SA-STUDENT

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"You have to ask yourself how badly do you want something? If you really, really want something then put in the work". -Lewis Hamilton

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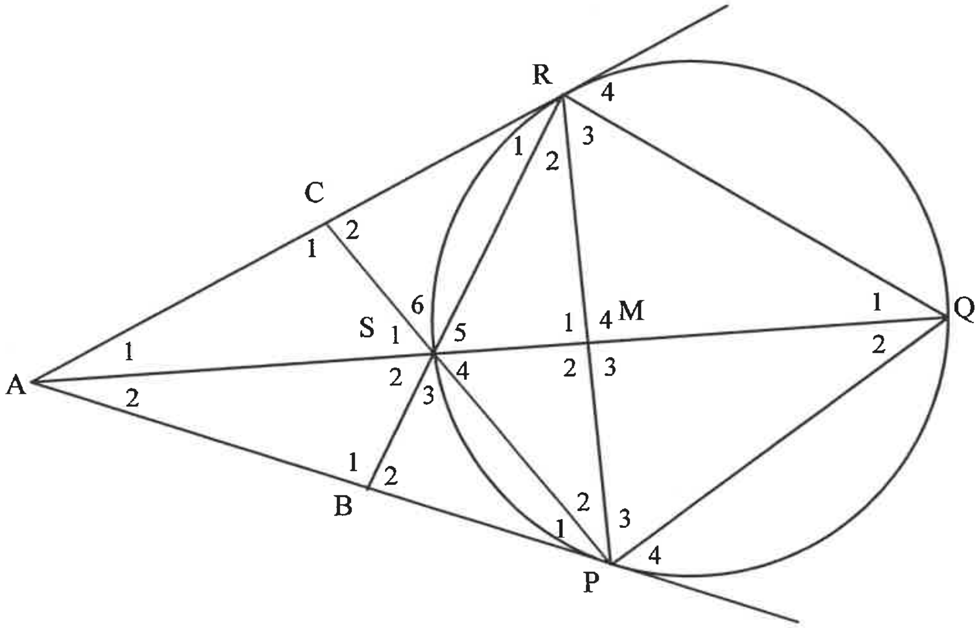
QUESTION 10

In the diagram, PQRS is a cyclic quadrilateral such that PQ = PR. The tangents to the circle through P and R meet QS produced at A. RS is produced to meet tangent AP at B. PS is produced to meet tangent AR at C. PR and QS intersect at M.



Prove, giving reasons, that:

10.1 $\hat{S}_3 = \hat{S}_4$ (5)

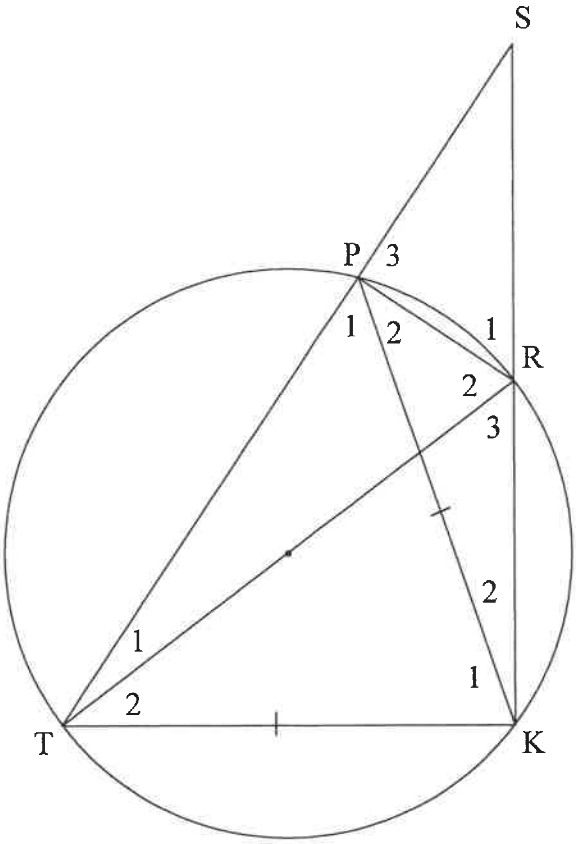
10.2 SMRC is a cyclic quadrilateral (4)

10.3 RP is a tangent to the circle passing through P, S and A at P (6)
[15]

TOTAL: 150

QUESTION 10

In the diagram, TR is a diameter of the circle. PRKT is a cyclic quadrilateral. Chords TP and KR are produced to intersect at S. Chord PK is drawn such that PK = TK.



10.1 Prove, giving reasons, that:

10.1.1 SR is a diameter of a circle passing through points S, P and R (4)

10.1.2 $\hat{S} = \hat{P}_2$ (5)

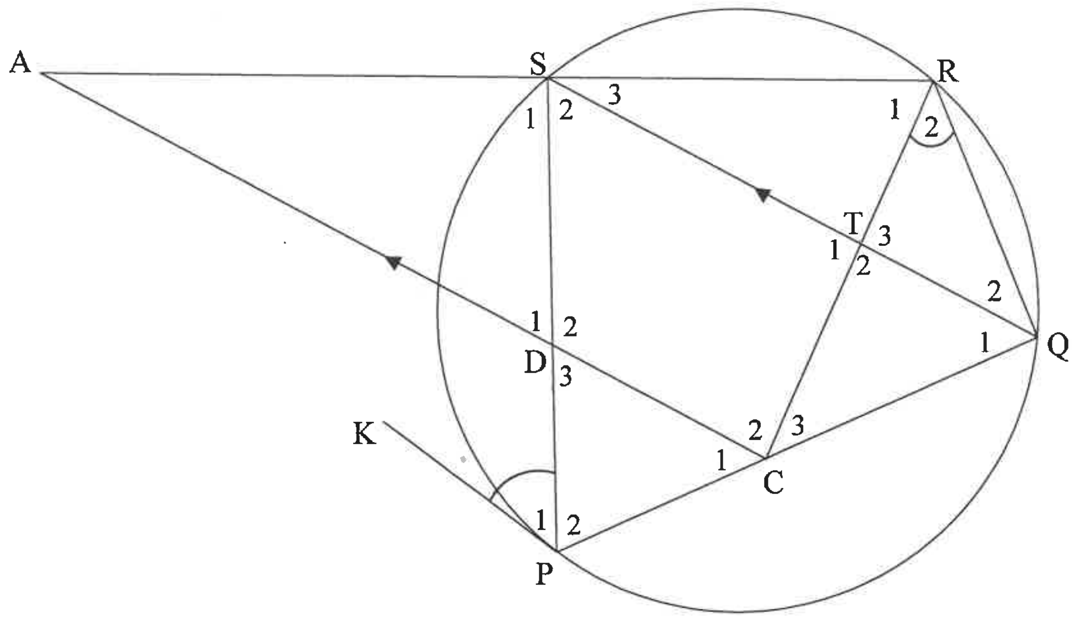
10.1.3 $\Delta SPK \parallel\!| \Delta PRK$ (3)

10.2 If it is further given that SR = RK, prove that $ST = \sqrt{6}RK$. (5)
[17]

TOTAL: 150

QUESTION 10

In the diagram, PQRS is a cyclic quadrilateral. KP is a tangent to the circle at P. C and D are points on chords PQ and PS respectively and CD produced meets RS produced at A. CA || QS. RC is drawn. $\hat{P}_1 = \hat{R}_2$.



Prove, giving reasons, that:

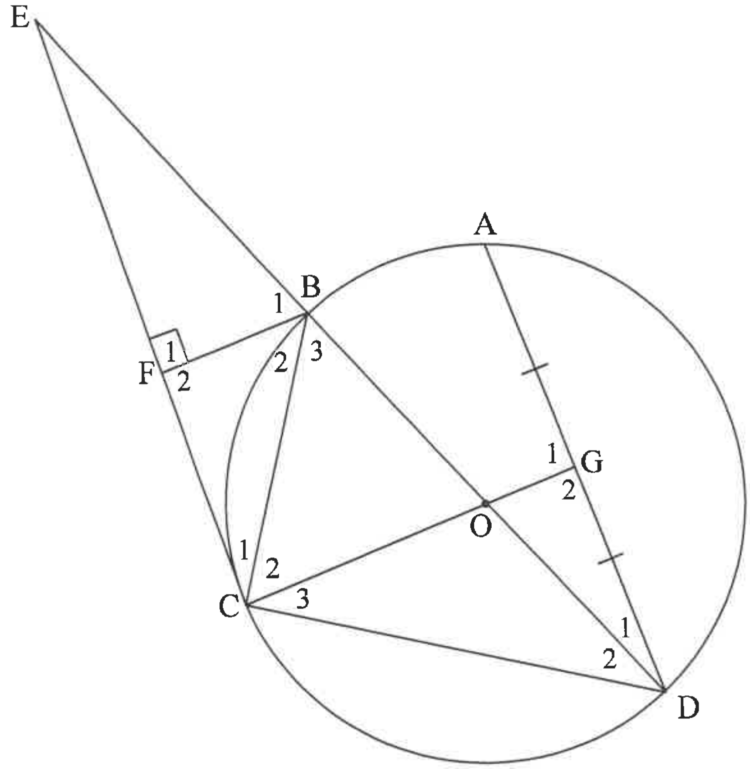
10.1 $\hat{S}_1 = \hat{T}_2$ (4)

10.2 $\dfrac{AD}{AR} = \dfrac{AS}{AC}$ (5)

10.3 $AC \times SD = AR \times TC$ (4)
[13]

TOTAL: 150

10.2 In the diagram, O is the centre of a circle passing through A, B, C and D. EC is a tangent to the circle at C. Diameter DB produced meets tangent EC at E. F is a point on EC such that BF ⊥ EC. Radius CO produced bisects AD at G. BC and CD are drawn.



10.2.1 Prove, with reasons, that:

(a) FB || CG (3)

(b) ΔFCB ||| ΔCDB (5)

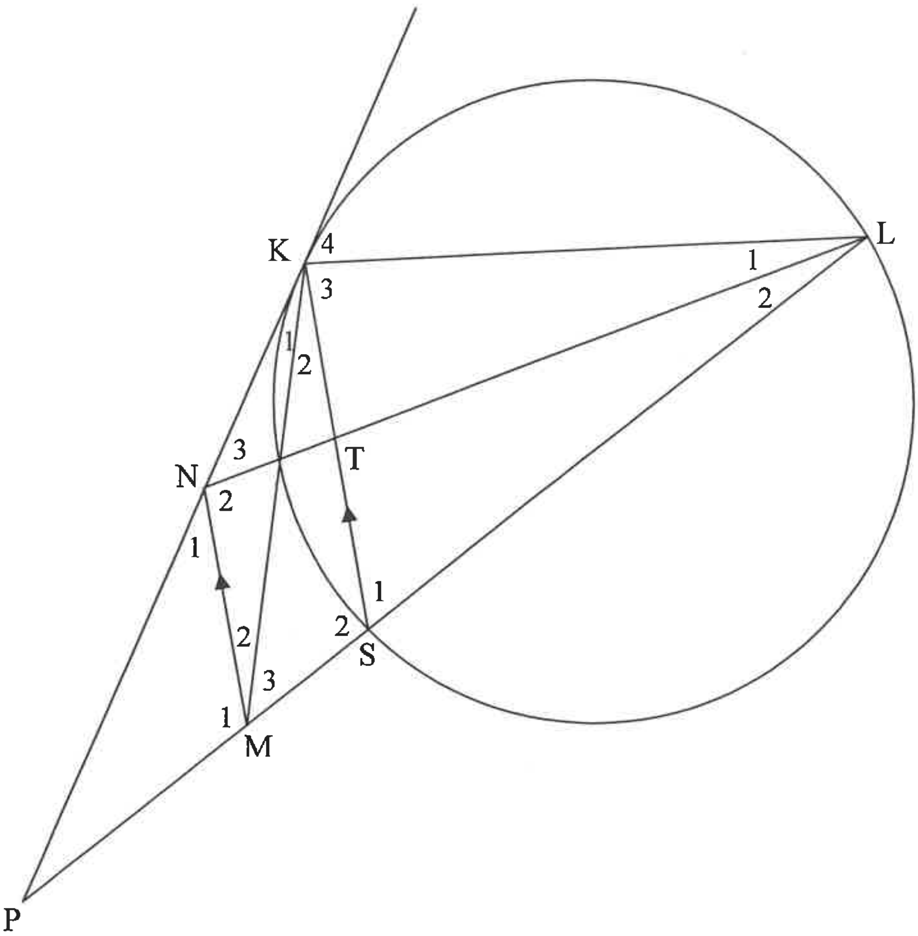
10.2.2 Give a reason why $\hat{G}_1 = 90°$. (1)

10.2.3 Prove, with reasons, that $CD^2 = CG.DB$. (5)

10.2.4 Hence, prove that DB = CG + FB. (5)
[25]

TOTAL: 150

11.2 In the diagram, PK is a tangent to the circle at K. Chord LS is produced to P. N and M are points on KP and SP respectively such that MN || SK. Chord KS and LN intersect at T.



11.2.1 Prove, giving reasons, that:

(a) $\hat{K}_4 = N\hat{M}L$ (4)

(b) KLMN is a cyclic quadrilateral (1)

11.2.2 Prove, giving reasons, that $\Delta LKN \;|||\; \Delta KSM$. (5)

11.2.3 If $LK = 12$ units and $3KN = 4SM$, determine the length of KS. (4)

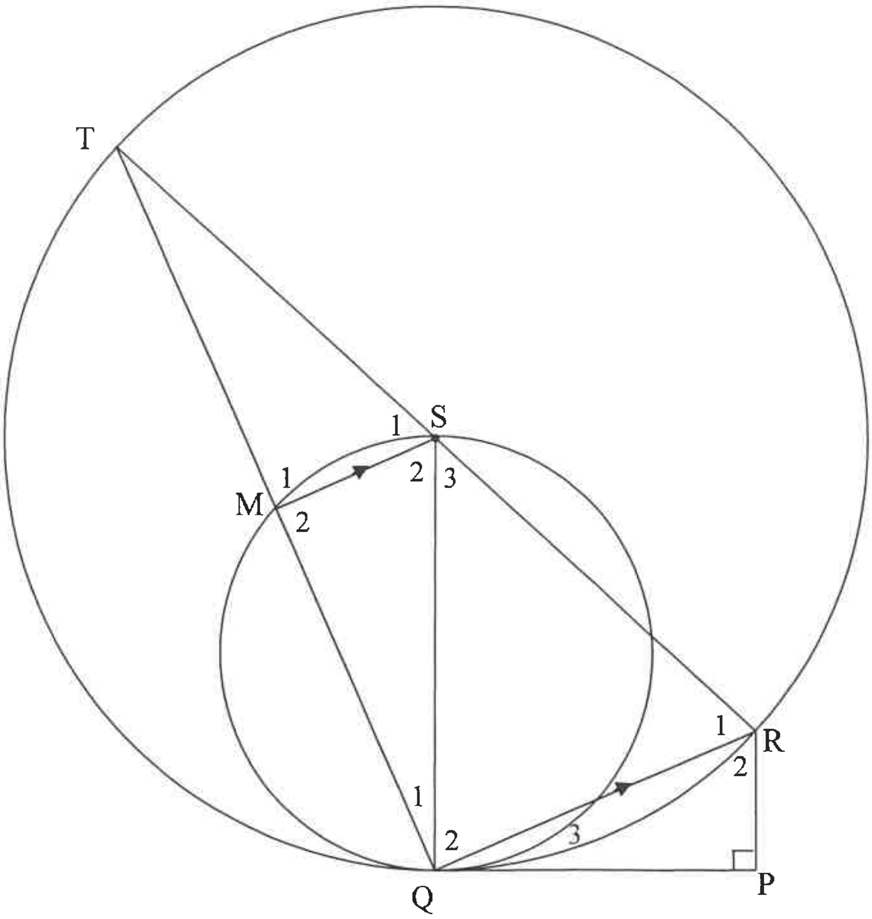
11.2.4 If it is further given that $NL = 16$ units, $LS = 13$ units and $KN = 8$ units, determine, with reasons, the length of LT. (4)

[23]

TOTAL: 150

QUESTION 10

In the diagram, TSR is a diameter of the larger circle having centre S. Chord TQ of the larger circle cuts the smaller circle at M. PQ is a common tangent to the two circles at Q. SQ is drawn.
RP ⊥ PQ and MS || QR.



10.1 Prove, giving reasons that:

10.1.1 SQ is the diameter of the smaller circle (3)

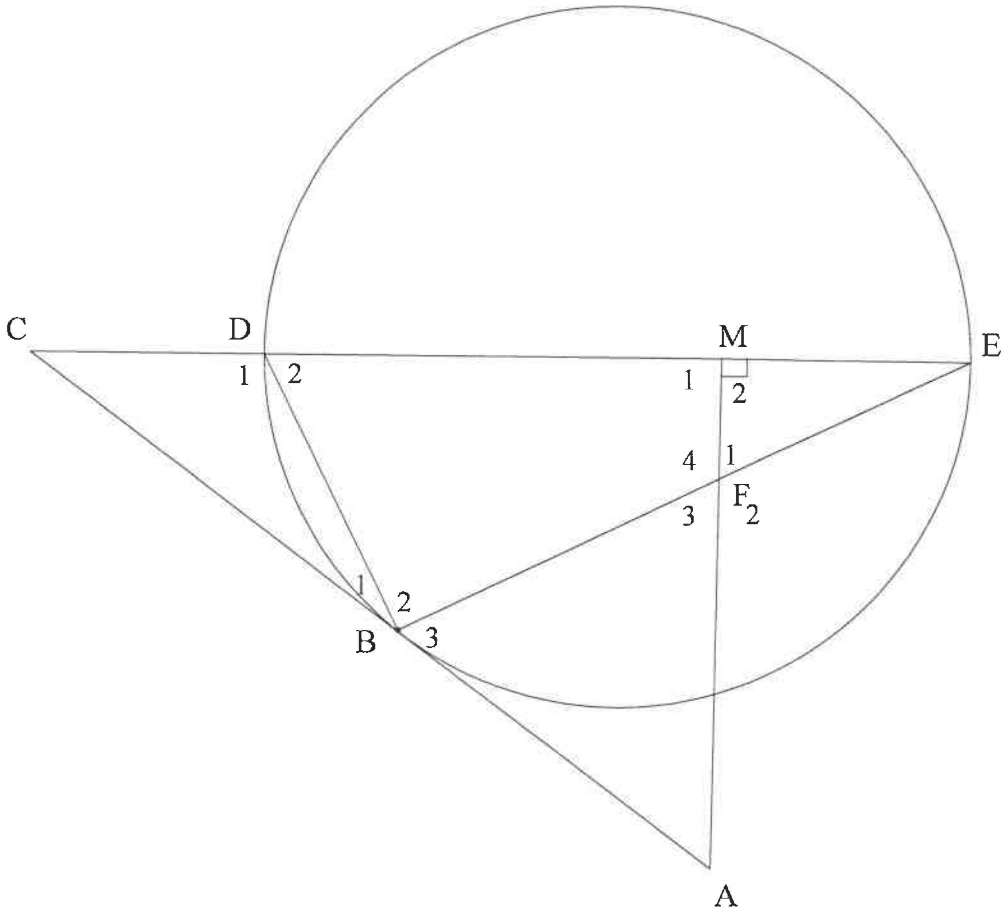
10.1.2 $RT = \dfrac{RQ^2}{RP}$ (6)

10.2 If MS = 14 units and $PQ = \sqrt{640}$ units, calculate, giving reasons, the length of the radius of the larger circle. (6)
[15]

TOTAL: 150

QUESTION 10

In the diagram, a circle passes through D, B and E. Diameter ED of the circle is produced to C and AC is a tangent to the circle at B. M is a point on DE such that AM ⊥ DE. AM and chord BE intersect at F.



10.1 Prove, giving reasons, that:

10.1.1 FBDM is a cyclic quadrilateral (3)

10.1.2 $\hat{B}_3 = \hat{F}_1$ (4)

10.1.3 $\Delta CDB \,|||\, \Delta CBE$ (3)

10.2 If it is further given that CD = 2 units and DE = 6 units, calculate the length of:

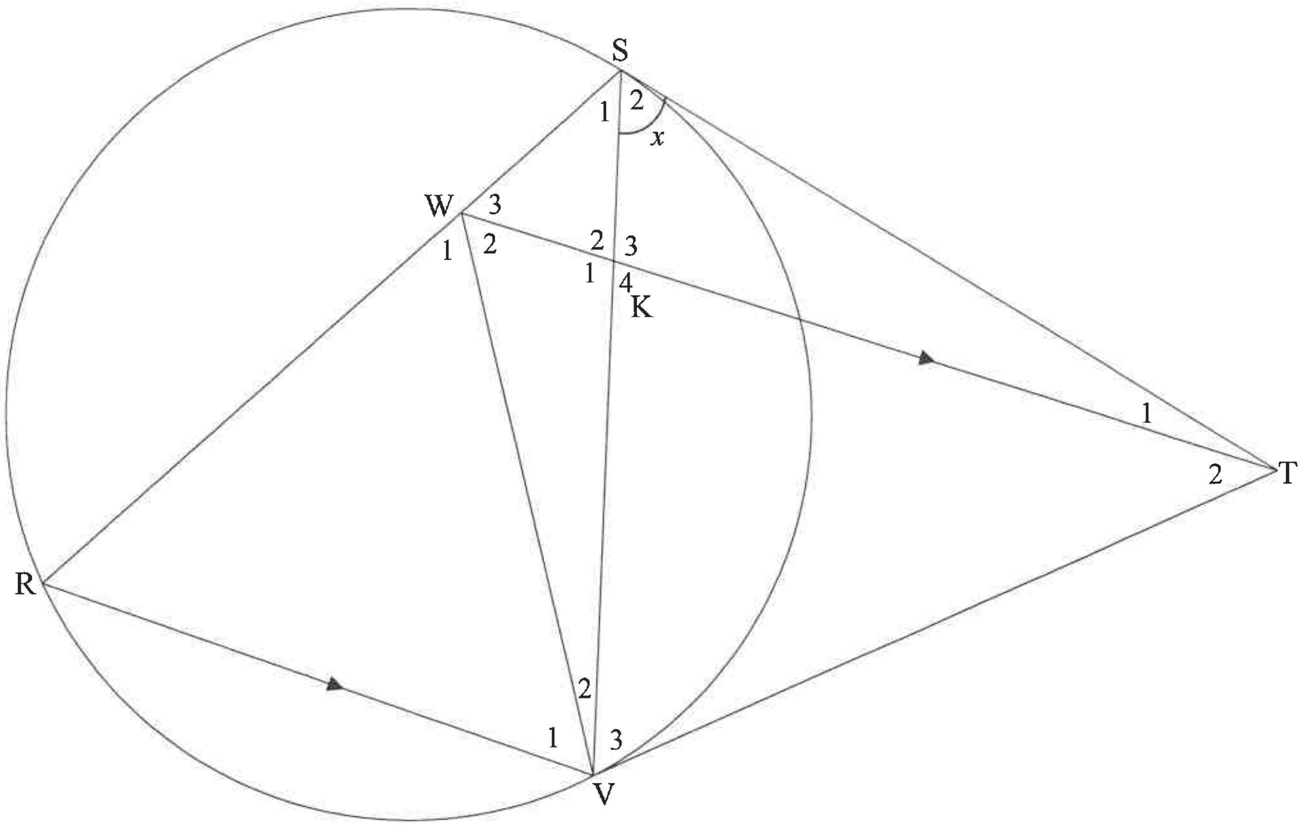
10.2.1 BC (3)

10.2.2 DB (4)

[17]

TOTAL: 150

10.2 In the diagram, ST and VT are tangents to the circle at S and V respectively. R is a point on the circle and W is a point on chord RS such that WT is parallel to RV. SV and WV are drawn. WT intersects SV at K. Let $\hat{S}_2 = x$.



10.2.1 Write down, with reasons, THREE other angles EACH equal to x. (6)

10.2.2 Prove, with reasons, that:

(a) WSTV is a cyclic quadrilateral (2)

(b) ΔWRV is isosceles (4)

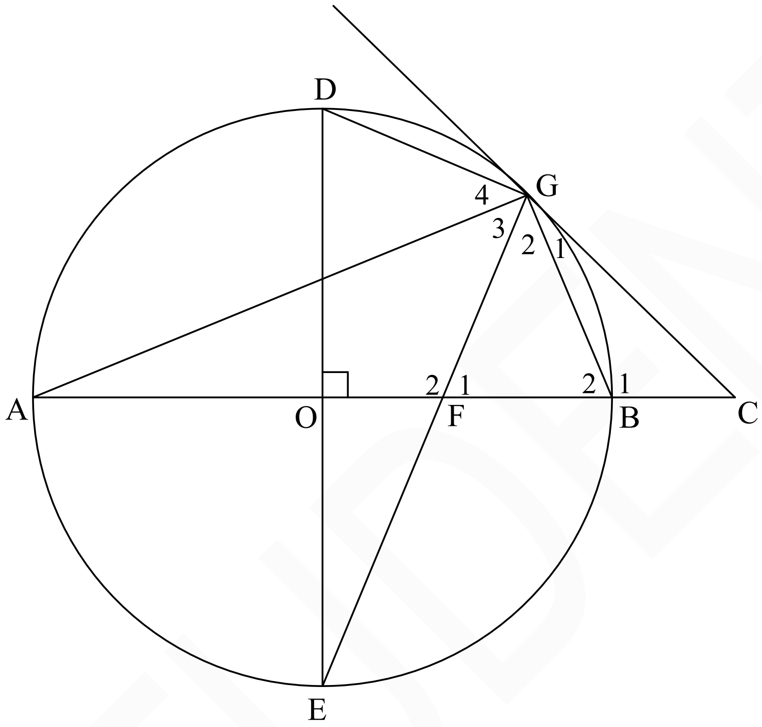
(c) $\Delta WRV \,|||\, \Delta TSV$ (3)

(d) $\dfrac{RV}{SR} = \dfrac{KV}{TS}$ (4)

[25]

TOTAL: 150

10.2 In the diagram, O is the centre of the circle and CG is a tangent to the circle at G. The straight line from C passing through O cuts the circle at A and B. Diameter DOE is perpendicular to CA. GE and CA intersect at F. Chords DG, BG and AG are drawn.



10.2.1 Prove that:

(a) DGFO is a cyclic quadrilateral (3)

(b) GC = CF (5)

10.2.2 If it is further given that CO = 11 units and DE = 14 units, calculate:

(a) The length of BC (3)

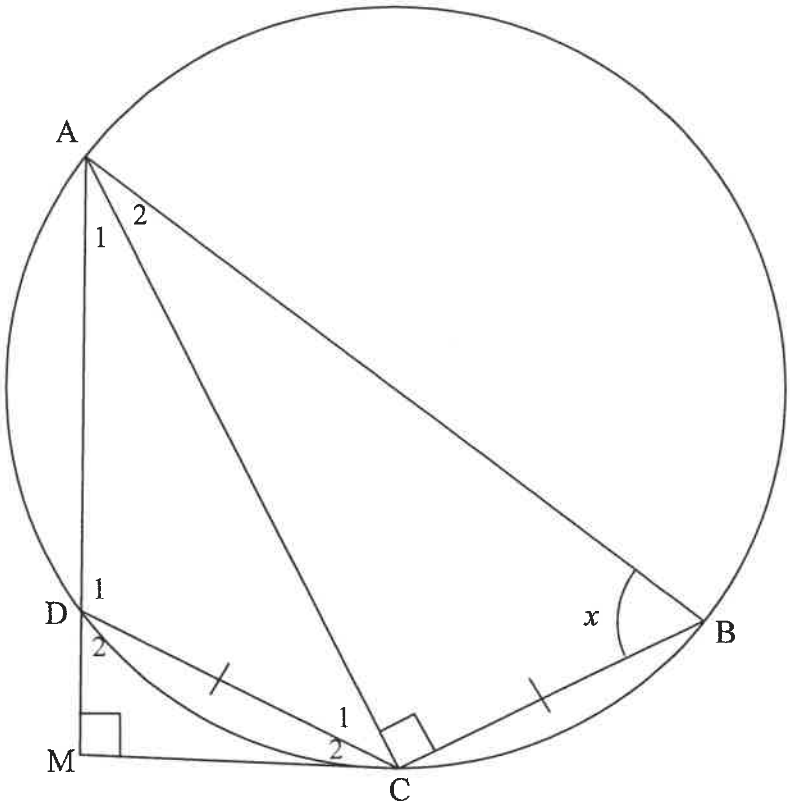
(b) The length of CG (5)

(c) The size of $\hat{E}$. (4)
[26]

TOTAL: 150

QUESTION 10

In the diagram, ABCD is a cyclic quadrilateral such that $AC \perp CB$ and $DC = CB$. AD is produced to M such that $AM \perp MC$. Let $\hat{B} = x$.



10.1 Prove that:

10.1.1 MC is a tangent to the circle at C (5)

10.1.2 $\Delta ACB\ |||\ \Delta CMD$ (3)

10.2 Hence, or otherwise, prove that:

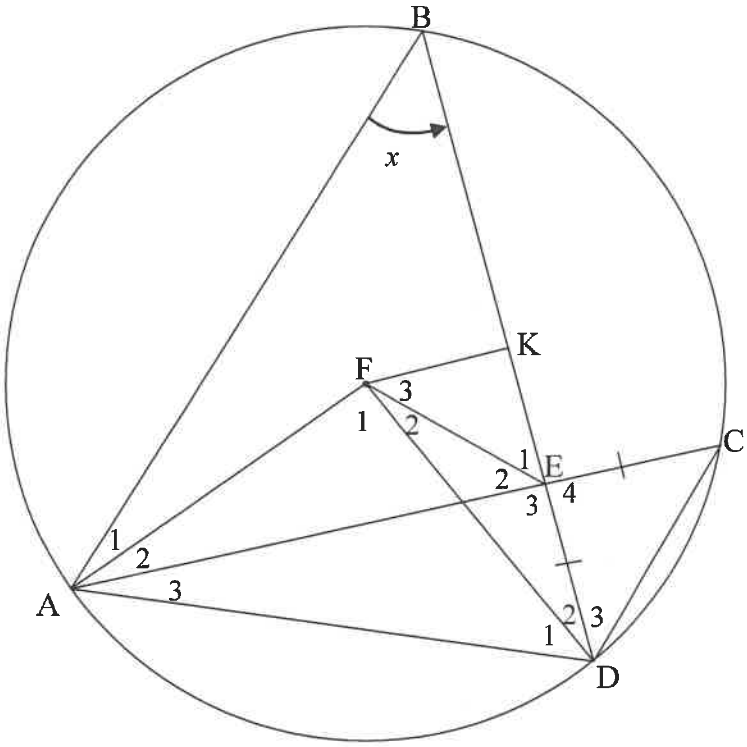
10.2.1 $\frac{CM^2}{DC^2} = \frac{AM}{AB}$ (6)

10.2.2 $\frac{AM}{AB} = \sin^2 x$ (2)

[16]

TOTAL: 150

10.2 In the diagram, the circle with centre F is drawn. Points A, B, C and D lie on the circle. Chords AC and BD intersect at E such that EC = ED. K is the midpoint of chord BD. FK, AB, CD, AF, FE and FD are drawn. Let $\hat{B} = x$.



10.2.1 Determine, with reasons, the size of EACH of the following in terms of x:

(a) $\hat{F}_1$ (2)

(b) $\hat{C}$ (2)

10.2.2 Prove, with reasons, that AFED is a cyclic quadrilateral. (4)

10.2.3 Prove, with reasons, that $\hat{F}_3 = x$. (6)

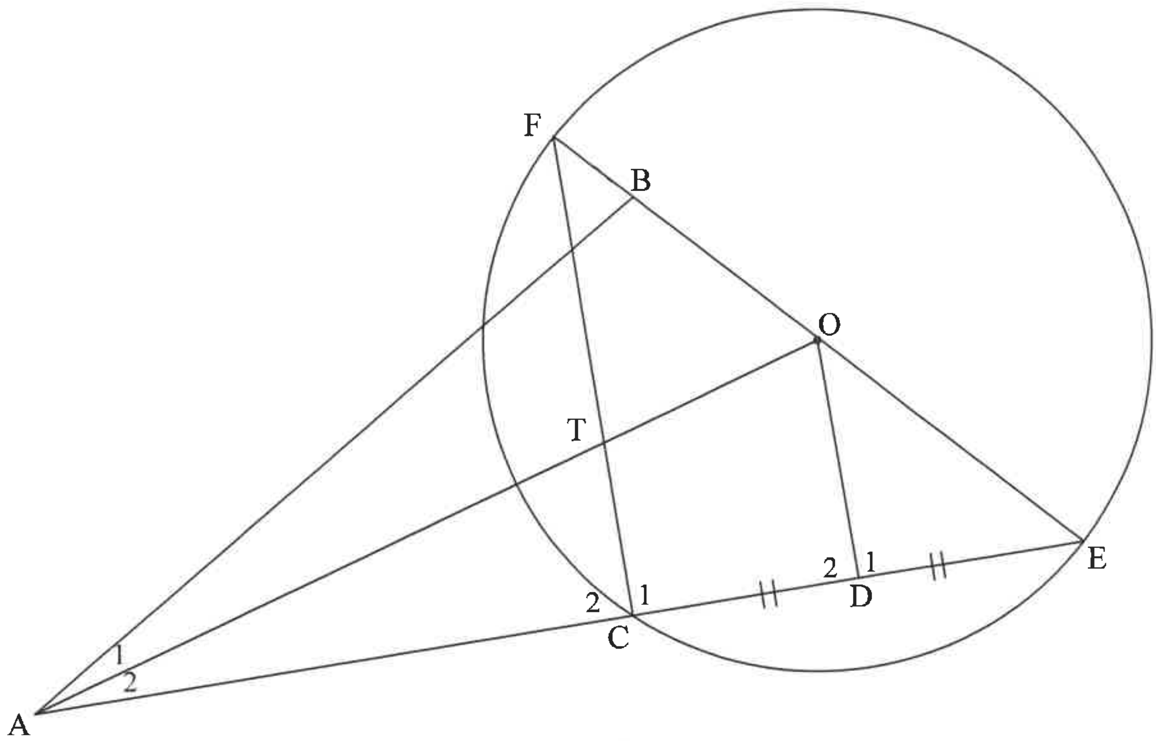
10.2.4 If area $\Delta AEB = 6{,}25 \times$ area ΔDEC, calculate $\frac{AE}{ED}$. (5)

[24]

TOTAL: 150

QUESTION 10

In the diagram, FBOE is a diameter of a circle with centre O. Chord EC produced meets line BA at A, outside the circle. D is the midpoint of CE. OD and FC are drawn. AFBC is a cyclic quadrilateral.



10.1 Prove, giving reasons, that:

10.1.1 FC || OD (5)

10.1.2 $D\hat{O}E = B\hat{A}E$ (4)

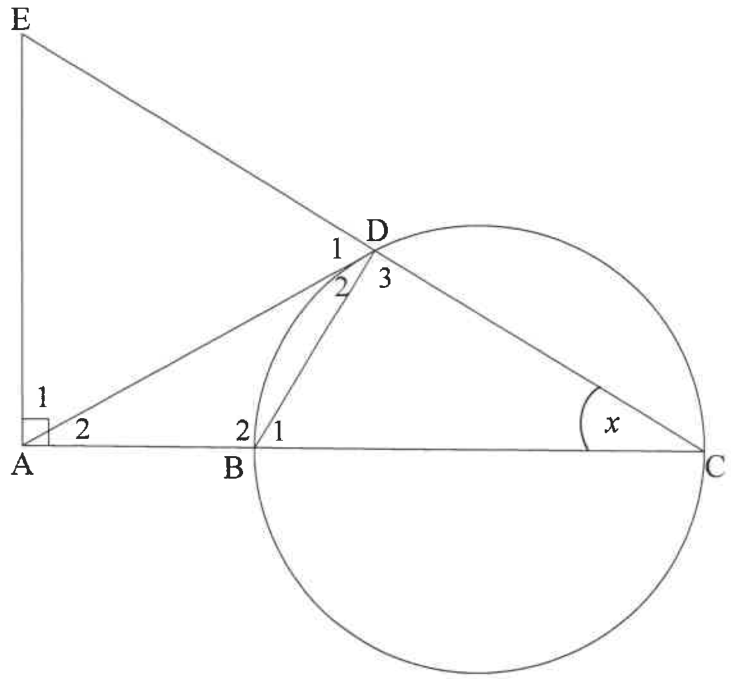
10.1.3 AB × OF = AE × OD (7)

10.2 If it is further given that AT = 3TO, prove that $5CE^2 = 2BE.FE$ (5)

[21]

TOTAL: 150

11.2 In the diagram, BC is a diameter of the circle. The tangent at point D on the circle meets CB produced at A. CD is produced to E such that EA $\perp$ AC. BD is drawn.
Let $\hat{C} = x$.



11.2.1 Give a reason why:

(a) $\hat{D}_3 = 90°$ (1)

(b) ABDE is a cyclic quadrilateral (1)

(c) $\hat{D}_2 = x$ (1)

11.2.2 Prove that:

(a) AD = AE (3)

(b) $\Delta ADB \ ||| \ \Delta ACD$ (3)

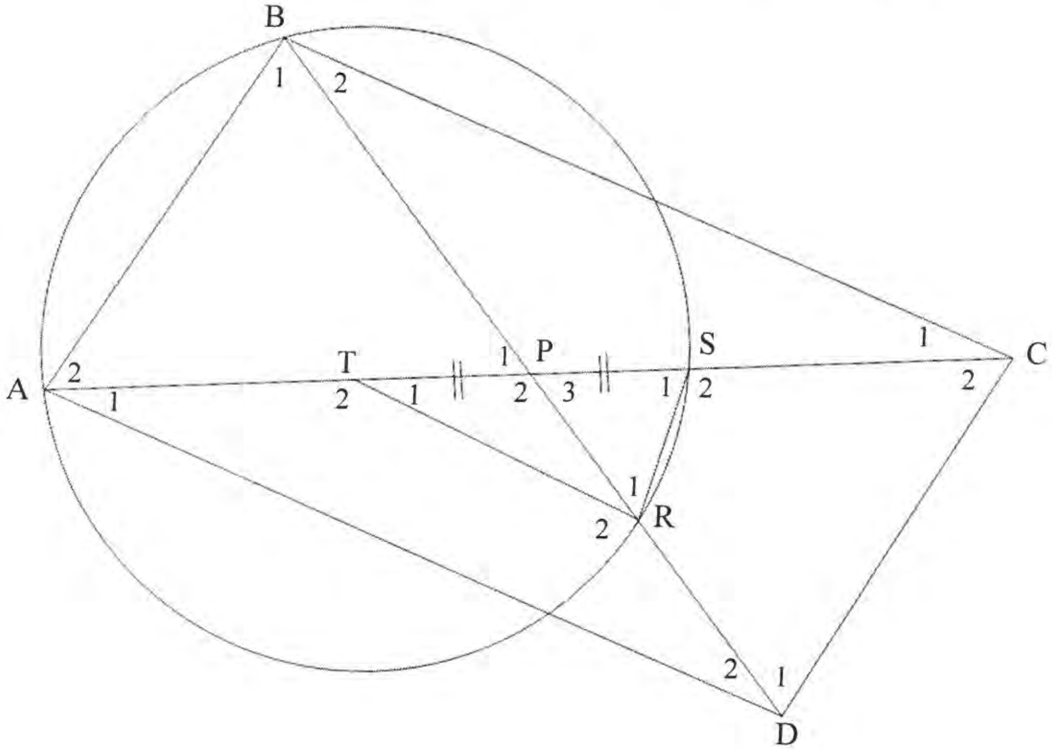
11.2.3 It is further given that $BC = 2AB = 2r$.

(a) Prove that $AD^2 = 3r^2$ (2)

(b) Hence, prove that ΔADE is equilateral. (4)
[20]

TOTAL: 150

11.2 In the diagram, ABCD is a parallelogram with A and B on the circle. The diagonals BD and AC intersect in P. PC and PD intersect the circle at S and R respectively. T is a point on AP such that TP = PS. TR is drawn.



11.2.1 Prove that:

(a) AT = SC (2)

(b) ΔPSR ||| ΔPBA (5)

11.2.2 If it is further given that $\frac{PR}{PA} = \frac{TR}{AD}$, prove that:

(a) ΔRPT ||| ΔAPD (3)

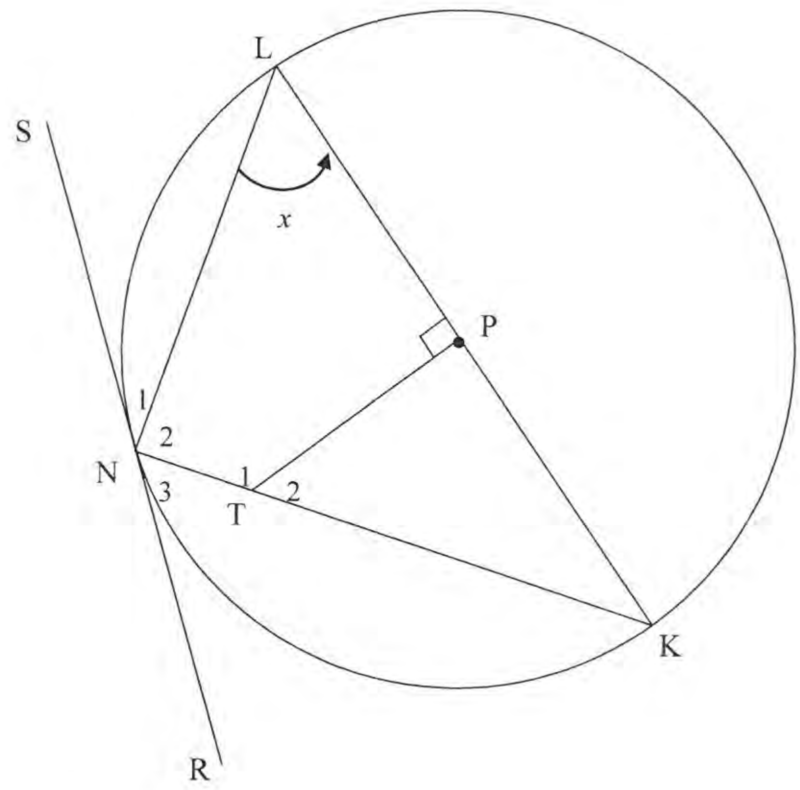
(b) ATRD is a cyclic quadrilateral (2)

[18]

TOTAL: 150

QUESTION 11

In the diagram, LK is a diameter of the circle with centre P. RNS is a tangent to the circle at N. T is a point on NK and $TP \perp KL$. $P\hat{L}N = x$.



11.1 Prove that TPLN is a cyclic quadrilateral. (3)

11.2 Determine, giving reasons, the size of $\hat{N}_1$ in terms of x. (3)

11.3 Prove that:

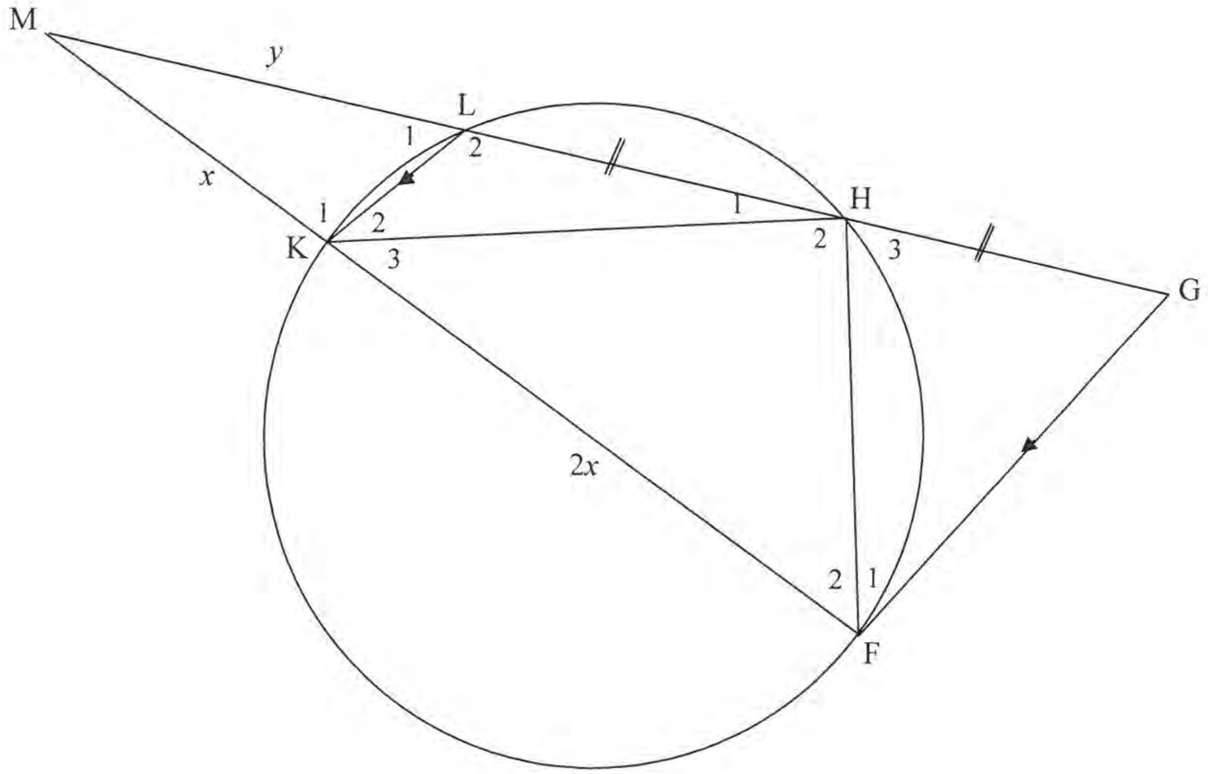
11.3.1 $\Delta KTP \,|||\, \Delta KLN$ (3)

11.3.2 $KT \cdot KN = 2KT^2 - 2TP^2$ (5)

[14]

TOTAL: 150

10.2 In the diagram HLKF is a cyclic quadrilateral. The chords HL and FK are produced to meet at M. The line through F parallel to KL meets MH produced at G. MK = x, KF = $2x$, ML = y and LH = HG.



10.2.1 Give a reason why $G\hat{F}M = L\hat{K}M$. (1)

10.2.2 Prove that:

(a) $GH = y$ (3)

(b) $\Delta MFH \,|||\, \Delta MGF$ (5)

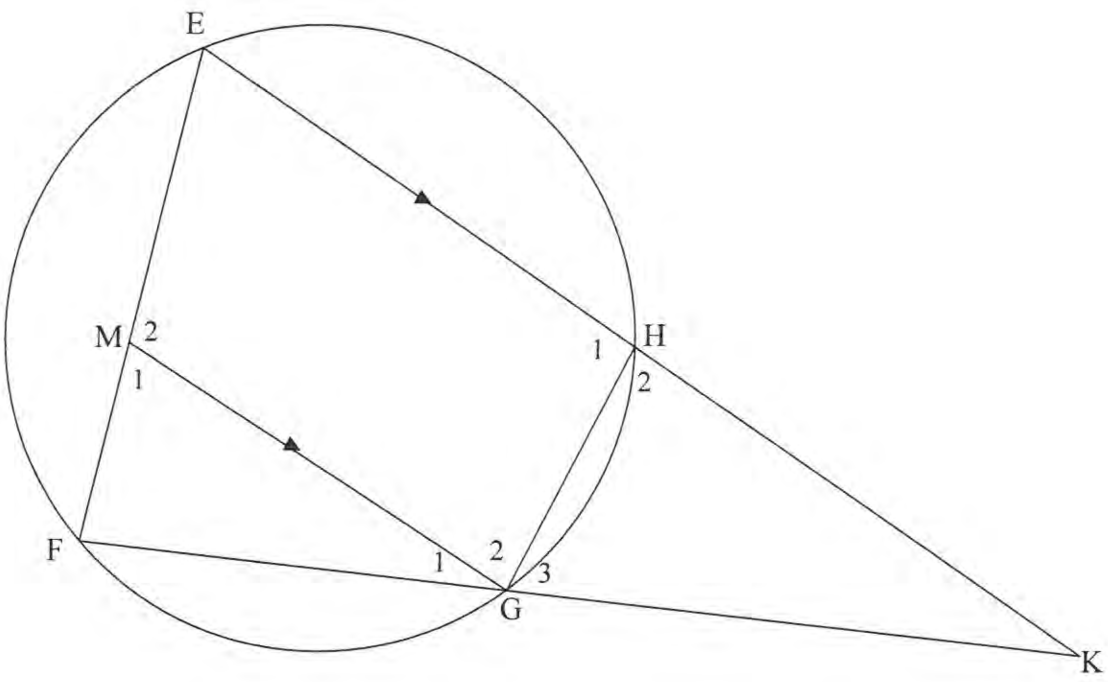
(c) $\dfrac{GF}{FH} = \dfrac{3x}{2y}$ (2)

10.2.3 Show that $\dfrac{y}{x} = \sqrt{\dfrac{3}{2}}$ (3)

[20]

TOTAL: 150

10.2 In the diagram below, cyclic quadrilateral EFGH is drawn. Chord EH produced and chord FG produced meet at K. M is a point on EF such that MG | | EK. Also KG = EF



10.2.1 Prove that:

(a) $\Delta KGH\ |||\ \Delta KEF$ (4)

(b) $EF^2 = KE \cdot GH$ (2)

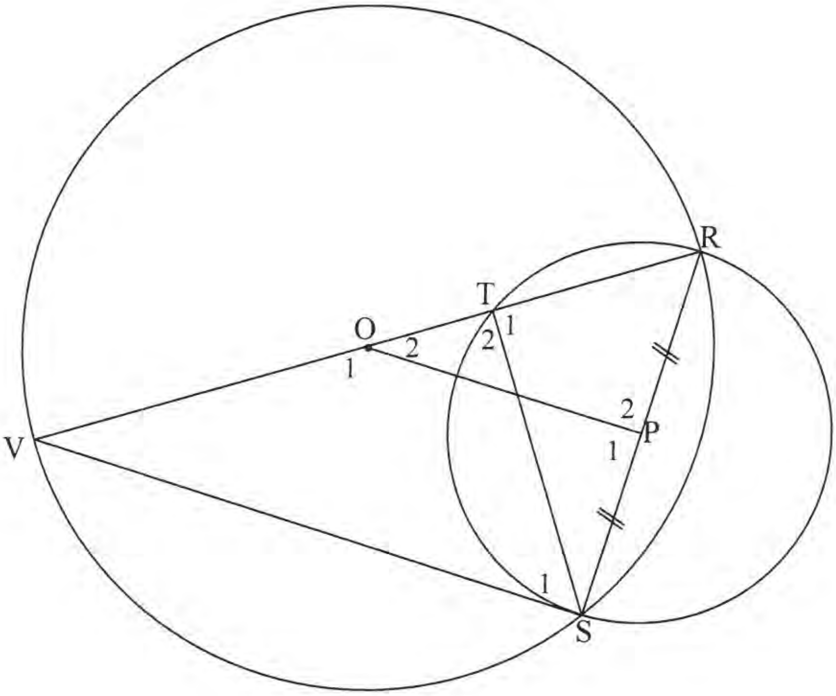
(c) $KG^2 = EM \cdot KF$ (3)

10.2.2 If it is given that KE = 20 units, KF = 16 units and GH = 4 units, calculate the length of EM. (3)

[19]

TOTAL: 150

10.2 In the diagram below, VR is a diameter of a circle with centre O. S is any point on the circumference. P is the midpoint of RS. The circle with RS as diameter cuts VR at T. ST, OP and SV are drawn.



10.2.1	Why is $OP \perp PS$?	(1)
10.2.2	Prove that $\Delta ROP \mid\mid\mid \Delta RVS$.	(4)
10.2.3	Prove that $\Delta RVS \mid\mid\mid \Delta RST$.	(3)
10.2.4	Prove that $ST^2 = VT \cdot TR$.	(6)
		[21]

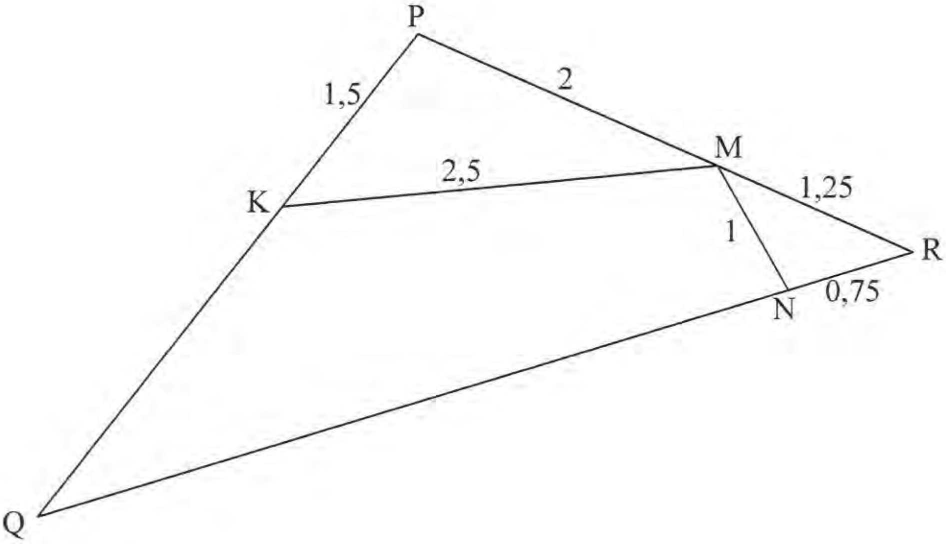
TOTAL: 150

QUESTION 11

11.1 Complete the following statement:

If the sides of two triangles are in the same proportion, then the triangles are ... (1)

11.2 In the diagram below, K, M and N respectively are points on sides PQ, PR and QR of ΔPQR. KP = 1,5; PM = 2; KM = 2,5; MN = 1; MR = 1,25 and NR = 0,75.



11.2.1 Prove that $\Delta KPM \mid\mid\mid \Delta RNM$. (3)

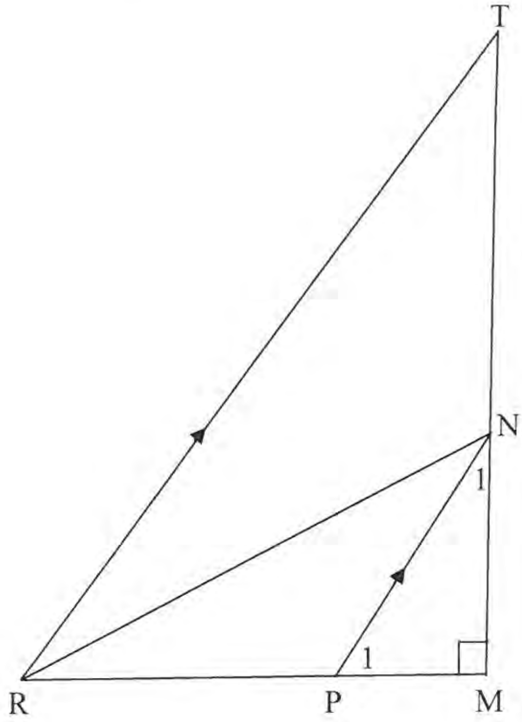
11.2.2 Determine the length of NQ. (6)

[10]

TOTAL: 150

QUESTION 10

In ΔTRM, $\hat{M} = 90°$. NP is drawn parallel to TR with N on TM and P on RM. It is further given that RT = 3PN.



10.1 Give reasons for the statements below.
Use DIAGRAM SHEET 5.

	Statement	**Reason**
	In ΔPNM and ΔRTM :	
10.1.1	$\hat{N}_1 = \hat{T}$	....
	$\hat{M}$ is common	
10.1.2	$\therefore \Delta PNM \mid\mid\mid \Delta RTM$	....

(2)

10.2 Prove that $\frac{PM}{RM} = \frac{1}{3}$. (2)

10.3 Show that $RN^2 - PN^2 = 2RP^2$. (4)

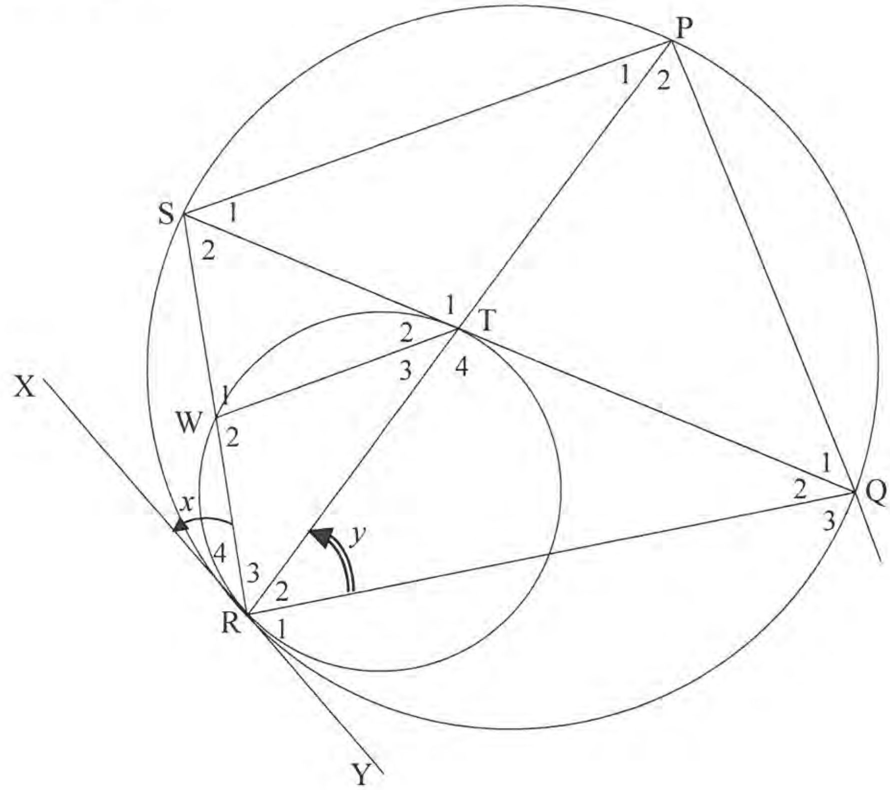
[8]

TOTAL: 150

QUESTION 10

The two circles in the diagram have a common tangent XRY at R. W is any point on the small circle. The straight line RWS meets the large circle at S. The chord STQ is a tangent to the small circle, where T is the point of contact. Chord RTP is drawn.

Let $\hat{R}_4 = x$ and $\hat{R}_2 = y$



10.1 Give reasons for the statements below.
Complete the table on DIAGRAM SHEET 6.

Let $\hat{R}_4 = x$ and $\hat{R}_2 = y$		
	Statement	**Reason**
10.1.1	$\hat{T}_3 = x$	
10.1.2	$\hat{P}_1 = x$	
10.1.3	WT | | SP	
10.1.4	$\hat{S}_1 = y$	
10.1.5	$\hat{T}_2 = y$	

(5)

10.2 Prove that $RT = \frac{WR.RP}{RS}$ (2)

10.3 Identify, with reasons, another TWO angles equal to y. (4)

10.4 Prove that $\hat{Q}_3 = \hat{W}_2$. (3)

10.5 Prove that $\Delta RTS \mid\mid\mid \Delta RQP$. (3)

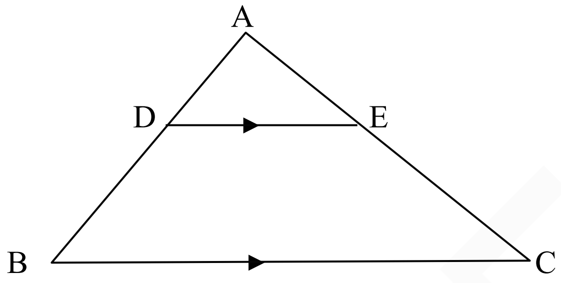
10.6 Hence, prove that $\frac{WR}{RQ} = \frac{RS^2}{RP^2}$. (3)

[20]

TOTAL: 150

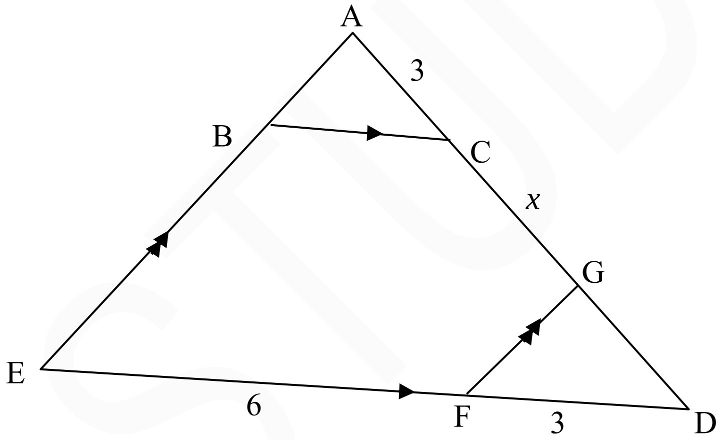
QUESTION 10

10.1 In the diagram, points D and E lie on sides AB and AC respectively of ΔABC such that DE | | BC. Use Euclidean Geometry methods to prove the theorem which states that $\frac{AD}{DB} = \frac{AE}{EC}$.



(6)

10.2 In the diagram, ADE is a triangle having BC | | ED and AE | | GF. It is also given that AB : BE = 1 : 3, AC = 3 units, EF = 6 units, FD = 3 units and CG = x units.



Calculate, giving reasons:

10.2.1 The length of CD (3)

10.2.2 The value of x (4)

10.2.3 The length of BC (5)

10.2.4 The value of $\frac{\text{area } \Delta ABC}{\text{area } \Delta GFD}$ (5)

[23]

TOTAL: 150