SA-STUDENT

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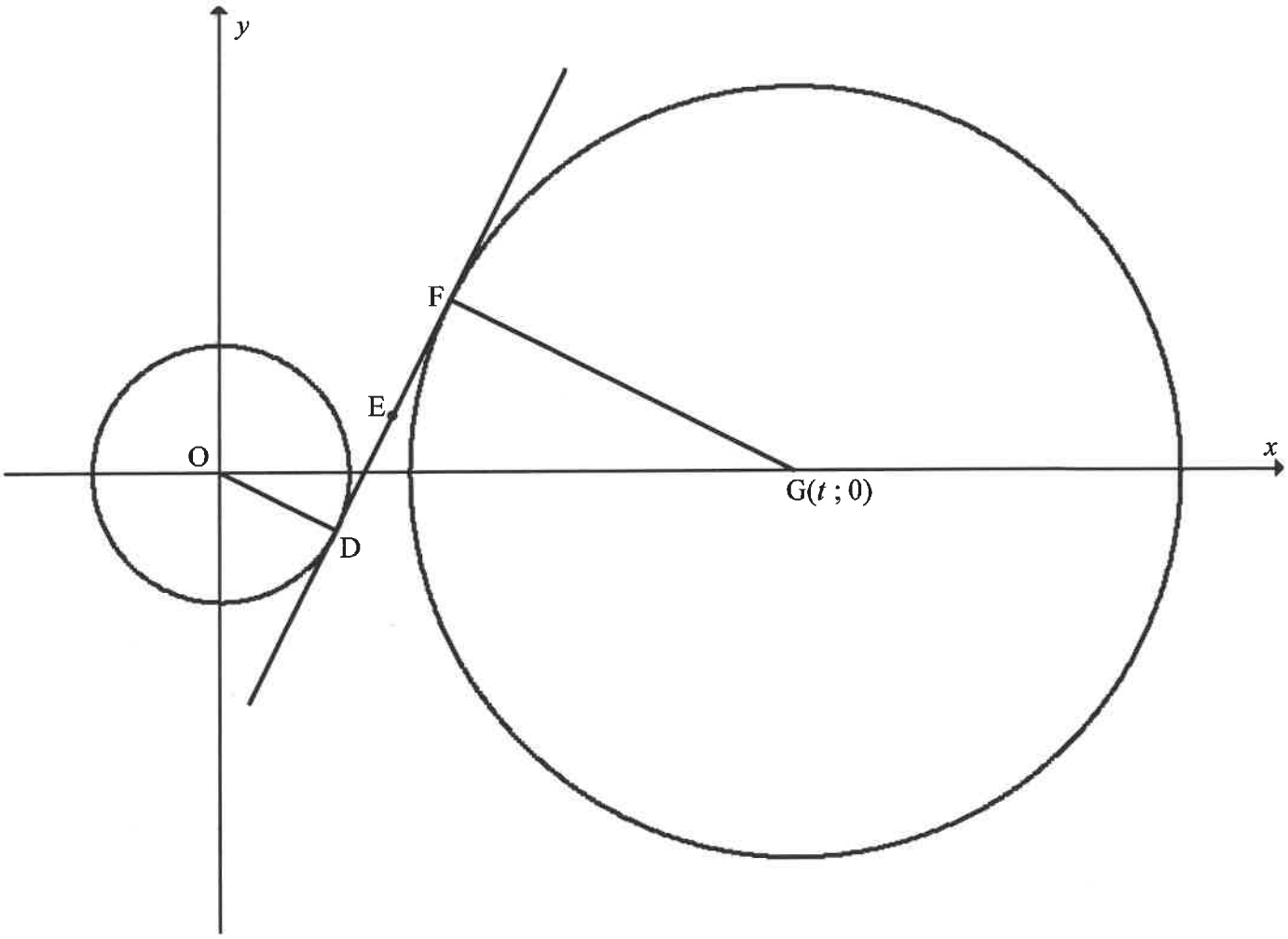
"You have to ask yourself how badly do you want something? If you really, really want something then put in the work". -Lewis Hamilton

QUESTION 4

In the diagram, the circle with centre O has the equation $x^2 + y^2 = 20$. $G(t\ ;\ 0)$ is the centre of the larger circle. A common tangent touches the circles at D and F respectively, such that D(p ; –2) lies in the 4th quadrant.



4.1 Given that D(p ; –2) lies on the smaller circle, show that $p = 4$. (2)

4.2 $E(6\ ;\ 2)$ is the midpoint of DF. Determine the coordinates of F. (3)

4.3 Determine the equation of the common tangent, DF, in the form $y = mx + c$. (4)

4.4 Calculate the value of t. Show ALL working. (3)

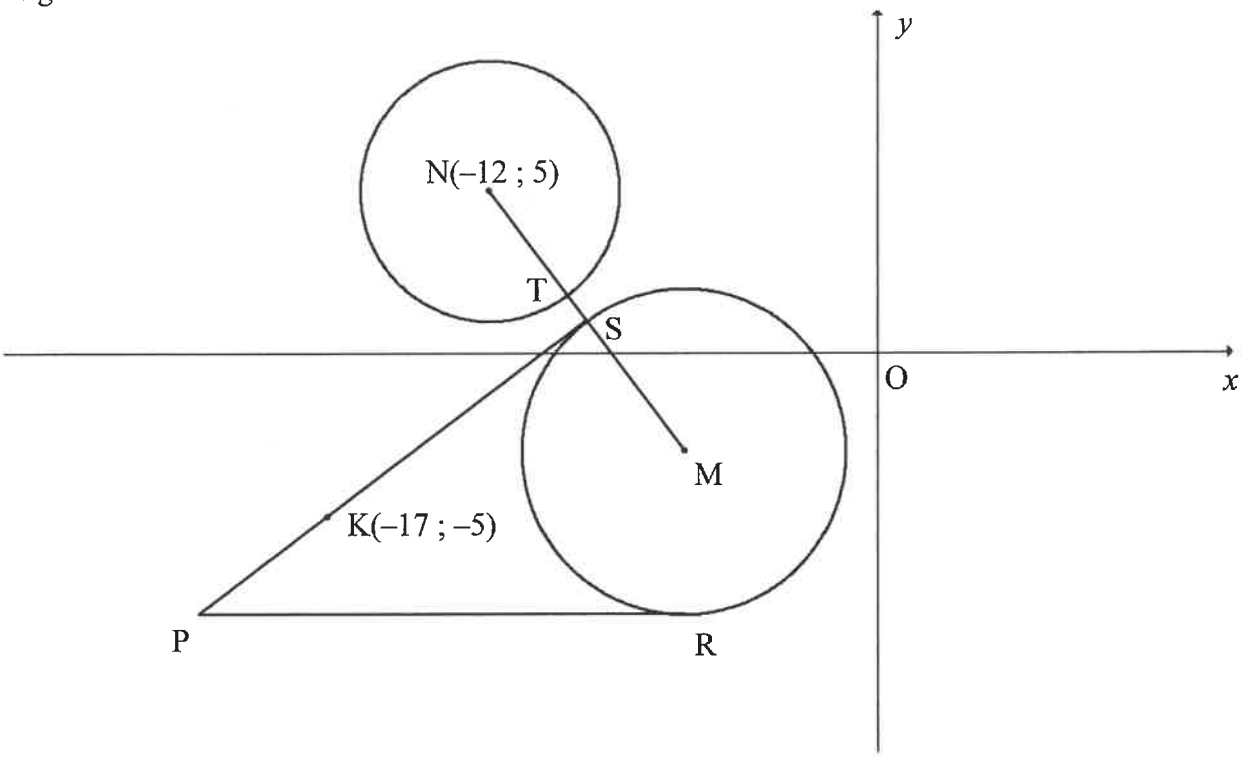
4.5 Determine the equation of the larger circle in the form $ax^2 + by^2 + cx + dy + e = 0$. (4)

4.6 The smaller circle must be translated by k units along the x-axis to touch the larger circle internally. Calculate the possible values of k. (4)

[20]

QUESTION 4

In the diagram, the equation of the circle centred at N(–12 ; 5) is $x^2 + y^2 + 24x - 10y + 153 = 0$. The equation of the circle centred at M is $(x+6)^2 + (y+3)^2 = 25$. PS and PR are tangents to the circle centred at M at S and R respectively. PR is parallel to the x-axis. K(–17 ; –5) is a point on PS. The straight line joining N and M cuts the smaller circle at T and the larger circle at S.



4.1 Write down the coordinates of M. (2)

4.2 Calculate the:

4.2.1 Length of the radius of the smaller circle (2)

4.2.2 Length of TS (4)

4.3 Determine the equation of the tangent:

4.3.1 PR (2)

4.3.2 PS, in the form $y = mx + c$ (5)

4.4 Quadrilateral PSMR is drawn. Calculate the:

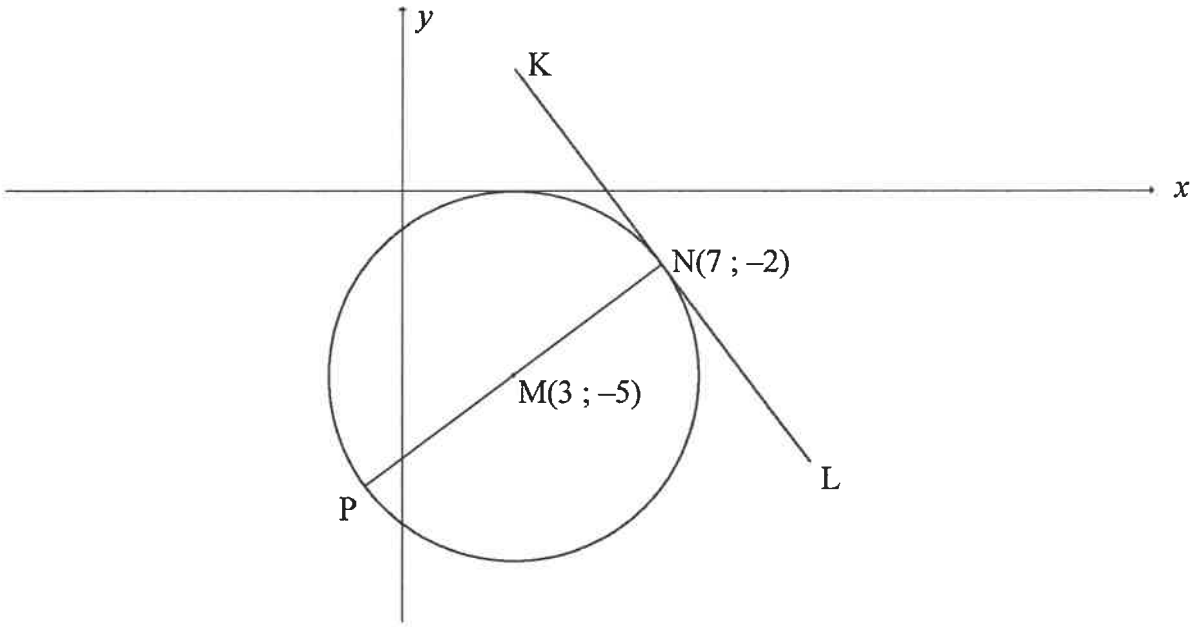
4.4.1 Perimeter of PSMR (5)

4.4.2 Ratio of $\dfrac{\text{area of } \Delta NPS}{\text{area of quadrilateral PSMR}}$ (2)

[22]

QUESTION 4

In the diagram, M(3 ; –5) is the centre of the circle having PN as its diameter. KL is a tangent to the circle at N(7 ; –2).



4.1 Calculate the coordinates of P. (2)

4.2 Determine the equation of:

4.2.1 The circle in the form $(x-a)^2+(y-b)^2=r^2$ (3)

4.2.2 KL in the form $y=mx+c$ (5)

4.3 For which values of k will $y=-\frac{4}{3}x+k$ be a secant to the circle? (4)

4.4 Points A(t ; t) and B are not shown on the diagram.

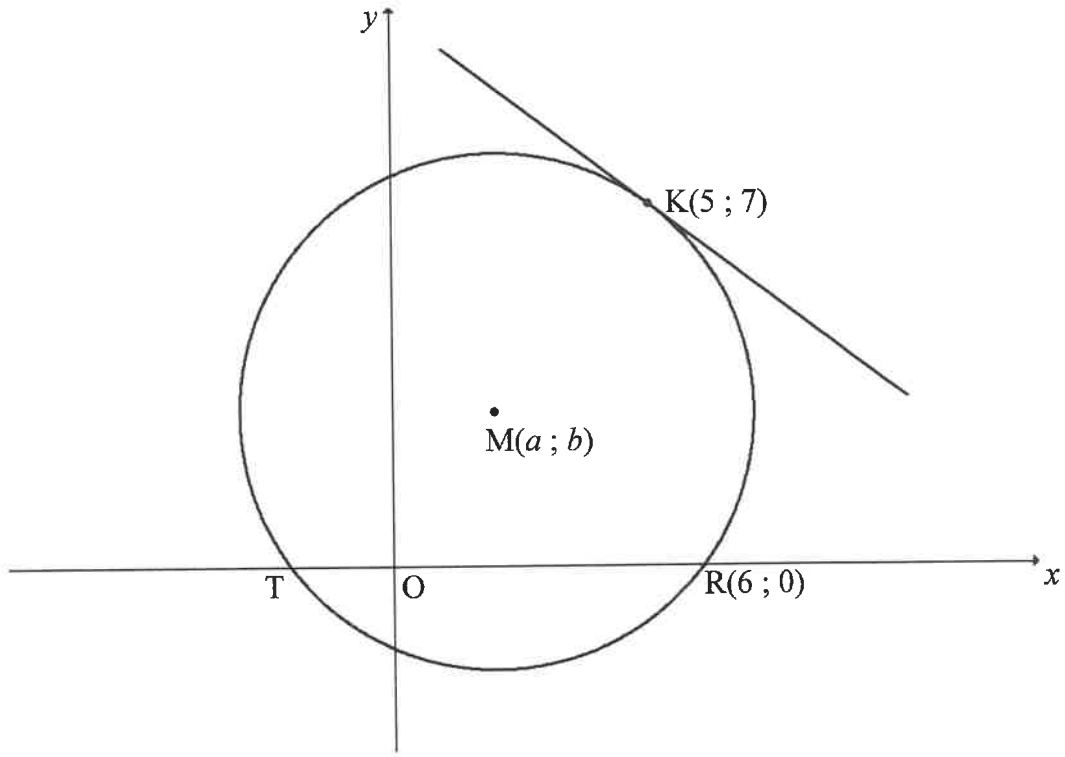
From point A, another tangent is drawn to touch the circle with centre M at B.

4.4.1 Show that the length of tangent AB is given by $\sqrt{2t^2+4t+9}$. (2)

4.4.2 Determine the minimum length of AB. (4)
[20]

QUESTION 4

In the diagram, the circle centred at M(a ; b) is drawn. T and R(6 ; 0) are the x-intercepts of the circle. A tangent is drawn to the circle at K(5 ; 7).



4.1 M is a point on the line $y = x + 1$.

4.1.1 Write b in terms of a. (1)

4.1.2 Calculate the coordinates of M. (5)

4.2 If the coordinates of M are (2 ; 3), calculate the length of:

4.2.1 The radius of the circle (2)

4.2.2 TR (2)

4.3 Determine the equation of the tangent to the circle at K. Write your answer in the form $y = mx + c$. (5)

4.4 A horizontal line is drawn as a tangent to the circle M at the point N(c ; d), where $d < 0$.

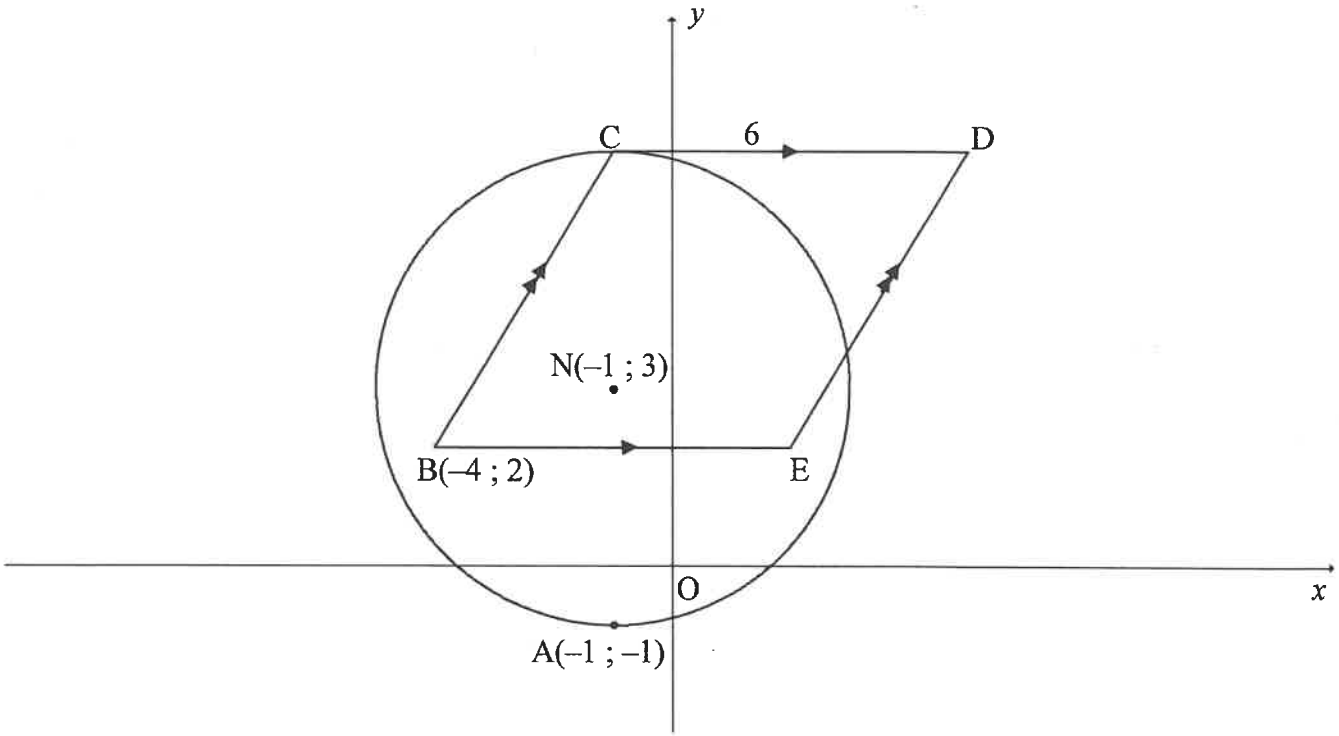
4.4.1 Write down the coordinates of N. (2)

4.4.2 Determine the equation of the circle centred at N and passing through T. Write your answer in the form $(x-a)^2 + (y-b)^2 = r^2$. (3)

[20]

QUESTION 4

In the diagram, the circle centred at N(–1 ; 3) passes through A(–1 ; –1) and C. B(–4 ; 2), C, D and E are joined to form a parallelogram such that BE is parallel to the x-axis. CD is a tangent to the circle at C and CD = 6 units.



4.1 Write down the length of the radius of the circle. (1)

4.2 Calculate the:

4.2.1 Coordinates of C (2)

4.2.2 Coordinates of D (2)

4.2.3 Area of ΔBCD (3)

4.3 The circle, centred at N, is reflected about the line $y = x$. M is the centre of the new circle which is formed. The two circles intersect at A and F.

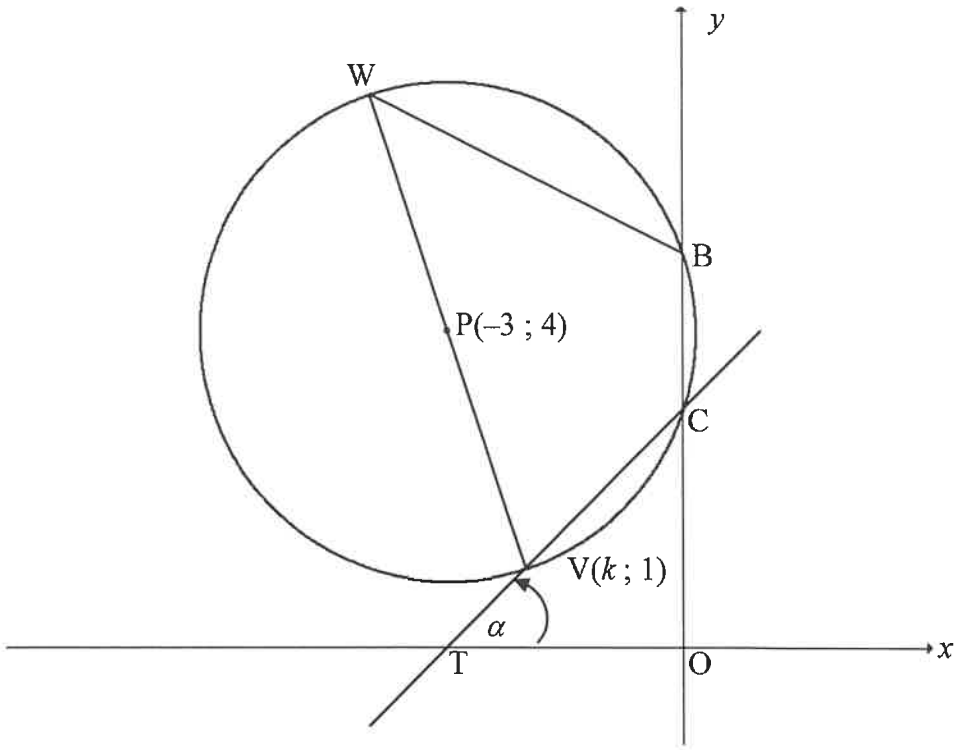
Calculate the:

4.3.1 Length of NM (3)

4.3.2 Midpoint of AF (4)
[15]

QUESTION 4

In the diagram, P(–3 ; 4) is the centre of the circle. V(k ; 1) and W are the endpoints of a diameter. The circle intersects the y-axis at B and C. BCVW is a cyclic quadrilateral. CV is produced to intersect the x-axis at T. $O\hat{T}C = \alpha$.



4.1 The radius of the circle is $\sqrt{10}$. Calculate the value of k if point V is to the right of point P. Clearly show ALL calculations. (5)

4.2 The equation of the circle is given as $x^2 + 6x + y^2 - 8y + 15 = 0$. Calculate the length of BC. (4)

4.3 If $k = -2$, calculate the size of:

4.3.1 α (3)

4.3.2 $V\hat{W}B$ (2)

4.4 A new circle is obtained when the given circle is reflected about the line $y = 1$.

Determine the:

4.4.1 Coordinates of Q, the centre of the new circle (2)

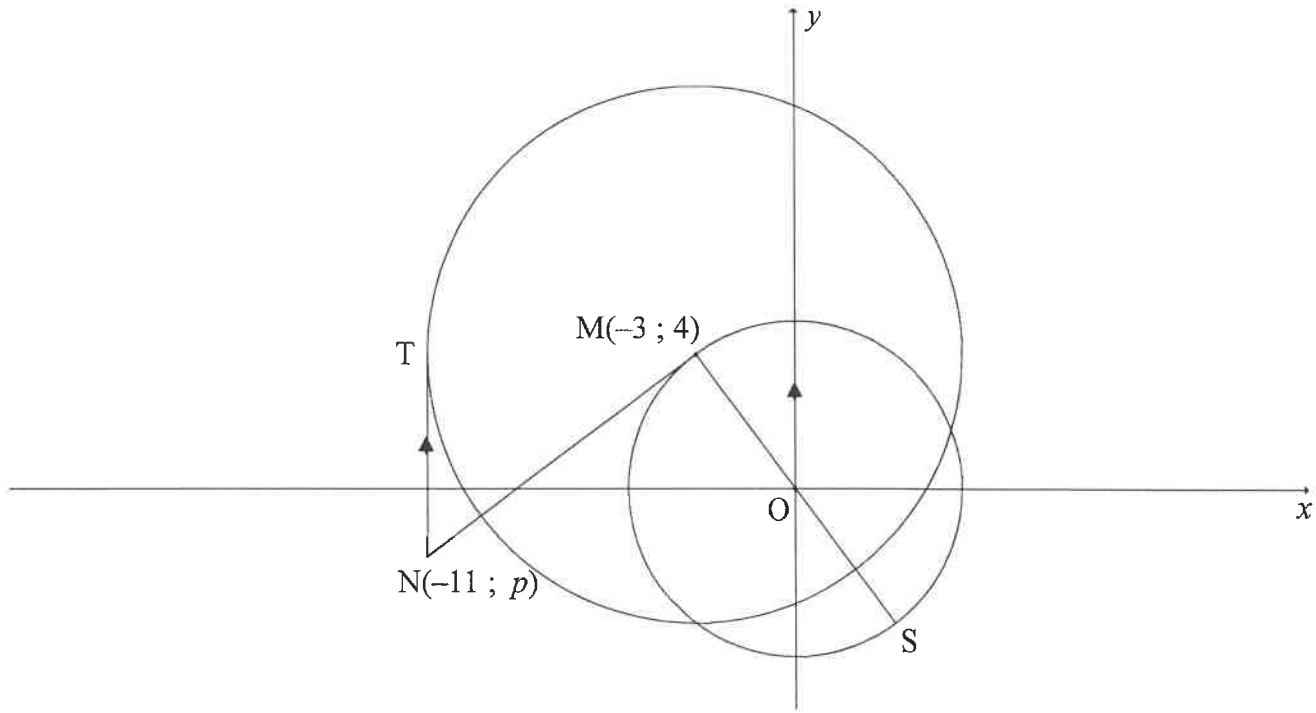
4.4.2 Equation of the new circle in the form $(x-a)^2 + (y-b)^2 = r^2$ (2)

4.4.3 Equations of the lines drawn parallel to the y-axis and passing through the points of intersection of the two circles (2)

[20]

QUESTION 4

M(–3 ; 4) is the centre of the large circle and a point on the small circle having centre O(0; 0). From N(–11 ; p), a tangent is drawn to touch the large circle at T with NT is parallel to the y-axis. NM is a tangent to the smaller circle at M with MOS a diameter.



4.1 Determine the equation of the small circle. (2)

4.2 Determine the equation of the circle centred at M in the form $(x-a)^2+(y-b)^2=r^2$ (3)

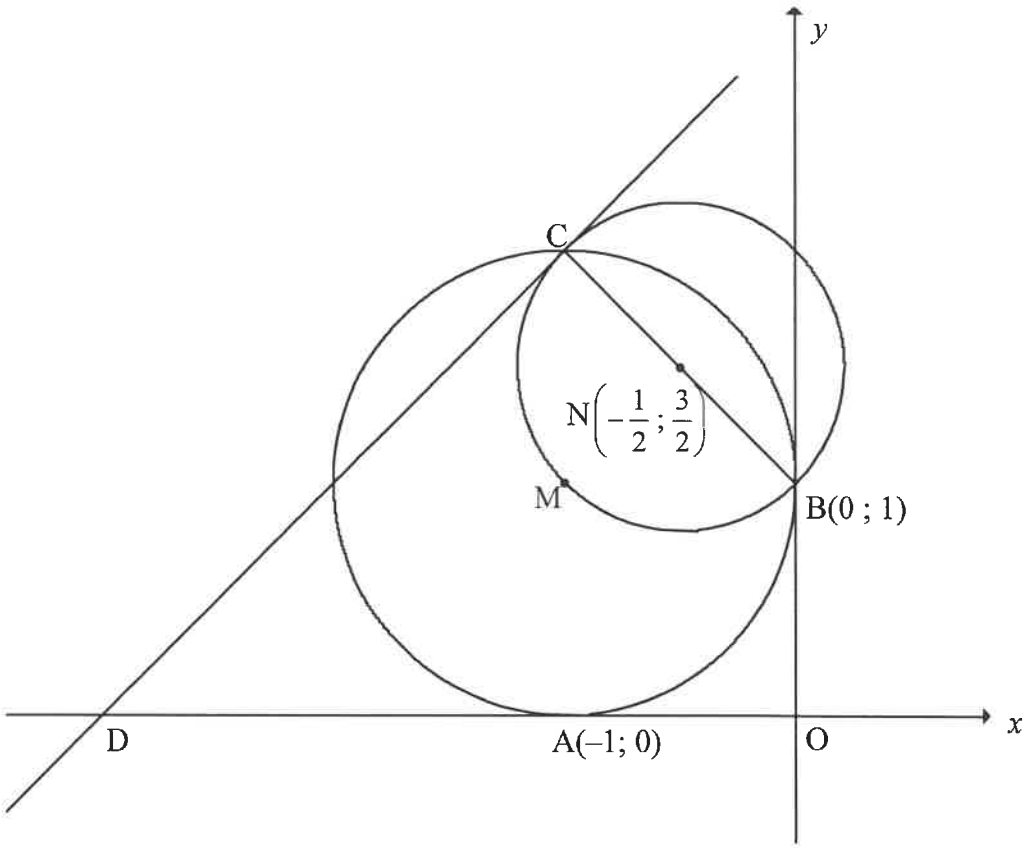
4.3 Determine the equation of NM in the form $y=mx+c$ (4)

4.4 Calculate the length of SN. (5)

4.5 If another circle with centre B(–2 ; 5) and radius k touches the circle centred at M, determine the value(s) of k, correct to ONE decimal place. (5)
[19]

QUESTION 4

In the diagram, a circle having centre M touches the x–axis at A(–1 ; 0) and the y-axis at B(0 ; 1). A smaller circle, centred at $N\left(-\frac{1}{2};\frac{3}{2}\right)$, passes through M and cuts the larger circle at B and C. BNC is a diameter of the smaller circle. A tangent drawn to the smaller circle at C, cuts the x-axis at D.



4.1 Determine the equation of the circle centred at M in the form $(x-a)^2+(y-b)^2=r^2$ (3)

4.2 Calculate the coordinates of C. (2)

4.3 Show that the equation of the tangent CD is $y-x=3$. (4)

4.4 Determine the values of t for which the line $y=x+t$ will NOT touch or cut the smaller circle. (3)

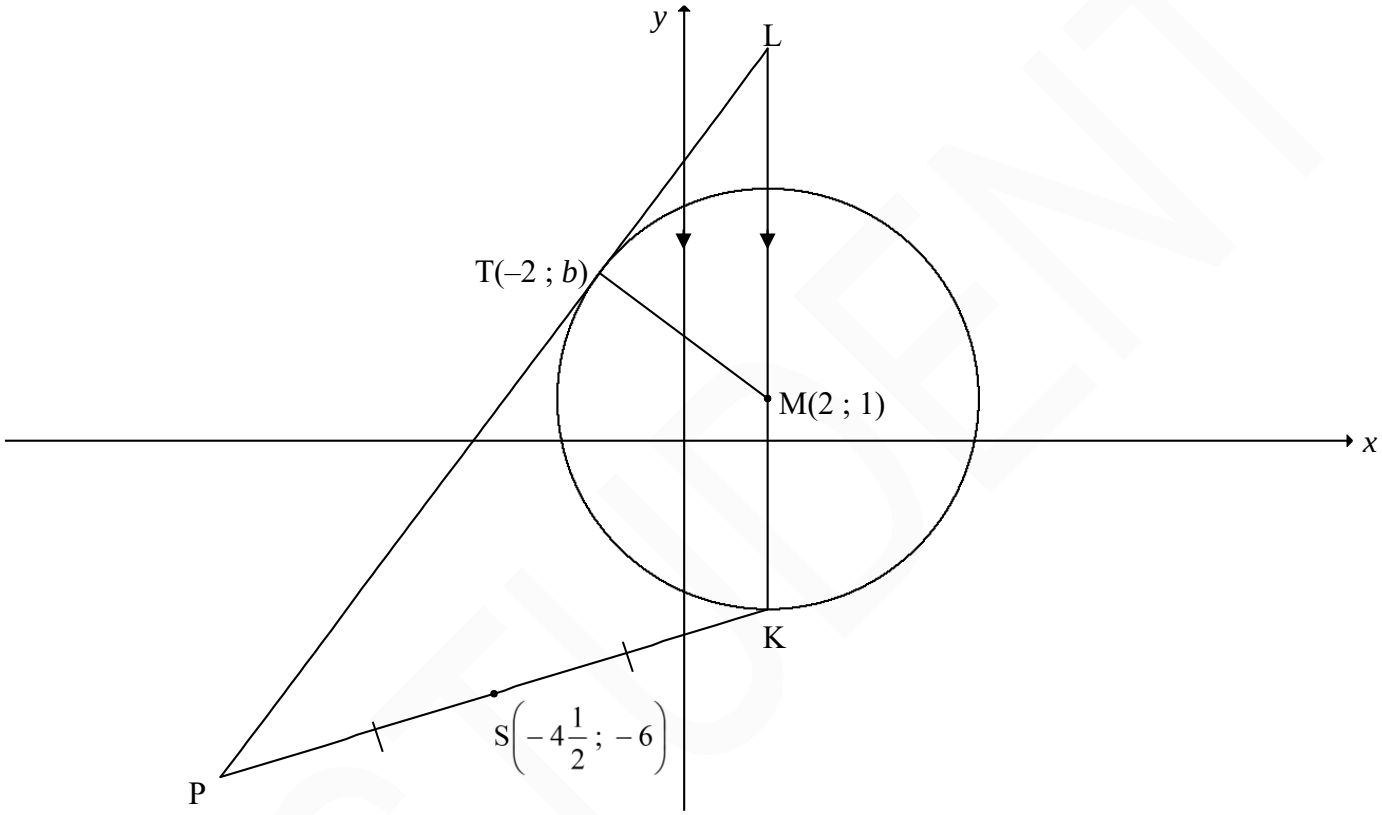
4.5 The smaller circle centred at N is transformed such that point C is translated along the tangent to D. Calculate the coordinates of E, the new centre of the smaller circle. (3)

4.6 If it is given that the area of quadrilateral OBCD is $2a^2$ square units and $a>0$, show that $a=\frac{\sqrt{7}}{2}$ units. (5)

[20]

QUESTION 4

In the diagram, the circle is centred at M(2 ; 1). Radius KM is produced to L, a point outside the circle, such that KML || *y*-axis. LTP is a tangent to the circle at T(–2 ; *b*). $S\left(-4\frac{1}{2};\ -6\right)$ is the midpoint of PK.



4.1 Given that the radius of the circle is 5 units, show that $b = 4$. (4)

4.2 Determine:

4.2.1 The coordinates of K (2)

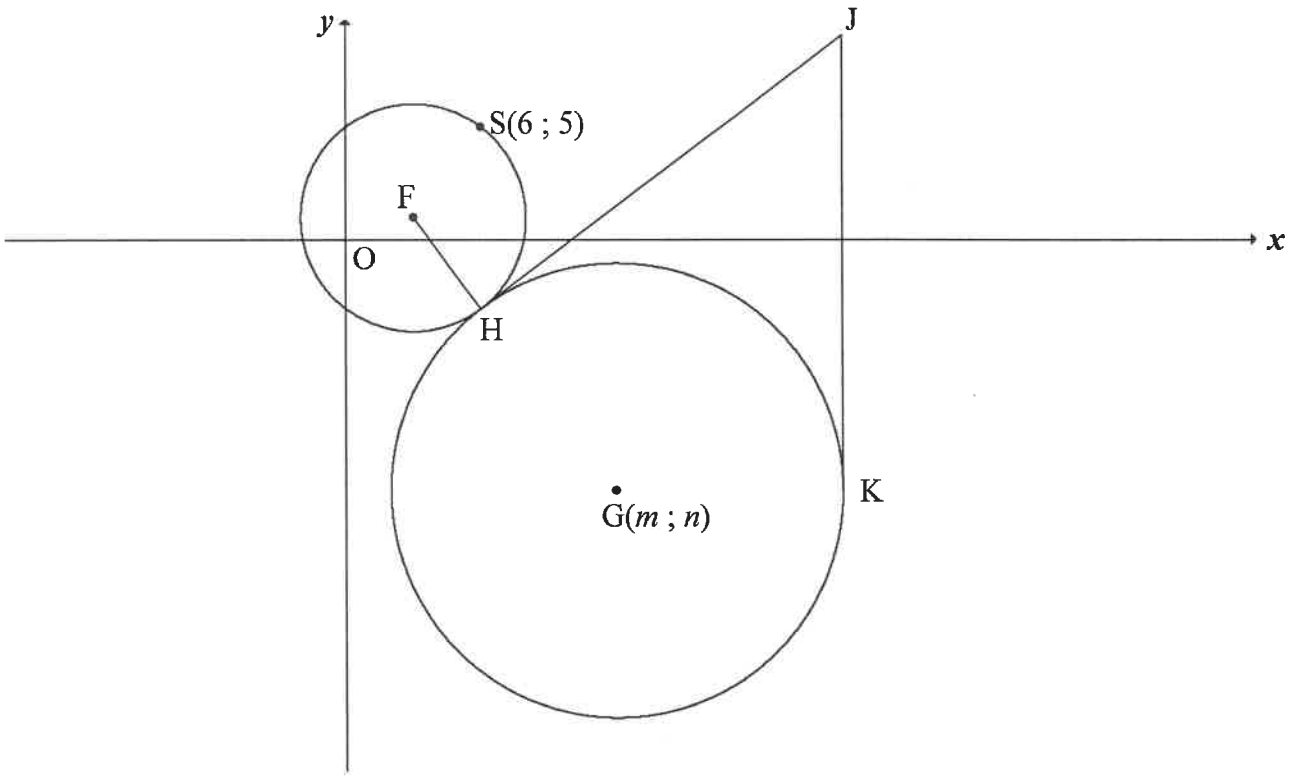
4.2.2 The equation of the tangent LTP in the form $y = mx + c$ (4)

4.2.3 The area of ΔLPK (7)

4.3 Another circle with equation $(x-2)^2 + (y-n)^2 = 25$ is drawn. Determine, with an explanation, the value(s) of n for which the two circles will touch each other externally. (4)
[21]

QUESTION 4

In the diagram, the equation of the circle with centre F is $(x-3)^2+(y-1)^2=r^2$. S(6 ; 5) is a point on the circle with centre F. Another circle with centre G$(m\,;\,n)$ in the 4th quadrant touches the circle with centre F, at H such that FH : HG = 1 : 2. The point J lies in the first quadrant such that HJ is a common tangent to both these circles. JK is a tangent to the larger circle at K.



4.1 Write down the coordinates of F. (2)

4.2 Calculate the length of FS. (2)

4.3 Write down the length of HG. (1)

4.4 Give a reason why JH = JK. (1)

4.5 Determine:

4.5.1 The distance FJ, with reasons, if it is given that JK = 20 (4)

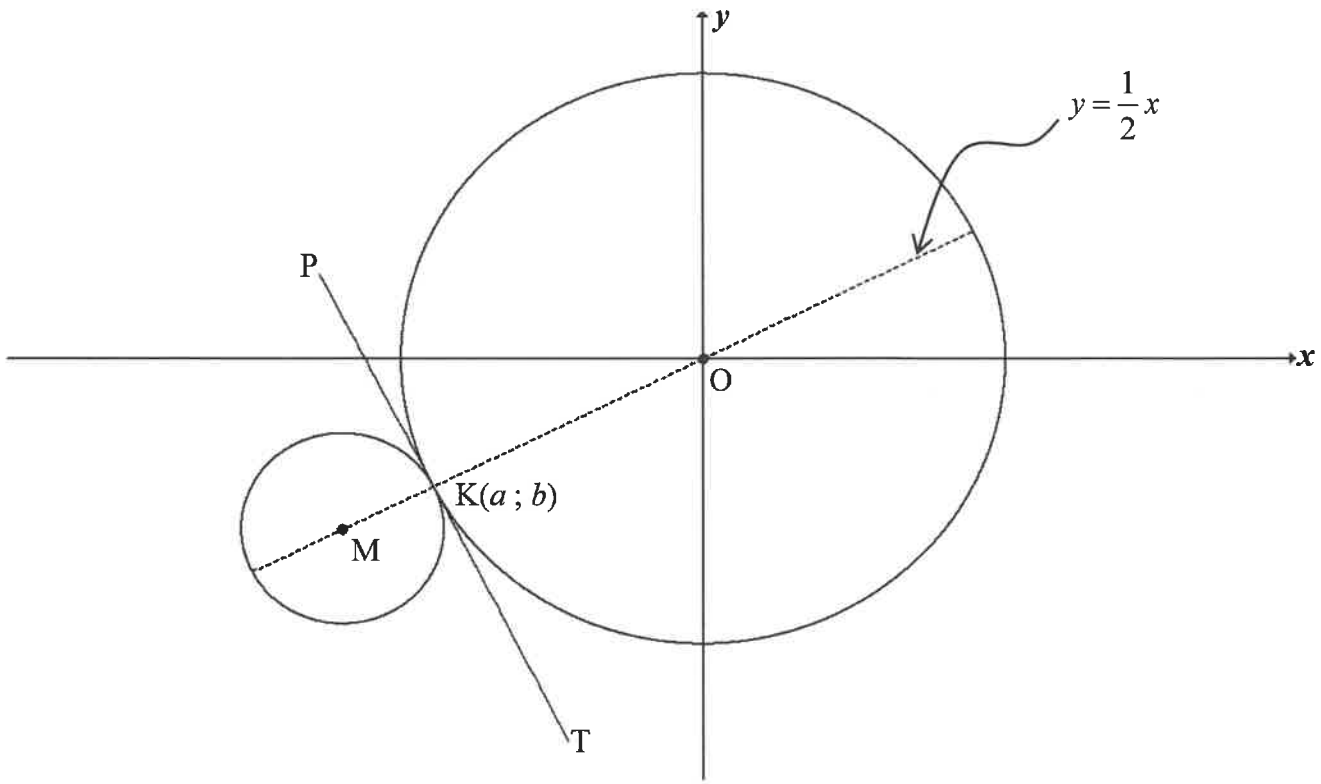
4.5.2 The equation of the circle with centre G in terms of m and n in the form $(x-a)^2+(y-b)^2=r^2$ (1)

4.5.3 The coordinates of G, if it is further given that the equation of tangent JK is $x=22$ (7)

[18]

QUESTION 4

In the diagram, PKT is a common tangent to both circles at K(a ; b). The centres of both circles lie on the line $y = \frac{1}{2}x$. The equation of the circle centred at O is $x^2 + y^2 = 180$. The radius of the circle is three times that of the circle centred at M.



4.1 Write down the length of OK **in surd form**. (1)

4.2 Show that K is the point (–12 ; –6). (4)

4.3 Determine:

4.3.1 The equation of the common tangent, PKT, in the form $y = mx + c$ (3)

4.3.2 The coordinates of M (6)

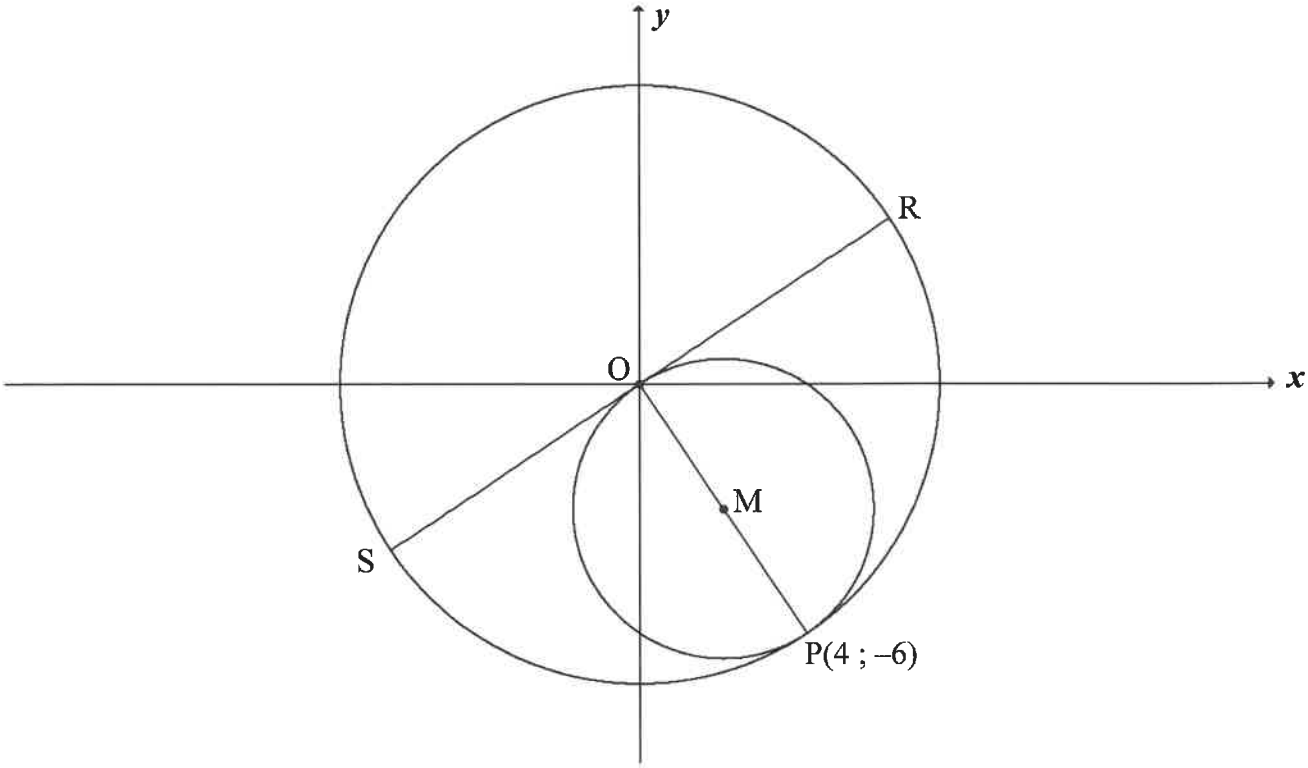
4.3.3 The equation of the smaller circle in the form $(x-a)^2 + (y-b)^2 = r^2$ (2)

4.4 For which value(s) of r will another circle, with equation $x^2 + y^2 = r^2$, intersect the circle centred at M at two distinct points? (3)

4.5 Another circle, $x^2 + y^2 + 32x + 16y + 240 = 0$, is drawn. Prove by calculation that this circle does NOT cut the circle with centre M(–16 ; –8). (5)
[24]

QUESTION 4

In the diagram, a circle having centre at the origin passes through P(4 ; – 6). PO is the diameter of a smaller circle having centre at M. The diameter RS of the larger circle is a tangent to the smaller circle at O.



4.1 Calculate the coordinates of M. (2)

4.2 Determine the equation of:

4.2.1 The large circle (2)

4.2.2 The small circle in the form $x^2 + y^2 + Cx + Dy + E = 0$ (3)

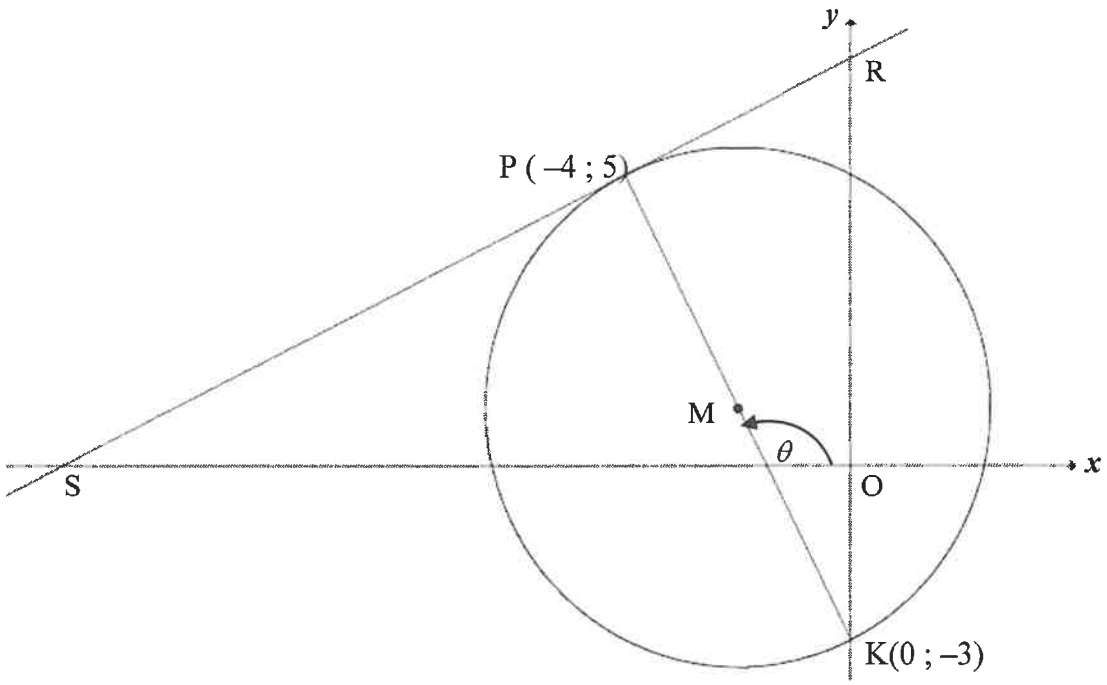
4.2.3 The equation of RS in the form $y = mx + c$ (3)

4.3 Determine the length of chord NR, where N is the reflection of R in the y-axis. (4)

4.4 The circle with centre at M is reflected about the x-axis to form another circle centred at K. Calculate the length of the common chord of these two circles. (3)
[17]

QUESTION 4

In the diagram, P(–4 ; 5) and K(0 ; –3) are the end points of the diameter of a circle with centre M. S and R are respectively the x- and y-intercept of the tangent to the circle at P. θ is the inclination of PK with the positive x-axis.



4.1 Determine:

4.1.1 The gradient of SR (4)

4.1.2 The equation of SR in the form $y = mx + c$ (2)

4.1.3 The equation of the circle in the form $(x-a)^2 + (y-b)^2 = r^2$ (4)

4.1.4 The size of $P\hat{K}R$ (3)

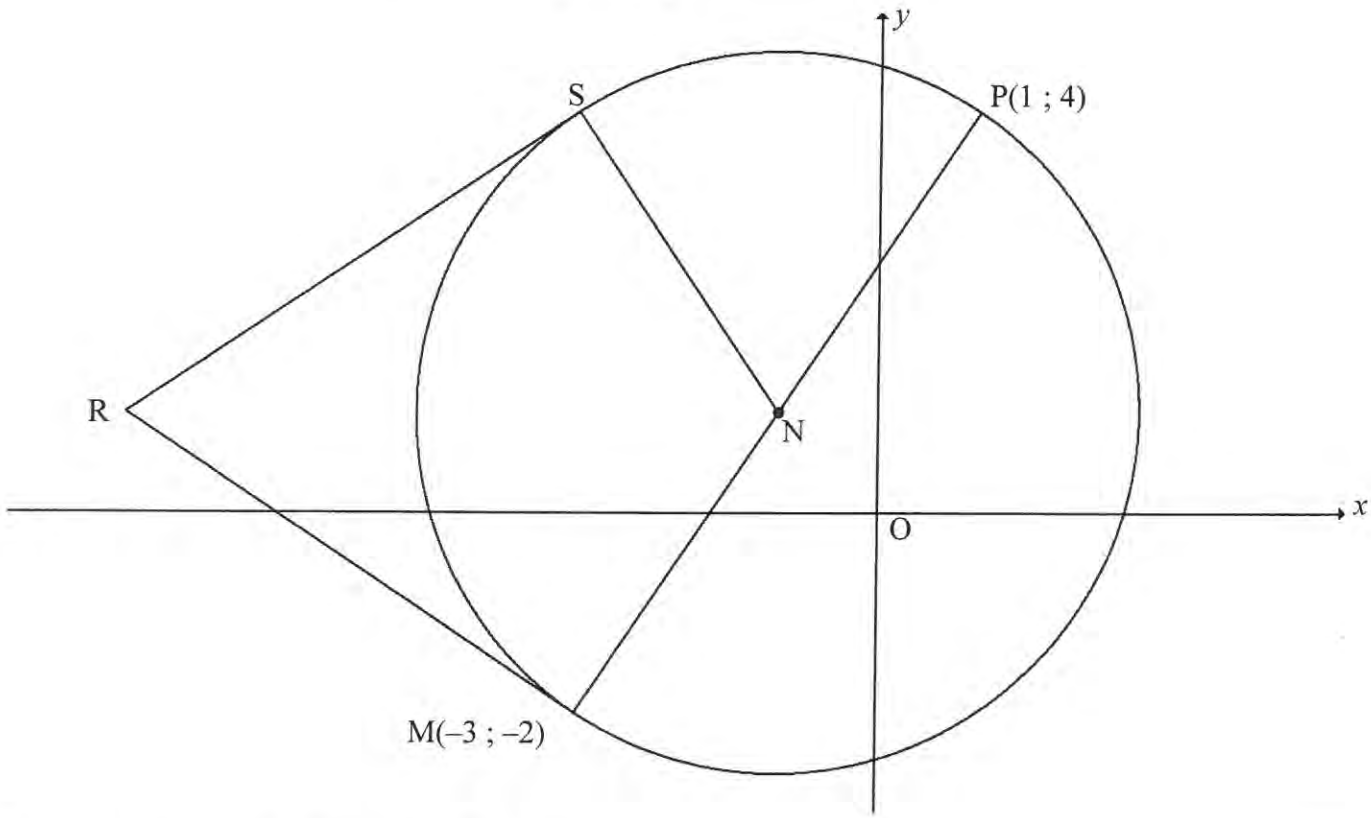
4.1.5 The equation of the tangent to the circle at K in the form $y = mx + c$ (2)

4.2 Determine the values of t such that the line $y = \frac{1}{2}x + t$ cuts the circle at two different points. (3)

4.3 Calculate the area of ΔSMK. (5)
[23]

QUESTION 4

In the diagram, N is the centre of the circle. M(–3 ; –2) and P(1 ; 4) are points on the circle. MNP is the diameter of the circle. Tangents drawn to circle N from point R, outside the circle, meet the circle at S and M respectively.



4.1 Determine the coordinates of N. (3)

4.2 Determine the equation of the circle in the form $(x-a)^2+(y-b)^2=r^2$. (4)

4.3 Determine the equation of the tangent RM in the form $y=mx+c$. (5)

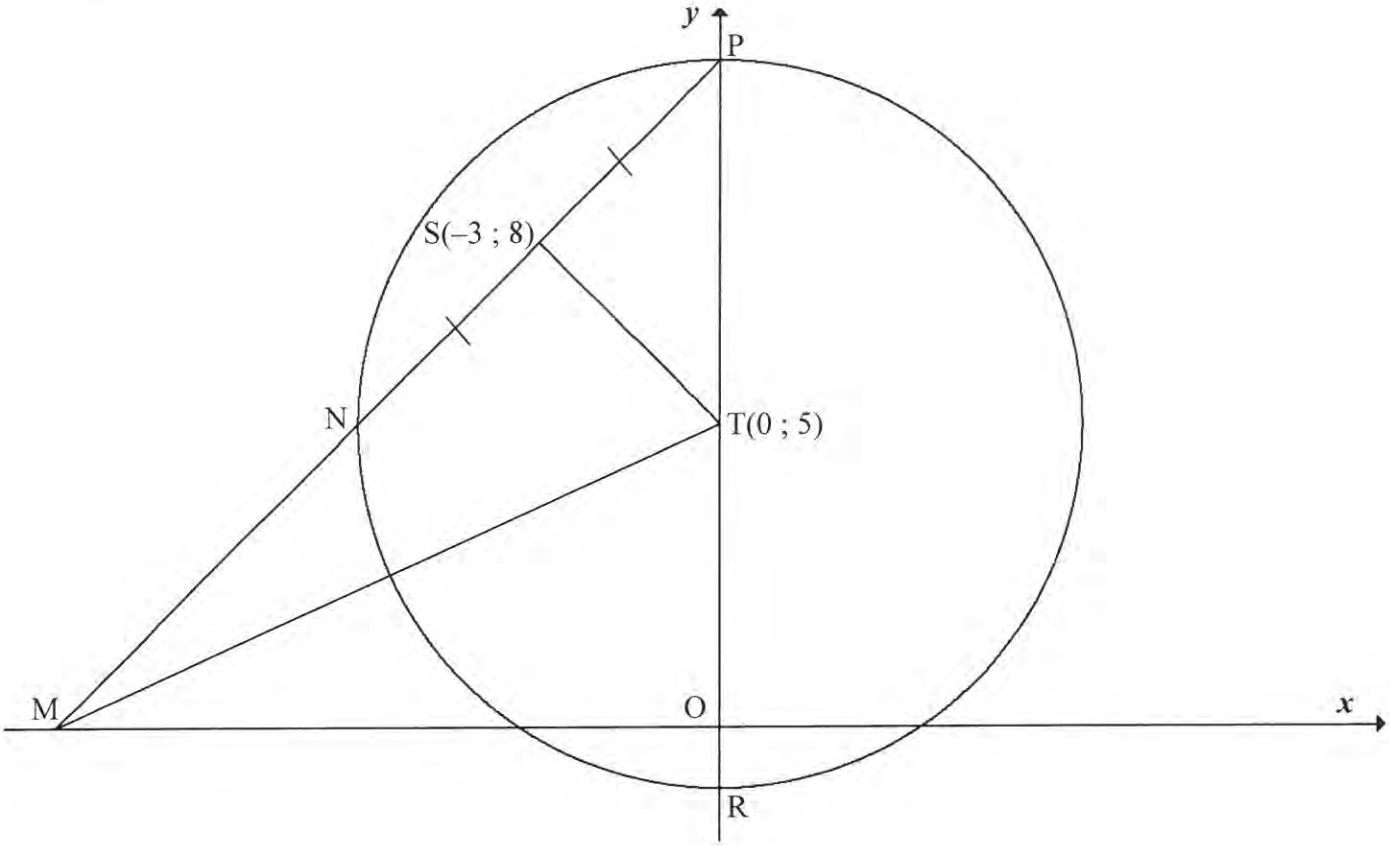
4.4 If it is given that the line joining S to M is perpendicular to the x-axis, determine the coordinates of S. (2)

4.5 Determine the coordinates of R, the common external point from which both tangents to the circle are drawn. (4)

4.6 Calculate the area of RSNM. (4)
[22]

QUESTION 4

In the diagram, the circle, having centre T(0 ; 5), cuts the y-axis at P and R. The line through P and S(–3 ; 8) intersects the circle at N and the x-axis at M. NS = PS. MT is drawn.



4.1 Give a reason why TS ⊥ NP. (1)

4.2 Determine the equation of the line passing through N and P in the form $y = mx + c$. (5)

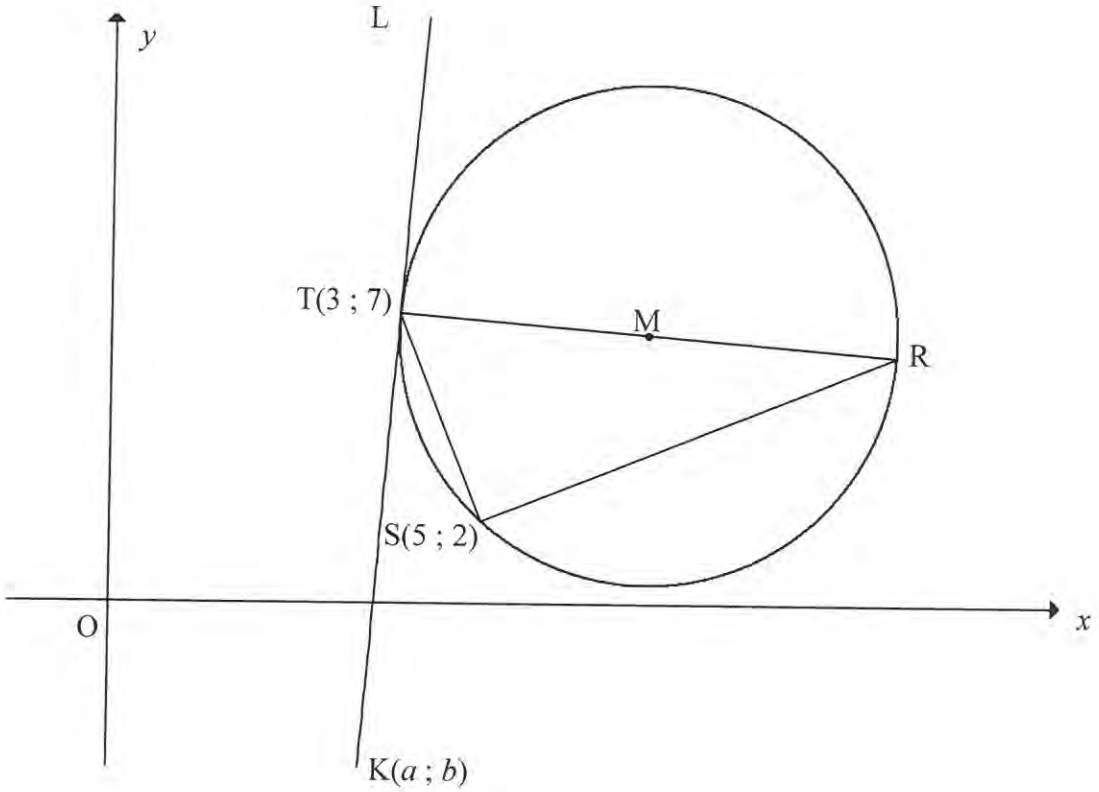
4.3 Determine the equations of the tangents to the circle that are parallel to the x-axis. (4)

4.4 Determine the length of MT. (4)

4.5 Another circle is drawn through the points S, T and M. Determine, with reasons, the equation of this circle STM in the form $(x-a)^2 + (y-b)^2 = r^2$. (5)
[19]

QUESTION 4

In the diagram, M is the centre of the circle passing through T(3 ; 7), R and S(5 ; 2). RT is a diameter of the circle. K(a ; b) is a point in the 4th quadrant such that KTL is a tangent to the circle at T.



4.1 Give a reason why $T\hat{S}R = 90°$. (1)

4.2 Calculate the gradient of TS. (2)

4.3 Determine the equation of the line SR in the form $y = mx + c$. (3)

4.4 The equation of the circle above is $(x-9)^2 + \left(y - 6\frac{1}{2}\right)^2 = 36\frac{1}{4}$.

4.4.1 Calculate the length of TR in surd form. (2)

4.4.2 Calculate the coordinates of R. (3)

4.4.3 Calculate $\sin R$. (3)

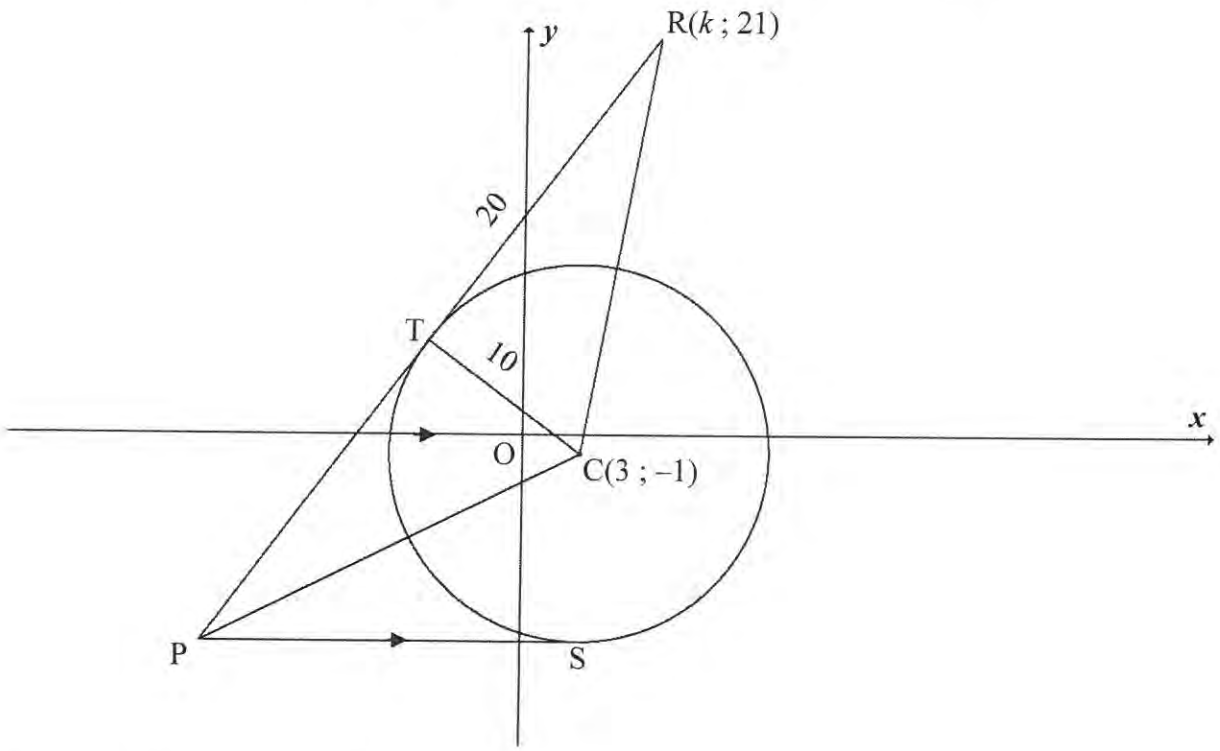
4.4.4 Show that $b = 12a - 29$. (3)

4.4.5 If TK = TR, calculate the coordinates of K. (6)

[23]

QUESTION 4

A circle having C(3 ; –1) as centre and a radius of 10 units is drawn. PTR is a tangent to this circle at T. R(k ; 21), C and P are the vertices of a triangle. TR = 20 units.



4.1 Give a reason why TC ⊥ TR. (1)

4.2 Calculate the length of RC. Leave your answer in surd form. (2)

4.3 Calculate the value of k if R lies in the first quadrant. (4)

4.4 Determine the equation of the circle having centre C and passing through T. Write your answer in the form $(x-a)^2+(y-b)^2=r^2$ (2)

4.5 PS, a tangent to the circle at S, is parallel to the x-axis. Determine the equation of PS. (2)

4.6 The equation of PTR is $3y-4x=35$

4.6.1 Calculate the coordinates of P. (2)

4.6.2 Calculate, giving a reason, the length of PT. (3)

4.7 Consider another circle with equation $(x-3)^2+(y+16)^2=16$ and having centre M.

4.7.1 Write down the coordinates of centre M. (1)

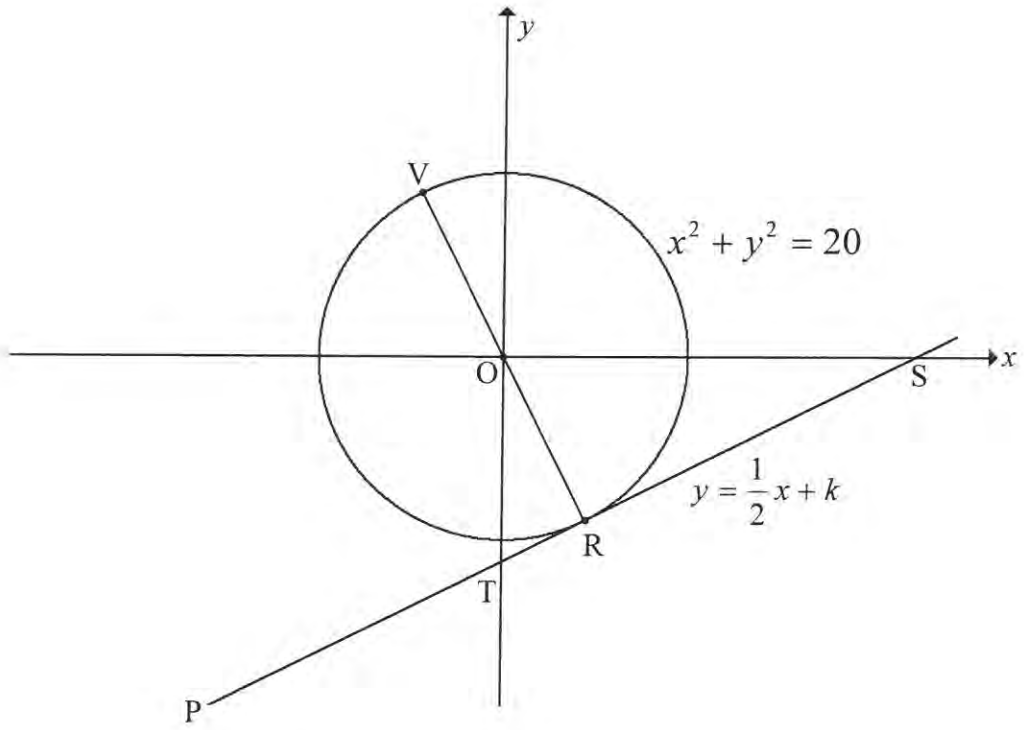
4.7.2 Write down the length of the radius of this circle. (1)

4.7.3 Prove that the circle with centre C and the circle with centre M do not intersect or touch. (3)

[21]

QUESTION 4

In the diagram below, the equation of the circle with centre O is $x^2 + y^2 = 20$. The tangent PRS to the circle at R has the equation $y = \frac{1}{2}x + k$. PRS cuts the y-axis at T and the x-axis at S.



4.1 Determine, giving reasons, the equation of OR in the form $y = mx + c$. (3)

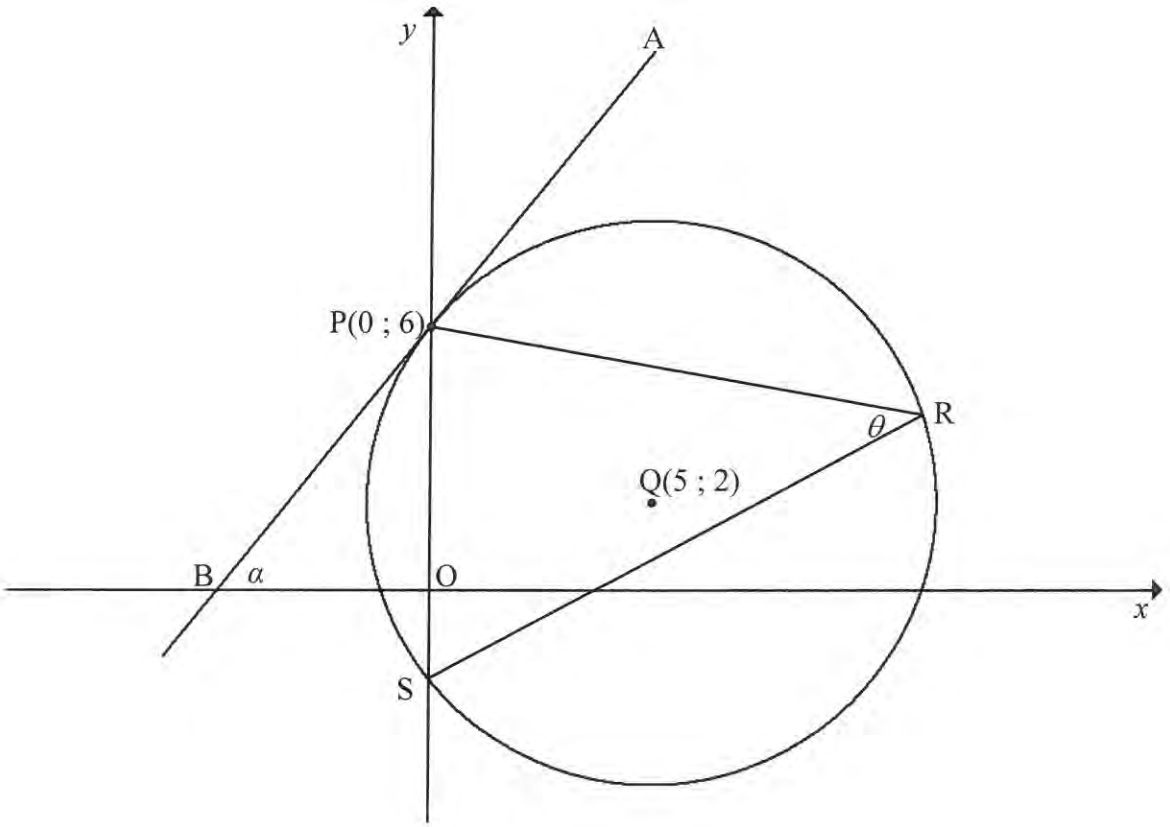
4.2 Determine the coordinates of R. (4)

4.3 Determine the area of ΔOTS, given that R(2 ; –4). (6)

4.4 Calculate the length of VT. (4)
[17]

QUESTION 4

In the diagram below, Q(5 ; 2) is the centre of a circle that intersects the y-axis at P(0 ; 6) and S. The tangent APB at P intersects the x-axis at B and makes the angle α with the positive x-axis. R is a point on the circle and $P\hat{R}S = \theta$.



4.1 Determine the equation of the circle in the form $(x-a)^2 + (y-b)^2 = r^2$. (3)

4.2 Calculate the coordinates of S. (3)

4.3 Determine the equation of the tangent APB in the form $y = mx + c$. (4)

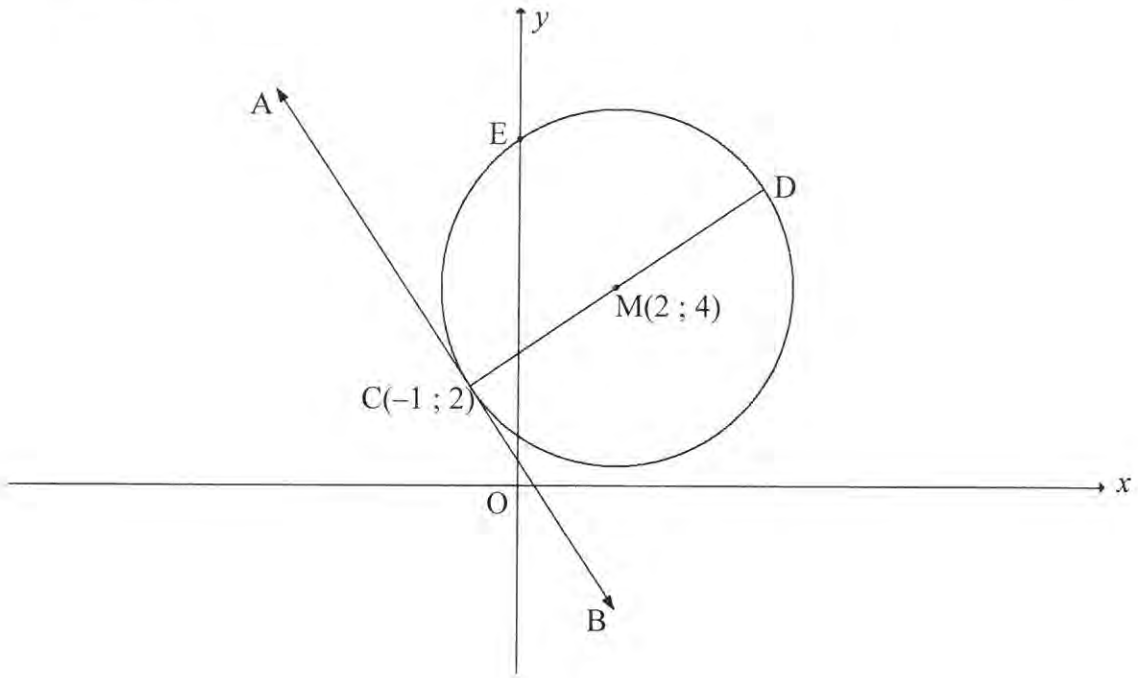
4.4 Calculate the size of α. (2)

4.5 Calculate, with reasons, the size of θ. (4)

4.6 Calculate the area of ΔPQS. (4)
[20]

QUESTION 4

4.1 In the diagram below, the circle centred at M(2 ; 4) passes through C(–1 ; 2) and cuts the y-axis at E. The diameter CMD is drawn and ACB is a tangent to the circle.



4.1.1 Determine the equation of the circle in the form $(x-a)^2+(y-b)^2=r^2$. (3)

4.1.2 Write down the coordinates of D. (2)

4.1.3 Determine the equation of AB in the form $y=mx+c$. (5)

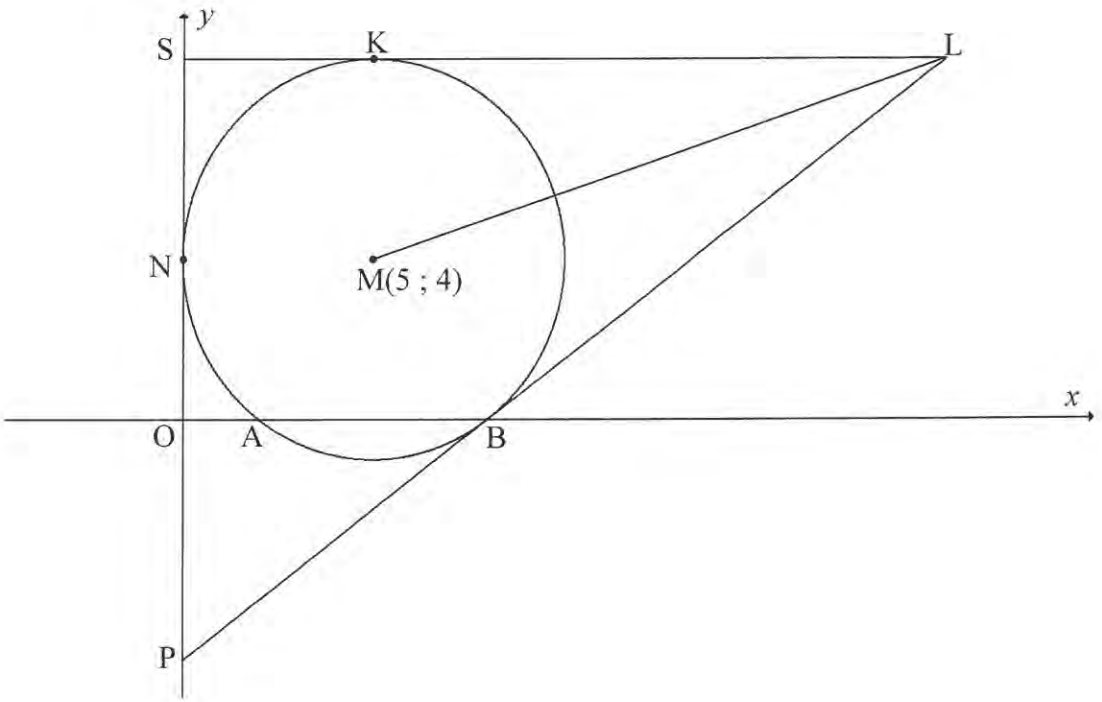
4.1.4 Calculate the coordinates of E. (4)

4.1.5 Show that EM is parallel to AB. (2)

4.2 Determine whether or not the circles having equations $(x+2)^2+(y-4)^2=25$ and $(x-5)^2+(y+1)^2=9$ will intersect. Show ALL calculations. (6)
[22]

QUESTION 3

In the diagram below, a circle with centre M(5 ; 4) touches the y-axis at N and intersects the x-axis at A and B. PBL and SKL are tangents to the circle where SKL is parallel to the x-axis and P and S are points on the y-axis. LM is drawn.



3.1 Write down the length of the radius of the circle having centre M. (1)

3.2 Write down the equation of the circle having centre M, in the form $(x-a)^2+(y-b)^2=r^2$. (1)

3.3 Calculate the coordinates of A. (3)

3.4 If the coordinates of B are (8 ; 0), calculate:

3.4.1 The gradient of MB (2)

3.4.2 The equation of the tangent PB in the form $y=mx+c$ (3)

3.5 Write down the equation of tangent SKL. (2)

3.6 Show that L is the point (20 ; 9). (2)

3.7 Calculate the length of ML in surd form. (2)

3.8 Determine the equation of the circle passing through points K, L and M in the form $(x-p)^2+(y-q)^2=c^2$ (5)

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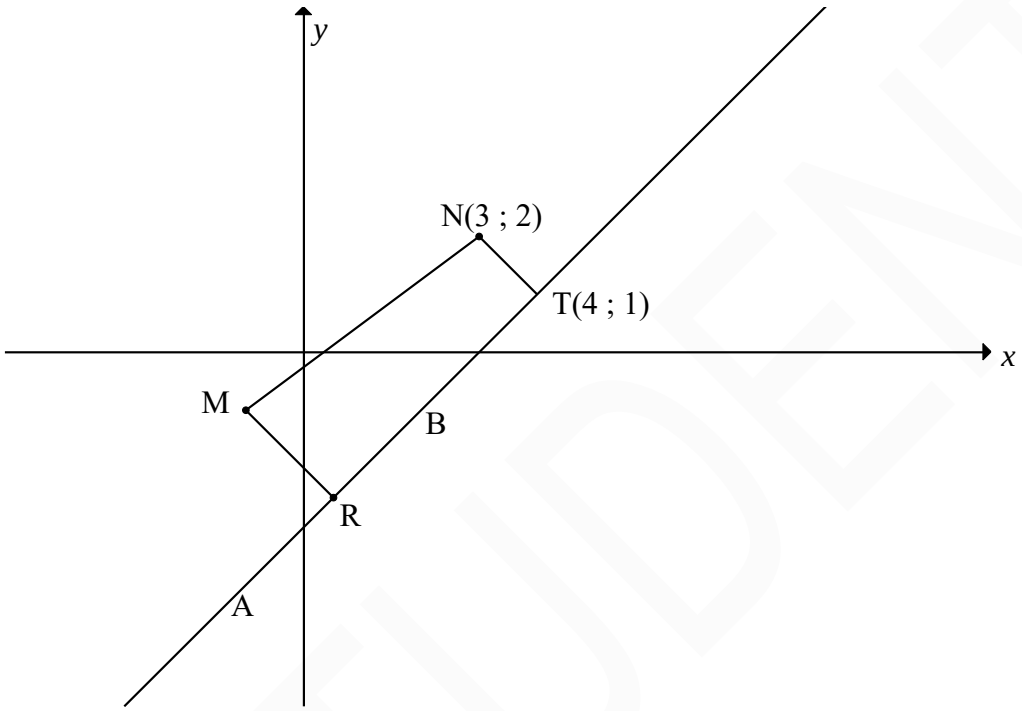
QUESTION 4

In the diagram below, the equation of the circle having centre M is $(x + 1)^2 + (y + 1)^2 = 9$. R is a point on chord AB such that MR bisects AB. ABT is a tangent to the circle having centre N(3 ; 2) at point T(4 ; 1).



4.1 Write down the coordinates of M. (1)

4.2 Determine the equation of AT in the form $y = mx + c$. (5)

4.3 If it is further given that MR = $\frac{\sqrt{10}}{2}$ units, calculate the length of AB.
Leave your answer in simplest surd form. (4)

4.4 Calculate the length of MN. (2)

4.5 Another circle having centre N touches the circle having centre M at point K. Determine the equation of the new circle. Write your answer in the form $x^2 + y^2 + Cx + Dy + E = 0$. (3)
[15]