SA-STUDENT

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"You have to ask yourself how badly do you want something? If you really, really want something then put in the work". -Lewis Hamilton

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QUESTION 5

5.1 Given: $\sin\beta = \frac{1}{3}$, where $\beta \in (90°\ ;\ 270°)$

Without using a calculator, determine each of the following:

5.1.1 $\cos\beta$ (3)

5.1.2 $\sin 2\beta$ (3)

5.1.3 $\cos(450° - \beta)$ (3)

5.2 Given: $\dfrac{\cos^4 x + \sin^2 x.\cos^2 x}{1 + \sin x}$

5.2.1 Prove that $\dfrac{\cos^4 x + \sin^2 x.\cos^2 x}{1 + \sin x} = 1 - \sin x$ (4)

5.2.2 For what value(s) of x in the interval $x \in [0°\ ;\ 360°]$ is $\dfrac{\cos^4 x + \sin^2 x.\cos^2 x}{1 + \sin x}$ undefined? (2)

5.2.3 Write down the minimum value of the function defined by $y = \dfrac{\cos^4 x + \sin^2 x.\cos^2 x}{1 + \sin x}$ (2)

5.3 Given: $\cos(A - B) = \cos A\cos B + \sin A\sin B$

5.3.1 Use the above identity to deduce that $\sin(A - B) = \sin A\cos B - \cos A\sin B$ (3)

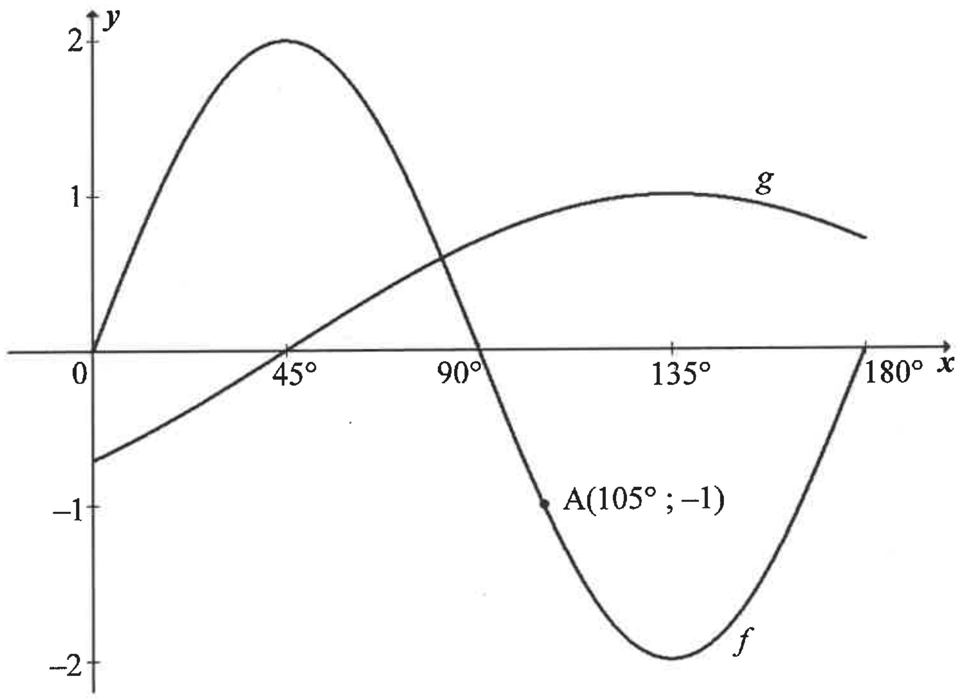
5.3.2 Hence, or otherwise, determine the general solution of the equation $\sin 48°\cos x - \cos 48°\sin x = \cos 2x$ (5)

5.4 Simplify $\dfrac{\sin 3x + \sin x}{\cos 2x + 1}$ to a single trigonometric ratio. (6)

[31]

QUESTION 6

In the diagram, the graphs of $f(x)=2\sin 2x$ and $g(x)=-\cos(x+45°)$ are drawn for the interval $x\in[0°;180°]$. A(105° ; –1) lies on f.



6.1 Write down the period of f. (1)

6.2 Determine the range of g in the interval $x \in [0° ; 180°]$. (2)

6.3 Determine the values of x, in the interval $x \in [0° ; 180°]$, for which:

6.3.1 $f(x).g(x)>0$ (2)

6.3.2 $f(x)+1\leq 0$ (2)

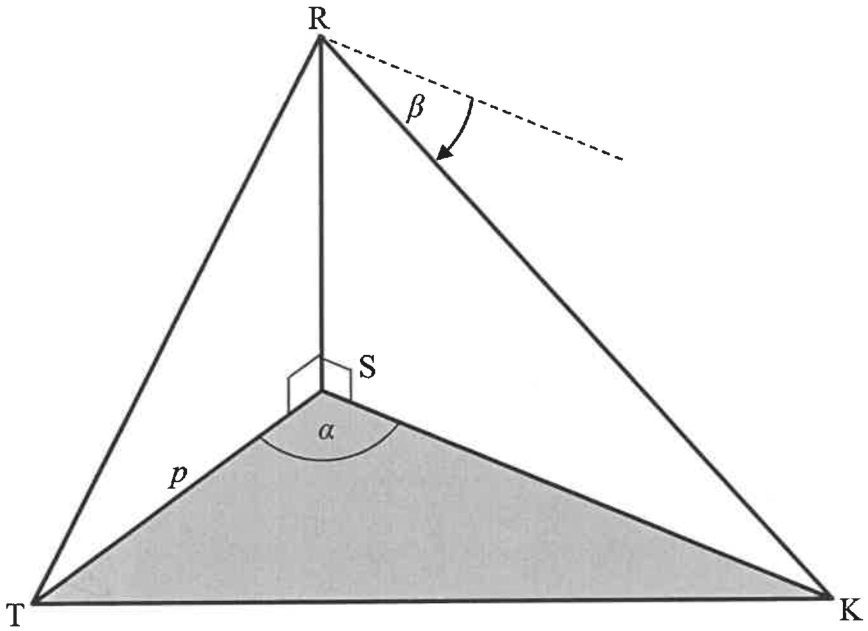
6.4 Another graph p is defined as $p(x) = -f(x)$. D(k ; –1) lies on p. Determine the value(s) of k in the interval $x \in [0° ; 180°]$. (3)

6.5 Graph h is obtained when g is translated 45° to the left. Determine the equation of h. Write your answer in its simplest form. (2)

[12]

QUESTION 7

In the diagram, S, T and K lie in the same horizontal plane. RS is a vertical tower. The angle of depression from R to K is β. $T\hat{S}K = \alpha$, TS $= p$ metres and the area of ΔSTK is q m^2.



7.1 Determine the length of SK in terms of p, q and α. (2)

7.2 Show that $RS = \dfrac{2q \tan \beta}{p \sin \alpha}$ (2)

7.3 Calculate the size of α if $\alpha < 90°$ and RS = 70 m, $p = 80$ m, $q = 2\ 500$ m^2 and $\beta = 42°$. (3)
[7]

QUESTION 5

5.1 **Without using a calculator**, simplify the following expression to a single trigonometry ratio:

$$\frac{1-\sin(-\theta)\cos(90°+\theta)}{\cos(\theta-360°)}$$ (5)

5.2 Given that $\cos 20° = p$

Without using a calculator, write EACH of the following in terms p:

5.2.1 $\cos 200°$ (2)

5.2.2 $\sin(-70°)$ (2)

5.2.3 $\sin 10°$ (3)

5.3 Determine, **without using a calculator**, the value of:

$$\cos(A+55°)\cos(A+10°)+\sin(A+55°)\sin(A+10°)$$ (3)

5.4 Consider: $\dfrac{\cos 2x+\sin 2x-\cos^2 x}{\sin x-2\cos x}=-\sin x$

5.4.1 Prove the above identity. (3)

5.4.2 Determine the value of $\dfrac{\cos 2x+\sin 2x-\cos^2 x}{-3\sin^2 x+6\sin x\cos x}$ (3)

5.5 Given: $3\tan 4x=-2\cos 4x$

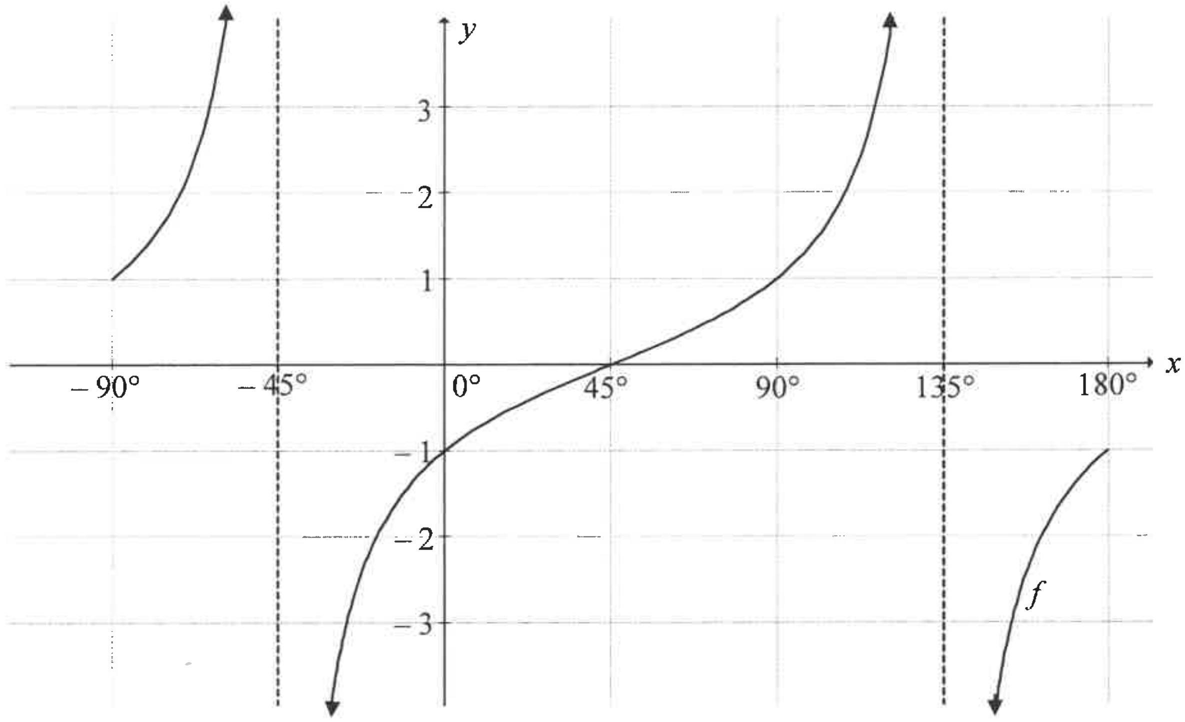
5.5.1 **Without using a calculator**, show that $\sin 4x=-0{,}5$ is the only solution to the above equation. (4)

5.5.2 Hence, determine the general solution of x in the equation $3\tan 4x=-2\cos 4x$ (3)

[28]

QUESTION 6

In the diagram below, the graph of $f(x) = \tan(x - 45°)$ is drawn for $x \in [-90° ; 180°]$.



6.1 Write down the period of *f*. (1)

6.2 Draw the graph of $g(x) = -\cos 2x$ for the interval $x \in [-90° ; 180°]$ on the grid given in the ANSWER BOOK. Show ALL intercepts with the axes, as well as the minimum and maximum points of the graph. (3)

6.3 Write down the range of g. (1)

6.4 The graph of g is shifted 45° to the left to form the graph of h. Determine the equation of h in its simplest form. (2)

6.5 Use the graph(s) to determine the values of x in the interval $x \in [-90° ; 90°]$ for which:

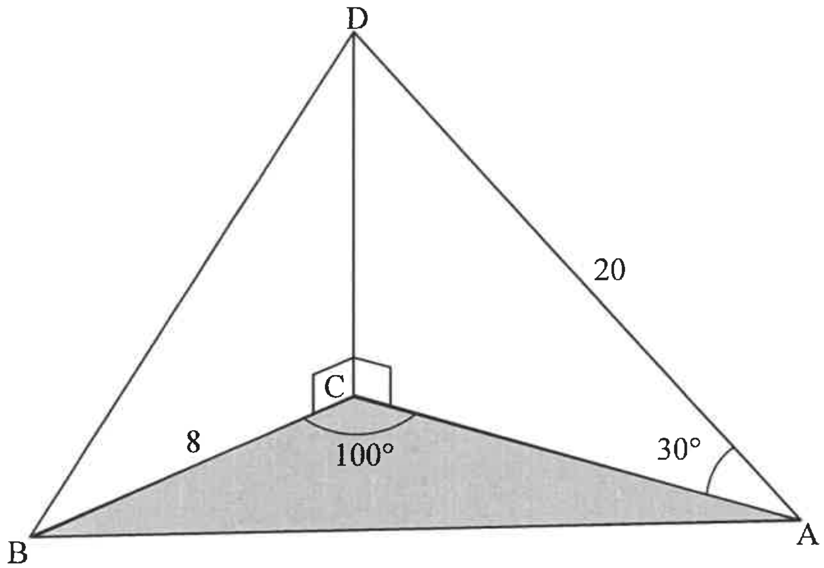
6.5.1 $f(x) > 1$ (2)

6.5.2 $2\cos 2x - 1 > 0$ (4)

[13]

QUESTION 7

In the diagram, A, B and C are points in the same horizontal plane. D is a point directly above C, that is $DC \perp AC$ and $DC \perp BC$. It is given that $A\hat{C}B = 100°$, $C\hat{A}D = 30°$, $AD = 20$ units and $BC = 8$ units.



7.1 Calculate the length of:

7.1.1 AC (2)

7.1.2 AB (3)

7.2 If it is further given that $A\hat{B}D = 73,4°$, calculate the size of $A\hat{D}B$. (3)

[8]

QUESTION 5

5.1 Given that $\sqrt{13}\sin x + 3 = 0$, where $x \in (90°\,;\,270°)$.

Without using a calculator, determine the value of:

5.1.1 $\sin(360° + x)$ (2)

5.1.2 $\tan x$ (3)

5.1.3 $\cos(180° + x)$ (2)

5.2 Determine the value of the following expression, **without using a calculator**:

$$\frac{\cos(90° + \theta)}{\sin(\theta - 180°) + 3\sin(-\theta)}$$ (5)

5.3 Determine the general solution of the following equation:

$(\cos x + 2\sin x)(3\sin 2x - 1) = 0$ (6)

5.4 Given the identity: $\cos(x + y).\cos(x - y) = 1 - \sin^2 x - \sin^2 y$

5.4.1 Prove the identity. (4)

5.4.2 Hence, determine the value of $1 - \sin^2 45° - \sin^2 15°$, **without using a calculator.** (3)

5.5 Consider the trigonometric expression: $16\sin x.\cos^3 x - 8\sin x.\cos x$

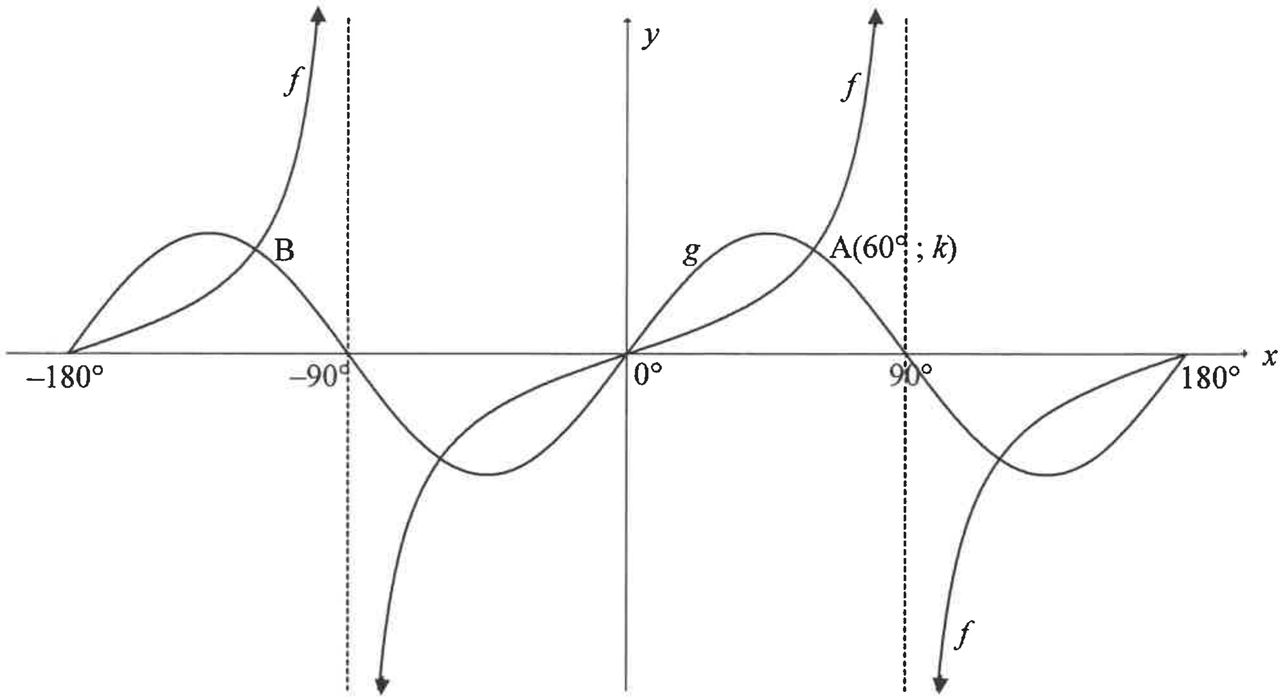
5.5.1 Rewrite the expression as a single trigonometric ratio. (4)

5.5.2 For which value of x in the interval $x \in [0°\,;\,90°]$ will $16\sin x.\cos^3 x - 8\sin x.\cos x$ have its minimum value? (1)

[30]

QUESTION 6

In the diagram below, the graphs of $f(x) = \tan x$ and $g(x) = 2\sin 2x$ are drawn for the interval $x \in [-180° ; 180°]$ A$(60° ; k)$ and B are two points of intersection of f and g.



6.1 Write down the period of g. (1)

6.2 Calculate the:

6.2.1 Value of k (1)

6.2.2 Coordinates of B (1)

6.3 Write down the range of $2g(x)$. (2)

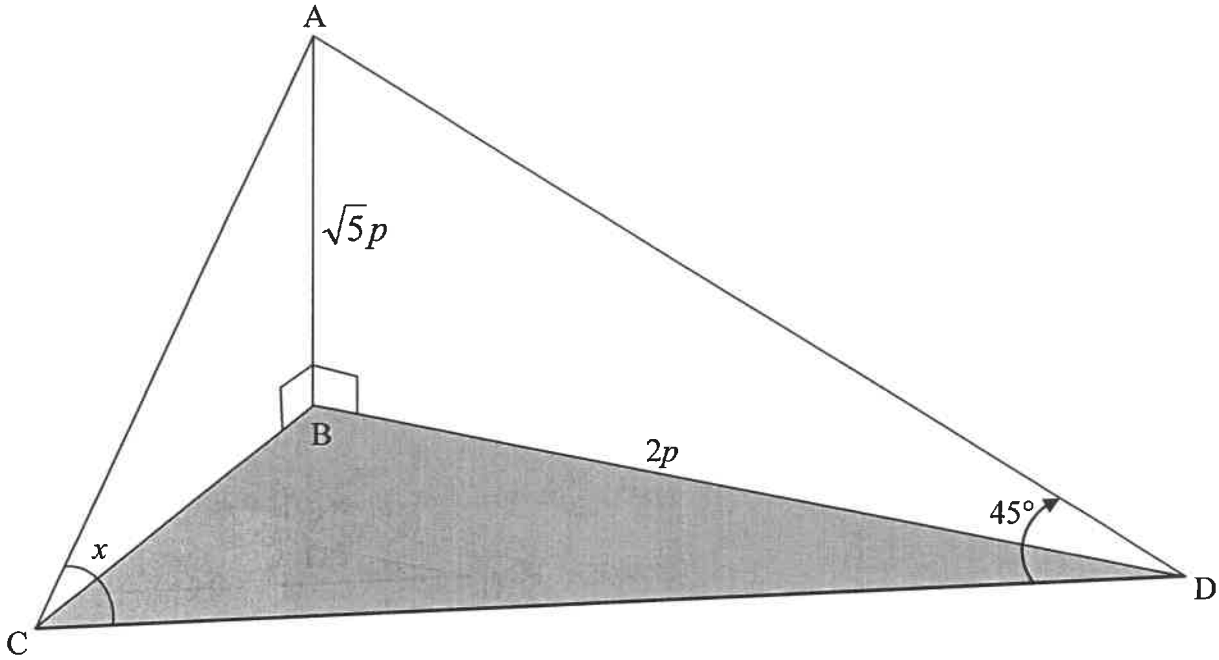
6.4 For which values of x will $g(x+5°) - f(x+5°) \leq 0$ in the interval $x \in [-90° ; 0°]$? (2)

6.5 Determine the values of p for which $\sin x.\cos x = p$ will have exactly two real roots in the interval $x \in [-180° ; 180°]$. (3)

[10]

QUESTION 7

AB is a vertical flagpole that is $\sqrt{5}p$ metres long. AC and AD are two cables anchoring the flagpole. B, C and D are in the same horizontal plane.
BD = $2p$ metres, $A\hat{C}D = x$ and $A\hat{D}C = 45°$.



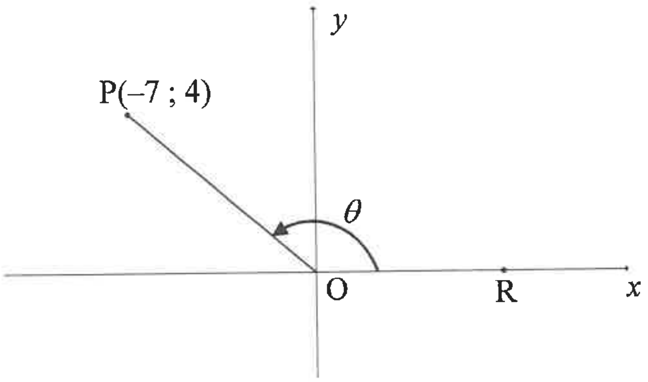
7.1 Determine the length of AD in terms of p. (2)

7.2 Show that the length of $CD = \dfrac{3p(\sin x + \cos x)}{\sqrt{2}\sin x}$. (5)

7.3 If it is further given that $p = 10$ and $x = 110°$, calculate the area of ΔADC. (3)
[10]

QUESTION 5

5.1 In the diagram below, P(–7 ; 4) is a point in the Cartesian plane. R is a point on the positive x-axis such that obtuse $P\hat{O}R = \theta$.



Calculate, **without using a calculator**, the:

5.1.1 Length OP (2)

5.1.2 Value of:

(a) $\tan\theta$ (1)

(b) $\cos(\theta - 180°)$ (2)

5.2 Determine the general solution of: $\sin x\cos x + \sin x = 3\cos^2 x + 3\cos x$ (7)

5.3 Given the identity: $\dfrac{\sin 3x}{1-\cos 3x} = \dfrac{1+\cos 3x}{\sin 3x}$

5.3.1 Prove the identity given above. (3)

5.3.2 Determine the values of x, in the interval $x \in [0° ; 60°]$, for which the identity will be undefined. (3)

[18]

QUESTION 6

6.1 **Without using a calculator**, simplify the following expression to a single trigonometric term:

$$\frac{\sin 10°}{\cos 440°} + \tan(360° - \theta).\sin 2\theta$$ (6)

6.2 Given: $\sin(60° + 2x) + \sin(60° - 2x)$

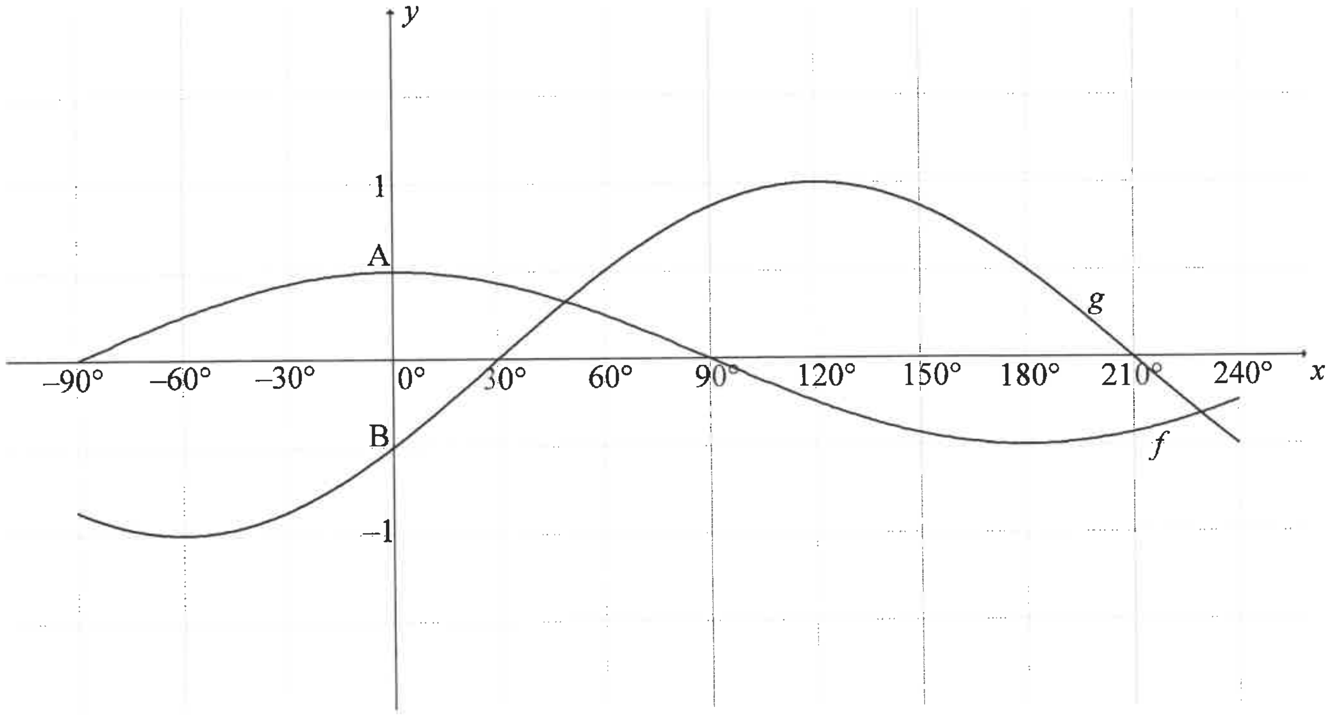
6.2.1 Calculate the value of k if $\sin(60° + 2x) + \sin(60° - 2x) = k\cos 2x$. (3)

6.2.2 If $\cos x = \sqrt{t}$, **without using a calculator**, determine the value of $\tan 60°[\sin(60° + 2x) + \sin(60° - 2x)]$ in terms of t. (3)

[12]

QUESTION 7

In the diagram below, the graphs of $f(x)=\frac{1}{2}\cos x$ and $g(x)=\sin(x-30°)$ are drawn for the interval $x\in[-90°\,;\,240°]$. A and B are the y-intercepts of f and g respectively.



7.1 Determine the length of AB. (2)

7.2 Write down the range of $3f(x)+2$. (2)

7.3 Read off from the graphs a value of x for which $g(x)-f(x)=\frac{\sqrt{3}}{2}$. (2)

7.4 For which values of x, in the interval $x\in[-90°\,;\,240°]$, will:

7.4.1 $f(x).g(x)>0$ (2)

7.4.2 $g'(x-5°)>0$ (2)

[10]

QUESTION 8

FIGURE I shows a ramp leading to the entrance of a building. B, C and D lie on the same horizontal plane. The perpendicular height (AC) of the ramp is 0,5 m and the angle of elevation from B to A is $15°$. The entrance of the building (AE) is 0,915 m wide.

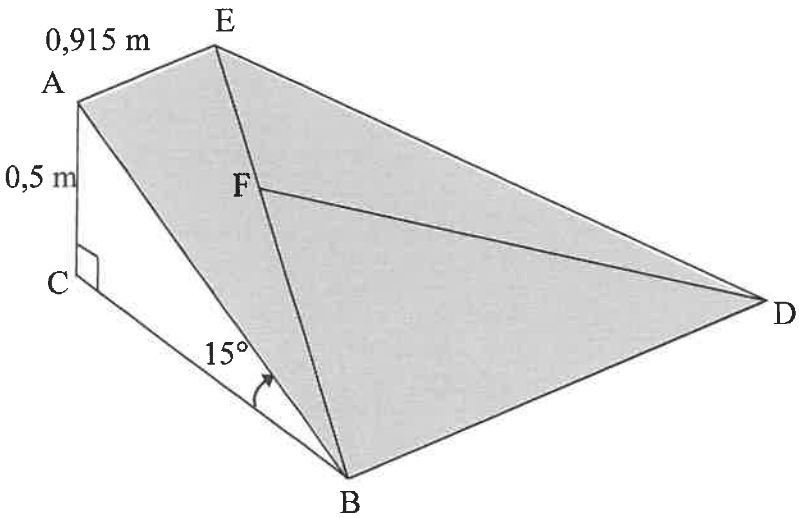


FIGURE I

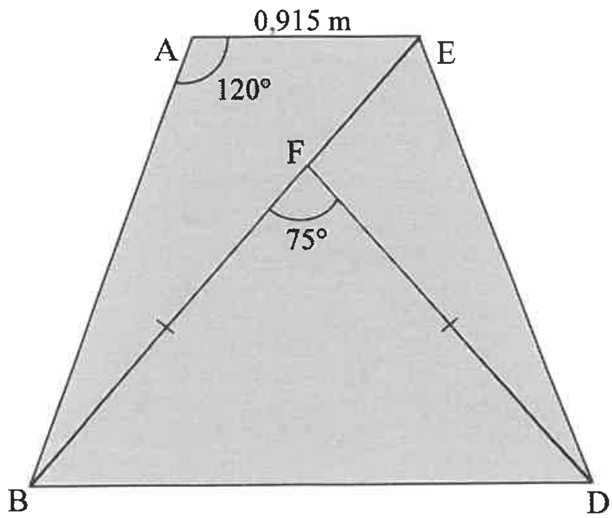


FIGURE II (top view)

8.1 Calculate the length of AB. (2)

8.2 Figure II shows the top view of the ramp. The area of the top of the ramp is divided into three triangles, as shown in the diagram.

If $B\hat{A}E = 120°$, calculate the length of BE. (3)

8.3 Calculate the area of ΔBFD if $B\hat{F}D = 75°$, $BF = FD$ and $BF = \frac{5}{7}BE$. (3)

[8]

QUESTION 5

5.1 **Without using a calculator**, simplify the following expression to ONE trigonometric ratio:

$$\frac{\sin 140°.\sin(360° - x)}{\cos 50°.\tan(-x)}$$ (6)

5.2 Prove the identity: $\frac{-2\sin^2 x + \cos x + 1}{1 - \cos(540° - x)} = 2\cos x - 1$ (4)

5.3 Given: $\sin 36° = \sqrt{1 - p^2}$

Without using a calculator, determine EACH of the following in terms of p:

5.3.1 $\tan 36°$ (3)

5.3.2 $\cos 108°$ (4)

[17]

QUESTION 6

6.1 Given: $\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

6.1.1 Use the given identity to derive a formula for $\cos(\alpha + \beta)$ (3)

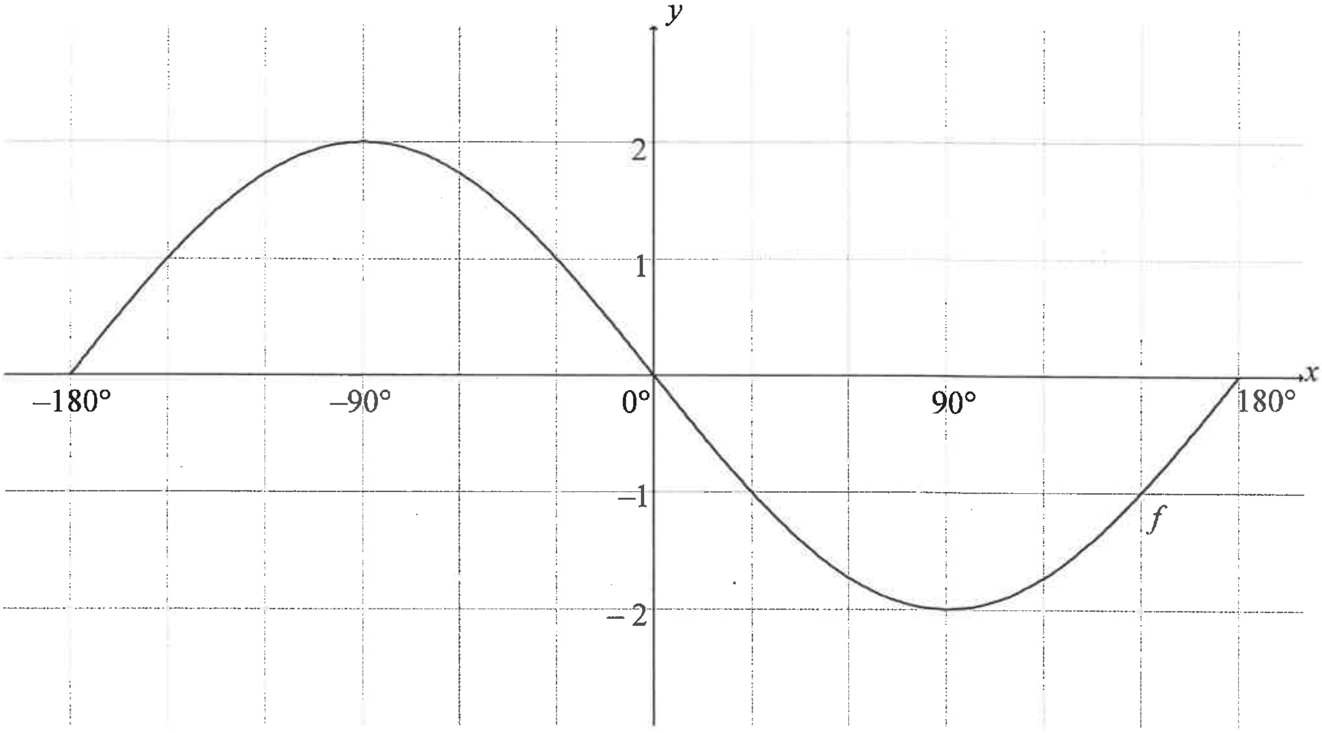
6.1.2 Simplify completely: $2\cos 6x\cos 4x - \cos 10x + 2\sin^2 x$ (5)

6.2 Determine the general solution of $\tan x = 2\sin 2x$ where $\cos x < 0$. (7)

[15]

QUESTION 7

In the diagram below, the graph of $f(x) = -2\sin x$ is drawn for the interval $x \in [-180°\,;\,180°]$.



7.1 On the grid provided in the ANSWER BOOK, draw the graph of $g(x) = \cos(x - 60°)$ for $x \in [-180°\,;\,180°]$. Clearly show ALL intercepts with the axes and turning points of the graph. (3)

7.2 Write down the period of $f(3x)$. (2)

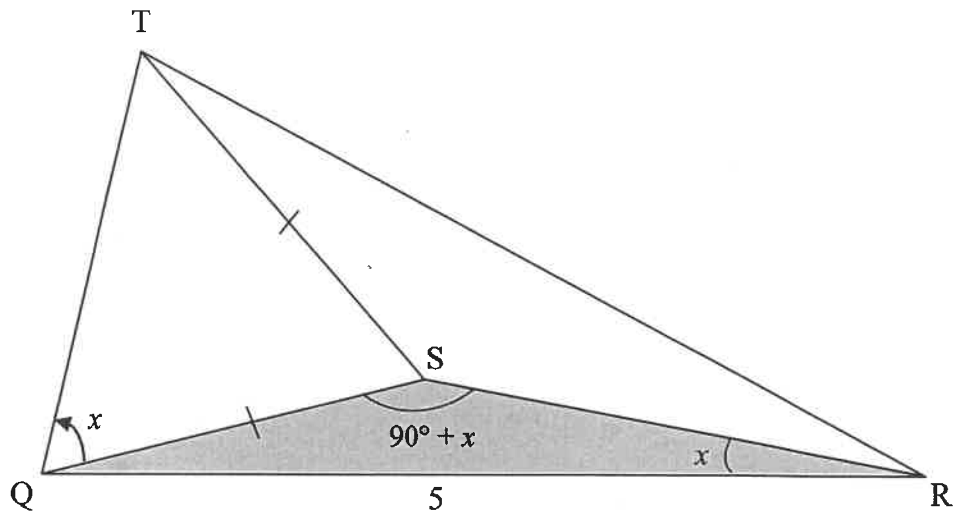
7.3 **Use the graphs** to determine the value of x in the interval $x \in [-180°\,;\,180°]$ for which $f(x) - g(x) = 1$. (1)

7.4 Write down the range of k, if $k(x) = \frac{1}{2}g(x) + 1$. (2)

[8]

QUESTION 8

In the diagram below, T is a hook on the ceiling of an art gallery. Points Q, S and R are on the same horizontal plane from where three people are observing the hook T. The angle of elevation from Q to T is x.
$Q\hat{S}R = 90° + x$, $Q\hat{R}S = x$, $QR = 5$ units and $TS = SQ$.



8.1 Prove that $QS = 5 \tan x$ (3)

8.2 Prove that the length of $QT = 10 \sin x$ (5)

8.3 Calculate the area of ΔTQR if $T\hat{Q}R = 70°$ and $x = 25°$. (2)
[10]

QUESTION 5

5.1 Simplify the expression to a **single trigonometric term:**

$\tan(-x).\cos x.\sin(x-180°)-1$ (5)

5.2 Given: $\cos 35° = m$

Without using a calculator, determine the value of EACH of the following in terms of m:

5.2.1 $\cos 215°$ (2)

5.2.2 $\sin 20°$ (3)

5.3 Determine the general solution of:

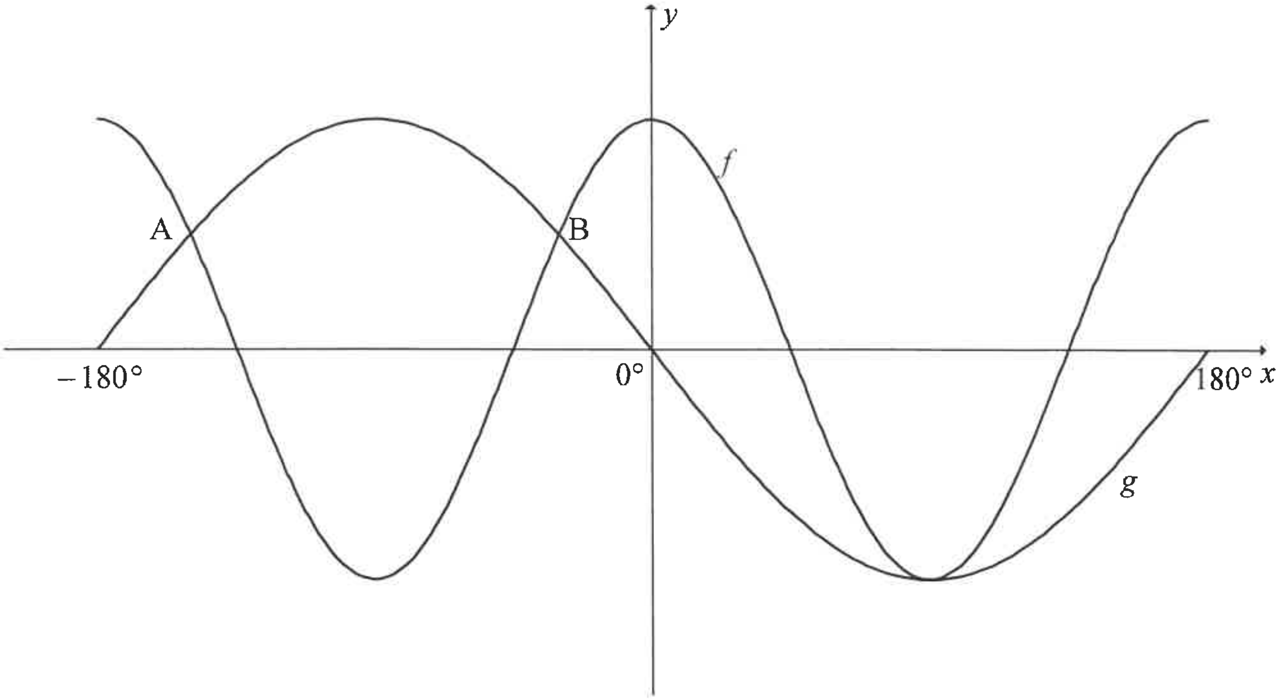
$\cos 4x.\cos x + \sin x.\sin 4x = -0{,}7$ (4)

5.4 Prove the identity: $\dfrac{\sin 4x.\cos 2x - 2\cos 4x.\sin x.\cos x}{\tan 2x} = \cos^2 x - \sin^2 x$ (4)

[18]

QUESTION 6

In the diagram below, the graphs of $f(x) = \cos 2x$ and $g(x) = -\sin x$ are drawn for the interval $x \in [-180° ; 180°]$ A and B are two points of intersection of f and g.



6.1 **Without using a calculator**, determine the values of x for which $\cos 2x = -\sin x$ in the interval $x \in [-180° ; 180°]$. (6)

6.2 Use the graphs above to answer the following questions:

6.2.1 How many degrees apart are points A and B from each other? (2)

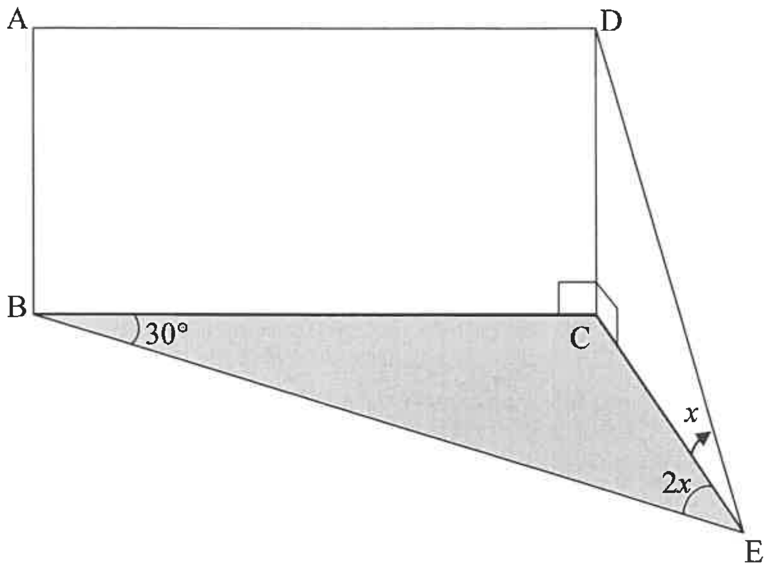
6.2.2 For which values of x in the given interval will $f'(x).g'(x) > 0$? (2)

6.2.3 Determine the values of k for which $\cos 2x + 3 = k$ will have no solution. (3)

[13]

QUESTION 7

Points B, C and E lie in the same horizontal plane. ABCD is a rectangular piece of board. CDE is a triangular piece of board having a right angle at C. Each piece of board is placed perpendicular to the horizontal plane and joined along DC, as shown in the diagram. The angle of elevation from E to D is x. $B\hat{E}C = 2x$ and $E\hat{B}C = 30°$.



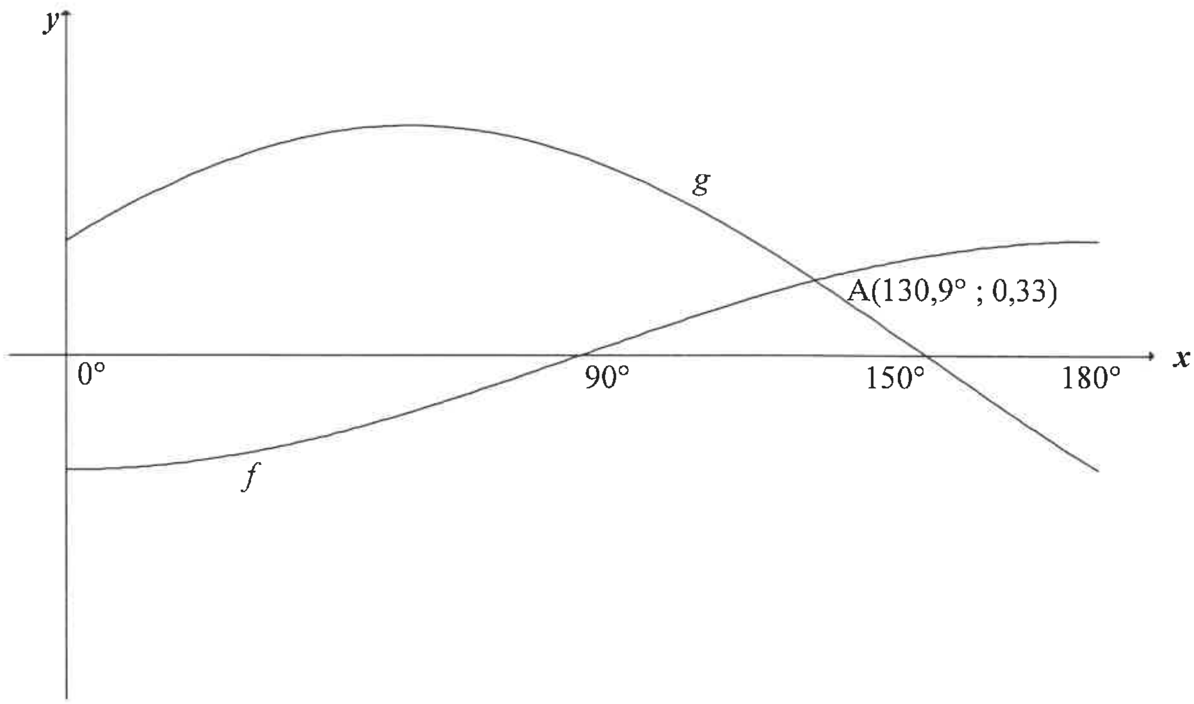
7.1 Show that $DC = \frac{BC}{4\cos^2 x}$ (6)

7.2 If $x = 30°$, show that the area of $ABCD = 3AB^2$. (3)

[9]

QUESTION 5

The graphs of $f(x) = -\frac{1}{2}\cos x$ and $g(x) = \sin(x + 30°)$, for the interval $x \in [0°; 180°]$, are drawn below. A(130,9° ; 0,33) is the approximate point of intersection of the two graphs.



5.1 Write down the period of g. (1)

5.2 Write down the amplitude of f (1)

5.3 Determine the value of $f(180°) - g(180°)$ (1)

5.4 Use the graphs to determine the values of x, in the interval $x \in [0° ; 180°]$, for which:

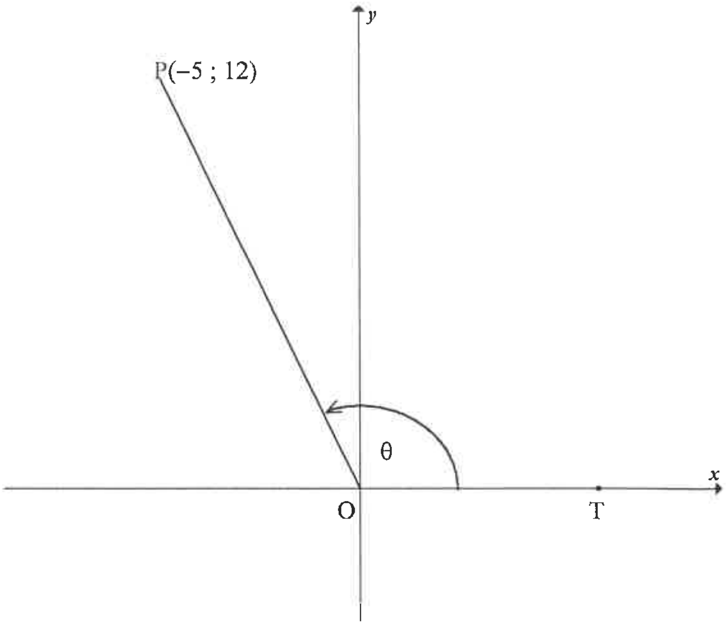
5.4.1 $f(x - 10°) = g(x - 10°)$ (1)

5.4.2 $\sqrt{3}\sin x + \cos x \geq 1$ (4)

[8]

QUESTION 6

6.1 In the diagram, P(–5 ; 12) and T lies on the positive x-axis. $\hat{POT} = \theta$



Answer the following **without using a calculator**:

6.1.1 Write down the value of $\tan\theta$ (1)

6.1.2 Calculate the value of $\cos\theta$ (3)

6.1.3 S(a ; b) is a point in the third quadrant such that $\hat{TOS} = \theta + 90°$ and OS = 6,5 units. Calculate the value of b. (4)

6.2 Determine, **without using a calculator**, the value of the following trigonometric expression:

$$\frac{\sin 2x.\cos(-x) + \cos 2x.\sin(360° - x)}{\sin(180° + x)}$$ (5)

6.3 Determine the general solution of the following equation:

$$6\sin^2 x + 7\cos x - 3 = 0$$ (6)

6.4 Given: $x + \frac{1}{x} = 3\cos A$ and $x^2 + \frac{1}{x^2} = 2$

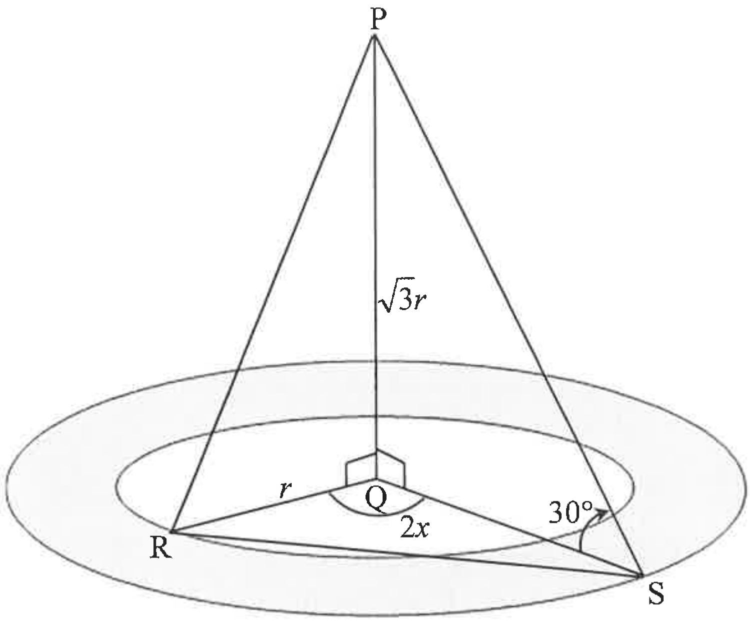
Determine the value of cos 2A **without using a calculator**. (5)

[24]

QUESTION 7

A landscape artist plans to plant flowers within two concentric circles around a vertical light pole PQ. R is a point on the inner circle and S is a point on the outer circle. R, Q and S lie in the same horizontal plane. RS is a pipe used for the irrigation system in the garden.

- The radius of the inner circle is r units and the radius of the outer circle is QS.
- The angle of elevation from S to P is 30°.
- $R\hat{Q}S = 2x$ and $PQ = \sqrt{3}r$



7.1	Show that $QS = 3r$	(3)
7.2	Determine, in terms of r, the area of the flower garden.	(2)
7.3	Show that $RS = r\sqrt{10 - 6\cos 2x}$	(3)
7.4	If $r = 10$ metres and $x = 56°$, calculate RS.	(2)
		[10]

QUESTION 5

5.1 Simplify the following expression to ONE trigonometric term:

$$\frac{\sin x}{\cos x.\tan x}+\sin(180°+x)\cos(90°-x)$$ (5)

5.2 **Without using a calculator**, determine the value of: $\dfrac{\sin^2 35°-\cos^2 35°}{4\sin 10°\cos 10°}$ (4)

5.3 Given: $\cos 26° = m$

Without using a calculator, determine $2\sin^2 77°$ in terms of m. (4)

5.4 Consider: $f(x)=\sin(x+25°)\cos 15°-\cos(x+25°)\sin 15°$

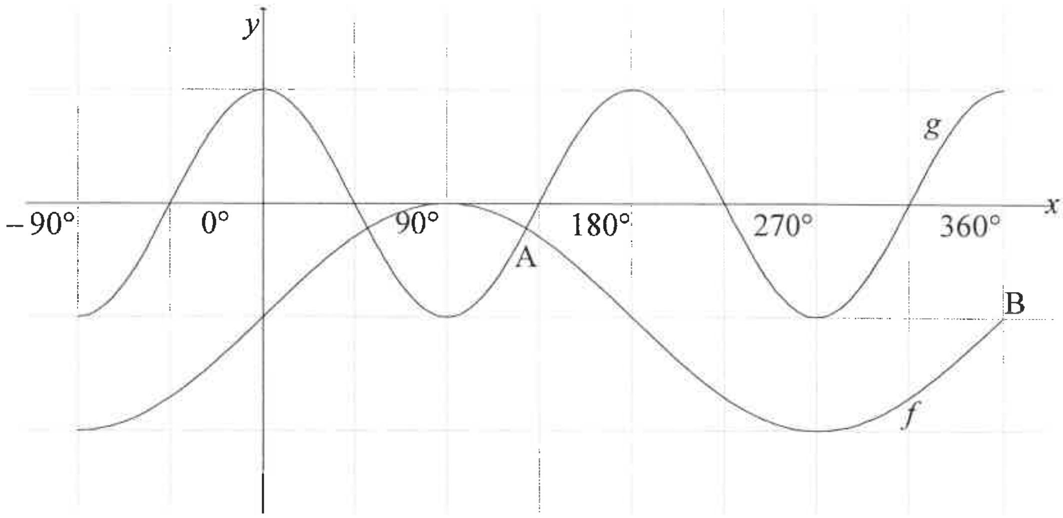
5.4.1 Determine the general solution of $f(x)=\tan 165°$ (6)

5.4.2 Determine the value(s) of x in the interval $x\in[0°;360°]$ for which $f(x)$ will have a minimum value. (3)

[22]

QUESTION 6

In the diagram, the graphs of $f(x) = \sin x - 1$ and $g(x) = \cos 2x$ are drawn for the interval $x \in [-90°; 360°]$. Graphs f and g intersect at A. B(360° ; –1) is a point on f.



6.1 Write down the range of f. (2)

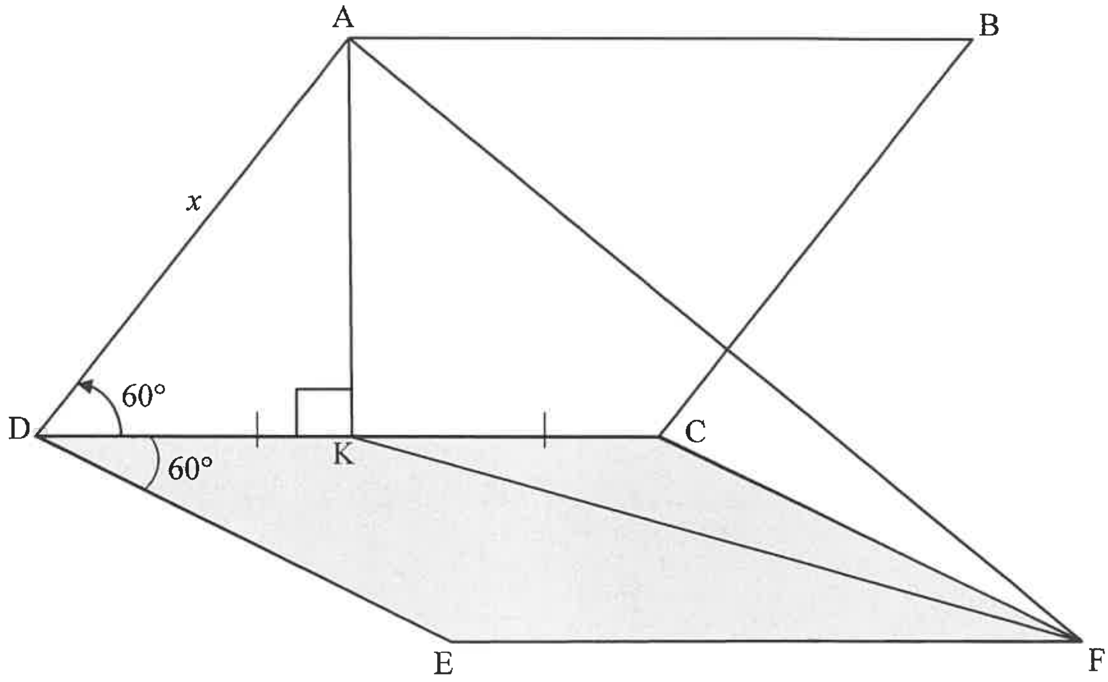
6.2 Write down the values of x in the interval $x \in [-90°; 360°]$ for which graph f is decreasing. (2)

6.3 P and Q are points on graphs g and f respectively such that PQ is parallel to the y-axis. If PQ lies between A and B, determine the value(s) of x for which PQ will be a maximum. (6)

[10]

QUESTION 7

The diagram below shows a solar panel, ABCD, which is fixed to a flat piece of concrete slab EFCD. ABCD and EFCD are two identical rhombuses. K is a point on DC such that DK = KC and AK ⊥ DC. AF and KF are drawn. $A\hat{D}C = C\hat{D}E = 60°$ and AD = x units.



7.1 Determine AK in terms of x. (2)

7.2 Write down the size of $K\hat{C}F$. (1)

7.3 It is further given that $A\hat{K}F$, the angle between the solar panel and the concrete slab, is y. Determine the area of ΔAKF in terms of x and y. (7)

[10]

QUESTION 5

5.1 **Without using a calculator,** write the following expressions in terms of $\sin 11°$:

5.1.1 $\sin 191°$ (1)

5.1.2 $\cos 22°$ (1)

5.2 Simplify $\cos(x - 180°) + \sqrt{2}\sin(x + 45°)$ to a single trigonometric ratio. (5)

5.3 Given: $\sin \text{P} + \sin \text{Q} = \frac{7}{5}$ and $\hat{\text{P}} + \hat{\text{Q}} = 90°$

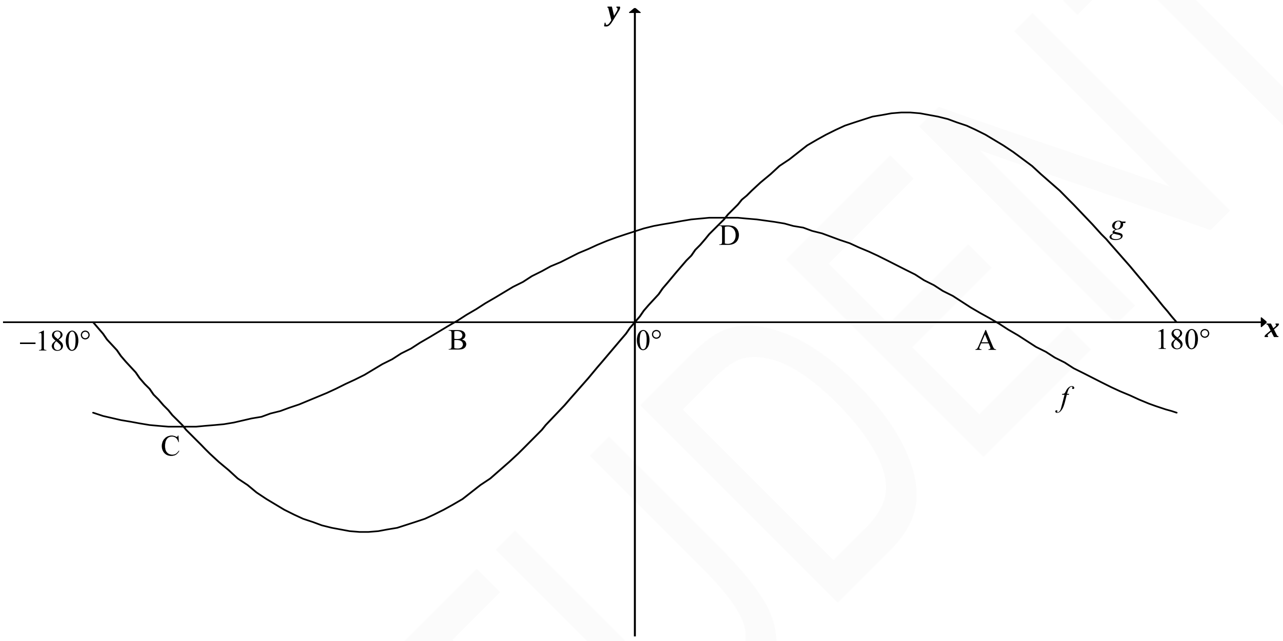
Without using a calculator, determine the value of $\sin 2\text{P}$. (5)

[12]

QUESTION 6

6.1 Determine the general solution of $\cos(x - 30°) = 2\sin x$. (6)

6.2 In the diagram, the graphs of $f(x) = \cos(x - 30°)$ and $g(x) = 2\sin x$ are drawn for the interval $x \in [-180°; 180°]$. A and B are the x–intercepts of f. The two graphs intersect at C and D, the minimum and maximum turning points respectively of f.



6.2.1 Write down the coordinates of:

(a) A (1)

(b) C (2)

6.2.2 Determine the values of x in the interval $x \in [-180°; 180°]$, for which:

(a) Both graphs are increasing (2)

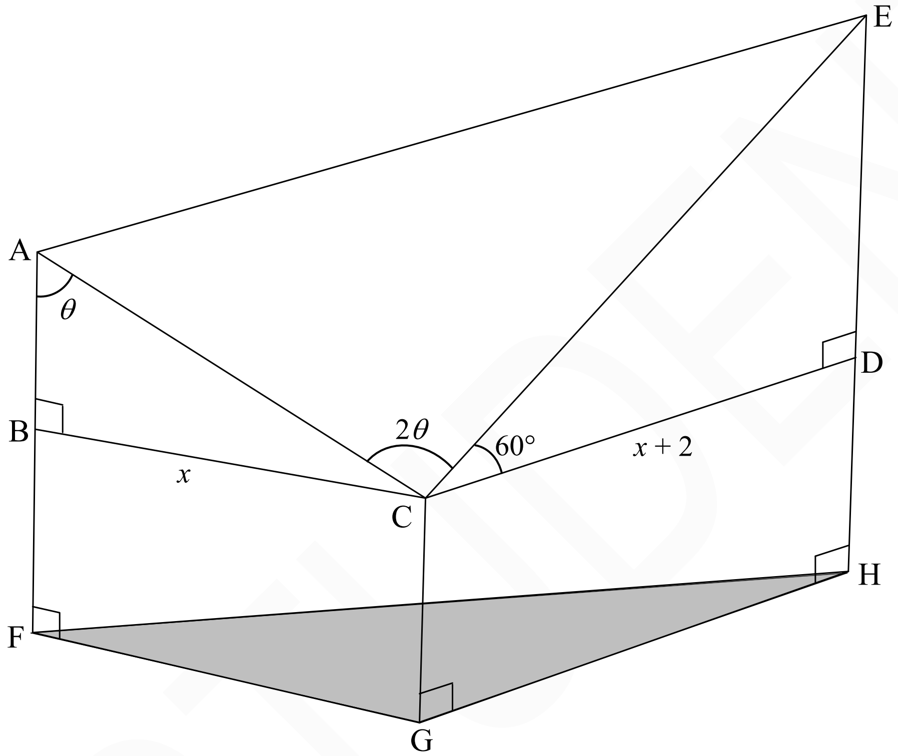
(b) $f(x + 10°) > g(x + 10°)$ (2)

6.2.3 Determine the range of $y = 2^{2\sin x + 3}$ (5)

[18]

QUESTION 7

In the diagram below, CGFB and CGHD are fixed walls that are rectangular in shape and vertical to the horizontal plane FGH. Steel poles erected along FB and HD extend to A and E respectively. ΔACE forms the roof of an entertainment centre.
$BC = x$, $CD = x + 2$, $B\hat{A}C = \theta$, $A\hat{C}E = 2\theta$ and $E\hat{C}D = 60°$



7.1 Calculate the length of:

7.1.1 AC in terms of x and θ (2)

7.1.2 CE in terms of x (2)

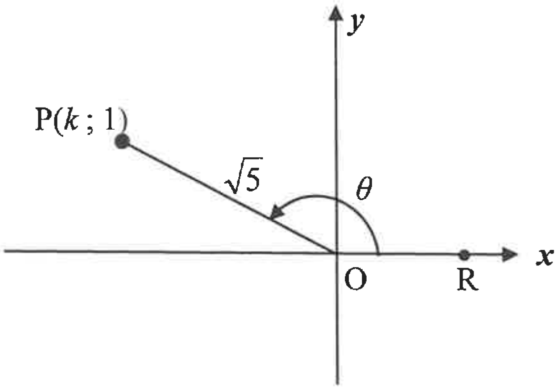
7.2 Show that the area of the roof ΔACE is given by $2x(x+2)\cos\theta$. (3)

7.3 If $\theta = 55°$ and BC = 12 metres, calculate the length of AE. (4)

[11]

QUESTION 5

5.1 In the diagram, P(k ; 1) is a point in the 2[nd] quadrant and is $\sqrt{5}$ units from the origin. R is a point on the positive x-axis and obtuse $R\hat{O}P = \theta$.



5.1.1 Calculate the value of k. (2)

5.1.2 **Without using a calculator**, calculate the value of:

(a) $\tan\theta$ (1)

(b) $\cos(180° + \theta)$ (2)

(c) $\sin(\theta + 60°)$ in the form $\dfrac{a+b}{\sqrt{20}}$ (5)

5.1.3 **Use a calculator** to calculate the value of $\tan(2\theta - 40°)$ correct to ONE decimal place. (3)

5.2 Prove the following identity: $\dfrac{\cos x + \sin x}{\cos x - \sin x} - \dfrac{\cos x - \sin x}{\cos x + \sin x} = 2\tan 2x$ (5)

5.3 Evaluate, **without using a calculator**: $\sum_{A=38°}^{52°} \cos^2 A$ (5)

[23]

QUESTION 6

Consider: $f(x) = -2\tan\frac{3}{2}x$

6.1 Write down the period of f. (1)

6.2 The point $A(t\,;2)$ lies on the graph. Determine the general solution of t. (3)

6.3 On the grid provided in the ANSWER BOOK, draw the graph of f for the interval $x \in [-120°\,;180°]$. Clearly show ALL asymptotes, intercepts with the axes and endpoint(s) of the graph. (4)

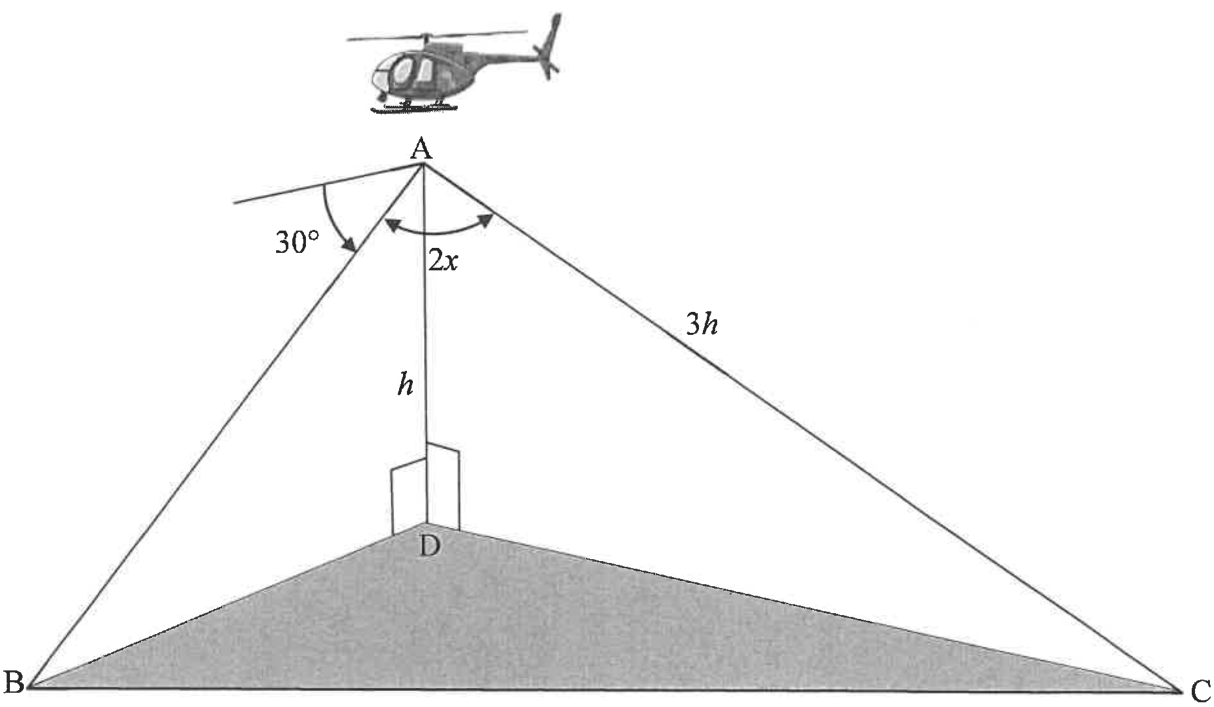
6.4 Use the graph to determine for which value(s) of x will $f(x) \geq 2$ for $x \in [-120°\,;180°]$. (3)

6.5 Describe the transformation of graph f to form the graph of $g(x) = -2\tan\left(\frac{3}{2}x + 60°\right)$. (2)

[13]

QUESTION 7

A pilot is flying in a helicopter. At point A, which is h metres directly above point D on the ground, he notices a strange object at point B. The pilot determines that the angle of depression from A to B is 30°. He also determines that the control room at point C is $3h$ metres from A and $B\hat{A}C = 2x$. Points B, C and D are in the same horizontal plane. This scenario is shown in the diagram below.



7.1 Determine the distance AB in terms of h. (2)

7.2 Show that the distance between the strange object at point B and the control room at point C is given by $BC = h\sqrt{25 - 24\cos^2 x}$. (4)

[6]

QUESTION 5

5.1 If $\cos 2\theta = -\frac{5}{6}$, where $2\theta \in [180°; 270°]$, calculate, **without using a calculator**, the values in simplest form of:

5.1.1 $\sin 2\theta$ (4)

5.1.2 $\sin^2 \theta$ (3)

5.2 Simplify $\sin(180° - x).\cos(-x) + \cos(90° + x).\cos(x - 180°)$ to a single trigonometric ratio. (6)

5.3 Determine the value of $\sin 3x.\cos y + \cos 3x.\sin y$ if $3x + y = 270°$. (2)

5.4 Given: $2\cos x = 3\tan x$

5.4.1 Show that the equation can be rewritten as $2\sin^2 x + 3\sin x - 2 = 0$. (3)

5.4.2 Determine the general solution of x if $2\cos x = 3\tan x$. (5)

5.4.3 Hence, determine two values of y, $144° \leq y \leq 216°$, that are solutions of $2\cos 5y = 3\tan 5y$. (4)

5.5 Consider: $g(x) = -4\cos(x + 30°)$

5.5.1 Write down the maximum value of $g(x)$. (1)

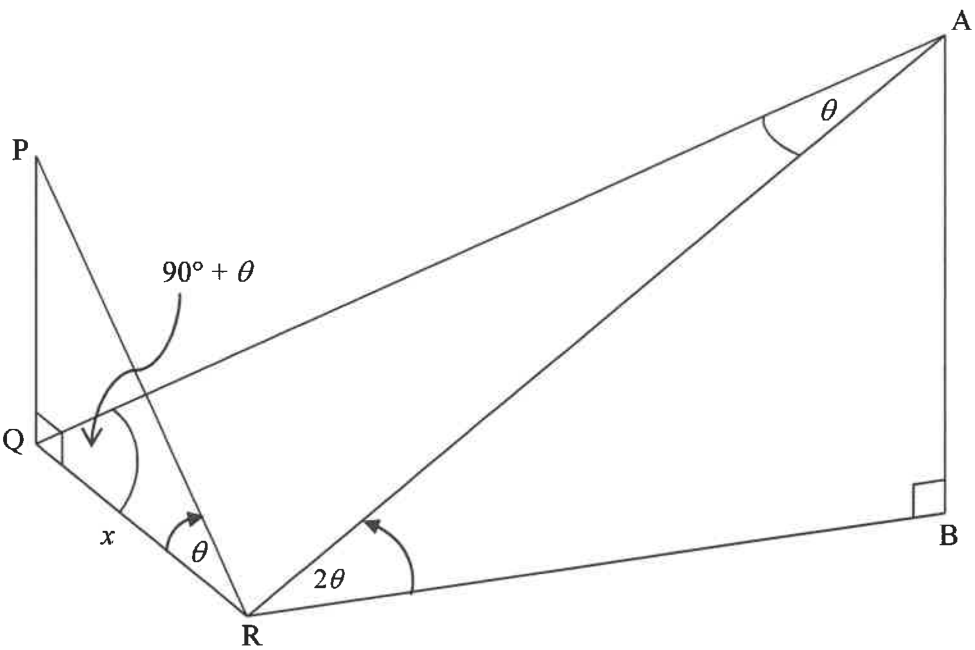
5.5.2 Determine the range of $g(x) + 1$. (2)

5.5.3 The graph of g is shifted $60°$ to the left and then reflected about the x-axis to form a new graph h. Determine the equation of h in its simplest form. (3)

[33]

QUESTION 6

PQ and AB are two vertical towers.
From a point R in the same horizontal plane as Q and B, the angles of elevation to P and A are θ and 2θ respectively.
$A\hat{Q}R = 90° + \theta$, $Q\hat{A}R = \theta$ and $QR = x$.



6.1 Determine in terms of x and θ:

6.1.1 QP (2)

6.1.2 AR (2)

6.2 Show that $AB = 2x\cos^2\theta$ (4)

6.3 Determine $\frac{AB}{QP}$ if $\theta = 12°$. (2)

[10]

QUESTION 5

5.1 In ΔMNP, $\hat{N} = 90°$ and $\sin M = \frac{15}{17}$.

Determine, **without using a calculator**:

5.1.1 $\tan M$ (3)

5.1.2 The length of NP if MP = 51 (2)

5.2 Simplify to a single term: $\cos(x - 360°).\sin(90° + x) + \cos^2(-x) - 1$ (4)

5.3 Consider: $\sin(2x + 40°)\cos(x + 30°) - \cos(2x + 40°)\sin(x + 30°)$

5.3.1 Write as a single trigonometric term in its simplest form. (2)

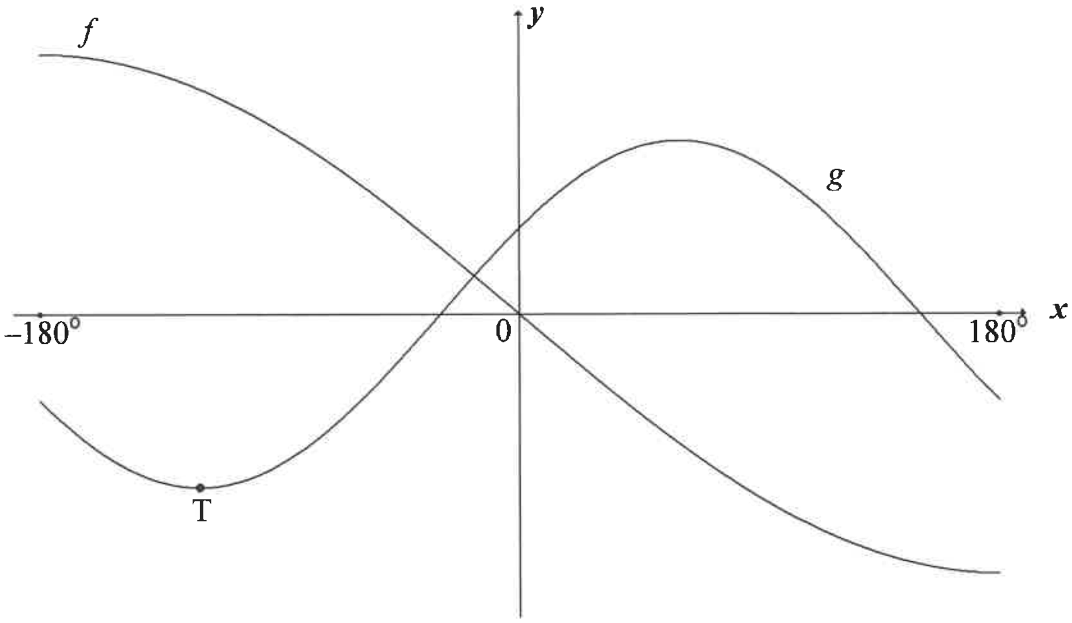
5.3.2 Determine the general solution of the following equation:

$$\sin(2x + 40°)\cos(x + 30°) - \cos(2x + 40°)\sin(x + 30°) = \cos(2x - 20°)$$ (7)

[18]

QUESTION 6

In the diagram, the graphs of $f(x) = -3\sin\frac{x}{2}$ and $g(x) = 2\cos(x - 60°)$ are drawn in the interval $x \in [-180°;180°]$. T(p ; q) is a turning point of g with $p < 0$.



6.1	Write down the period of f.	(1)
6.2	Write down the range of g.	(2)
6.3	Calculate $f(p) - g(p)$.	(3)

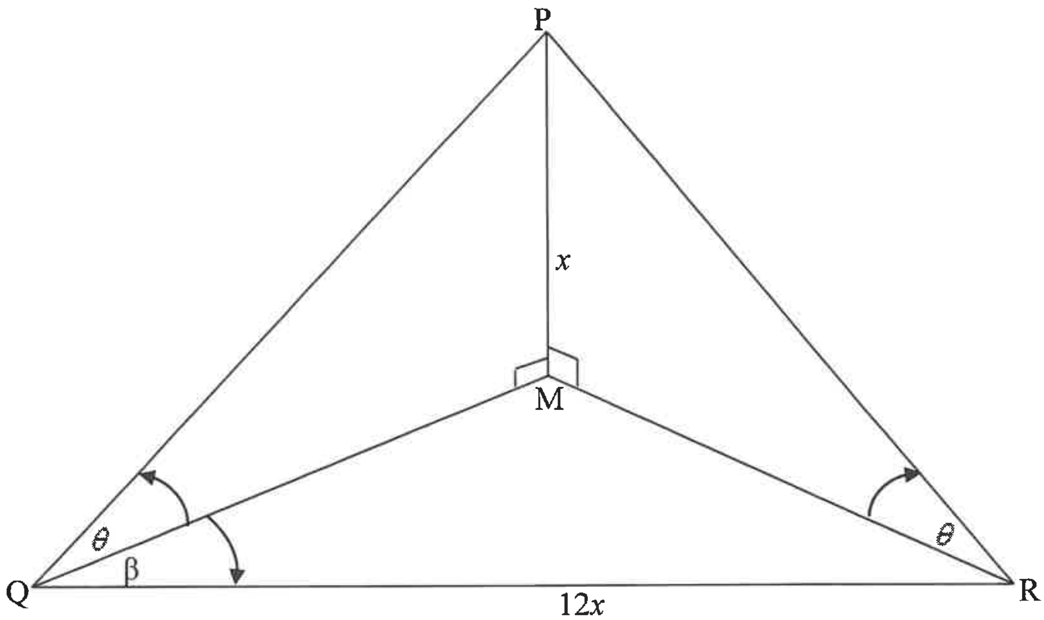
6.4 Use the graphs to determine the value(s) of x in the interval $x \in [-180°;180°]$ for which:

6.4.1	$g(x) > 0$	(3)
6.4.2	$g(x).g'(x) > 0$	(4)

[13]

QUESTION 7

The captain of a boat at sea, at point Q, notices a lighthouse PM directly north of his position. He determines that the angle of elevation of P, the top of the lighthouse, from Q is θ and the height of the lighthouse is x metres. From point Q the captain sails $12x$ metres in a direction β degrees east of north to point R. From point R, he notices that the angle of elevation of P is also θ. Q, M and R lie in the same horizontal plane.



7.1 Write QM in terms of x and θ. (2)

7.2 Prove that $\tan\theta = \dfrac{\cos\beta}{6}$. (4)

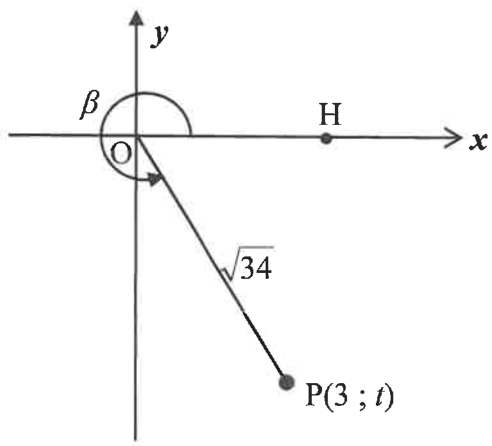
7.3 If $\beta = 40°$ and QM = 60 metres, calculate the height of the lighthouse **to the nearest metre**. (3)
[9]

QUESTION 5

5.1 Given: $\dfrac{\sin(A-360°).\cos(90°+A)}{\cos(90°-A).\tan(-A)}$

Simplify the expression to a single trigonometric ratio. (6)

5.2 In the diagram, P(3 ; t) is a point in the Cartesian plane. OP $=\sqrt{34}$ and $\hat{HOP}=\beta$ is a reflex angle.



Without using a calculator, determine the value of:

5.2.1 t (2)

5.2.2 $\tan\beta$ (1)

5.2.3 $\cos 2\beta$ (4)

5.3 Prove:

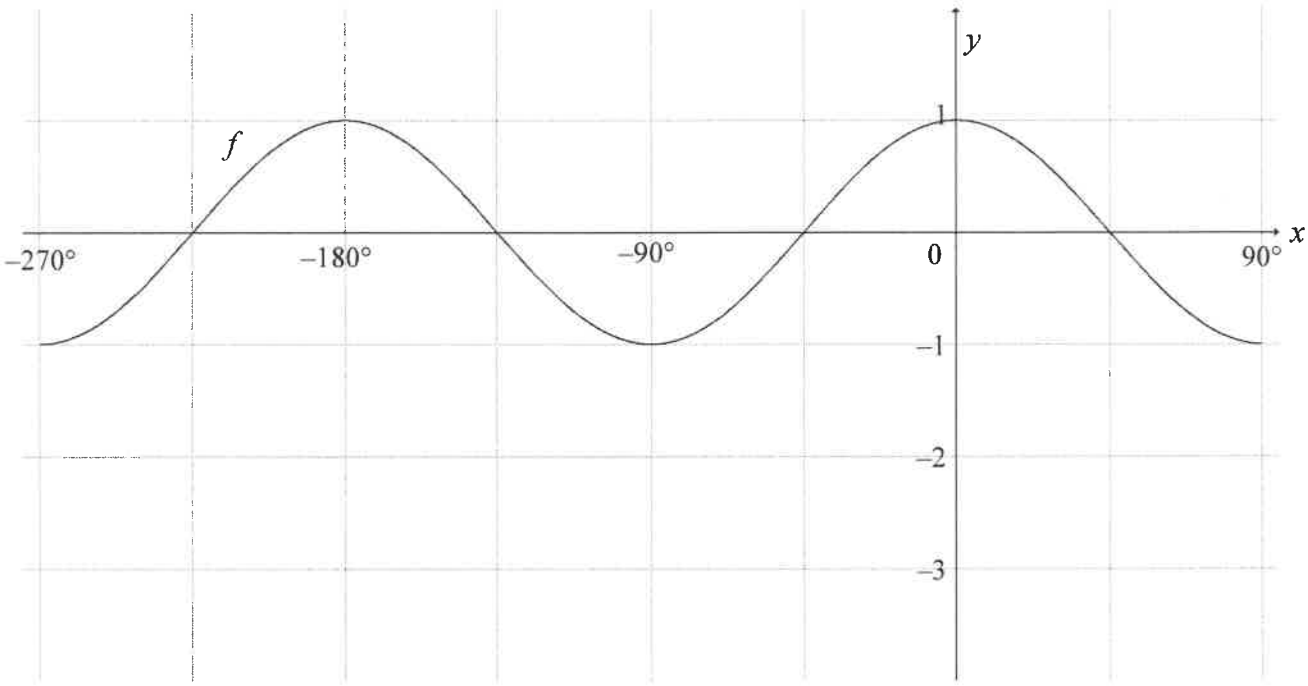
5.3.1 $\sin(A+B)-\sin(A-B)=2\cos A.\sin B$ (2)

5.3.2 **Without using a calculator,** that $\sin 77°-\sin 43°=\sin 17°$ (4)

[19]

QUESTION 6

In the diagram, the graph of $f(x) = \cos 2x$ is drawn for the interval $x \in [-270°; 90°]$.



6.1 Draw the graph of $g(x) = 2\sin x - 1$ for the interval $x \in [-270°; 90°]$ on the grid given in your ANSWER BOOK. Show ALL the intercepts with the axes, as well as the turning points. (4)

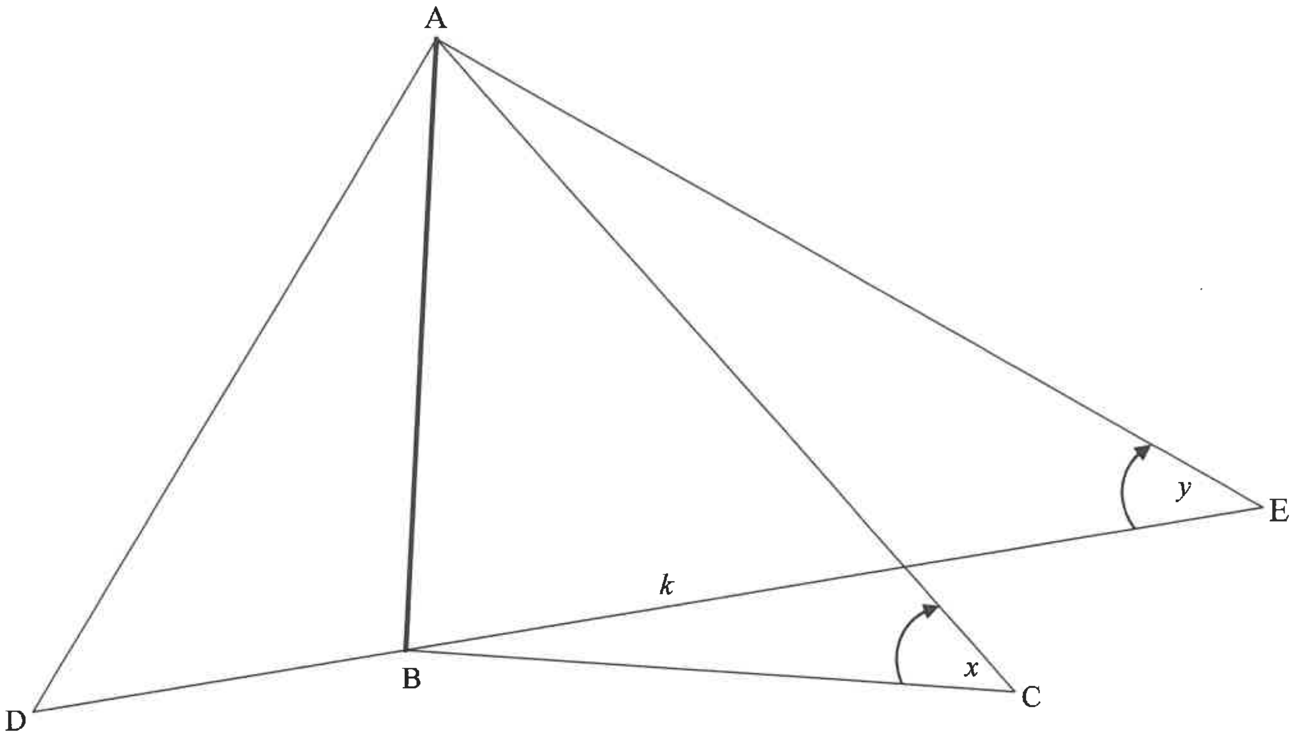
6.2 Let A be a point of intersection of the graphs of f and g. Show that the x-coordinate of A satisfies the equation $\sin x = \frac{-1+\sqrt{5}}{2}$. (4)

6.3 Hence, calculate the coordinates of the points of intersection of graphs of f and g for the interval $x \in [-270°; 90°]$. (4)

[12]

QUESTION 7

AB represents a vertical netball pole. Two players are positioned on either side of the netball pole at points D and E such that D, B and E are on the same straight line. A third player is positioned at C. The points B, C, D and E are in the same horizontal plane. The angles of elevation from C to A and from E to A are x and y respectively. The distance from B to E is k.



7.1 Write down the size of $\hat{ABC}$. (1)

7.2 Show that $AC = \dfrac{k.\tan y}{\sin x}$ (4)

7.3 If it is further given that $D\hat{A}C = 2x$ and AD = AC, show that the distance DC between the players at D and C is $2k \tan y$. (5)

[10]

QUESTION 5

5.1 Given: $\sin A = 2p$ and $\cos A = p$

5.1.1 Determine the value of $\tan A$. (2)

5.1.2 **Without using a calculator,** determine the value of p, if $A \in [180° ; 270°]$. (3)

5.2 Determine the general solution of $2\sin^2 x - 5\sin x + 2 = 0$ (6)

5.3 5.3.1 Expand $\sin(x + 300°)$ using an appropriate compound angle formula. (1)

5.3.2 **Without using a calculator,** determine the value of $\sin(x + 300°) - \cos(x - 150°)$. (5)

5.4 Prove the identity: $\dfrac{\tan x + 1}{\sin x \tan x + \cos x} = \sin x + \cos x$. (5)

5.5 Consider: $\sin x + \cos x = \sqrt{1 + k}$

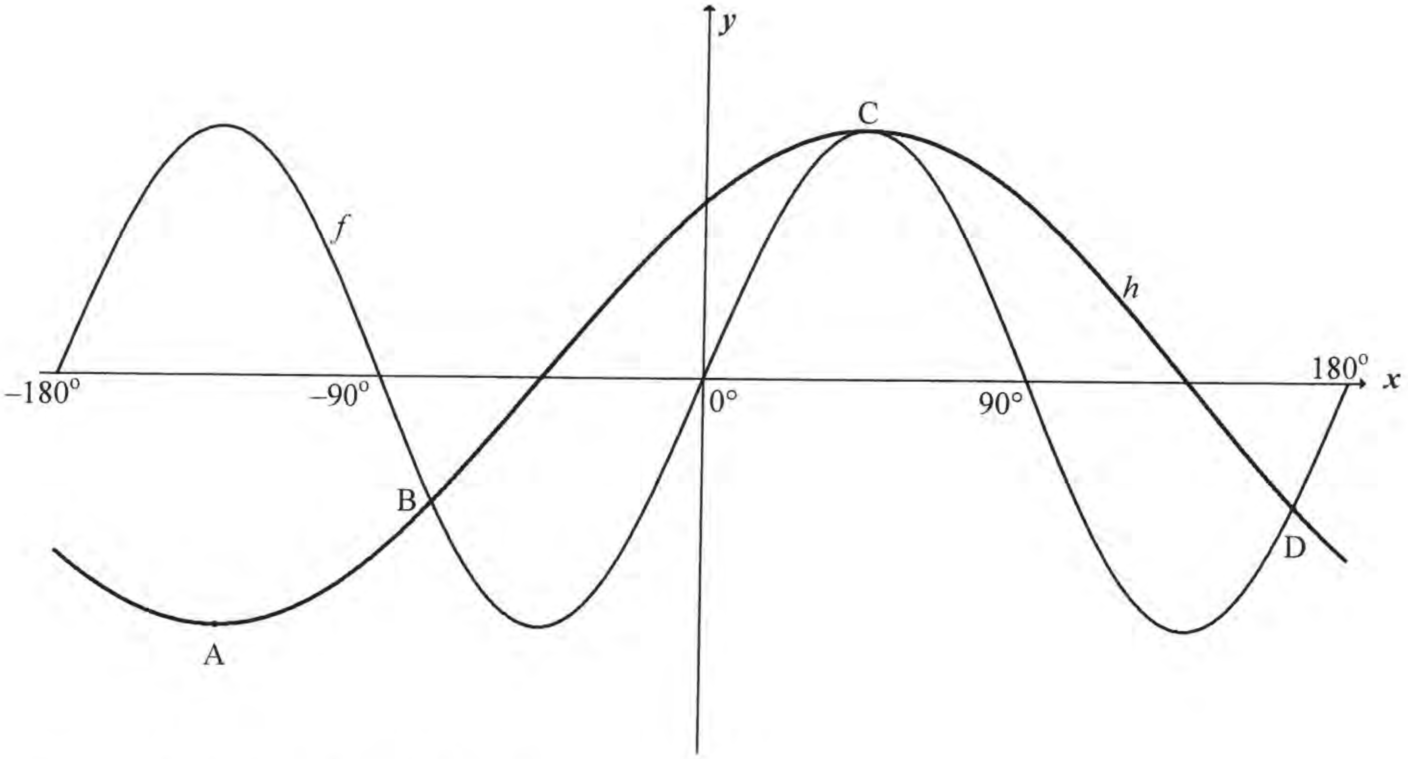
5.5.1 Determine k as a single trigonometric ratio. (3)

5.5.2 Hence, determine the maximum value of $\sin x + \cos x$. (2)

[27]

QUESTION 6

In the diagram are the graphs of $f(x) = \sin 2x$ and $h(x) = \cos(x - 45°)$ for the interval $x \in [-180° ; 180°]$. $A(-135° ; -1)$ is a minimum point on graph h and $C(45° ; 1)$ is a maximum point on both graphs. The two graphs intersect at B, C and $D\left(165°; -\frac{1}{2}\right)$.



6.1 Write down the period of *f*. (1)

6.2 Determine the *x*-coordinate of B. (1)

6.3 Use the graphs to solve $2\sin x.\cos x \leq \frac{1}{\sqrt{2}}(\cos x + \sin x)$ for the interval $x \in [-180° ; 180°]$. Show ALL working. (4)

[6]

QUESTION 7

A rectangular box with lid ABCD is given in FIGURE (i) below. The lid is opened through $60°$ to position HKCD, as shown in the FIGURE (ii) below. EF = 12 cm, FG = 6 cm and BG = 8 cm.

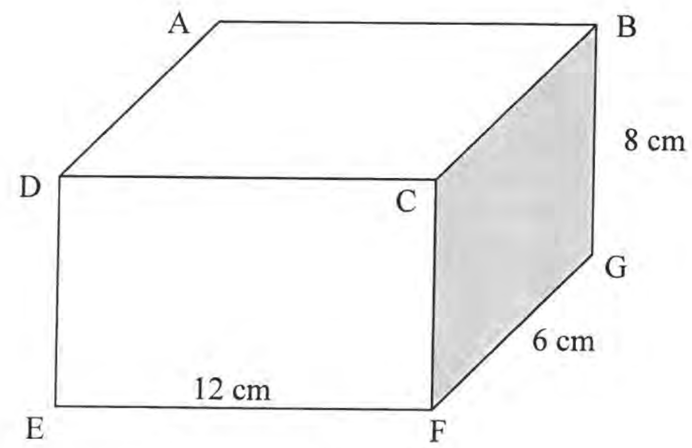


FIGURE (I)

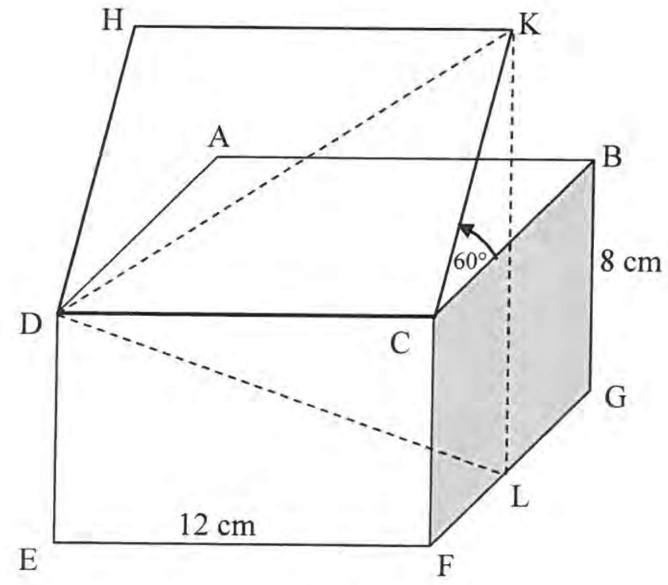


FIGURE (II)

7.1 Write down the length of KC. (1)

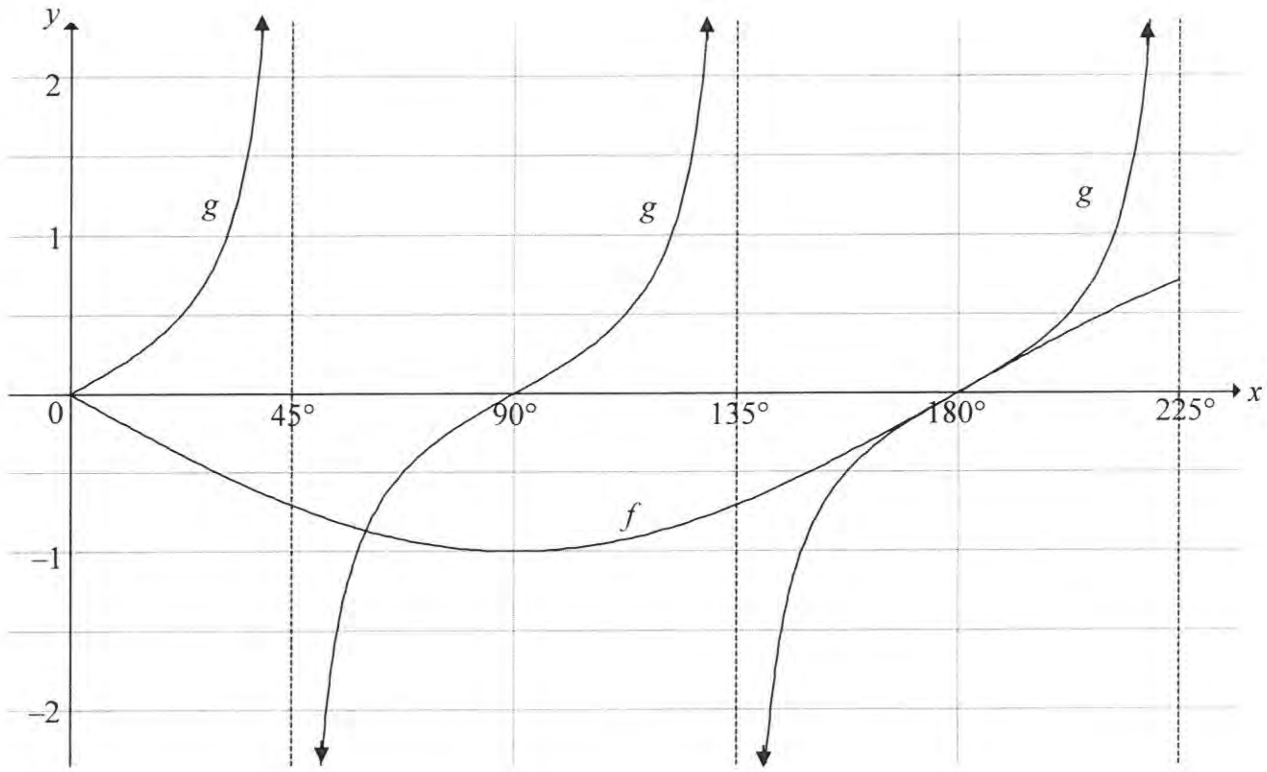
7.2 Determine KL, the perpendicular height of K, above the base of the box. (3)

7.3 Hence, determine the value of $\dfrac{\sin K\hat{D}L}{\sin D\hat{L}K}$. (4)

[8]

QUESTION 5

In the diagram, the graphs of the functions $f(x) = a\sin x$ and $g(x) = \tan bx$ are drawn on the same system of axes for the interval $0° \leq x \leq 225°$.



5.1 Write down the values of a and b. (2)

5.2 Write down the period of $f(3x)$. (2)

5.3 Determine the values of x in the interval $90° \leq x \leq 225°$ for which $f(x).g(x) \leq 0$. (3)

[7]

QUESTION 6

6.1 **Without using a calculator**, determine the following in terms of $\sin 36°$:

6.1.1 $\sin 324°$ (1)

6.1.2 $\cos 72°$ (2)

6.2 Prove the identity: $1-\dfrac{\tan^2\theta}{1+\tan^2\theta}=\cos^2\theta$ (4)

6.3 Use QUESTION 6.2 to determine the general solution of:

$$1-\frac{\tan^2\frac{1}{2}x}{1+\tan^2\frac{1}{2}x}=\frac{1}{4}$$

(6)

6.4 Given: $\cos(A-B)=\cos A\cos B+\sin A\sin B$

6.4.1 Use the formula for $\cos(A-B)$ to derive a formula for $\sin(A-B)$. (4)

6.4.2 **Without using a calculator**, show that

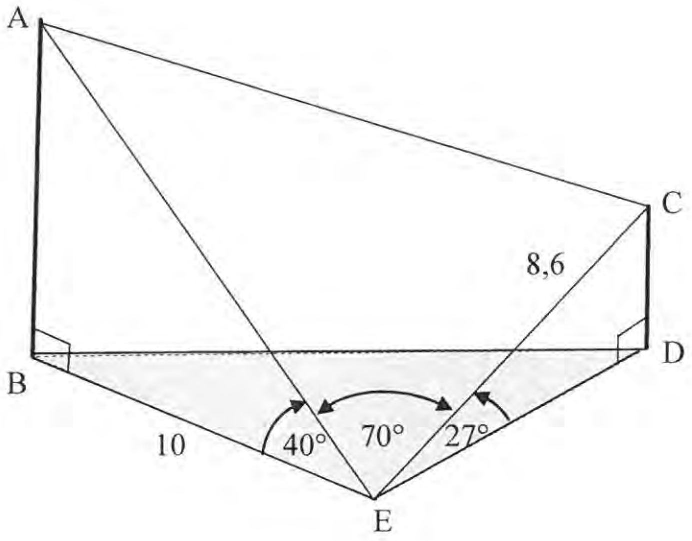
$$\sin(x+64°)\cos(x+379°)+\sin(x+19°)\cos(x+244°)=\frac{1}{\sqrt{2}}$$

for all values of x. (6)

[23]

QUESTION 7

In the diagram, B, E and D are points in the same horizontal plane. AB and CD are vertical poles. Steel cables AE and CE anchor the poles at E. Another steel cable connects A and C. CE = 8,6 m, BE = 10 m, $A\hat{E}B = 40°$, $A\hat{E}C = 70°$ and $C\hat{E}D = 27°$.



Calculate the:

7.1 Height of pole CD (2)

7.2 Length of cable AE (2)

7.3 Length of cable AC (4)
[8]

QUESTION 5

5.1 Given: $\sin 16° = p$
Determine the following in terms of p, **without using a calculator**.

5.1.1 $\sin 196°$ (2)

5.1.2 $\cos 16°$ (2)

5.2 Given: $\cos(A - B) = \cos A \cos B + \sin A \sin B$

Use the formula for $\cos(A - B)$ to derive a formula for $\sin(A + B)$ (3)

5.3 Simplify $\dfrac{\sqrt{1 - \cos^2 2A}}{\cos(-A).\cos(90° + A)}$ completely, given that $0° < A < 90°$. (5)

5.4 Given: $\cos 2B = \dfrac{3}{5}$ and $0° \leq B \leq 90°$

Determine, **without using a calculator**, the value of EACH of the following in its simplest form:

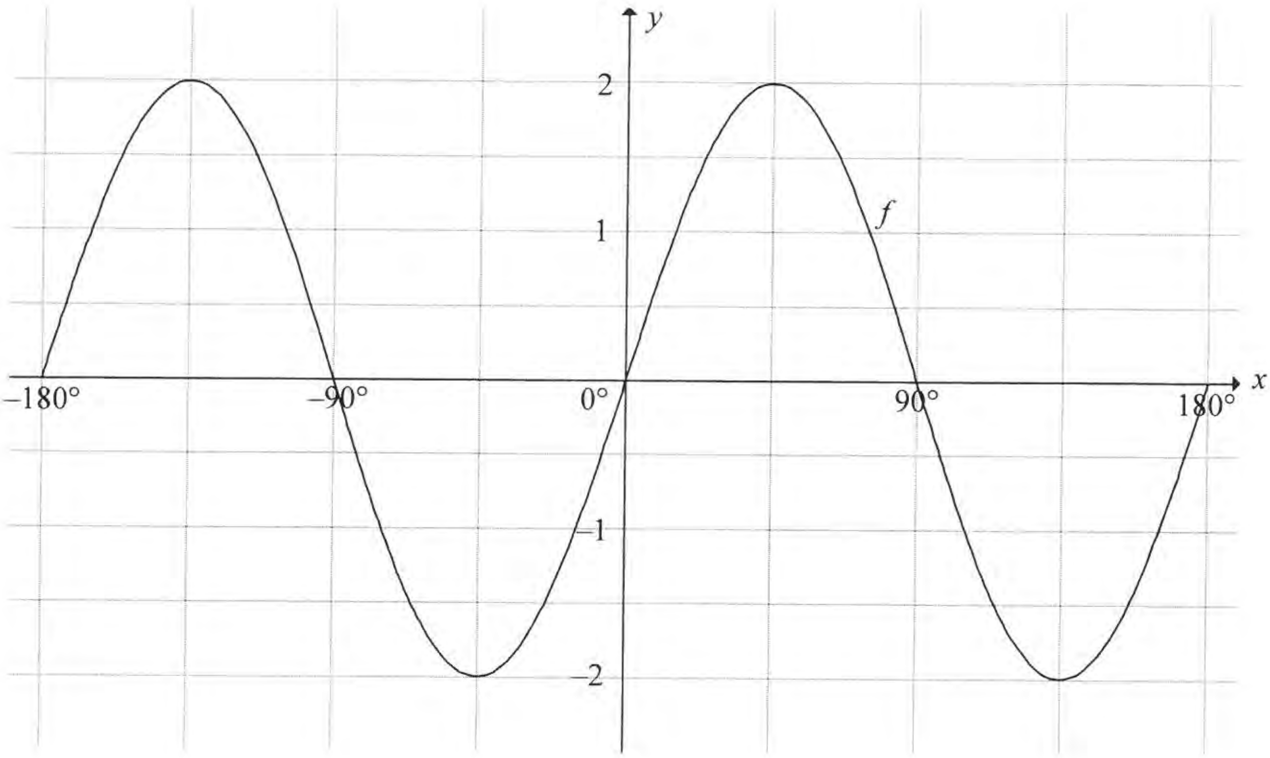
5.4.1 $\cos B$ (3)

5.4.2 $\sin B$ (2)

5.4.3 $\cos (B + 45°)$ (4)
[21]

QUESTION 6

In the diagram the graph of $f(x) = 2\sin 2x$ is drawn for the interval $x \in [-180° ; 180°]$.



6.1 On the system of axes on which f is drawn in the ANSWER BOOK, draw the graph of $g(x) = -\cos 2x$ for $x \in [-180° ; 180°]$. Clearly show all intercepts with the axes, the coordinates of the turning points and end points of the graph. (3)

6.2 Write down the maximum value of $f(x) - 3$. (2)

6.3 Determine the general solution of $f(x) = g(x)$. (4)

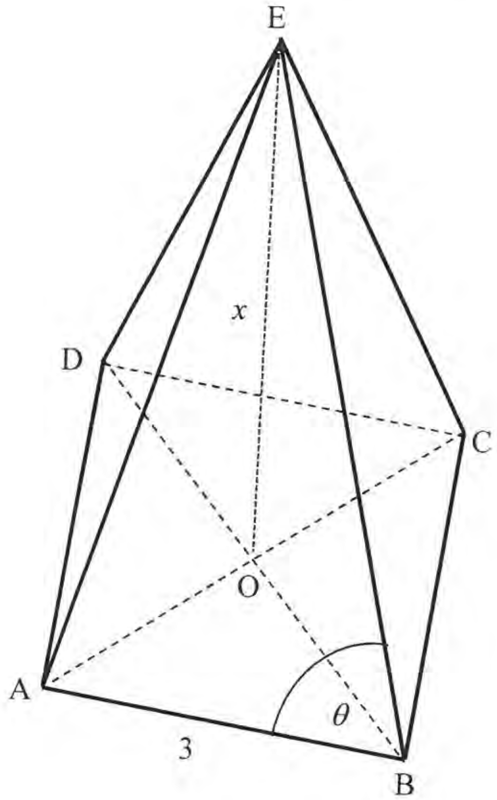
6.4 Hence, determine the values of x for which $f(x) < g(x)$ in the interval $x \in [-180° ; 0°]$. (3)

[12]

QUESTION 7

E is the apex of a pyramid having a square base ABCD. O is the centre of the base. $E\hat{B}A = \theta$, AB = 3 m and EO, the perpendicular height of the pyramid, is x.

Volume of pyramid $= \frac{1}{3}(\text{area of base}) \times (\perp \text{ height})$



7.1 Calculate the length of OB. (3)

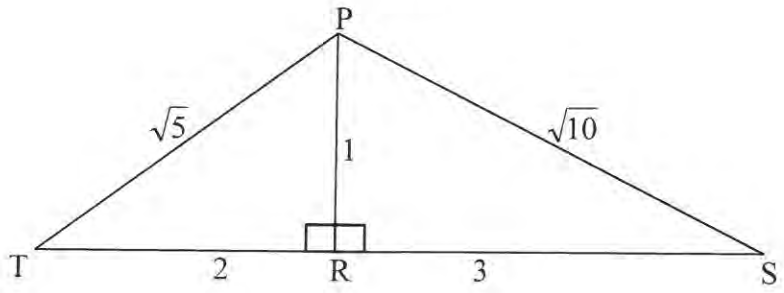
7.2 Show that $\cos\theta = \dfrac{3}{2\sqrt{x^2 + \frac{9}{2}}}$ (5)

7.3 If the volume of the pyramid is 15 m^3, calculate the value of θ. (4)

[12]

QUESTION 5

5.1 In the diagram PR ⊥ TS in obtuse triangle PTS.
PT = $\sqrt{5}$; TR = 2; PR = 1; PS = $\sqrt{10}$ and RS = 3



5.1.1 Write down the value of:

(a) $\sin \hat{T}$ (1)

(b) $\cos \hat{S}$ (1)

5.1.2 Calculate, WITHOUT using a calculator, the value of $\cos(\hat{T} + \hat{S})$ (5)

5.2 Determine the value of:

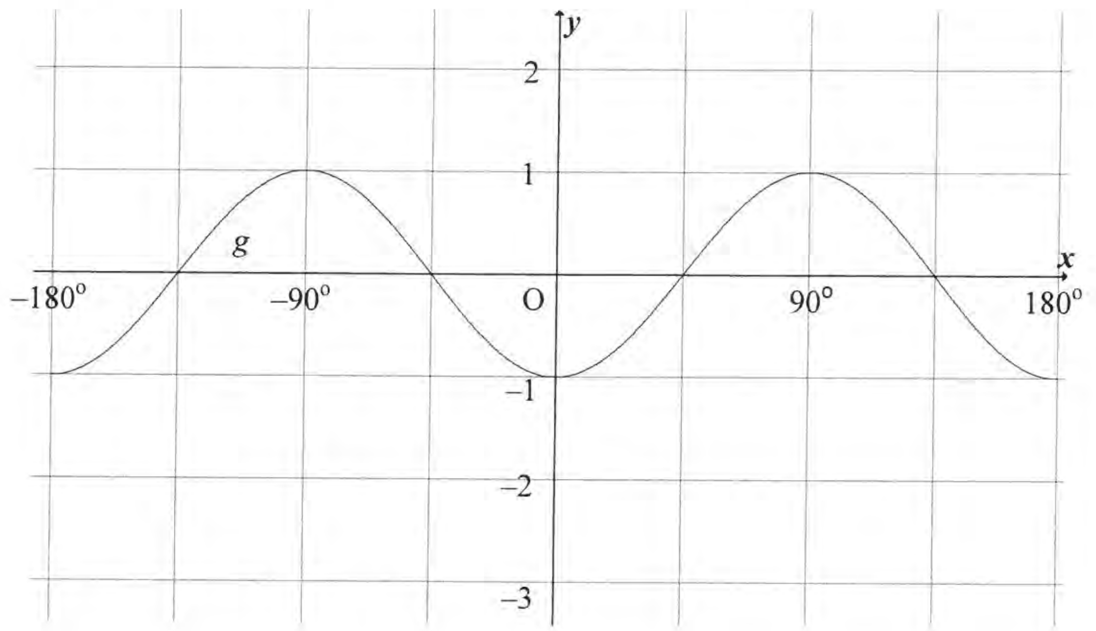
$$\frac{1}{\cos(360° - \theta).\sin(90° - \theta)} - \tan^2(180° + \theta)$$ (6)

5.3 If $\sin x - \cos x = \frac{3}{4}$, calculate the value of $\sin 2x$ WITHOUT using a calculator. (5)
[18]

QUESTION 6

6.1 Determine the general solution of $4\sin x + 2\cos 2x = 2$ (6)

6.2 The graph of $g(x) = -\cos 2x$ for $x \in [-180°\ ;\ 180°]$ is drawn below.



6.2.1 Draw the graph of $f(x) = 2\sin x - 1$ for $x \in [-180°\ ;\ 180°]$ on the set of axes provided in the ANSWER BOOK. (3)

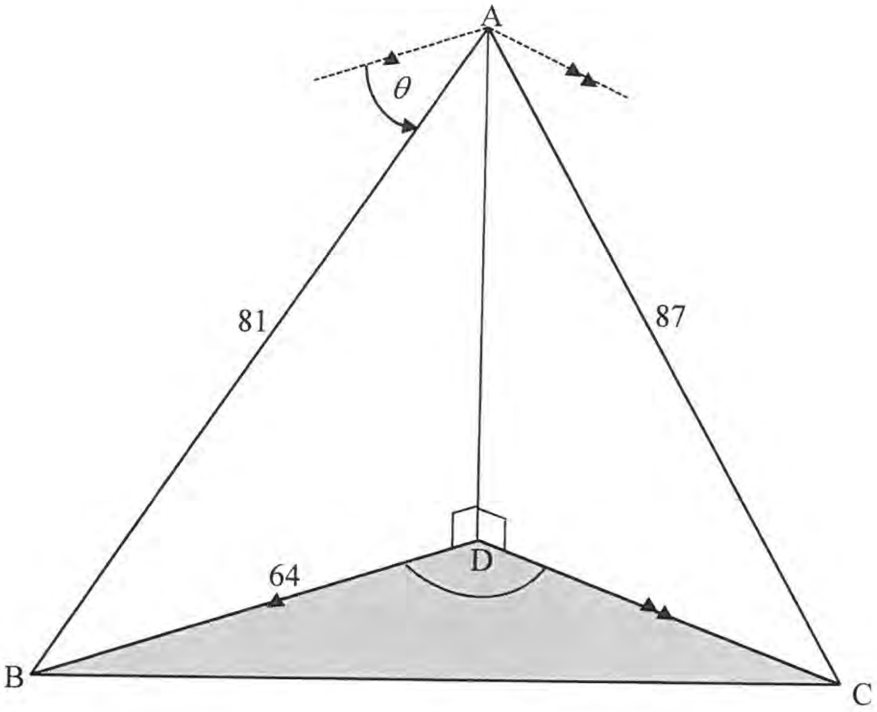
6.2.2 Write down the values of x for which g is strictly decreasing in the interval $x \in [-180°\ ;\ 0°]$ (2)

6.2.3 Write down the value(s) of x for which $f(x + 30°) - g(x + 30°) = 0$ for $x \in [-180°\ ;\ 180°]$ (2)

[13]

QUESTION 7

From point A an observer spots two boats, B and C, at anchor. The angle of depression of boat B from A is θ. D is a point directly below A and is on the same horizontal plane as B and C. BD = 64 m, AB = 81 m and AC = 87 m.



7.1 Calculate the size of θ to the nearest degree. (3)

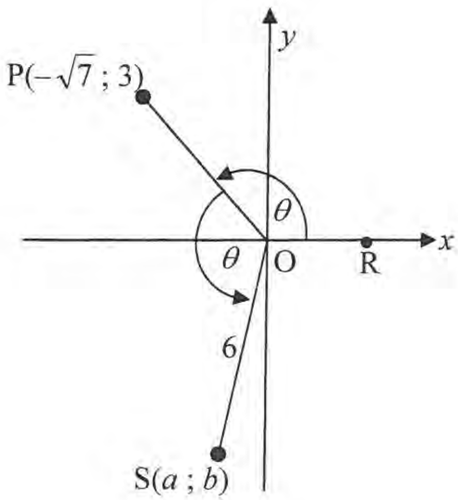
7.2 If it is given that $B\hat{A}C = 82,6°$, calculate BC, the distance between the boats. (3)

7.3 If $B\hat{D}C = 110°$, calculate the size of $D\hat{C}B$. (3)

[9]

QUESTION 5

5.1 $P(-\sqrt{7}\,;3)$ and $S(a\,;b)$ are points on the Cartesian plane, as shown in the diagram below. $P\hat{O}R = P\hat{O}S = \theta$ and $OS = 6$.



Determine, WITHOUT using a calculator, the value of:

5.1.1 $\tan\theta$ (1)

5.1.2 $\sin(-\theta)$ (3)

5.1.3 a (4)

5.2 5.2.1 Simplify $\dfrac{4\sin x\cos x}{2\sin^2 x-1}$ to a single trigonometric ratio. (3)

5.2.2 Hence, calculate the value of $\dfrac{4\sin 15^\circ\cos 15^\circ}{2\sin^2 15^\circ-1}$ WITHOUT using a calculator. (Leave your answer in simplest surd form.) (2)

[13]

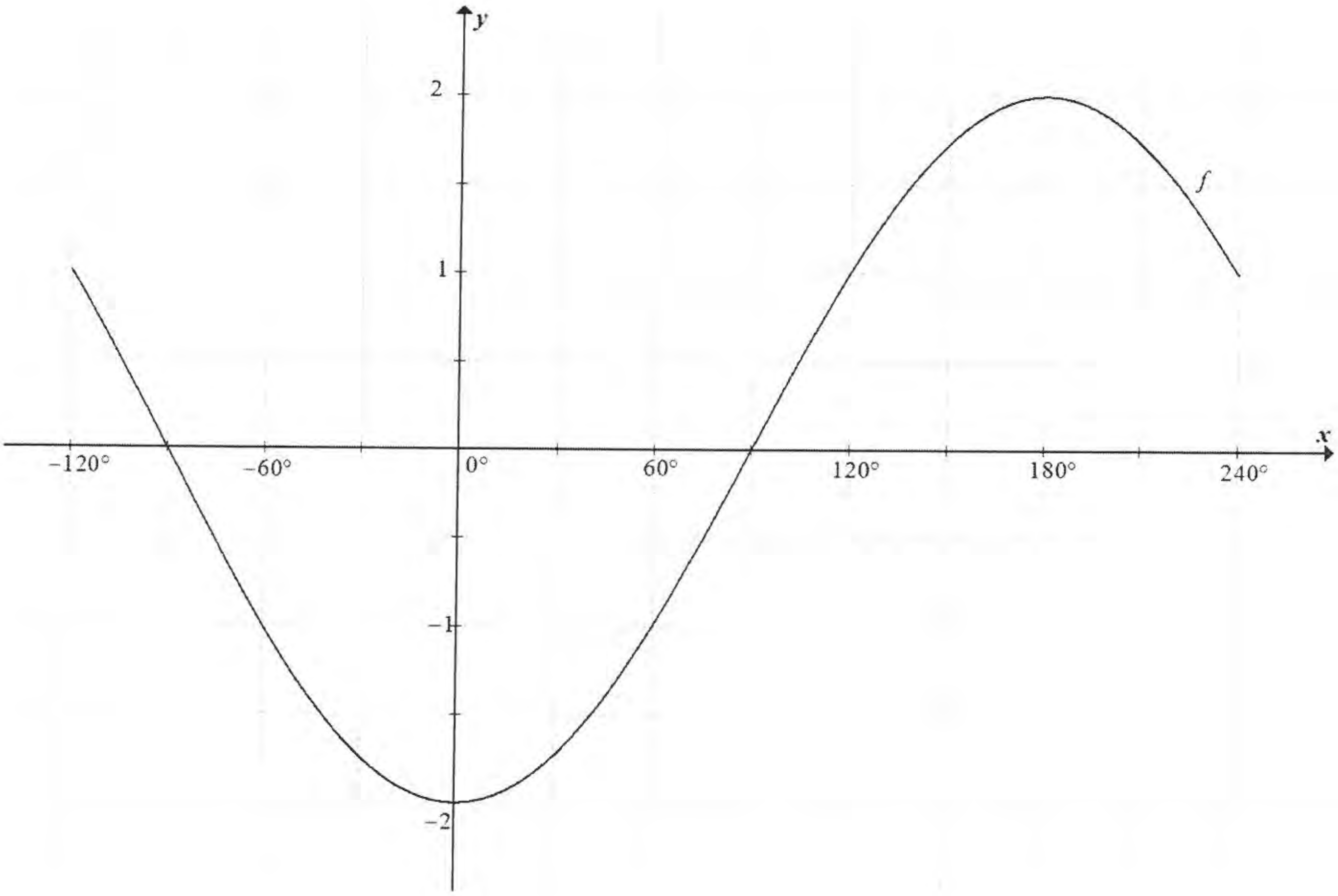
QUESTION 6

Given the equation: $\sin(x + 60°) + 2\cos x = 0$

6.1 Show that the equation can be rewritten as $\tan x = -4 - \sqrt{3}$. (4)

6.2 Determine the solutions of the equation $\sin(x + 60°) + 2\cos x = 0$ in the interval $-180° \leq x \leq 180°$. (3)

6.3 In the diagram below, the graph of $f(x) = -2\cos x$ is drawn for $-120° \leq x \leq 240°$.



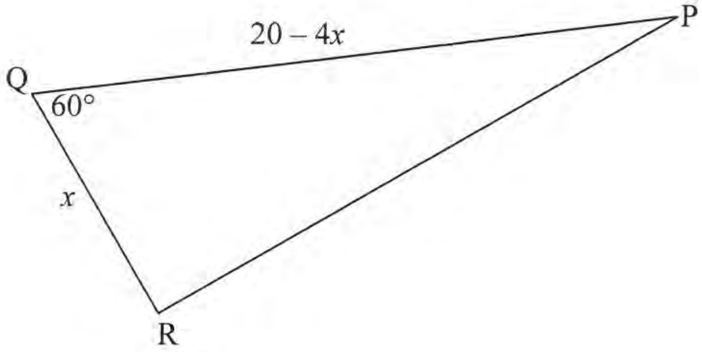
6.3.1 Draw the graph of $g(x) = \sin(x + 60°)$ for $-120° \leq x \leq 240°$ on the grid provided in the ANSWER BOOK. (3)

6.3.2 Determine the values of x in the interval $-120° \leq x \leq 240°$ for which $\sin(x + 60°) + 2\cos x > 0$. (3)

[13]

QUESTION 7

7.1 In the diagram below, ΔPQR is drawn with $PQ = 20 - 4x$, $RQ = x$ and $\hat{Q} = 60°$.

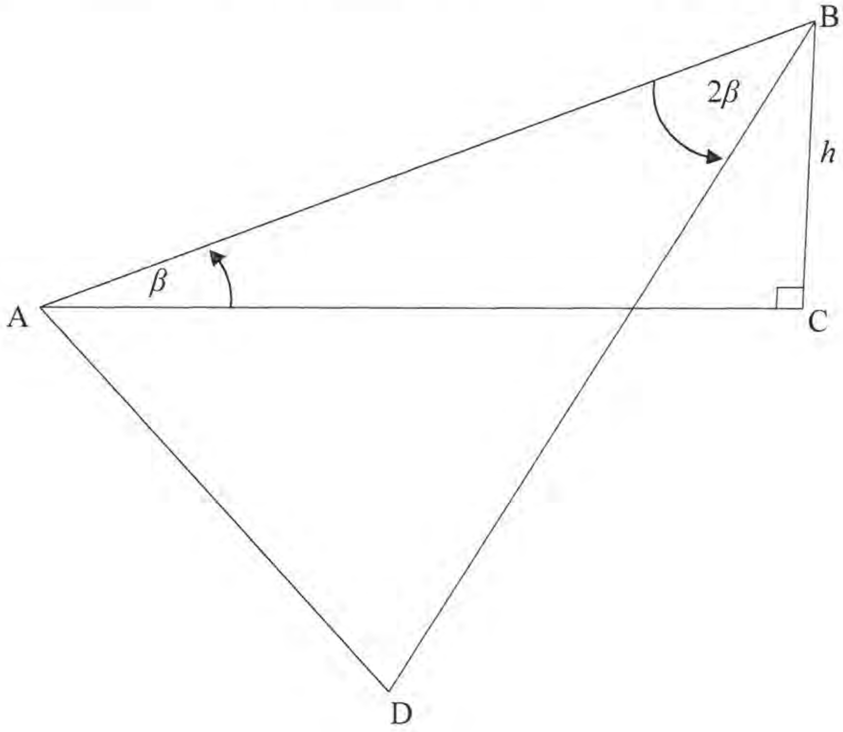


7.1.1 Show that the area of $\Delta PQR = 5\sqrt{3}x - \sqrt{3}x^2$. (2)

7.1.2 Determine the value of x for which the area of ΔPQR will be a maximum. (3)

7.1.3 Calculate the length of PR if the area of ΔPQR is a maximum. (3)

7.2 In the diagram below, BC is a pole anchored by two cables at A and D. A, D and C are in the same horizontal plane. The height of the pole is h and the angle of elevation from A to the top of the pole, B, is β. $A\hat{B}D = 2\beta$ and $BA = BD$.



Determine the distance AD between the two anchors in terms of h. (7)

[15]

QUESTION 5

5.1 Given that $\sin 23° = \sqrt{k}$, determine, in its simplest form, the value of each of the following in terms of k, WITHOUT using a calculator:

5.1.1 $\sin 203°$ (2)

5.1.2 $\cos 23°$ (3)

5.1.3 $\tan(-23°)$ (2)

5.2 Simplify the following expression to a single trigonometric function:

$$\frac{4\cos(-x).\cos(90° + x)}{\sin(30° - x).\cos x + \cos(30° - x).\sin x}$$ (6)

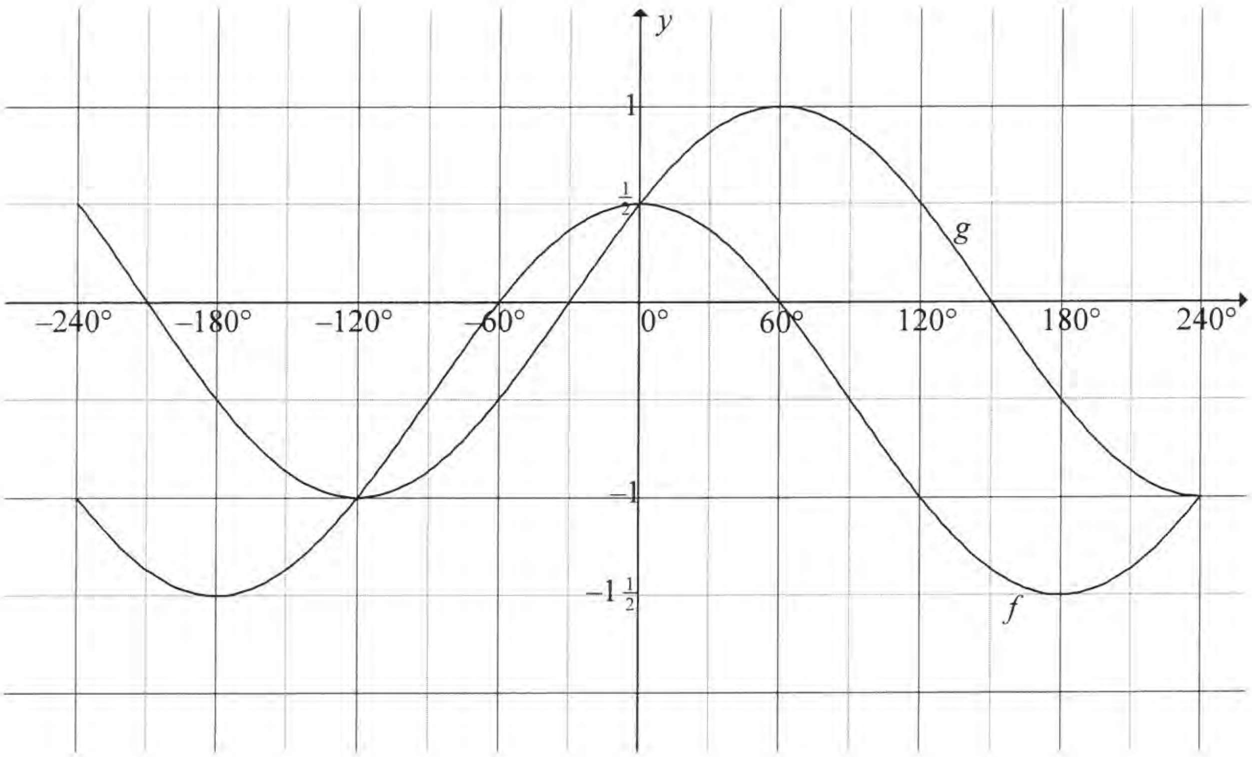
5.3 Determine the general solution of $\cos 2x - 7\cos x - 3 = 0$. (6)

5.4 Given that $\sin\theta = \frac{1}{3}$, calculate the numerical value of $\sin 3\theta$, WITHOUT using a calculator. (5)

[24]

QUESTION 6

In the diagram below, the graphs of $f(x) = \cos x + q$ and $g(x) = \sin(x + p)$ are drawn on the same system of axes for $-240° \le x \le 240°$. The graphs intersect at $\left(0° \,;\, \frac{1}{2}\right)$, $(-120° \,;\, -1)$ and $(240° \,;\, -1)$.



6.1 Determine the values of p and q. (4)

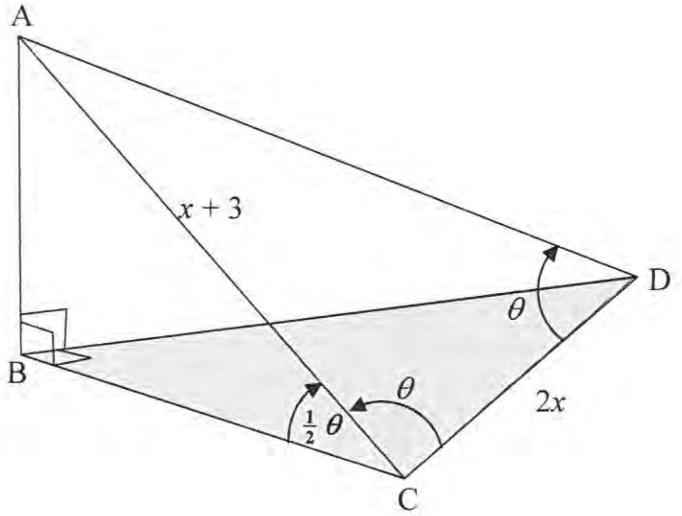
6.2 Determine the values of x in the interval $-240° \le x \le 240°$ for which $f(x) > g(x)$. (2)

6.3 Describe a transformation that the graph of g has to undergo to form the graph of h, where $h(x) = -\cos x$. (2)

[8]

QUESTION 7

A corner of a rectangular block of wood is cut off and shown in the diagram below.
The inclined plane, that is, ΔACD, is an isosceles triangle having $A\hat{D}C = A\hat{C}D = \theta$.
Also $A\hat{C}B = \frac{1}{2}\theta$, AC = $x + 3$ and CD = $2x$.



7.1 Determine an expression for $C\hat{A}D$ in terms of θ. (1)

7.2 Prove that $\cos\theta = \dfrac{x}{x+3}$. (4)

7.3 If it is given that $x = 2$, calculate AB, the height of the piece of wood. (5)
[10]

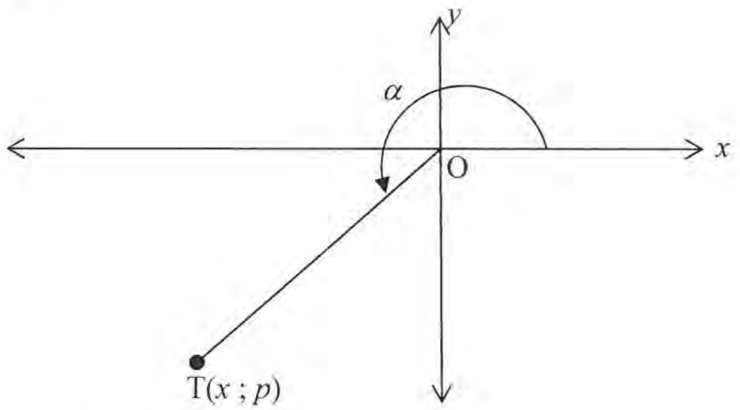
QUESTION 5

5.1 If $x = 3 \sin \theta$ and $y = 3 \cos \theta$, determine the value of $x^2 + y^2$. (3)

5.2 Simplify to a single term:

$$\sin(540° - x).\sin(-x) - \cos(180° - x).\sin(90° + x)$$ (6)

5.3 In the diagram below, T$(x ; p)$ is a point in the third quadrant and it is given that $\sin\alpha = \frac{p}{\sqrt{1+p^2}}$.



5.3.1 Show that $x = -1$. (3)

5.3.2 Write $\cos(180° + \alpha)$ in terms of p in its simplest form. (2)

5.3.3 Show that $\cos 2\alpha$ can be written as $\frac{1-p^2}{1+p^2}$. (3)

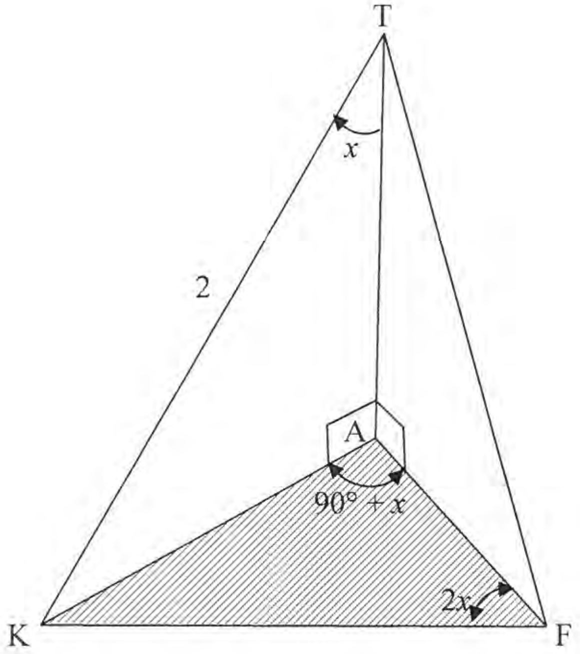
5.4 5.4.1 For which value(s) of x will $\frac{2\tan x - \sin 2x}{2\sin^2 x}$ be undefined in the interval $0° \leq x \leq 180°$? (3)

5.4.2 Prove the identity: $\frac{2\tan x - \sin 2x}{2\sin^2 x} = \tan x$ (6)

[26]

QUESTION 6

6.1 In the figure, points K, A and F lie in the same horizontal plane and TA represents a vertical tower. $A\hat{T}K = x$, $K\hat{A}F = 90° + x$ and $K\hat{F}A = 2x$ where $0° < x < 30°$. TK = 2 units.

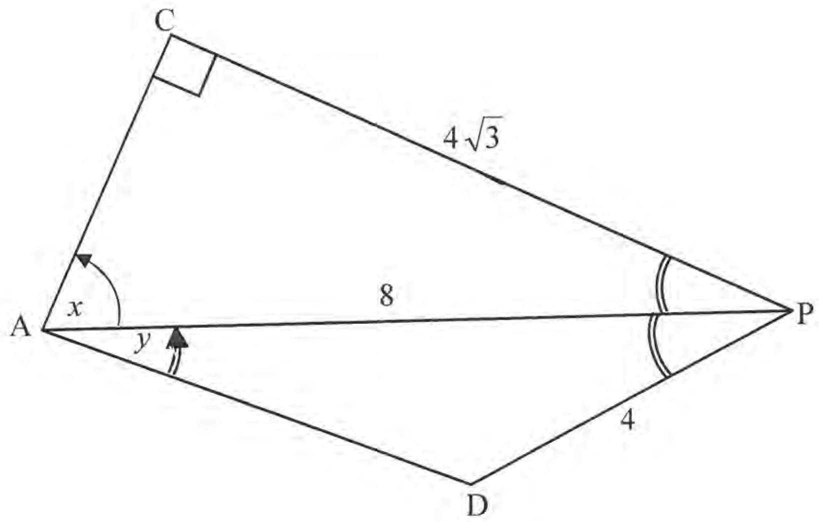


6.1.1 Express AK in terms of $\sin x$. (2)

6.1.2 Calculate the numerical value of KF. (5)

QUESTION 5

In the figure below, ACP and ADP are triangles with $\hat{C} = 90°$, $CP = 4\sqrt{3}$, $AP = 8$ and $DP = 4$. PA bisects $D\hat{P}C$. Let $C\hat{A}P = x$ and $D\hat{A}P = y$.



5.1 Show, by calculation, that $x = 60°$. (2)

5.2 Calculate the length of AD. (4)

5.3 Determine y. (3)
[9]

QUESTION 6

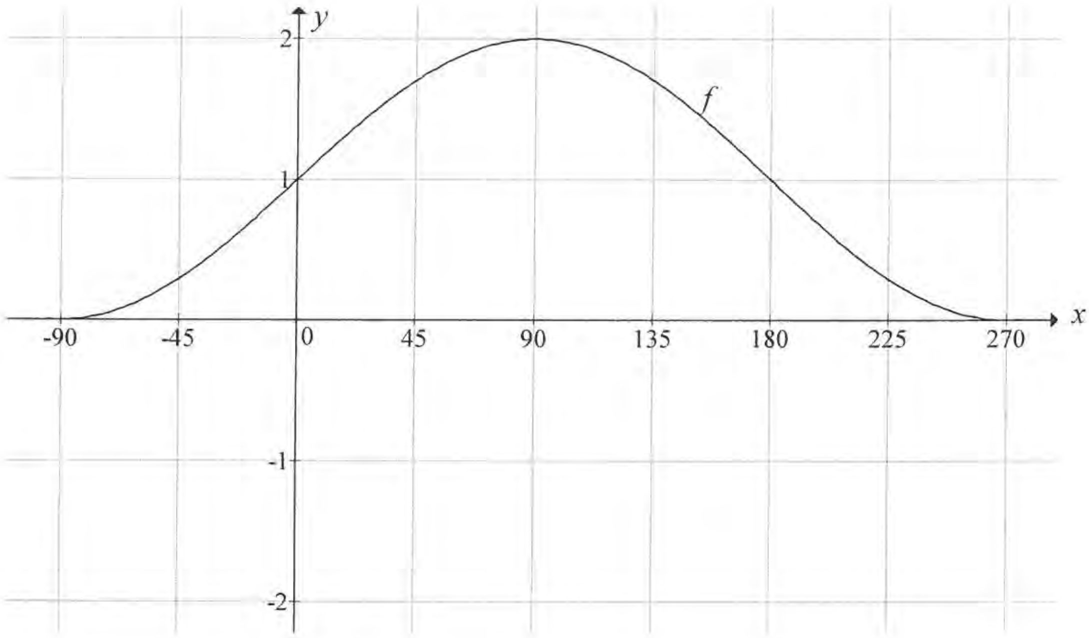
6.1 Prove the identity: $\cos^2(180° + x) + \tan(x - 180°)\sin(720° - x)\cos x = \cos 2x$ (5)

6.2 Use $\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$ to derive the formula for $\sin(\alpha - \beta)$. (3)

6.3 If $\sin 76° = x$ and $\cos 76° = y$, show that $x^2 - y^2 = \sin 62°$. (4)
[12]

QUESTION 7

In the diagram below, the graph of $f(x) = \sin x + 1$ is drawn for $-90° \leq x \leq 270°$.



7.1 Write down the range of f. (2)

7.2 Show that $\sin x + 1 = \cos 2x$ can be rewritten as $(2\sin x + 1)\sin x = 0$. (2)

7.3 Hence, or otherwise, determine the general solution of $\sin x + 1 = \cos 2x$. (4)

7.4 Use the grid on DIAGRAM SHEET 2 to draw the graph of $g(x) = \cos 2x$ for $-90° \leq x \leq 270°$. (3)

7.5 Determine the value(s) of x for which $f(x + 30°) = g(x + 30°)$ in the interval $-90° \leq x \leq 270°$. (3)

7.6 Consider the following geometric series:

$1 + 2\cos 2x + 4\cos^2 2x + \ldots$

Use the graph of g to determine the value(s) of x in the interval $0° \leq x \leq 90°$ for which this series will converge. (5)

[19]

QUESTION 5

5.1 Given that $\sin\alpha = -\frac{4}{5}$ and $90° < \alpha < 270°$.

WITHOUT using a calculator, determine the value of each of the following in its simplest form:

5.1.1 $\sin(-\alpha)$ (2)

5.1.2 $\cos\alpha$ (2)

5.1.3 $\sin(\alpha - 45°)$ (3)

5.2 Consider the identity: $\frac{8\sin(180° - x)\cos(x - 360°)}{\sin^2 x - \sin^2(90° + x)} = -4\tan 2x$

5.2.1 Prove the identity. (6)

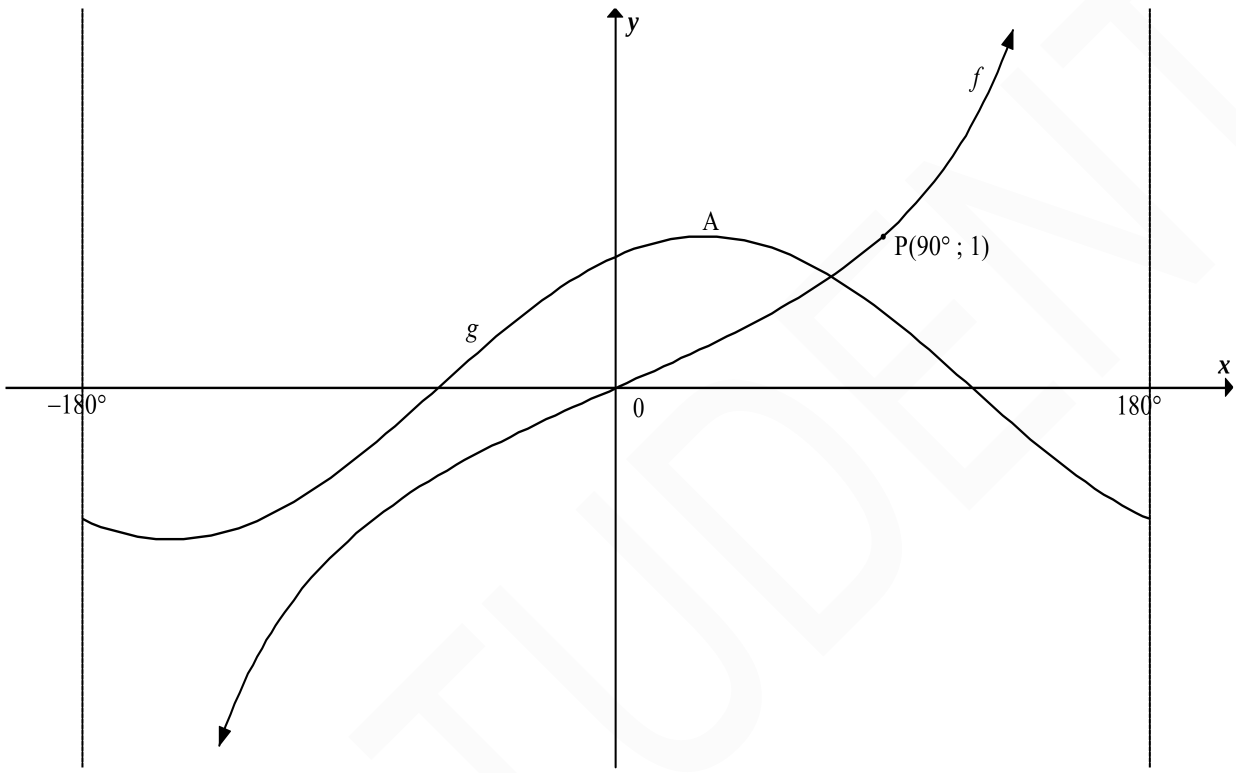
5.2.2 For which value(s) of x in the interval $0° < x < 180°$ will the identity be undefined? (2)

5.3 Determine the general solution of $\cos 2\theta + 4\sin^2\theta - 5\sin\theta - 4 = 0$. (7)

[22]

QUESTION 6

In the diagram below, the graphs of $f(x) = \tan bx$ and $g(x) = \cos(x - 30°)$ are drawn on the same system of axes for $-180° \leq x \leq 180°$. The point P(90° ; 1) lies on *f*. Use the diagram to answer the following questions.



6.1 Determine the value of b. (1)

6.2 Write down the coordinates of A, a turning point of g. (2)

6.3 Write down the equation of the asymptote(s) of $y = \tan b(x + 20°)$ for $x \in [-180°; 180°]$. (1)

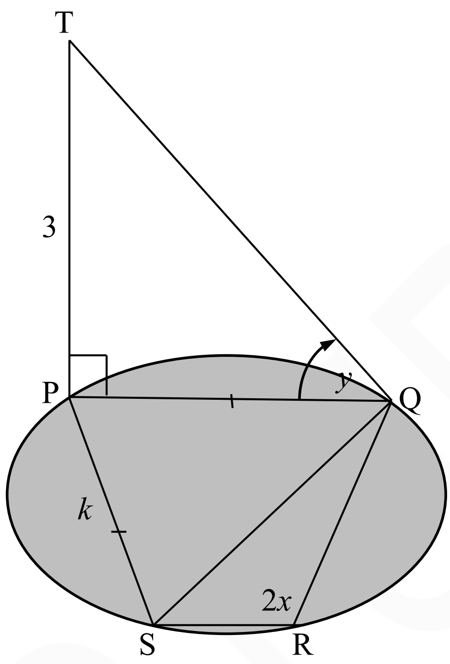
6.4 Determine the range of h if $h(x) = 2g(x) + 1$. (2)

[6]

QUESTION 7

7.1 Prove that in any acute-angled ΔABC, $\dfrac{\sin A}{a} = \dfrac{\sin B}{b}$. (5)

7.2 The framework for a construction consists of a cyclic quadrilateral PQRS in the horizontal plane and a vertical post TP as shown in the figure. From Q the angle of elevation of T is $y°$. $PQ = PS = k$ units, $TP = 3$ units and $S\hat{R}Q = 2x°$.



7.2.1 Show, giving reasons, that $P\hat{S}Q = x$. (2)

7.2.2 Prove that $SQ = 2k\cos x$. (4)

7.2.3 Hence, prove that $SQ = \dfrac{6\cos x}{\tan y}$. (2)

[13]