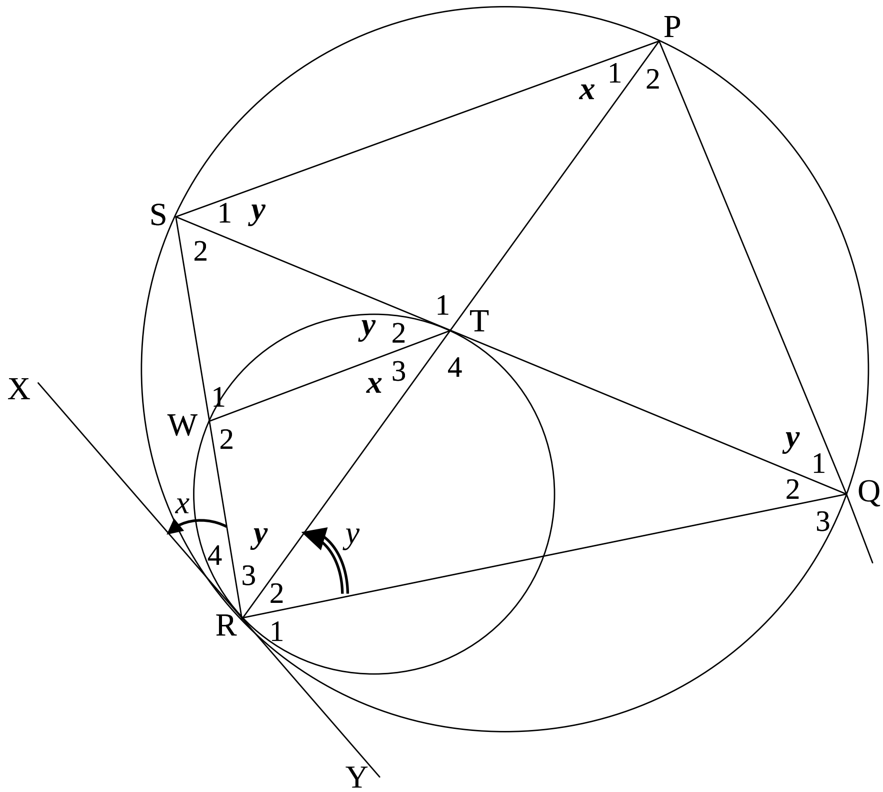


9.2.1	$\frac{EM}{AM}=\frac{FD}{AD}$ (Line parallel one side of Δ **OR** prop th; EF | | BD) (*Lyn ewewydig aan sy v Δ* ***OF*** *eweredigst; EF ||BD)* $\frac{EM}{AM}=\frac{3}{7}$	✓ S ✓R ✓ answer/*antw* (3)
9.2.2	CM = AM (diags of parm bisect/*hoekl parm halv*) $\frac{CM}{ME}=\frac{AM}{ME}=\frac{7}{3}$ (from 9.2.1/*vanaf 9.2.1*)	✓ S ✓R ✓ answer/*antw* (3)
9.2.3	*h* of ΔFDC = *h* of ΔBDC (AD | | BC) $\frac{\text{area }\Delta FDC}{\text{area }\Delta BDC}=\frac{\frac{1}{2}FD.h}{\frac{1}{2}BC.h}$ $=\frac{FD}{AD}$ (opp sides of parm =) *(tos sye v parm =)* $=\frac{3}{7}$ **OR/*OF*** $\frac{\text{area }\Delta FDC}{\text{area }\Delta ADC}=\frac{FD}{AD}=\frac{3}{7}$ (same heights) *(dieselfde hoogtes)* But Area ΔADC = Area ΔBDC (diags of parm bisect area) *(hoekl v parm halv opp)* $\frac{\text{area }\Delta FDC}{\text{area }\Delta BDC}=\frac{3}{7}$	✓ AD | | BC ✓ subst into area form/ *subst in opp formule* ✓ S ✓ answer/*antw* (4) ✓ S ✓ R ✓ S ✓ answer/*antw* (4) **[16]**

QUESTION/VRAAG 10



10.1.1	Tangent chord theorem/*Raaklyn-koordstelling*	✓ R (1)
10.1.2	Tangent chord theorem/*Raaklyn-koordstelling*	✓ R (1)
10.1.3	Corresponding angles equal/*Ooreenkomstige ∠e gelyk*	✓ R (1)
10.1.4	∠s subtended by chord PQ **OR** ∠s in same segment *∠e onderspan deur dieselfde koord* ***OF*** *∠e in dieselfde segment*	✓ R (1)
10.1.5	alternate ∠s/*verwisselende ∠e* ; WT | | SP	✓ R (1)
10.2	$\frac{RW}{RS} = \frac{RT}{RP}$ (Line parallel one side of Δ **OR** prop th; WT | | SP) (*Lyn ewewydig aan sy v Δ* ***OF*** *eweredighst: WT | | SP)* $\therefore RT = \frac{WR.RP}{RS}$ **OR/*OF*** ΔRTW | | | ΔRPS (∠; ∠; ∠) $\therefore \frac{RW}{RS} = \frac{RT}{RP}$ (ΔRTW | | | ΔRPS) $\therefore RT = \frac{RW.RP}{RS}$	✓ S ✓ R (2) ✓ S ✓ S (2)
10.3	$y = \hat{T}_2 = \hat{R}_3$ (tan chord theorem/*Rkl-koordst*) $y = \hat{R}_3 = \hat{Q}_1$ (∠s in same segment/*∠e in dieselfde segment*)	✓ S ✓ R ✓ S ✓ R (4)



10.4	$\hat{Q}_3 = P\hat{S}R$ (ext ∠ of cyc quad/*buite∠ v kdvh*) $P\hat{S}R = \hat{W}_2$ (corresp∠s/*ooreenk ∠e* ; WT | | SP) $\therefore \hat{Q}_3 = \hat{W}_2$ **OR/OF** $\hat{Q}_2 = x$ (∠s in same segment/*∠e in dies segment*) $\hat{Q}_3 = 180° - (x + y)$ (∠s on straight line/*∠e op reguitlyn*) $\hat{W}_2 = 180° - (x + y)$ (∠s of ΔWRT/*∠e v ΔWRT*) $\therefore \hat{Q}_3 = \hat{W}_2$	✓ S ✓ R ✓ S (3) ✓ R ✓ S ✓ S (3)
10.5	In ΔRTS and ΔRQP: $\hat{R}_3 = \hat{R}_2 = y$ (proven above/*hierbo bewys*) $\hat{S}_2 = \hat{P}_2$ (∠s in same segment/*∠e in dies segment*) $R\hat{T}S = R\hat{Q}P$ (3^{rd} angle of Δ) $\therefore$ΔRTS | | | ΔRQP (∠; ∠; ∠)	✓ S ✓ S/R ✓ S **OR/OF** (∠; ∠; ∠) (3)

10.6	$\frac{RT}{RQ}=\frac{RS}{RP}$ (ΔRTS | | | ΔRQP) $\frac{RS}{RP}\times\frac{RS}{RP}=\frac{RT}{RQ}\times\frac{RS}{RP}$ $\left(\frac{RS}{RP}\right)^2=\left(\frac{RT}{RP}\right)\left(\frac{RS}{RQ}\right)$ $=\left(\frac{RW}{RS}\right)\left(\frac{RS}{RQ}\right)$ (proven in 10.2/*bewys in 10.2*) $=\frac{RW}{RQ}$	✓ S ✓ $\times\frac{RS}{RP}$ on both sides ✓ $\left(\frac{RT}{RP}\right)\left(\frac{RS}{RQ}\right)$ (3)
	OR/OF $\frac{RT}{RQ}=\frac{RS}{RP}$ (ΔRTS | | | ΔRQP) But $RT=\frac{WR.RP}{RS}$ (proven in 10.2/*bewys in 10.2*) $\therefore\frac{RT}{RQ}=\frac{WR.RP}{RQ.RS}=\frac{RS}{RP}$ $WR.RP^2=RQ.RS^2$ $\therefore\frac{WR}{RQ}=\frac{RS^2}{RP^2}$	✓ S ✓ $RT=\frac{WR.RP}{RS}$ ✓multiplication/ *vermenigvuldig* (3)
	OR/OF $\frac{RT}{RS}=\frac{RQ}{RP}$ (ΔRTS | | | ΔRQP) $RQ=\frac{RT.RP}{RS}$ and $WR=\frac{RT.RS}{RP}$ (proven in 10.2/*bewys in 10.2*) $\frac{WR}{RQ}=\frac{\frac{RT.RS}{RP}}{\frac{RT.RP}{RS}}$ $=\frac{RT.RS}{RP}\times\frac{RS}{RT.RP}$ $=\frac{RS^2}{RP^2}$	✓ S ✓ $WR=\frac{RT.RS}{RP}$ ✓ simplification/ *vereenvoudiging* (3) **[20]**

TOTAL/TOTAAL: 150

basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE 12

MATHEMATICS P2

EXEMPLAR 2014

MARKS: 150

TIME: 3 hours

This question paper consists of 12 pages, 3 diagram sheets and 1 information sheet.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 10 questions.

2. Answer ALL the questions.

3. Clearly show ALL calculations, diagrams, graphs, et cetera which you have used in determining your answers.

4. Answers only will NOT necessarily be awarded full marks.

5. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.

6. If necessary, round off answers to TWO decimal places, unless stated otherwise.

7. THREE diagram sheets for QUESTION 2.1, QUESTION 8.2, QUESTION 9, QUESTION 10.1, and QUESTION 10.2 are attached at the end of this question paper. Write your centre number and examination number on these sheets in the spaces provided and insert them inside the back cover of your ANSWER BOOK.

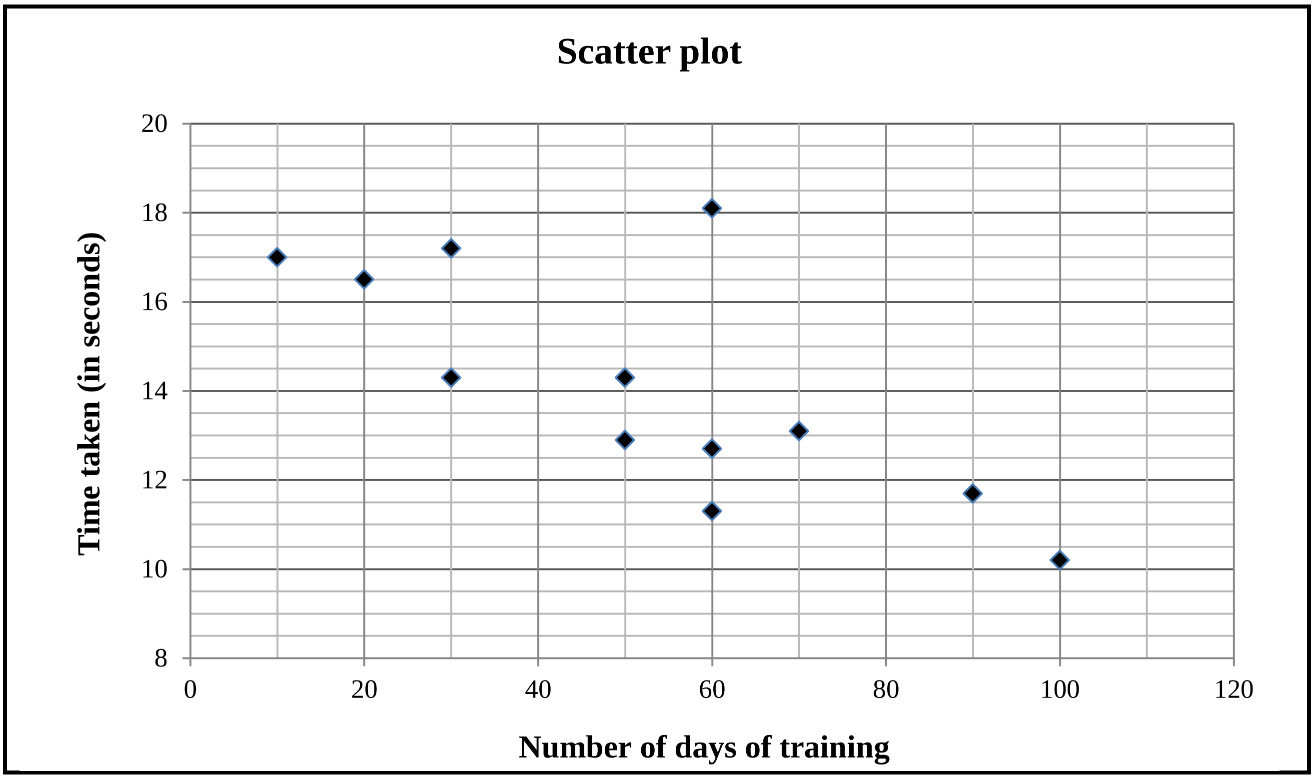
8. Number the answers correctly according to the numbering system used in this question paper.

9. Write neatly and legibly.

QUESTION 1

Twelve athletes trained to run the 100 m sprint event at the local athletics club trials. Some of them took their training more seriously than others. The following table and scatter plot shows the number of days that an athlete trained and the time taken to run the event. The time taken, in seconds, is rounded to one decimal place.

Number of days of training	50	70	10	60	60	20	50	90	100	60	30	30
Time taken (in seconds)	12,9	13,1	17,0	11,3	18,1	16,5	14,3	11,7	10,2	12,7	17,2	14,3



1.1 Discuss the trend of the data collected. (1)

1.2 Identify any outlier(s) in the data. (1)

1.3 Calculate the equation of the least squares regression line. (4)

1.4 Predict the time taken to run the 100 m sprint for an athlete training for 45 days. (2)

1.5 Calculate the correlation coefficient. (2)

1.6 Comment on the strength of the relationship between the variables. (1)
[11]

QUESTION 2

The table below shows the amount of time (in hours) that learners aged between 14 and 18 spent watching television during 3 weeks of the holiday.

Time (hours)	**Cumulative frequency**
$0 \leq t < 20$	25
$20 \leq t < 40$	69
$40 \leq t < 60$	129
$60 \leq t < 80$	157
$80 \leq t < 100$	166
$100 \leq t < 120$	172

2.1 Draw an ogive (cumulative frequency curve) on DIAGRAM SHEET 1 to represent the above data. (3)

2.2 Write down the modal class of the data. (1)

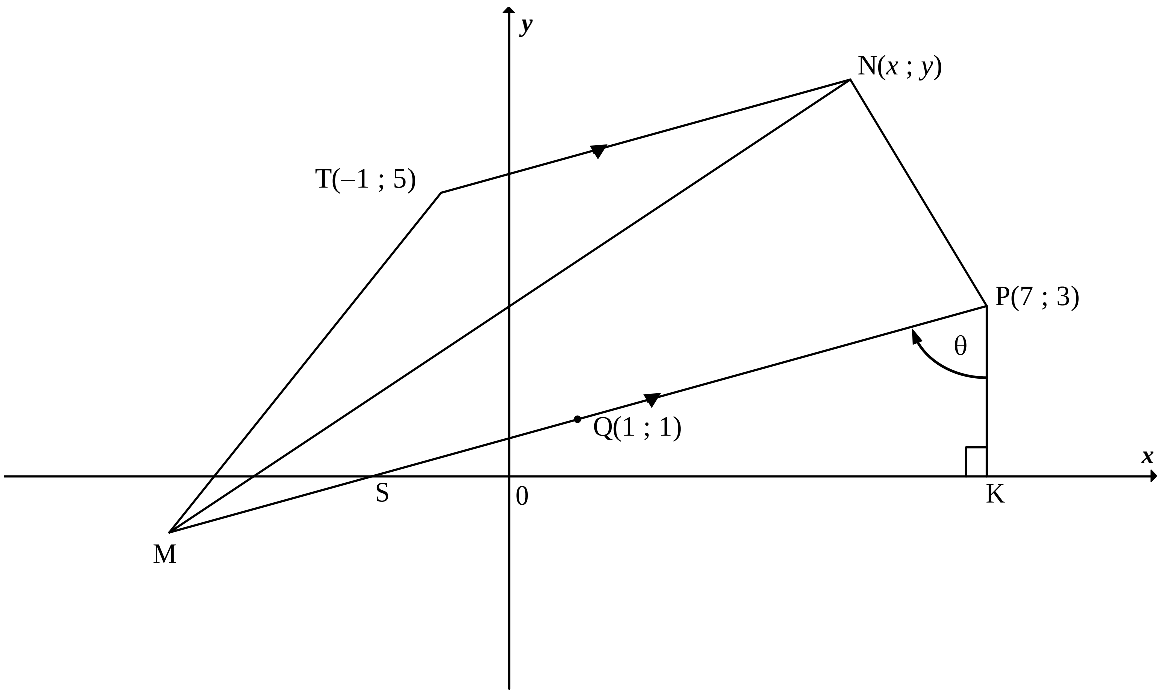
2.3 Use the ogive (cumulative frequency curve) to estimate the number of learners who watched television more than 80% of the time. (2)

2.4 Estimate the mean time (in hours) that learners spent watching television during 3 weeks of the holiday. (4)

[10]

QUESTION 3

In the diagram below, M, T(–1 ; 5), N(x ; y) and P(7 ; 3) are vertices of trapezium MTNP having TN | | MP. Q(1 ; 1) is the midpoint of MP. PK is a vertical line and $S\hat{P}K = \theta$. The equation of NP is $y = -2x + 17$.



3.1 Write down the coordinates of K. (1)

3.2 Determine the coordinates of M. (2)

3.3 Determine the gradient of PM. (2)

3.4 Calculate the size of θ. (3)

3.5 Hence, or otherwise, determine the length of PS. (3)

3.6 Determine the coordinates of N. (5)

3.7 If A(a ; 5) lies in the Cartesian plane:

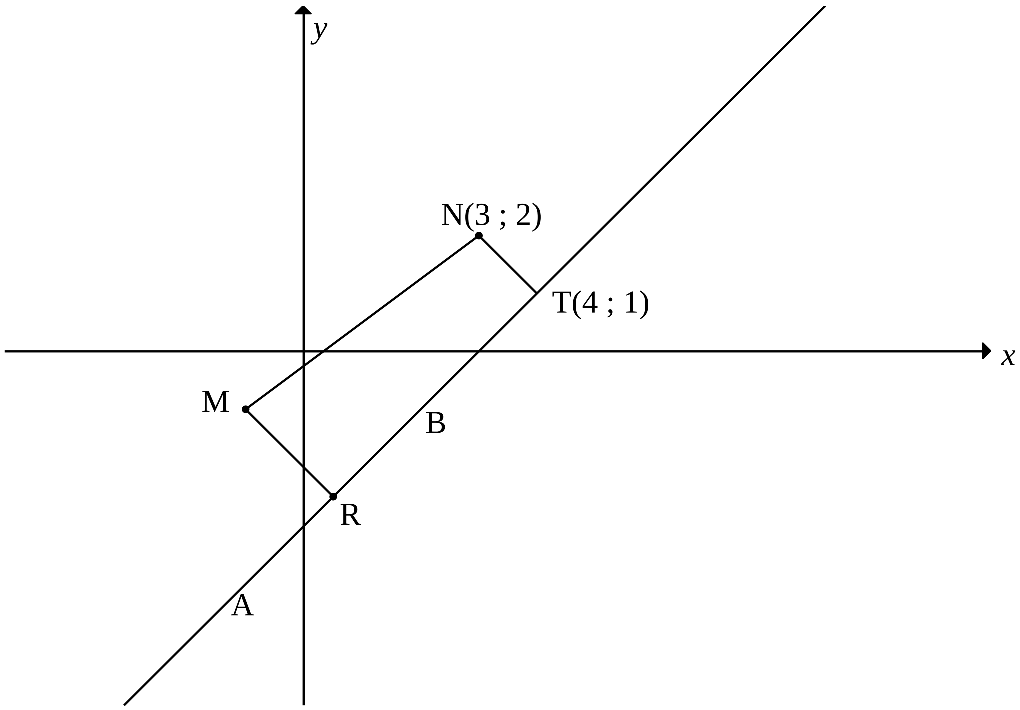
3.7.1 Write down the equation of the straight line representing the possible positions of A. (1)

3.7.2 Hence, or otherwise, calculate the value(s) of a for which $T\hat{A}Q = 45°$. (5)

[22]

QUESTION 4

In the diagram below, the equation of the circle having centre M is $(x + 1)^2 + (y + 1)^2 = 9$. R is a point on chord AB such that MR bisects AB. ABT is a tangent to the circle having centre N(3 ; 2) at point T(4 ; 1).



4.1 Write down the coordinates of M. (1)

4.2 Determine the equation of AT in the form $y = mx + c$. (5)

4.3 If it is further given that $MR = \frac{\sqrt{10}}{2}$ units, calculate the length of AB. Leave your answer in simplest surd form. (4)

4.4 Calculate the length of MN. (2)

4.5 Another circle having centre N touches the circle having centre M at point K. Determine the equation of the new circle. Write your answer in the form $x^2 + y^2 + Cx + Dy + E = 0$. (3)

[15]

QUESTION 5

5.1 Given that $\sin\alpha = -\frac{4}{5}$ and $90° < \alpha < 270°$.

WITHOUT using a calculator, determine the value of each of the following in its simplest form:

5.1.1 $\sin(-\alpha)$ (2)

5.1.2 $\cos\alpha$ (2)

5.1.3 $\sin(\alpha - 45°)$ (3)

5.2 Consider the identity: $\dfrac{8\sin(180° - x)\cos(x - 360°)}{\sin^2 x - \sin^2(90° + x)} = -4\tan 2x$

5.2.1 Prove the identity. (6)

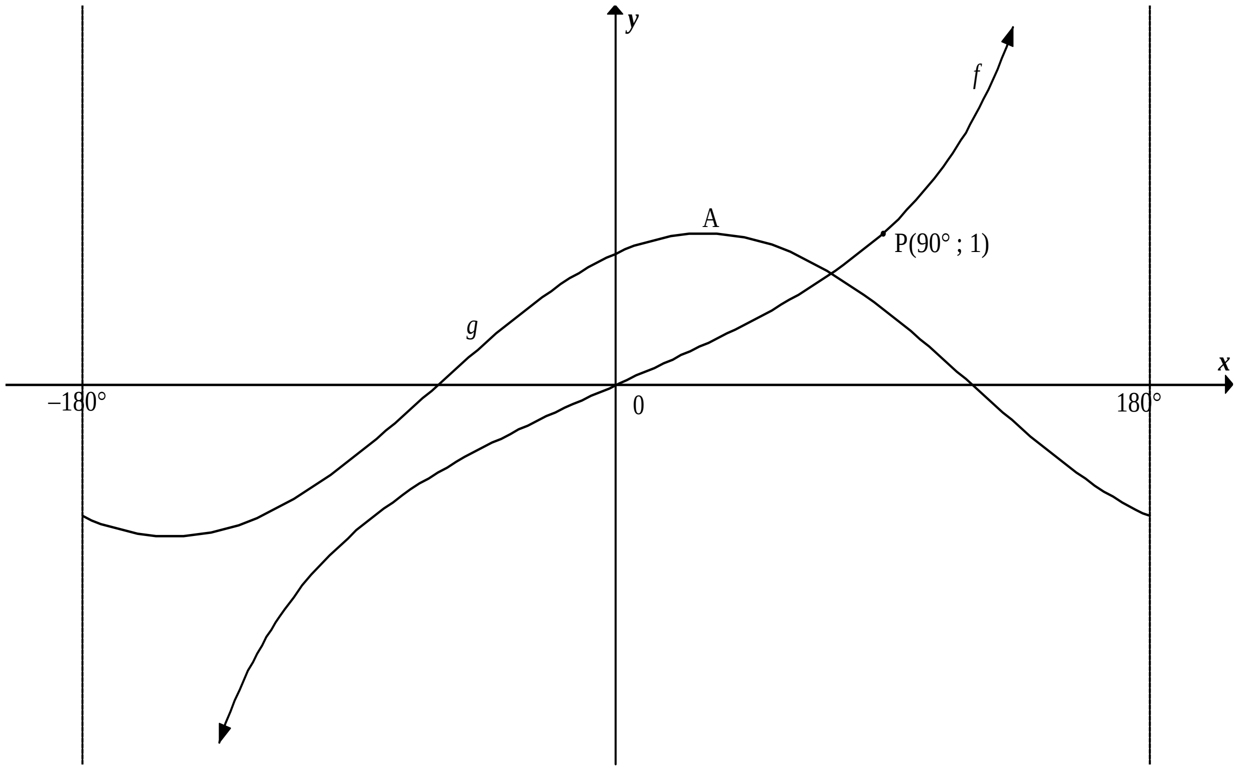
5.2.2 For which value(s) of x in the interval $0° < x < 180°$ will the identity be undefined? (2)

5.3 Determine the general solution of $\cos 2\theta + 4\sin^2\theta - 5\sin\theta - 4 = 0$. (7)

[22]

QUESTION 6

In the diagram below, the graphs of $f(x) = \tan bx$ and $g(x) = \cos(x - 30°)$ are drawn on the same system of axes for $-180° \leq x \leq 180°$. The point P(90° ; 1) lies on f. Use the diagram to answer the following questions.



6.1 Determine the value of b. (1)

6.2 Write down the coordinates of A, a turning point of g. (2)

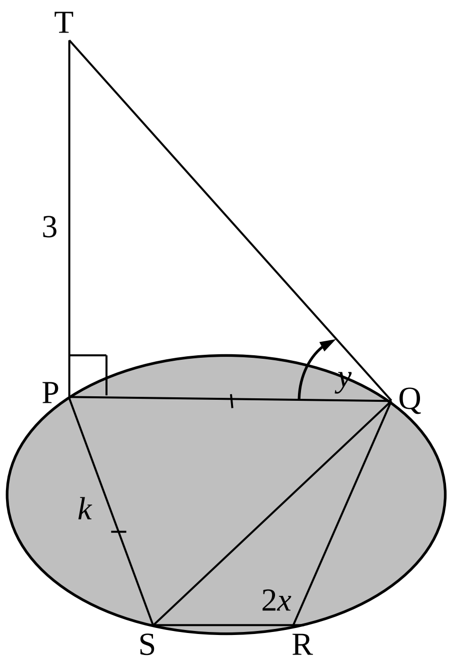
6.3 Write down the equation of the asymptote(s) of $y = \tan b(x + 20°)$ for $x \in [-180°; 180°]$. (1)

6.4 Determine the range of h if $h(x) = 2g(x) + 1$. (2)
[6]

QUESTION 7

7.1 Prove that in any acute-angled ΔABC, $\dfrac{\sin A}{a}=\dfrac{\sin B}{b}$. (5)

7.2 The framework for a construction consists of a cyclic quadrilateral PQRS in the horizontal plane and a vertical post TP as shown in the figure. From Q the angle of elevation of T is $y°$. $PQ = PS = k$ units, $TP = 3$ units and $S\hat{R}Q = 2x°$.



7.2.1 Show, giving reasons, that $P\hat{S}Q = x$. (2)

7.2.2 Prove that $SQ = 2k\cos x$. (4)

7.2.3 Hence, prove that $SQ = \dfrac{6\cos x}{\tan y}$. (2)

[13]

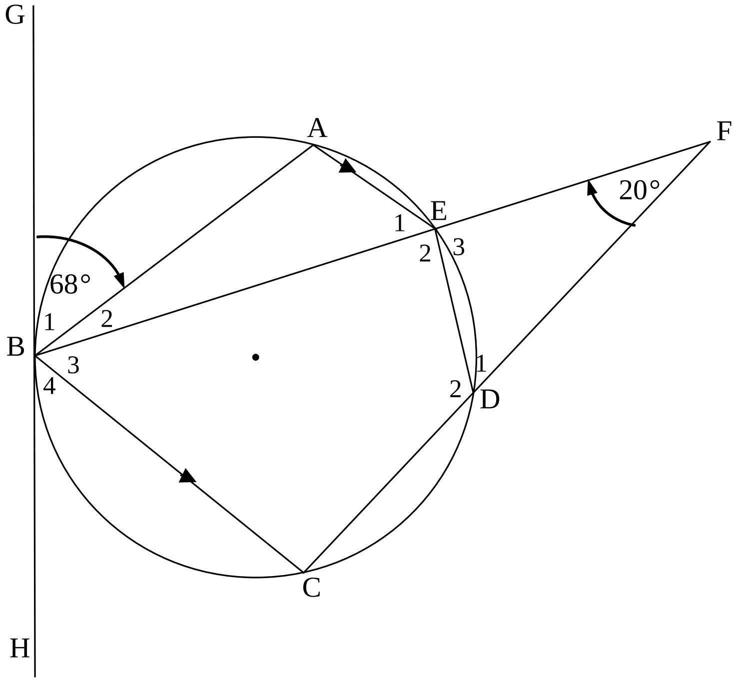
Give reasons for your statements in QUESTIONS 8, 9 and 10.

QUESTION 8

8.1 Complete the following statement:

The angle between the tangent and the chord at the point of contact is equal to ... (1)

8.2 In the diagram, A, B, C, D and E are points on the circumference of the circle such that AE | | BC. BE and CD produced meet in F. GBH is a tangent to the circle at B. $\hat{B}_1 = 68°$ and $\hat{F} = 20°$.



Determine the size of each of the following:

8.2.1 $\hat{E}_1$ (2)

8.2.2 $\hat{B}_3$ (1)

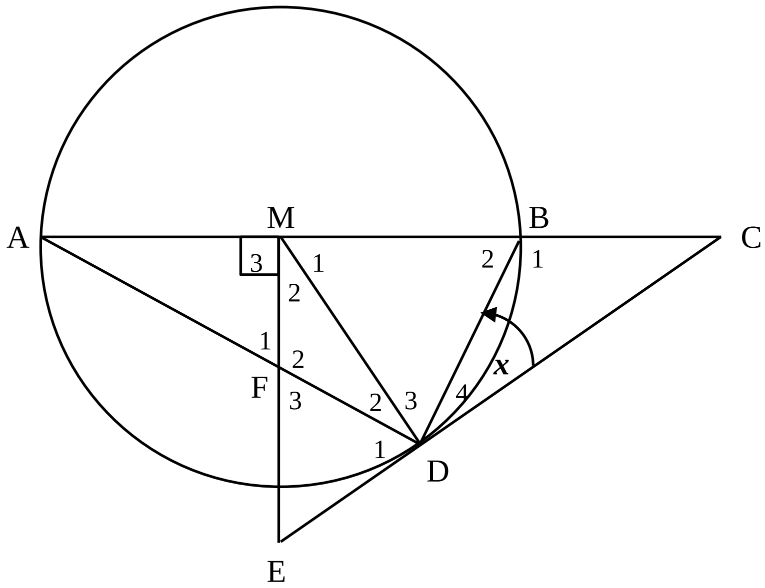
8.2.3 $\hat{D}_1$ (2)

8.2.4 $\hat{E}_2$ (1)

8.2.5 $\hat{C}$ (2)
[9]

QUESTION 9

In the diagram, M is the centre of the circle and diameter AB is produced to C. ME is drawn perpendicular to AC such that CDE is a tangent to the circle at D. ME and chord AD intersect at F. MB = 2BC.



9.1 If $\hat{D}_4 = x$, write down, with reasons, TWO other angles each equal to x. (3)

9.2 Prove that CM is a tangent at M to the circle passing through M, E and D. (4)

9.3 Prove that FMBD is a cyclic quadrilateral. (3)

9.4 Prove that $DC^2 = 5BC^2$. (3)

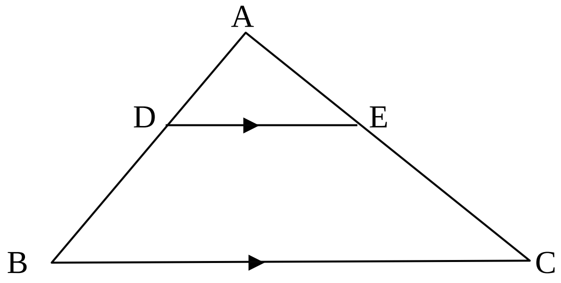
9.5 Prove that $\Delta DBC \mid\mid\mid \Delta DFM$. (4)

9.6 Hence, determine the value of $\frac{DM}{FM}$. (2)

[19]

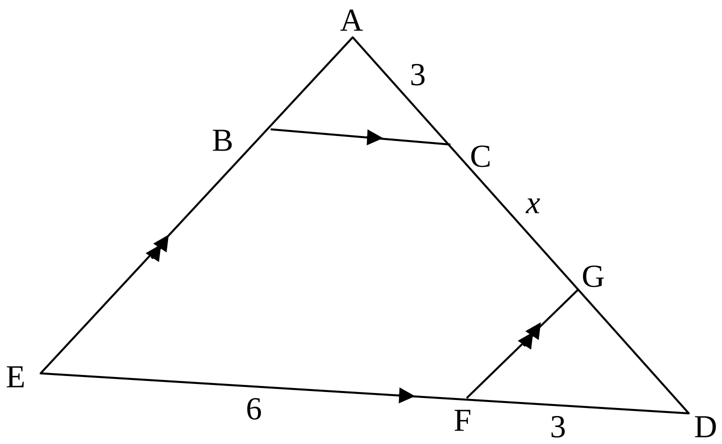
QUESTION 10

10.1 In the diagram, points D and E lie on sides AB and AC respectively of ΔABC such that DE | | BC. Use Euclidean Geometry methods to prove the theorem which states that $\frac{AD}{DB} = \frac{AE}{EC}$.



(6)

10.2 In the diagram, ADE is a triangle having BC | | ED and AE | | GF. It is also given that AB : BE = 1 : 3, AC = 3 units, EF = 6 units, FD = 3 units and CG = x units.



Calculate, giving reasons:

10.2.1 The length of CD (3)

10.2.2 The value of x (4)

10.2.3 The length of BC (5)

10.2.4 The value of $\frac{\text{area } \Delta ABC}{\text{area } \Delta GFD}$ (5)

[23]

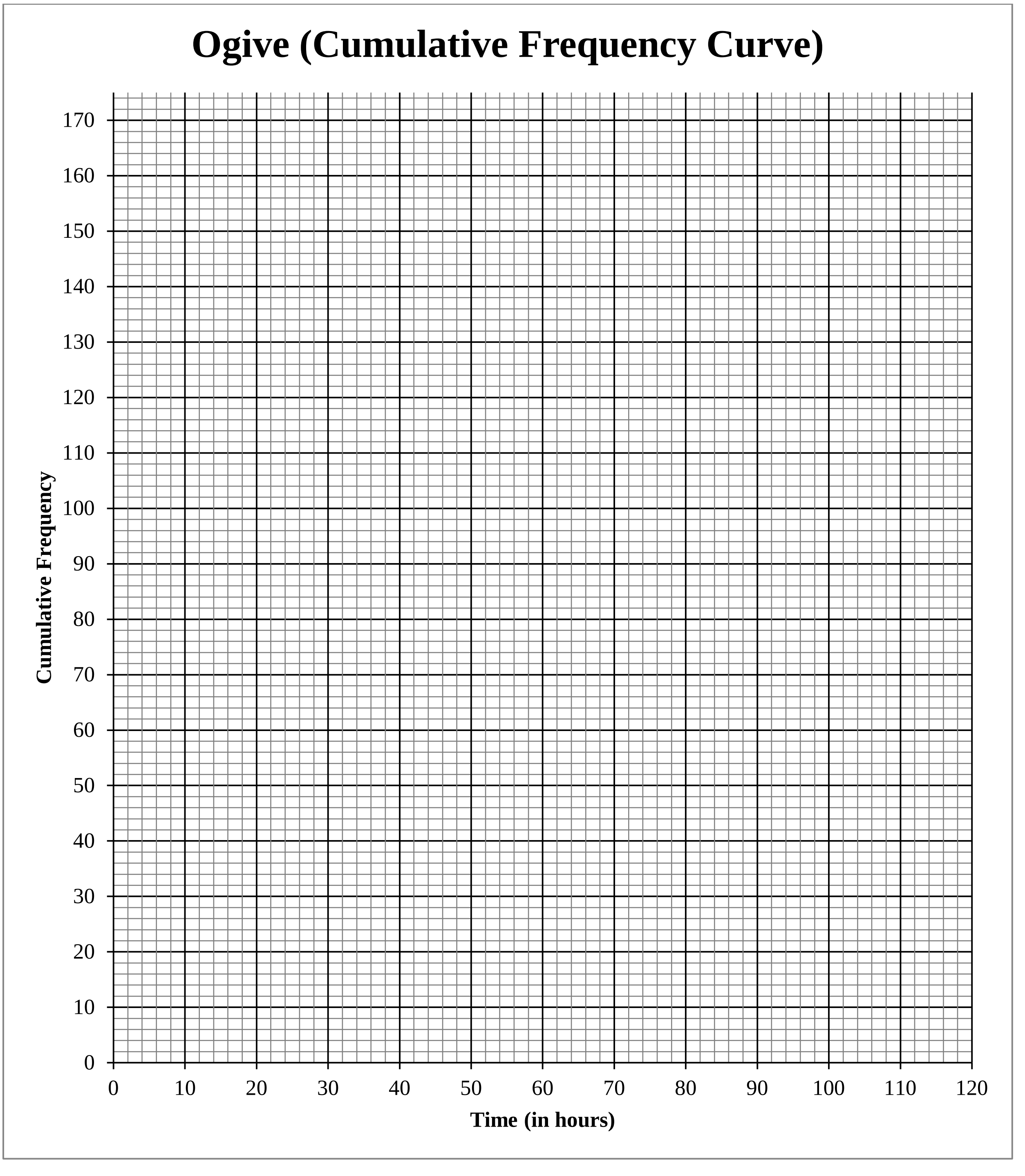
TOTAL: 150

NAME:

GRADE/CLASS:

DIAGRAM SHEET 1

QUESTION 2.1

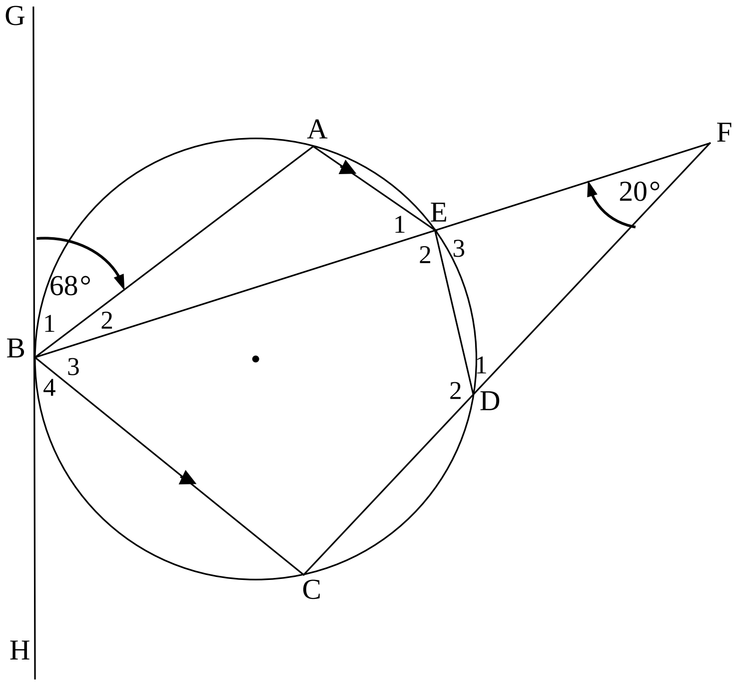


NAME:

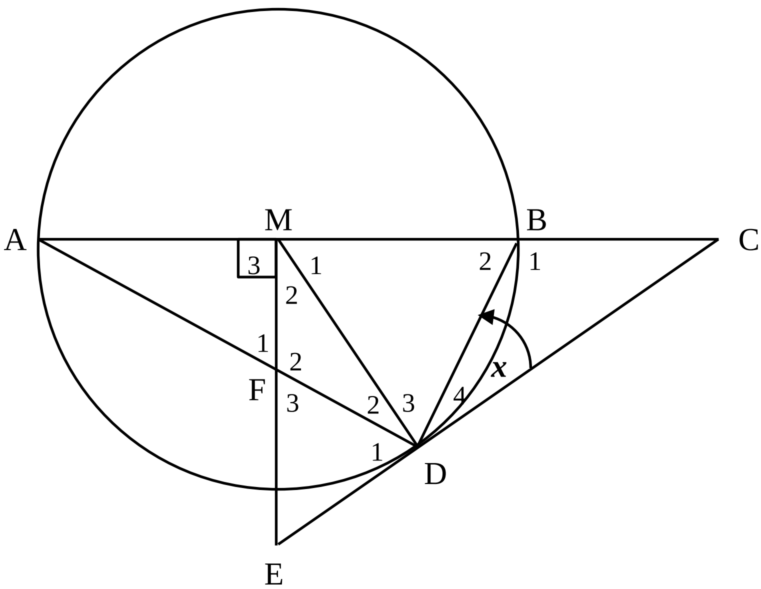
GRADE/CLASS:

DIAGRAM SHEET 2

QUESTION 8.2



QUESTION 9

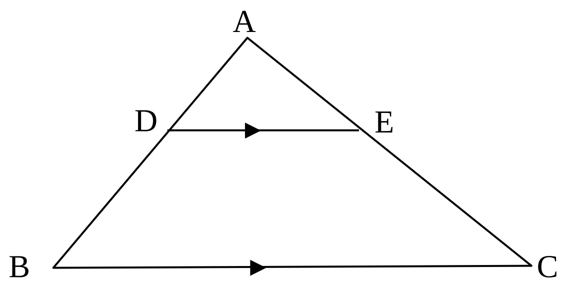


NAME:

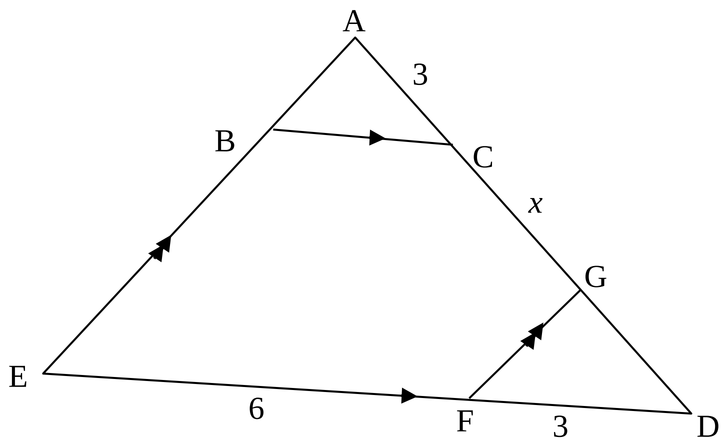
GRADE/CLASS:

DIAGRAM SHEET 3

QUESTION 10.1



QUESTION 10.2



INFORMATION SHEET: MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1+ni)$ $\quad A = P(1-ni)$ $\quad A = P(1-i)^n$ $\quad A = P(1+i)^n$

$T_n = a + (n-1)d$ $\quad S_n = \frac{n}{2}[2a + (n-1)d]$

$T_n = ar^{n-1}$ $\quad S_n = \frac{a(r^n - 1)}{r-1}$; $r \neq 1$ $\quad S_\infty = \frac{a}{1-r}$; $-1 < r < 1$

$F = \frac{x[(1+i)^n - 1]}{i}$ $\quad P = \frac{x[1-(1+i)^{-n}]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\quad \mathrm{M}\left(\frac{x_1 + x_2}{2} ; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $\quad y - y_1 = m(x - x_1)$ $\quad m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad m = \tan\theta$

$(x-a)^2 + (y-b)^2 = r^2$

In ΔABC: $\frac{a}{\sin \mathrm{A}} = \frac{b}{\sin \mathrm{B}} = \frac{c}{\sin \mathrm{C}}$ $\quad a^2 = b^2 + c^2 - 2bc.\cos A$ $\quad area\,\Delta \mathrm{ABC} = \frac{1}{2} ab.\sin \mathrm{C}$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\quad \sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\quad \cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$ $\quad \sin 2\alpha = 2\sin\alpha.\cos\alpha$

$\bar{x} = \frac{\sum fx}{n}$ $\quad \sigma^2 = \frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n}$

$P(A) = \frac{n(A)}{n(S)}$ $\quad P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$\hat{y} = a + bx$ $\quad b = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2}$