SA-STUDENT

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"You have to ask yourself how badly do you want something? If you really, really want something then put in the work". -Lewis Hamilton

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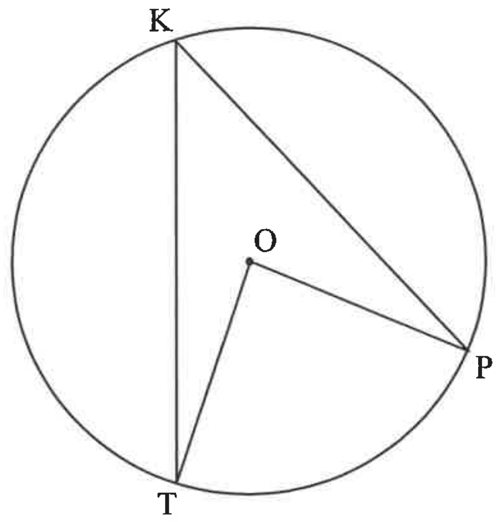
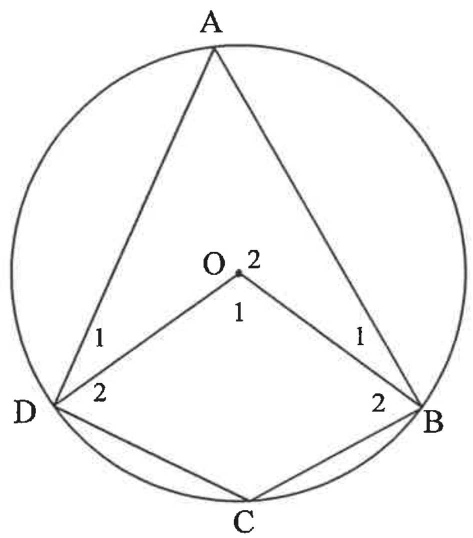
QUESTION 8

8.1 In the diagram, O is the centre of the circle.



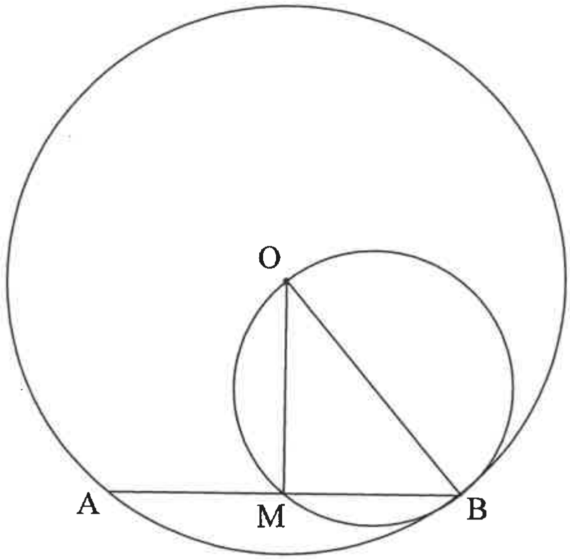
Use the diagram above to prove the theorem which states that the angle subtended by a chord at the centre of the circle is equal to twice the angle subtended by the same chord at the circumference, that is, prove that $T\hat{O}P = 2T\hat{K}P$. (5)

8.2 In the diagram, O is the centre of the circle and ABCD is a cyclic quadrilateral. OB and OD are drawn.



If $\hat{O}_1 = 4x + 100°$ and $\hat{C} = x + 34°$, calculate, giving reasons, the size of x. (5)

8.3 In the diagram, O is the centre of the larger circle. OB is a diameter of the smaller circle. Chord AB of the larger circle intersects the smaller circle at M and B.



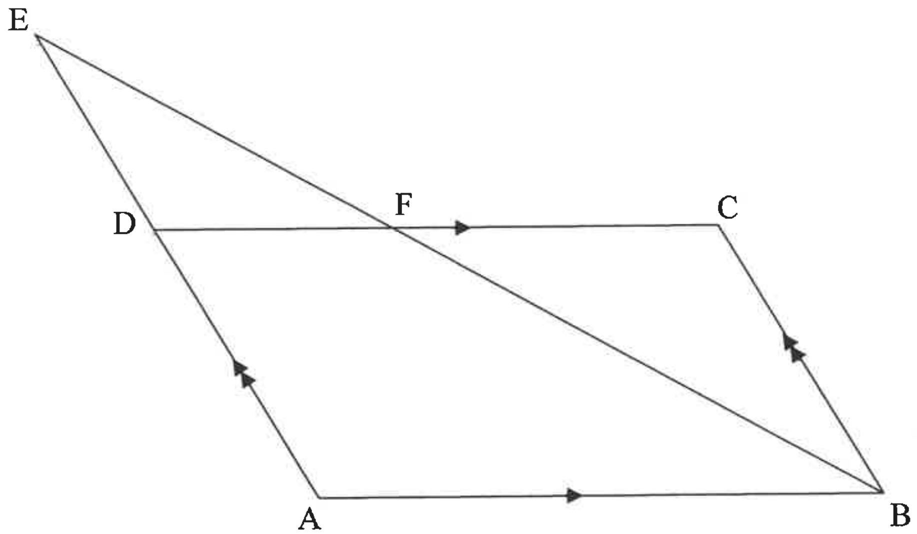
8.3.1 Write down the size of $O\hat{M}B$. Provide a reason. (2)

8.3.2 If $AB = \sqrt{300}$ units and $OM = 5$ units, calculate, giving reasons, the length of OB. (4)

[16]

QUESTION 9

In the diagram, ABCD is a parallelogram with AB = 14 units. AD is produced to E such that AD : DE = 4 : 3. EB intersects DC in F. EB = 21 units.



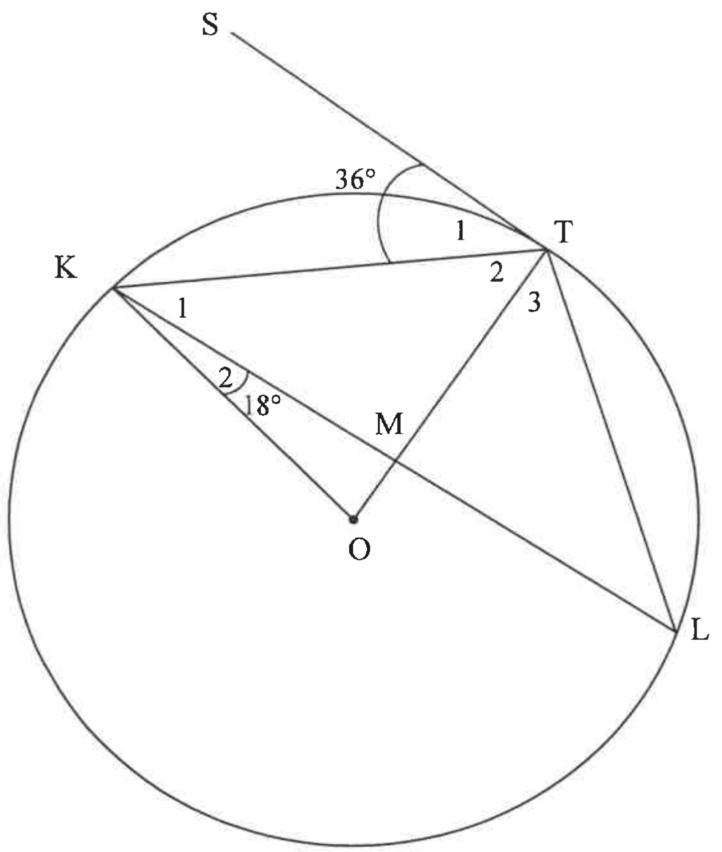
9.1 Calculate, with reasons, the length of FB. (3)

9.2 Prove, with reasons, that $\Delta EDF\ |||\ \Delta EAB$. (3)

9.3 Calculate, with reasons, the length of FC. (3)
[9]

QUESTION 8

8.1 In the diagram, O is the centre of the circle. K, T and L are points on the circle. KT, TL, KL, OK and OT are drawn. OT intersects KL at M. ST is a tangent to the circle at T. $S\hat{T}K = 36°$ and $O\hat{K}L = 18°$.



8.1.1 Determine, giving reasons, the size of:

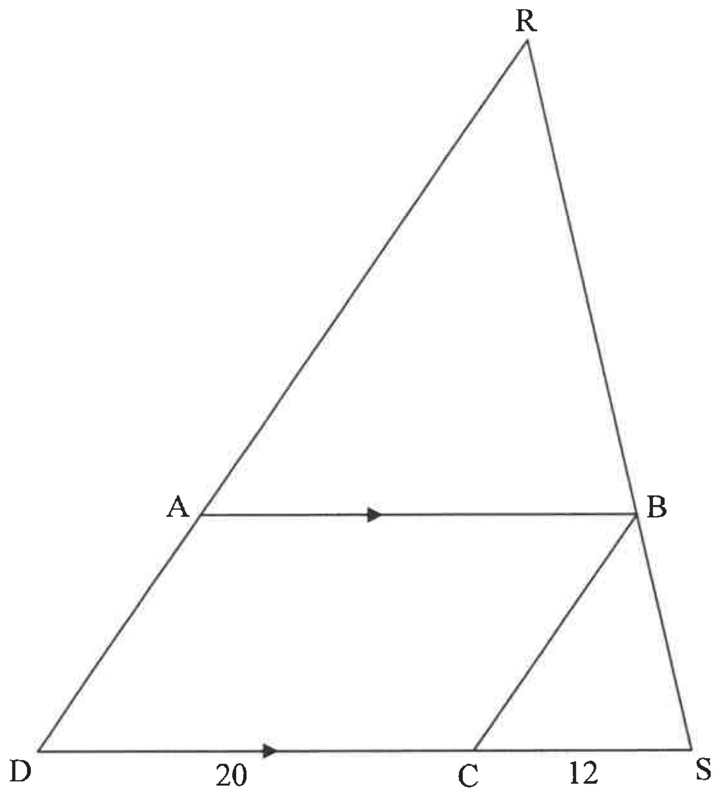
(a) $\hat{T}_2$ (2)

(b) $\hat{L}$ (2)

(c) $K\hat{O}T$ (2)

8.1.2 Prove, giving reasons, that KM = ML. (3)

8.2 In the diagram, ΔRDS is drawn. A, B and C are points on RD, RS and DS respectively such that $AB \parallel DS$ and $RB : BS = 5 : 3$. $DC = 20$ units and $CS = 12$ units.



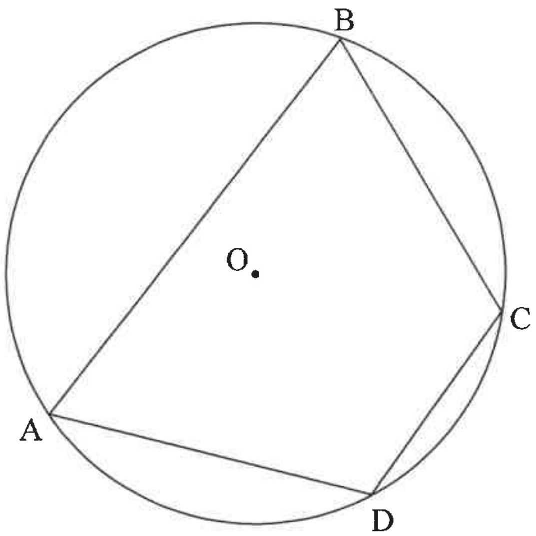
8.2.1 Prove, giving reasons, that $BC \parallel AD$. (3)

8.2.2 If it is further given that $RD = 48$ units, calculate, giving reasons, the value of the ratio $AD : AB$. (3)

[15]

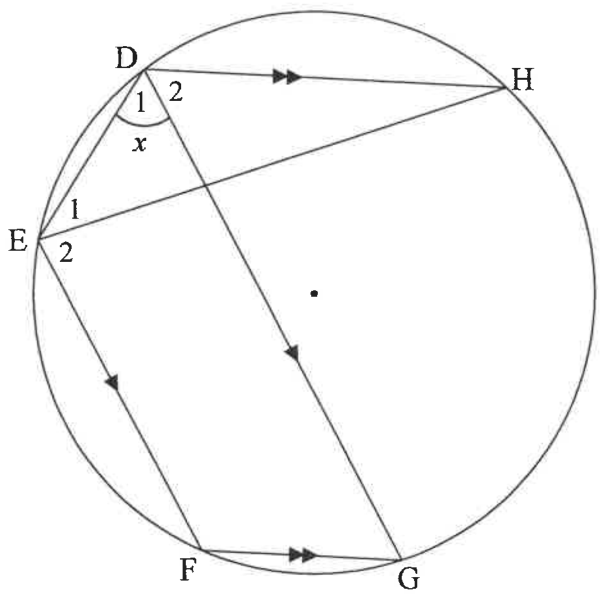
QUESTION 9

9.1 In the diagram, O is the centre of the circle. ABCD is a cyclic quadrilateral.



Use the diagram in the ANSWER BOOK to prove the theorem which states that the opposite angles of a cyclic quadrilateral are supplementary, that is prove that $\hat{B}+\hat{D}=180°$. (5)

9.2 In the diagram, DEFG is a cyclic quadrilateral such that EF || DG. H is another point on the circle such that DH || FG. Chord EH is drawn. Let $\hat{D}_1 = x$.

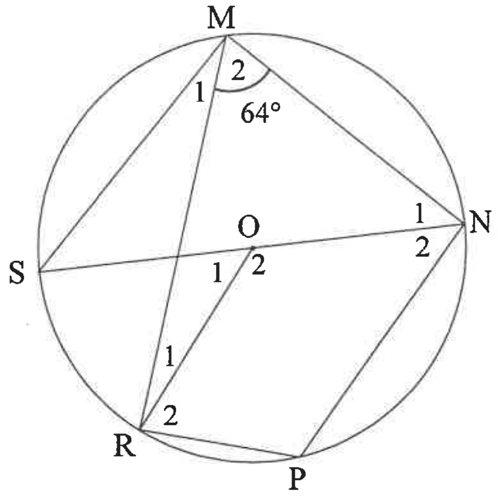


Prove, giving reasons, that $\hat{D}_1 = \hat{D}_2$. (4)

[9]

QUESTION 8

8.1 In the diagram, O is the centre of the circle. MNPR is a cyclic quadrilateral and SN is a diameter of the circle. Chord MS and radius OR are drawn. $\hat{M}_2 = 64°$.



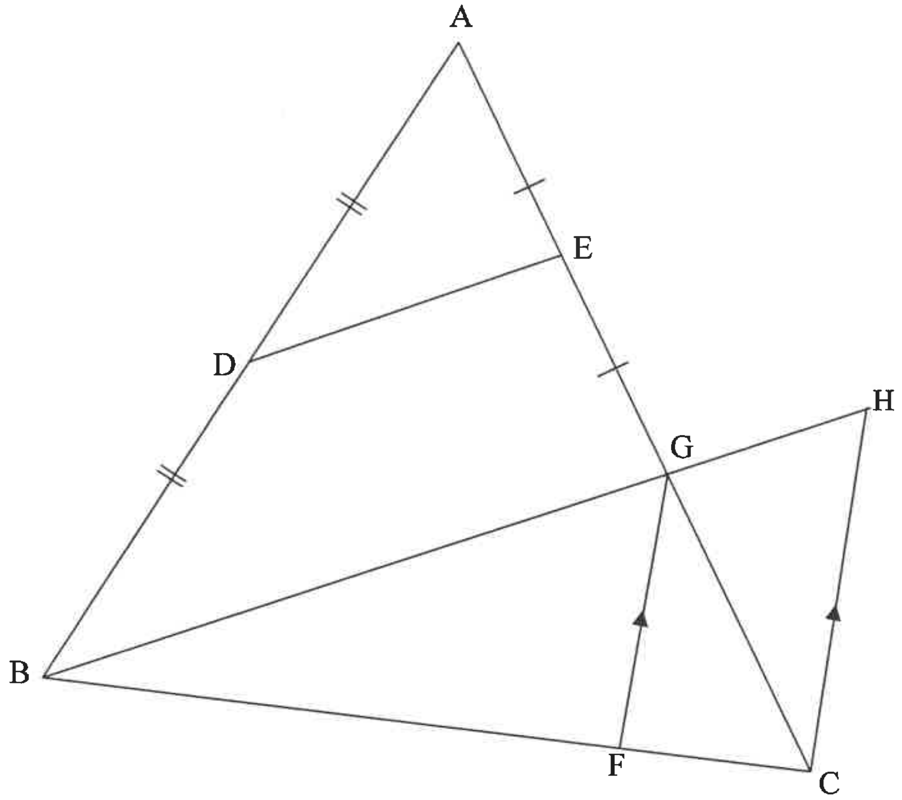
Determine, giving reasons, the size of the following angles:

8.1.1 $\hat{P}$ (2)

8.1.2 $\hat{M}_1$ (2)

8.1.3 $\hat{O}_1$ (2)

8.2 In the diagram, ΔABG is drawn. D and E are midpoints of AB and AG respectively. AG and BG are produced to C and H respectively. F is a point on BC such that FG $\parallel$ CH.



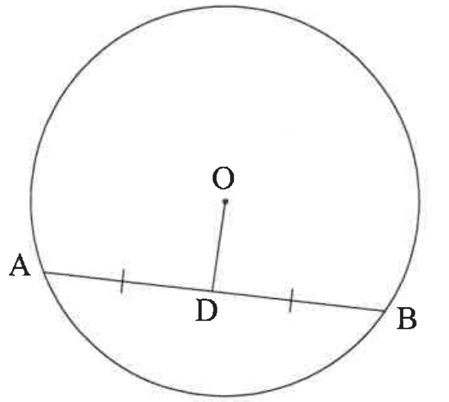
8.2.1 Give a reason why DE $\parallel$ BH. (1)

8.2.2 If it is further given that $\frac{FC}{BF} = \frac{1}{4}$, DE $= 3x - 1$ and GH $= x + 1$, calculate, giving reasons, the value of x. (6)

[13]

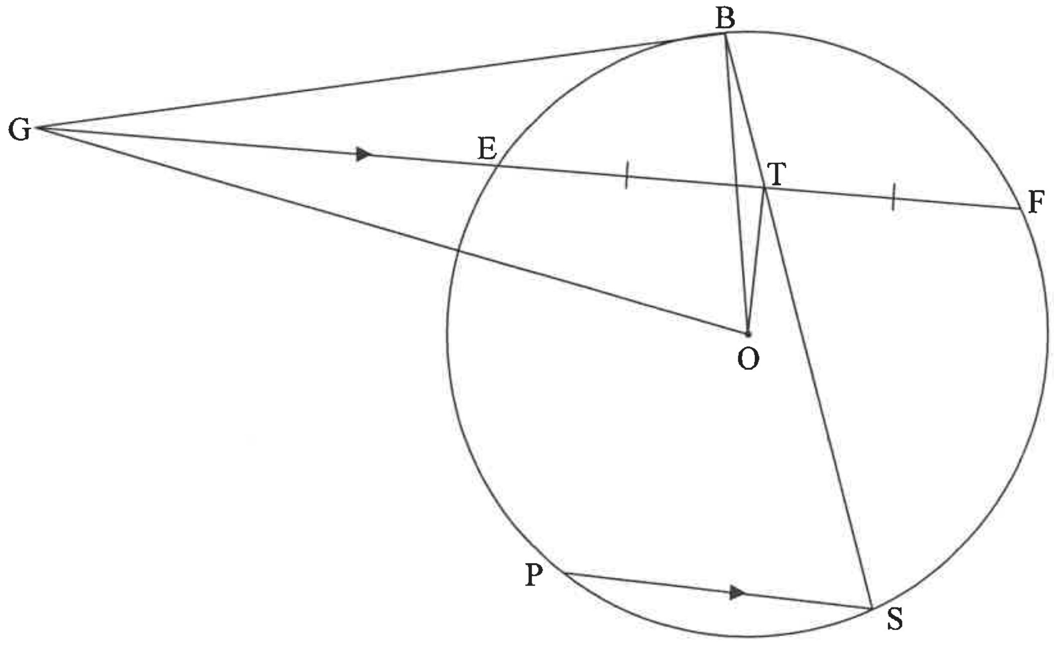
QUESTION 9

9.1 In the diagram, O is the centre of a circle. OD bisects chord AB.



Prove the theorem that states that the line from the centre of a circle that bisects a chord is perpendicular to the chord, i.e. $OD \perp AB$. (5)

9.2 In the diagram, E, B, F, S and P are points on the circle centred at O. GB is a tangent to the circle at B. FE is produced to meet the tangent at G. OT is drawn such that T is the midpoint of EF. GO and BO are drawn. BS is drawn through T. PS || GF.



Prove, giving reasons, that:

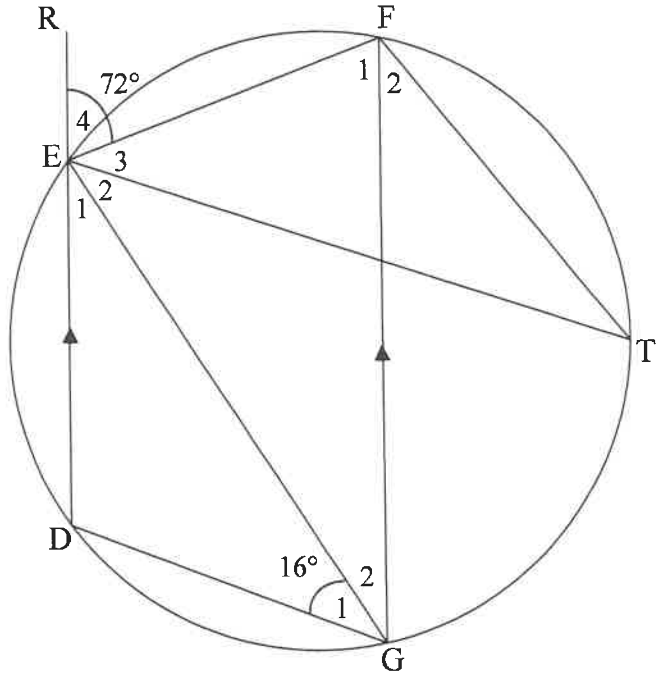
9.2.1 OTBG is a cyclic quadrilateral (5)

9.2.2 $G\hat{O}B = \hat{S}$ (4)

[14]

QUESTION 9

9.1 In the diagram, DEFG is a cyclic quadrilateral with DE || GF. DE is produced to R. T is another point on the circle. EG, FT and ET are drawn. $\hat{E}_4 = 72°$ and $\hat{G}_1 = 16°$.



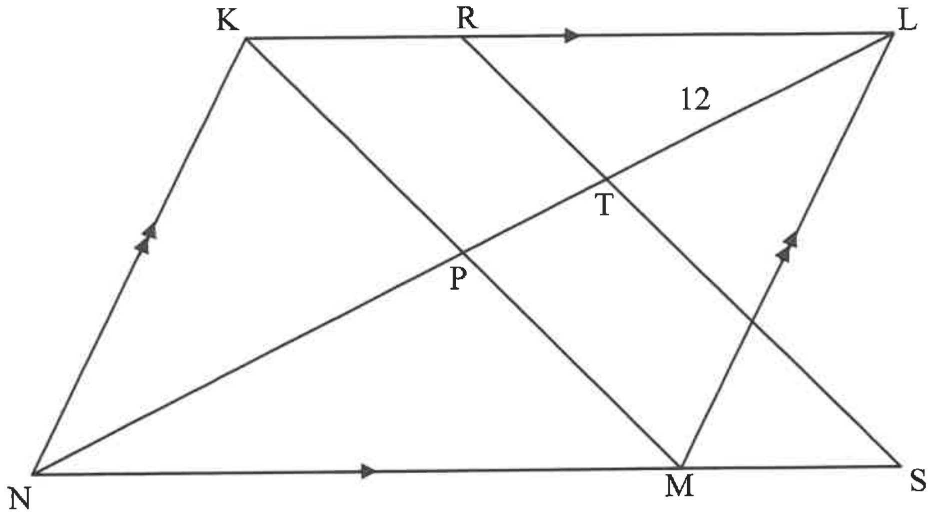
Determine, with reasons, the size of the following angles:

9.1.1 $D\hat{G}F$ (2)

9.1.2 $\hat{T}$ (2)

9.1.3 $G\hat{E}F$ (2)

9.2 In the diagram, the diagonals of parallelogram KLMN intersect at P. NM is produced to S. R is a point on KL and RS cuts PL at T.
NM : MS = 4 : 1, NL = 32 units and TL = 12 units.



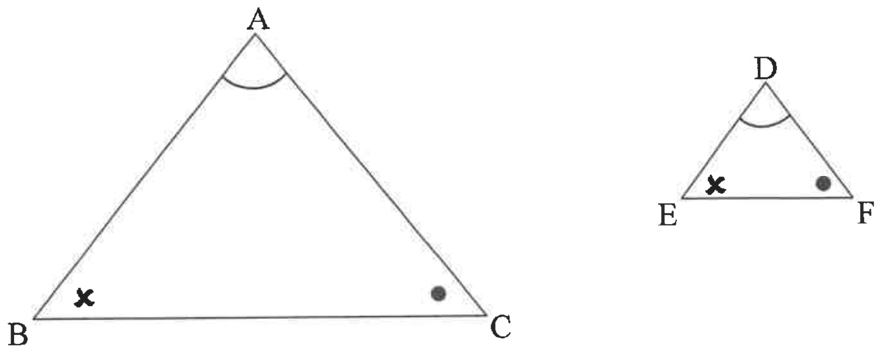
9.2.1 Determine, with reasons, the value of the ratio NP : PT in simplest form. (4)

9.2.2 Prove, with reasons, that KM || RS. (2)

9.2.3 If NM = 21 units, determine, with reasons, the length of RL. (4)
[16]

QUESTION 10

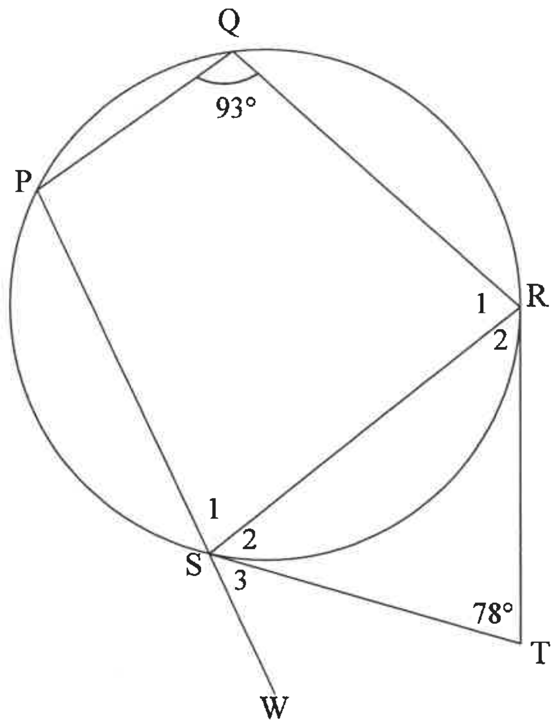
10.1 In the diagram, ΔABC and ΔDEF are drawn such that $\hat{A} = \hat{D}$, $\hat{B} = \hat{E}$ and $\hat{C} = \hat{F}$.



Use the diagram in the ANSWER BOOK to prove the theorem which states that if two triangles are equiangular, then the corresponding sides are in proportion, i.e. $\frac{AB}{DE} = \frac{AC}{DF}$. (6)

QUESTION 9

In the diagram, PQRS is a cyclic quadrilateral. PS is produced to W. TR and TS are tangents to the circle at R and S respectively. $\hat{T}=78°$ and $\hat{Q}=93°$.



9.1 Give a reason why ST = TR. (1)

9.2 Calculate, giving reasons, the size of:

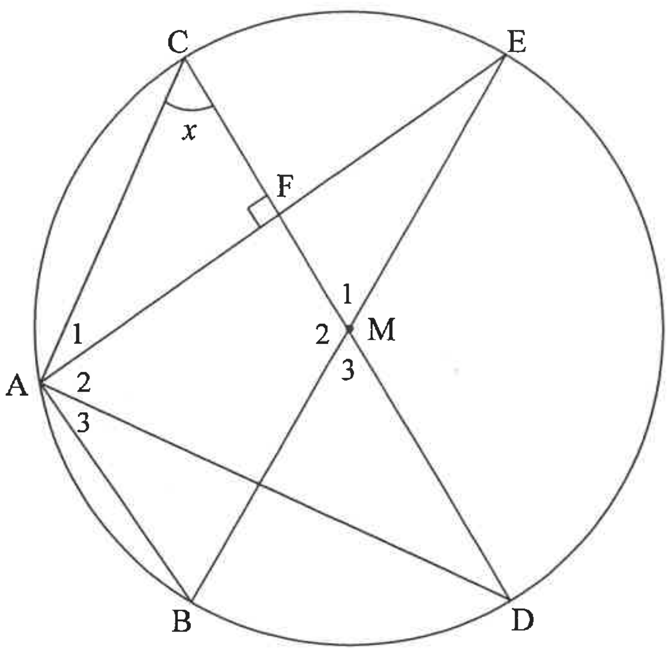
9.2.1 $\hat{S}_2$ (2)

9.2.2 $\hat{S}_3$ (2)

[5]

QUESTION 10

In the diagram, BE and CD are diameters of a circle having M as centre. Chord AE is drawn to cut CD at F. $AE \perp CD$. Let $\hat{C} = x$.



10.1 Give a reason why AF = FE. (1)

10.2 Determine, giving reasons, the size of $\hat{M}_1$ in terms of x. (3)

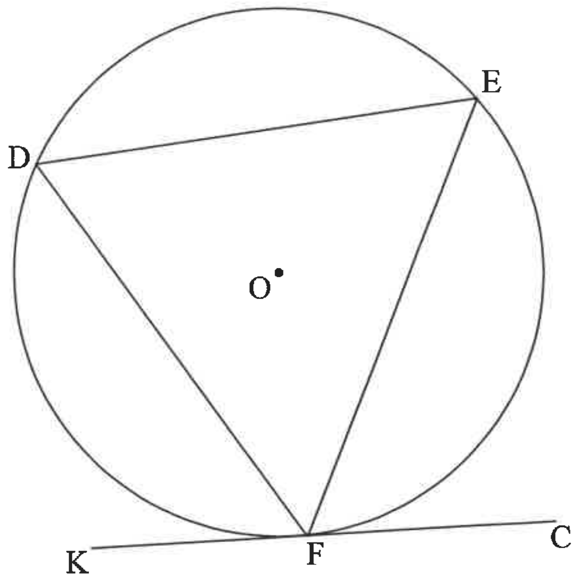
10.3 Prove, giving reasons, that AD is a tangent to the circle passing through A, C and F. (4)

10.4 Given that CF = 6 units and AB = 24 units, calculate, giving reasons, the length of AE. (5)

[13]

QUESTION 11

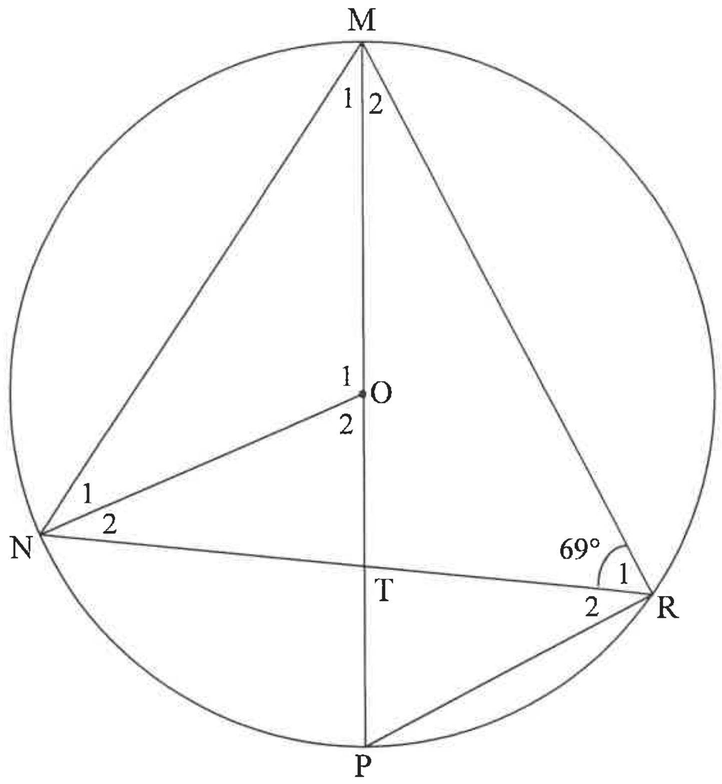
11.1 In the diagram, chords DE, EF and DF are drawn in the circle with centre O. KFC is a tangent to the circle at F.



Prove the theorem which states that $D\hat{F}K = \hat{E}$. (5)

QUESTION 8

8.1 In the diagram, MP is a diameter of a circle centered at O. MP cuts the chord NR at T. Radius NO and chords PR, MN and MR are drawn. $\hat{R}_1 = 69°$.



Determine, giving reasons, the size of:

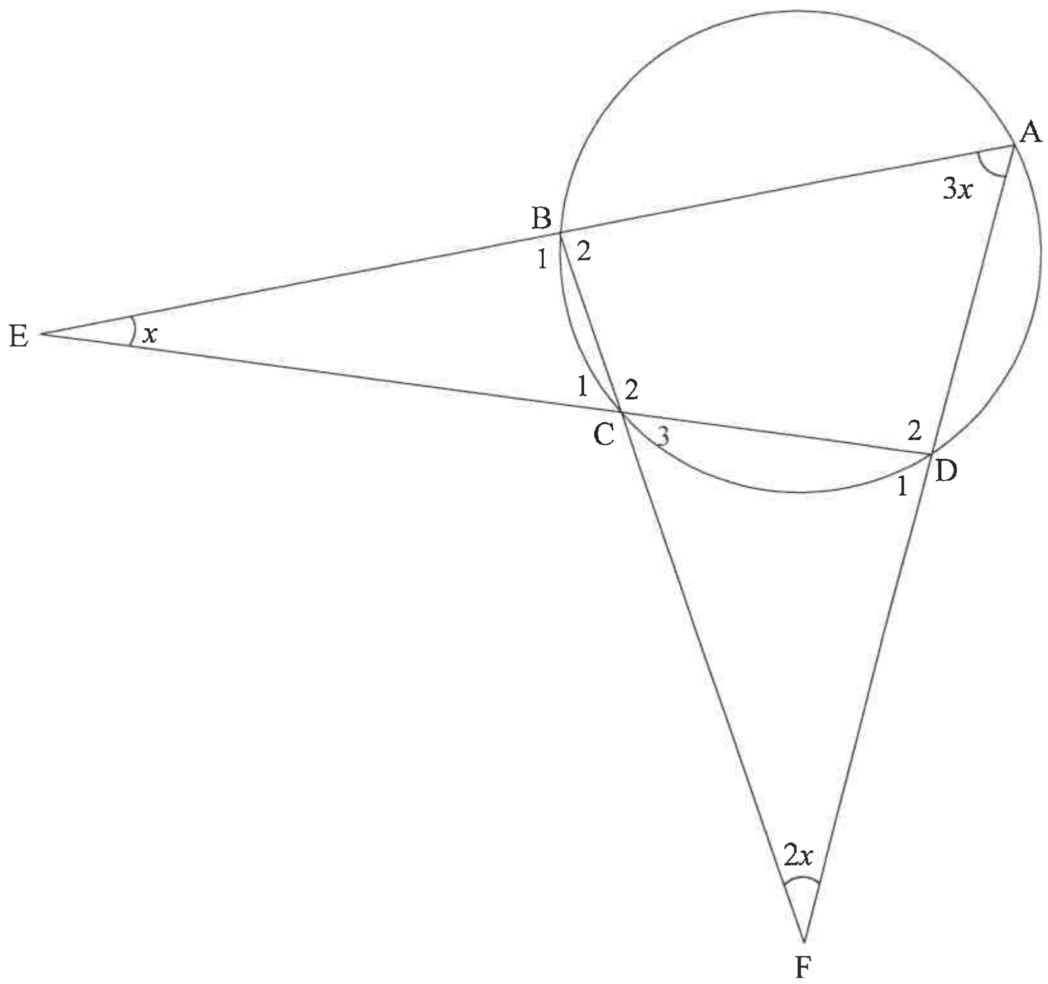
8.1.1 $\hat{R}_2$ (2)

8.1.2 $\hat{O}_1$ (2)

8.1.3 $\hat{M}_1$ (2)

8.1.4 $\hat{M}_2$, if it is further given that NO || PR (4)

8.2 In the diagram below, ABCD is a cyclic quadrilateral. AB and DC are produced to meet at E. AD and BC are produced to meet at F. $A\hat{F}B = 2x$, $D\hat{A}B = 3x$ and $A\hat{E}D = x$.

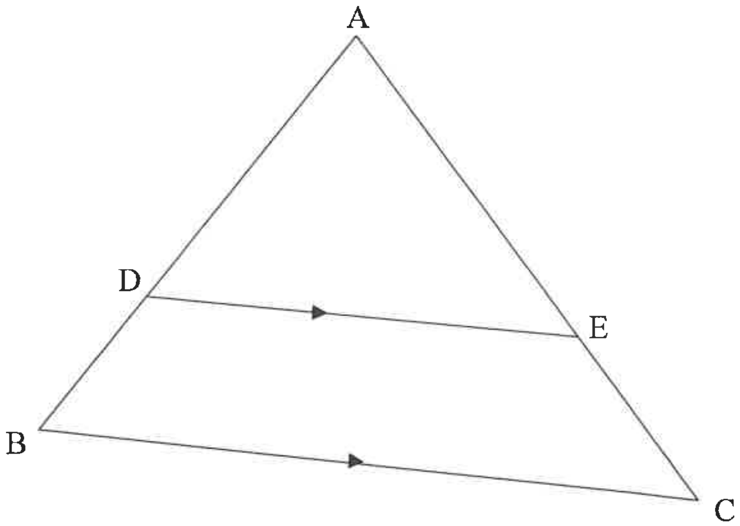


Determine, giving reasons, the value of x. (6)

[16]

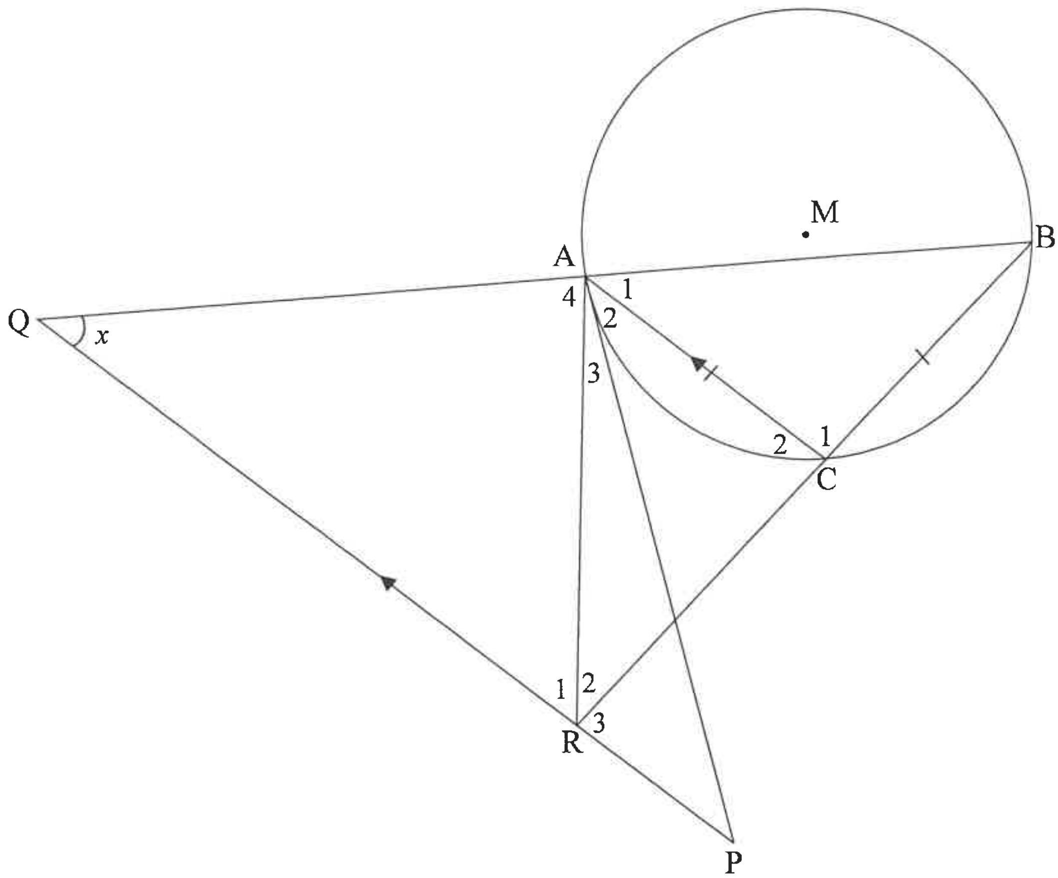
QUESTION 9

9.1 In the diagram, ABC is a triangle. D and E are points on sides AB and AC respectively such that DE || BC .



Use the diagram above to prove the theorem which states that a line drawn parallel to one side of a triangle divides the other two sides proportionally, i.e. prove that $\frac{AD}{DB}=\frac{AE}{EC}$. (6)

9.2 In the diagram, M is the centre of the circle. A, B and C are points on the circle such that AC = BC. PA is a tangent to the circle at A. PQ is drawn parallel to CA to meet BA produced at Q. BC produced meets PQ at R and AR is drawn.
Let $\hat{Q} = x$.



9.2.1 Determine, giving reasons, FOUR other angles EACH equal to x. (6)

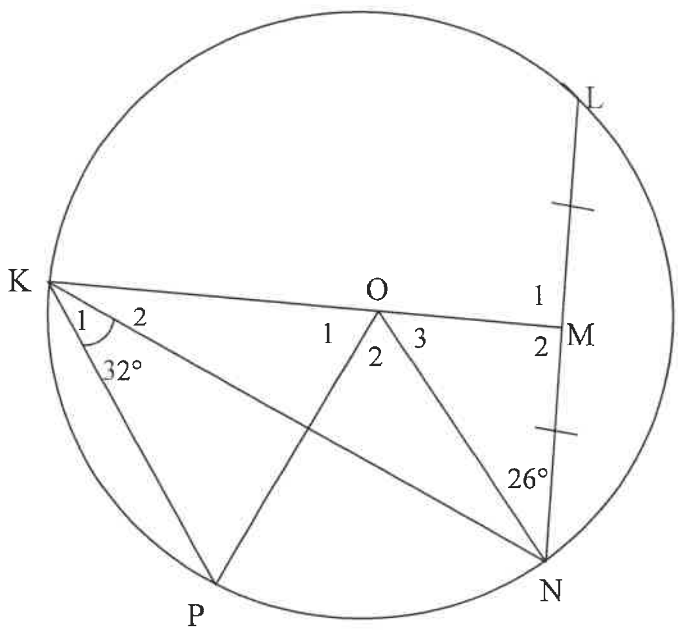
9.2.2 Prove that ABPR is a cyclic quadrilateral. (2)

9.2.3 Prove that $\frac{BA}{BQ} = \frac{BC}{QR}$. (3)

[17]

QUESTION 8

8.1 O is the centre of the circle.. KOM bisects chord LN and $M\hat{N}O = 26°$. K and P are points on the circle with $N\hat{K}P = 32°$. OP is drawn.



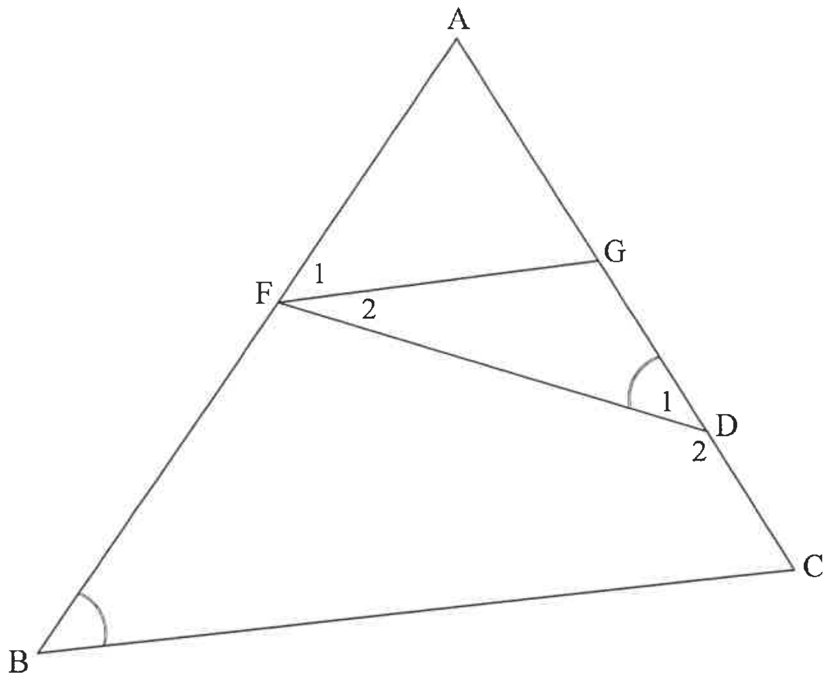
8.1.1 Determine, giving reasons, the size of:

(a) $\hat{O}_2$ (2)

(b) $\hat{O}_1$ (4)

8.1.2 Prove, giving reasons, that KN bisects $O\hat{K}P$. (3)

8.2 In ΔABC, F and G are points on sides AB and AC respectively. D is a point on GC such that $\hat{D}_1 = \hat{B}$.



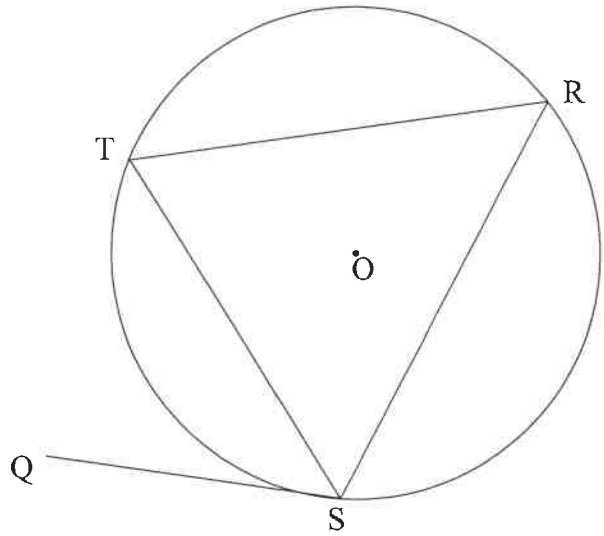
8.2.1 If AF is a tangent to the circle passing through points F, G and D, then prove, giving reasons, that FG || BC. (4)

8.2.2 If it is further given that $\frac{AF}{FB} = \frac{2}{5}$, $AC = 2x - 6$ and $GC = x + 9$, then calculate the value of x. (4)

[17]

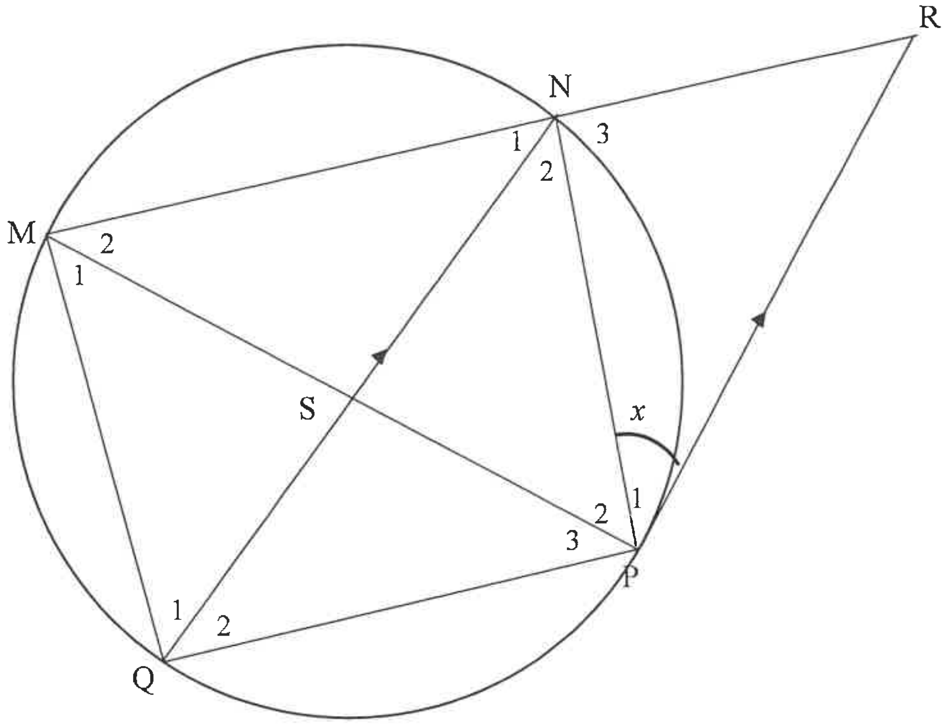
QUESTION 9

9.1 In the diagram, O is the centre of the circle. Points S, T and R lie on the circle. Chords ST, SR and TR are drawn in the circle. QS is a tangent to the circle at S.



Use the diagram to prove the theorem which states that $Q\hat{S}T = \hat{R}$. (5)

9.2 Chord QN bisects $M\hat{N}P$ and intersects chord MP at S. The tangent at P meets MN produced at R such that QN || PR. Let $\hat{P}_1 = x$.



9.2.1 Determine the following angles in terms of x. Give reasons

(a) $\hat{N}_2$ (2)

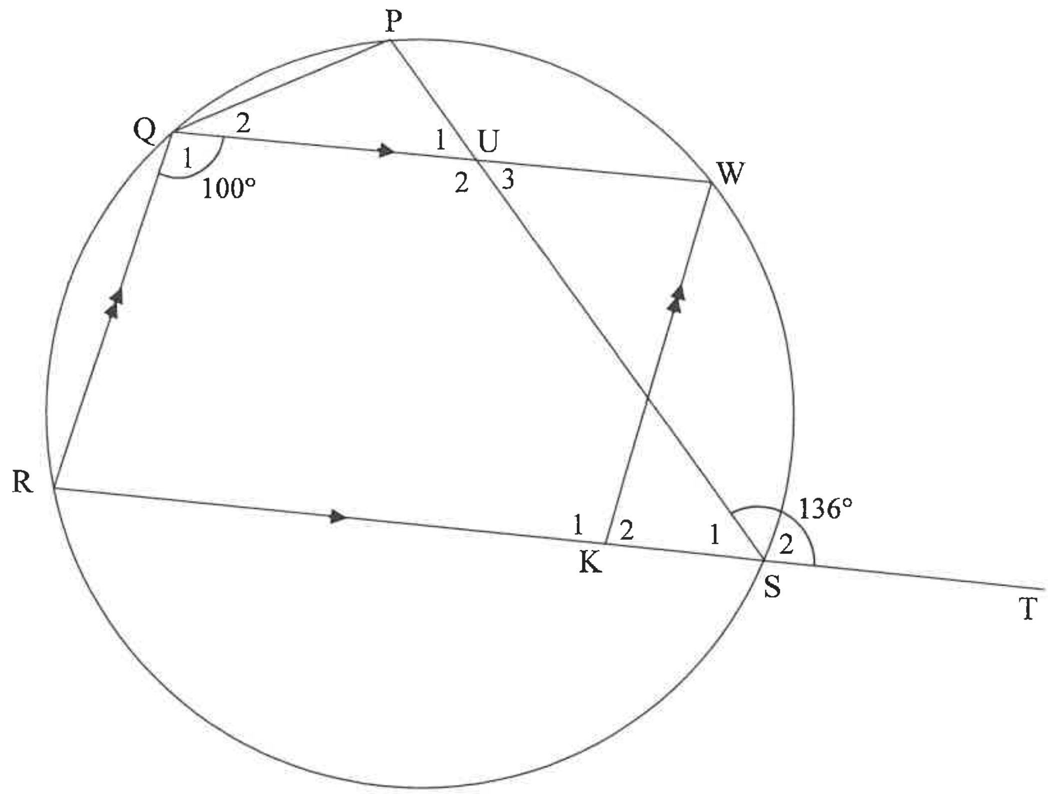
(b) $\hat{Q}_2$ (2)

9.2.2 Prove, giving reasons, that $\frac{MN}{NR} = \frac{MS}{SQ}$ (6)

[15]

QUESTION 8

8.1 In the diagram, PQRS is a cyclic quadrilateral. Chord RS is produced to T. K is a point on RS and W is a point on the circle such that QRKW is a parallelogram. PS and QW intersect at U. $P\hat{S}T = 136°$ and $\hat{Q}_1 = 100°$.



Determine, with reasons, the size of:

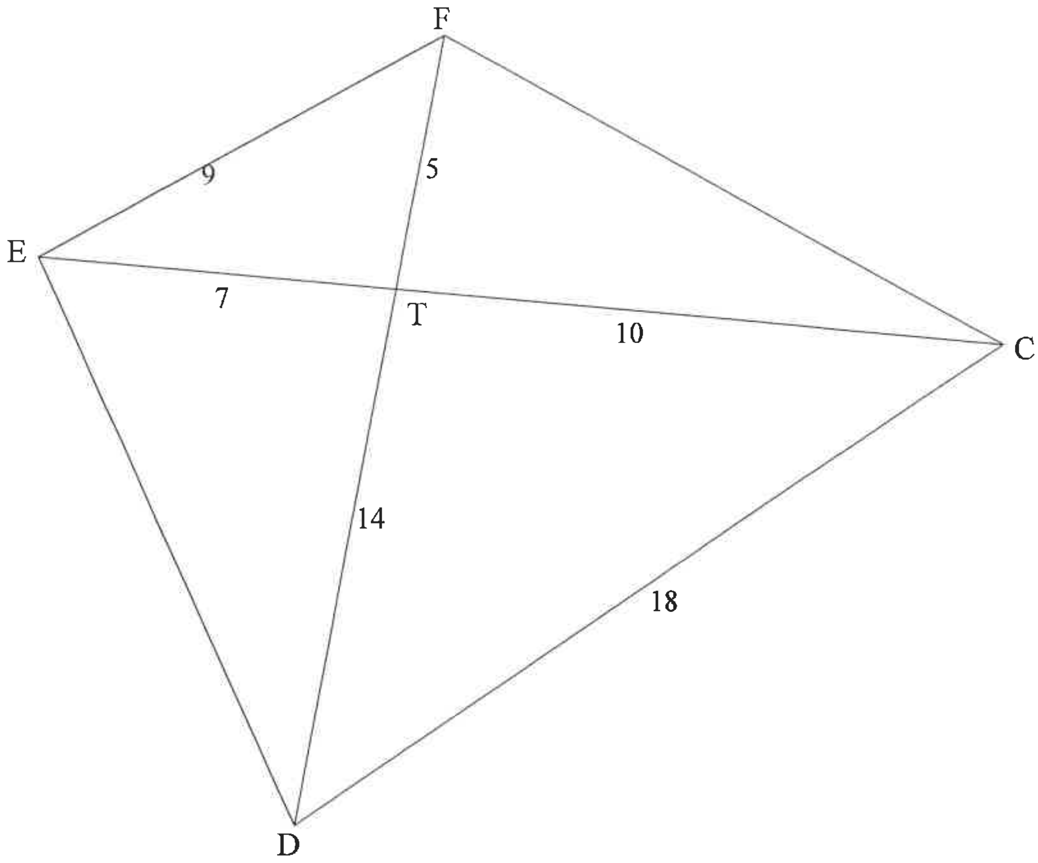
8.1.1 $\hat{R}$ (2)

8.1.2 $\hat{P}$ (2)

8.1.3 $P\hat{Q}W$ (3)

8.1.4 $\hat{U}_2$ (2)

8.2 In the diagram, the diagonals of quadrilateral CDEF intersect at T.
EF = 9 units, DC = 18 units, ET = 7 units, TC = 10 units, FT = 5 units and TD = 14 units.



Prove, with reasons, that:

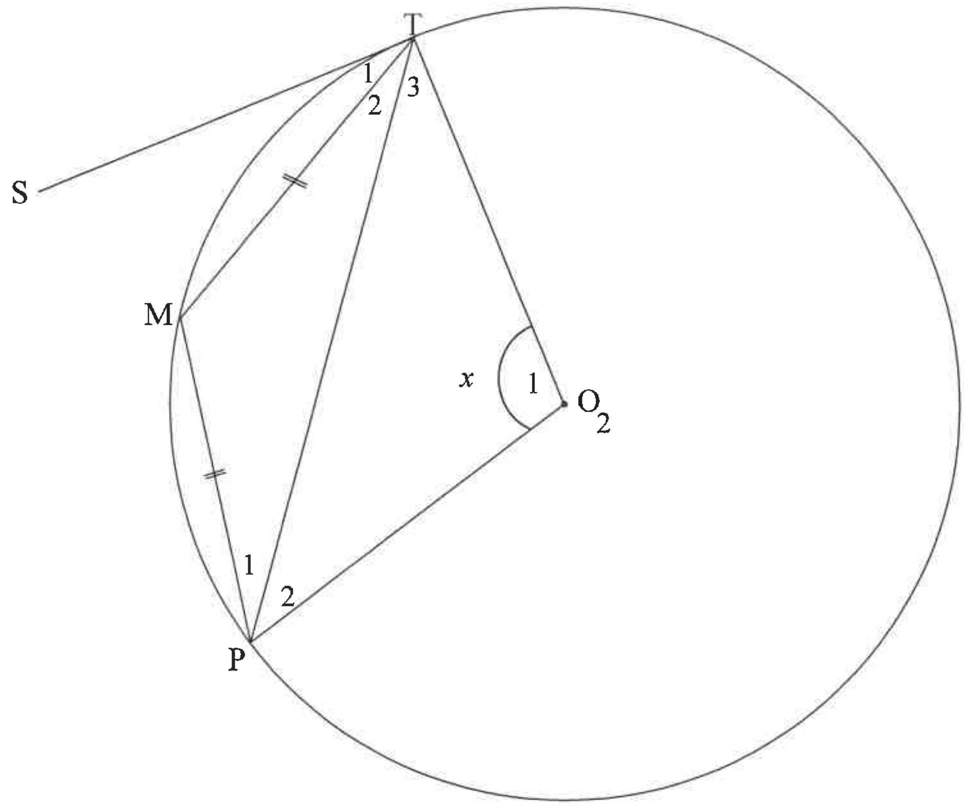
8.2.1 $E\hat{F}D = E\hat{C}D$ (4)

8.2.2 $D\hat{F}C = D\hat{E}C$ (3)

[16]

QUESTION 9

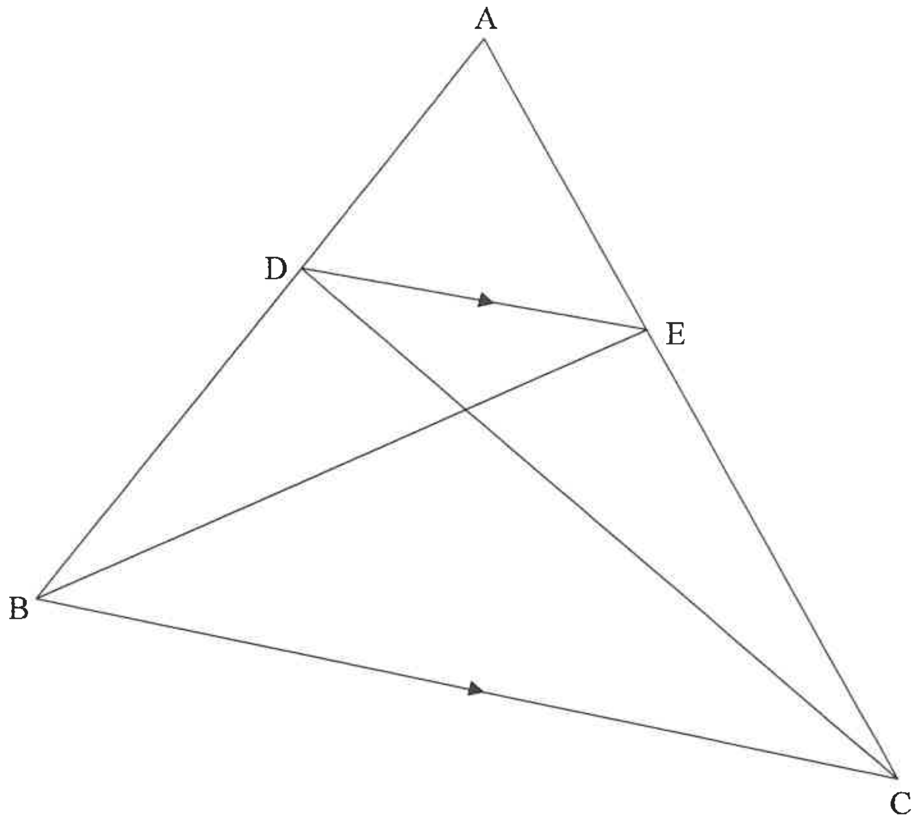
In the diagram, O is the centre of the circle. ST is a tangent to the circle at T. M and P are points on the circle such that TM = MP. OT, OP and TP are drawn. Let $\hat{O}_1 = x$.



Prove, with reasons, that $S\hat{T}M = \frac{1}{4}x$. **[7]**

QUESTION 10

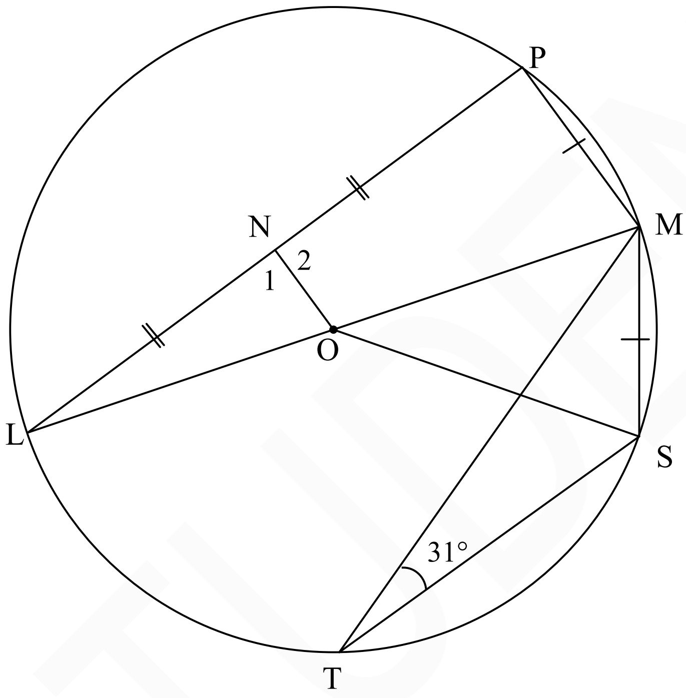
10.1 In the diagram, ΔABC is drawn. D is a point on AB and E is a point on AC such that $DE \parallel BC$. BE and DC are drawn.



Use the diagram to prove the theorem which states that a line drawn parallel to one side of a triangle divides the other two sides proportionally, in other words prove that $\dfrac{AD}{DB} = \dfrac{AE}{EC}$ (6)

QUESTION 8

8.1 In the diagram, O is the centre of the circle and LOM is a diameter of the circle. ON bisects chord LP at N. T and S are points on the circle on the other side of LM with respect to P. Chords PM, MS, MT and ST are drawn. PM = MS and $M\hat{T}S = 31°$



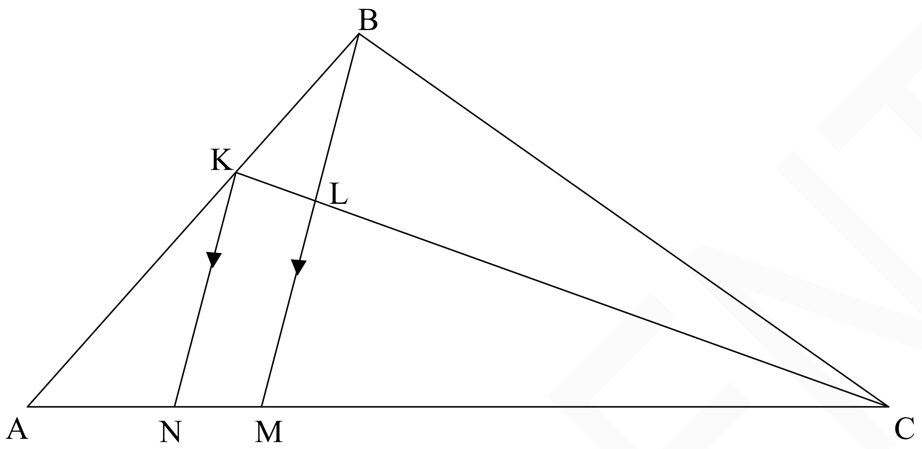
8.1.1 Determine, with reasons, the size of each of the following angles:

(a) $M\hat{O}S$ (2)

(b) $\hat{L}$ (2)

8.1.2 Prove that $ON = \frac{1}{2}MS$. (4)

8.2 In ΔABC in the diagram, K is a point on AB such that AK : KB = 3 : 2. N and M are points on AC such that KN || BM. BM intersects KC at L. AM : MC = 10 : 23.



Determine, with reasons, the ratio of:

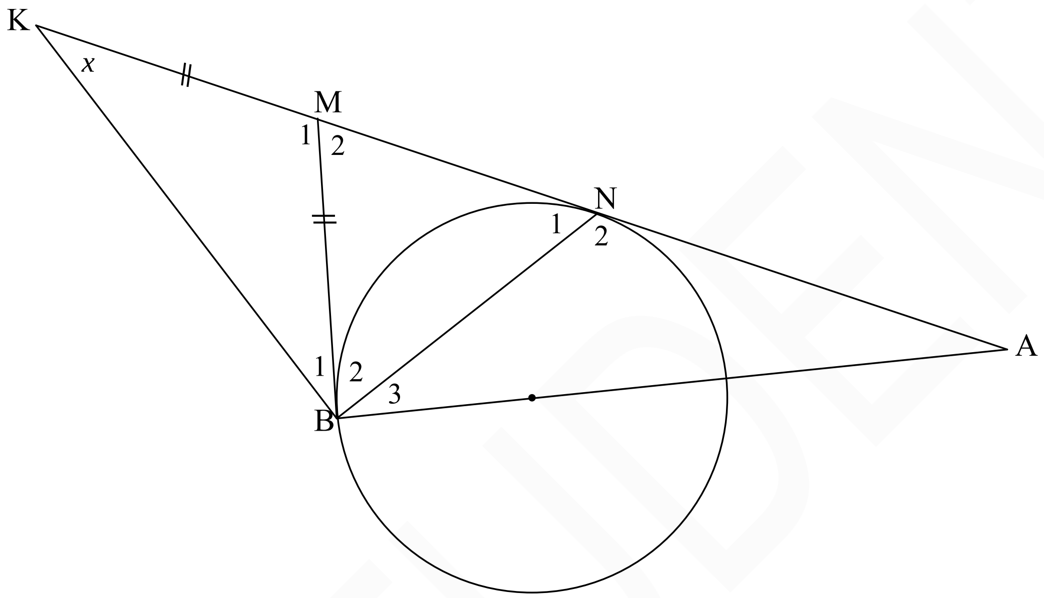
8.2.1 $\dfrac{AN}{AM}$ (2)

8.2.2 $\dfrac{CL}{LK}$ (3)

[13]

QUESTION 9

In the diagram, tangents are drawn from point M outside the circle, to touch the circle at B and N. The straight line from B passing through the centre of the circle meets MN produced in A. NM is produced to K such that BM = MK. BK and BN are drawn.
Let $\hat{K} = x$.

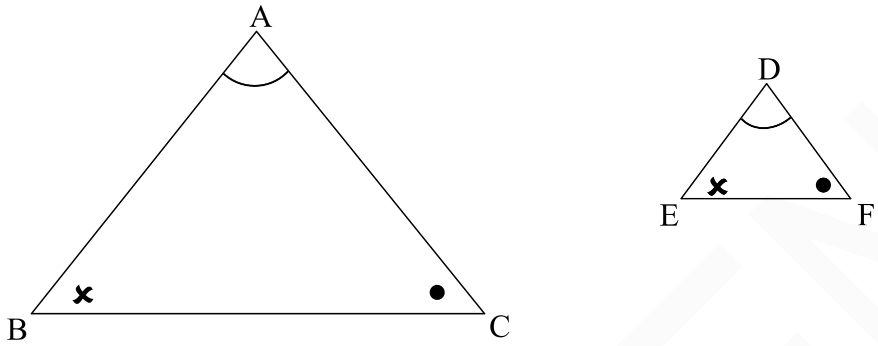


9.1 Determine, with reasons, the size of $\hat{N}_1$ in terms of x. (6)

9.2 Prove that BA is a tangent to the circle passing through K, B and N. (5)
[11]

QUESTION 10

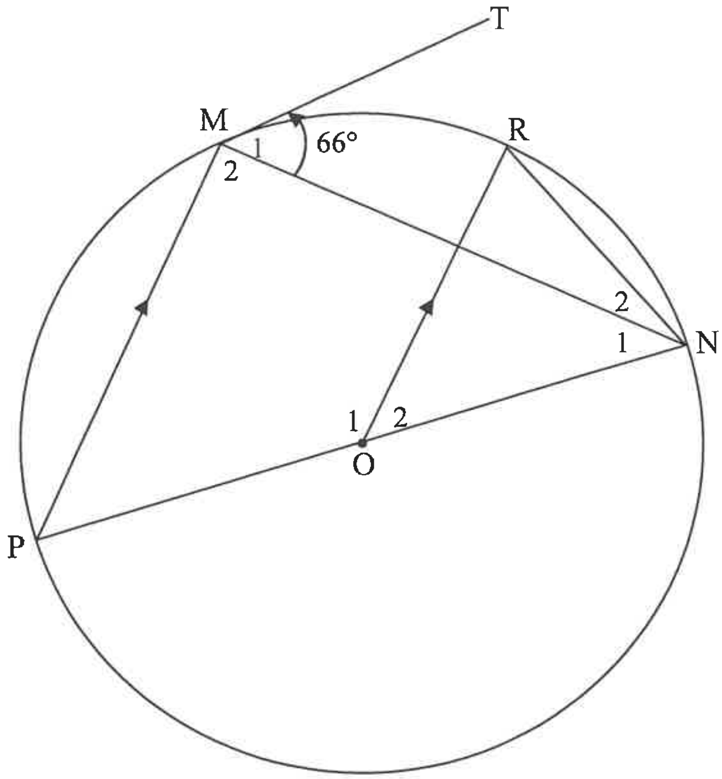
10.1 In the diagram, ΔABC and ΔDEF are drawn such that $\hat{A} = \hat{D}$, $\hat{B} = \hat{E}$ and $\hat{C} = \hat{F}$.



Use the diagram in the ANSWER BOOK to prove the theorem which states that if two triangles are equiangular, then the corresponding sides are in proportion, that is $\frac{AB}{DE} = \frac{AC}{DF}$. (6)

QUESTION 8

8.1 PON is a diameter of the circle centred at O. TM is a tangent to the circle at M, a point on the circle. R is another point on the circle such that OR || PM. NR and MN are drawn. Let $\hat{M}_1 = 66°$.



Calculate, with reasons, the size of EACH of the following angles:

8.1.1 $\hat{P}$ (2)

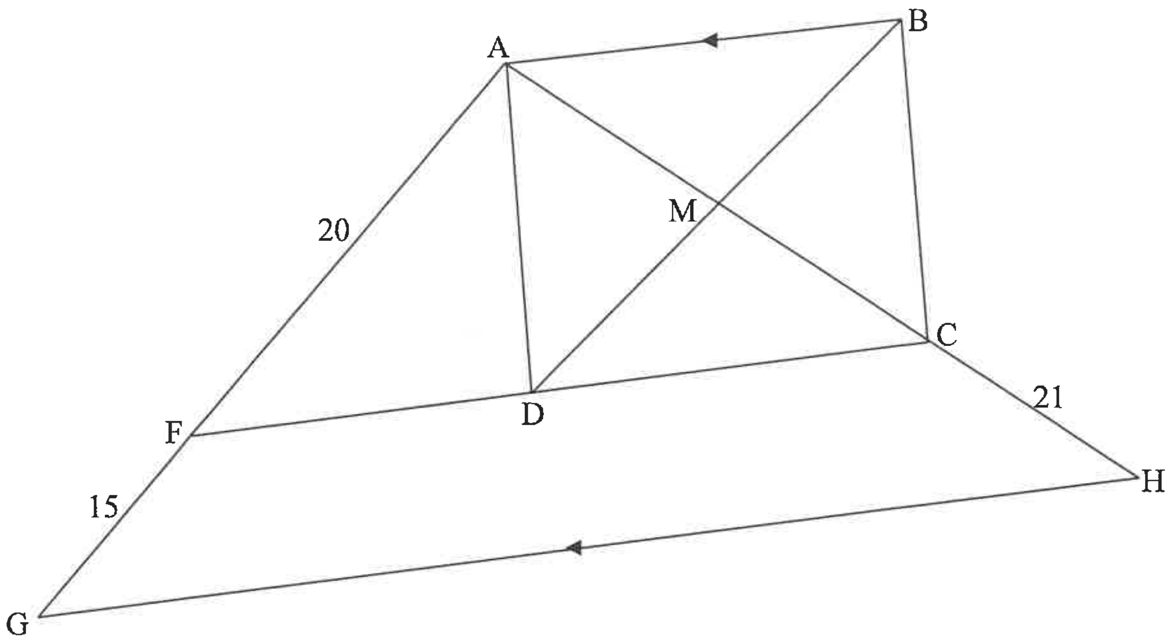
8.1.2 $\hat{M}_2$ (2)

8.1.3 $\hat{N}_1$ (1)

8.1.4 $\hat{O}_2$ (2)

8.1.5 $\hat{N}_2$ (3)

8.2 In the diagram, ΔAGH is drawn. F and C are points on AG and AH respectively such that AF = 20 units, FG = 15 units and CH = 21 units. D is a point on FC such that ABCD is a rectangle with AB also parallel to GH. The diagonals of ABCD intersect at M, a point on AH.



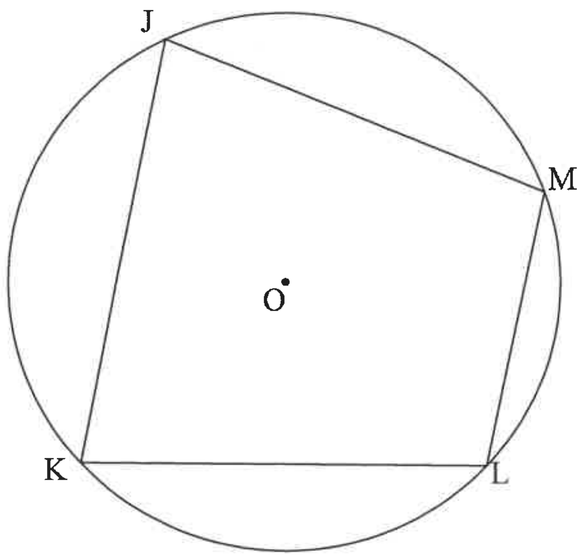
8.2.1 Explain why FC || GH. (1)

8.2.2 Calculate, with reasons, the length of DM. (5)

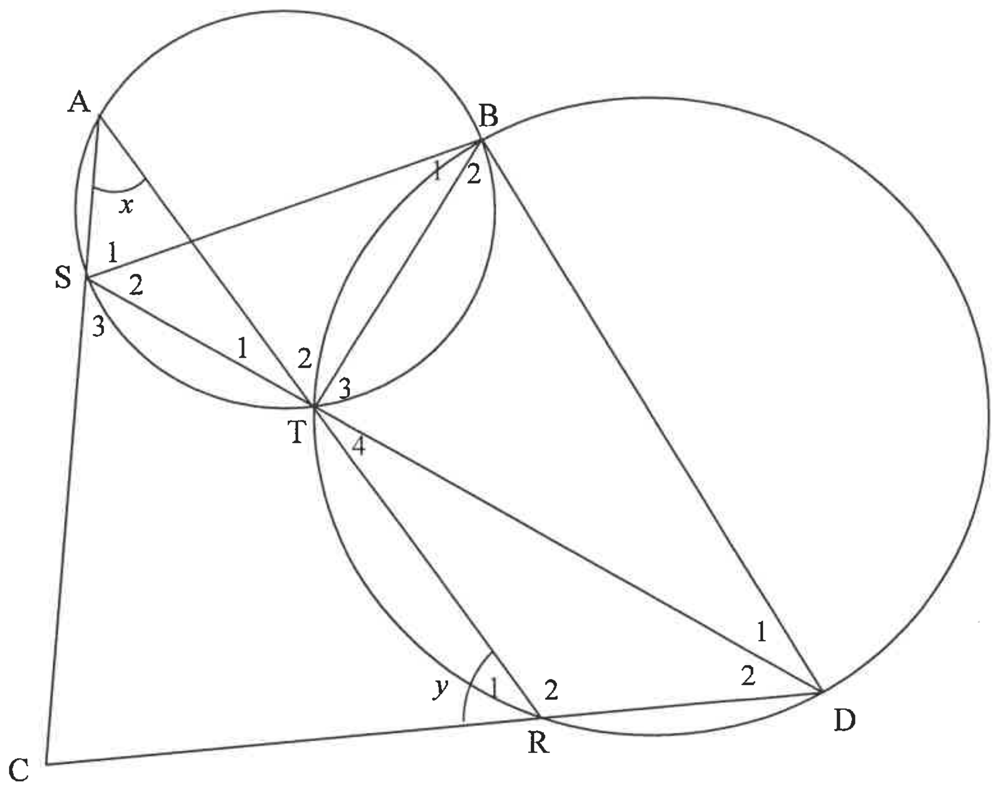
[16]

QUESTION 9

9.1 In the diagram, JKLM is a cyclic quadrilateral and the circle has centre O.
Prove the theorem which states that $\hat{J} + \hat{L} = 180°$. (5)



9.2 In the diagram, a smaller circle ABTS and a bigger circle BDRT are given. BT is a common chord. Straight lines STD and ATR are drawn. Chords AS and DR are produced to meet in C, a point outside the two circles. BS and BD are drawn. $\hat{A} = x$ and $\hat{R}_1 = y$.



9.2.1 Name, giving a reason, another angle equal to:

(a) x (2)

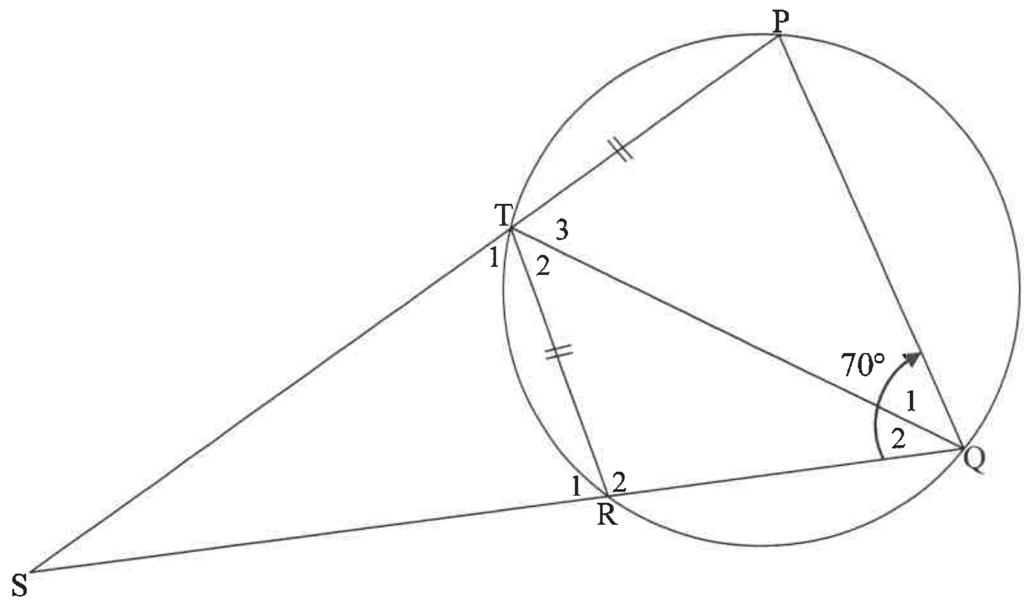
(b) y (2)

9.2.2 Prove that SCDB is a cyclic quadrilateral. (3)

9.2.3 It is further given that $\hat{D}_2 = 30°$ and $A\hat{S}T = 100°$.
Prove that SD is not a diameter of circle BDS. (4)
[16]

QUESTION 7

In the diagram, PQRT is a cyclic quadrilateral in a circle such that PT = TR. PT and QR are produced to meet in S. TQ is drawn. $\hat{SQP} = 70°$



7.1 Calculate, with reasons, the size of:

7.1.1 $\hat{T}_1$ (2)

7.1.2 $\hat{Q}_1$ (2)

7.2 If it is further given that PQ || TR:

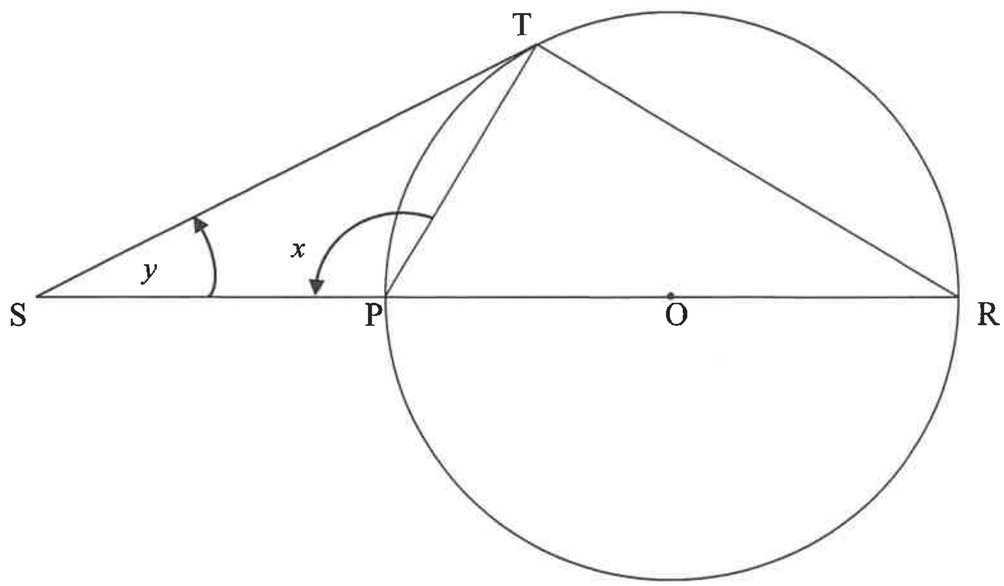
7.2.1 Calculate, with reasons, the size of $\hat{T}_2$ (2)

7.2.2 Prove that $\frac{TR}{TS} = \frac{RQ}{RS}$ (2)

[8]

QUESTION 8

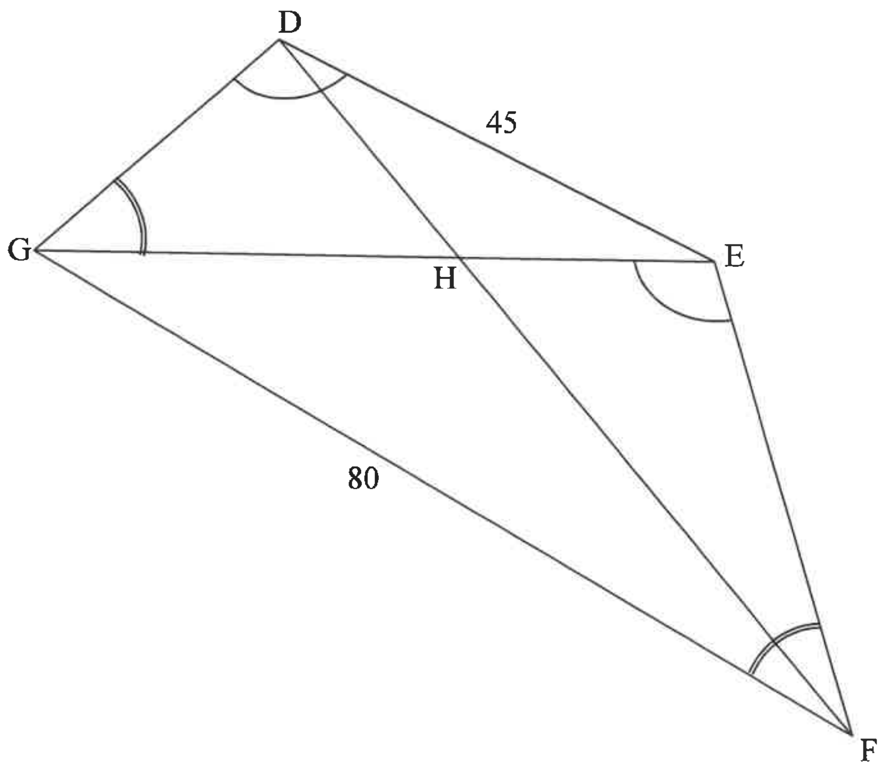
In the diagram, PR is a diameter of the circle with centre O. ST is a tangent to the circle at T and meets RP produced at S. $S\hat{P}T = x$ and $\hat{S} = y$.



Determine, with reasons, y in terms of x. **[6]**

QUESTION 9

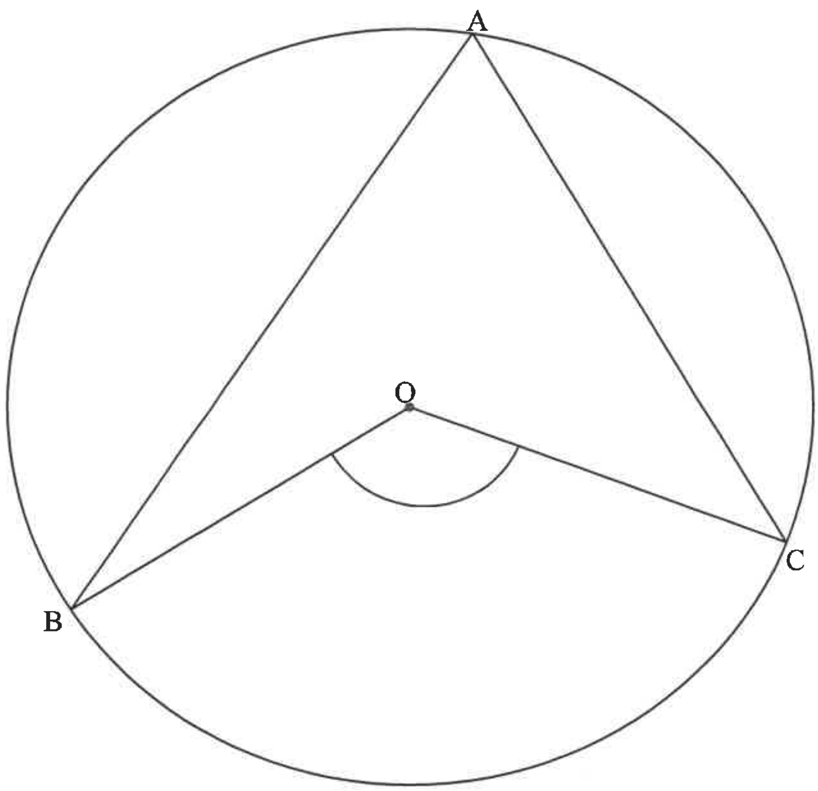
In the diagram, DEFG is a quadrilateral with $DE = 45$ and $GF = 80$. The diagonals GE and DF meet in H. $G\hat{D}E = F\hat{E}G$ and $D\hat{G}E = E\hat{F}G$.



9.1	Give a reason why $\Delta DEG \,|||\, \Delta EGF$.	(1)
9.2	Calculate the length of GE.	(3)
9.3	Prove that $\Delta DEH \,|||\, \Delta FGH$.	(3)
9.4	Hence, calculate the length of GH.	(3) **[10]**

QUESTION 10

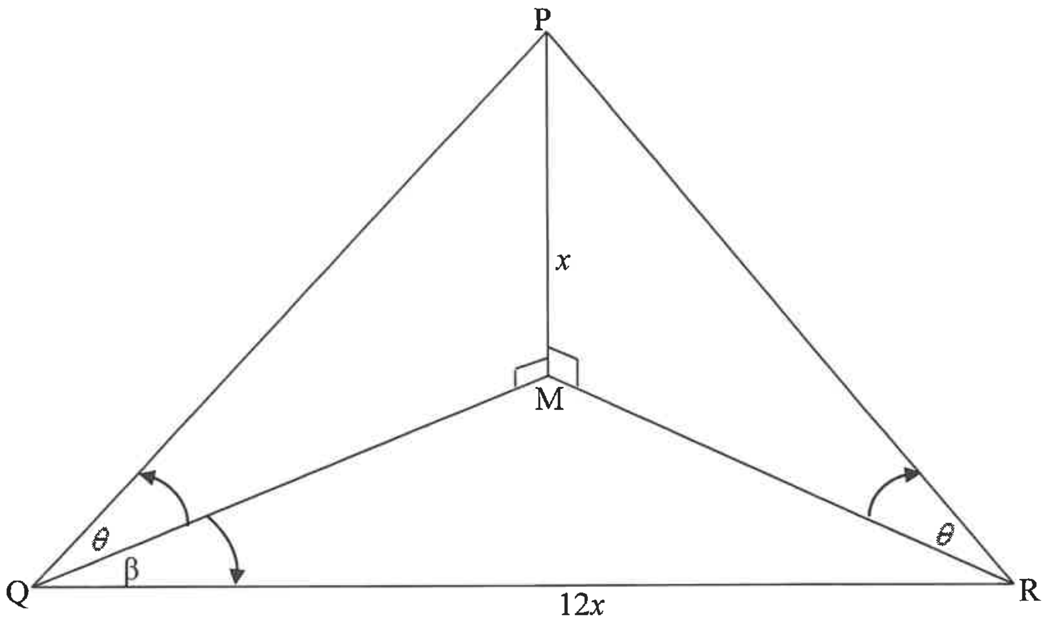
10.1 In the diagram, O is the centre of the circle with A, B and C drawn on the circle.



Prove the theorem which states that $B\hat{O}C = 2\hat{A}$. (5)

QUESTION 7

The captain of a boat at sea, at point Q, notices a lighthouse PM directly north of his position. He determines that the angle of elevation of P, the top of the lighthouse, from Q is θ and the height of the lighthouse is x metres. From point Q the captain sails $12x$ metres in a direction β degrees east of north to point R. From point R, he notices that the angle of elevation of P is also θ. Q, M and R lie in the same horizontal plane.



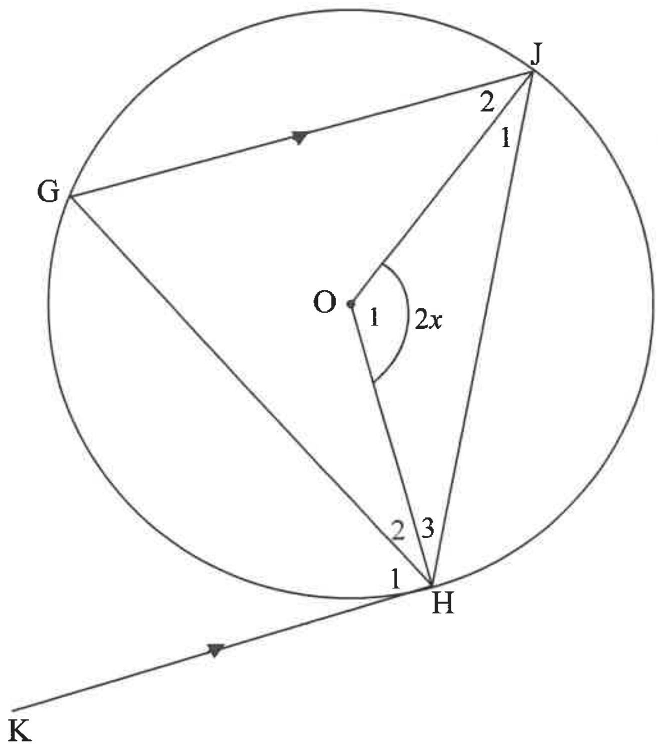
7.1 Write QM in terms of x and θ. (2)

7.2 Prove that $\tan\theta = \dfrac{\cos\beta}{6}$. (4)

7.3 If $\beta = 40°$ and QM = 60 metres, calculate the height of the lighthouse **to the nearest metre**. (3)
[9]

QUESTION 8

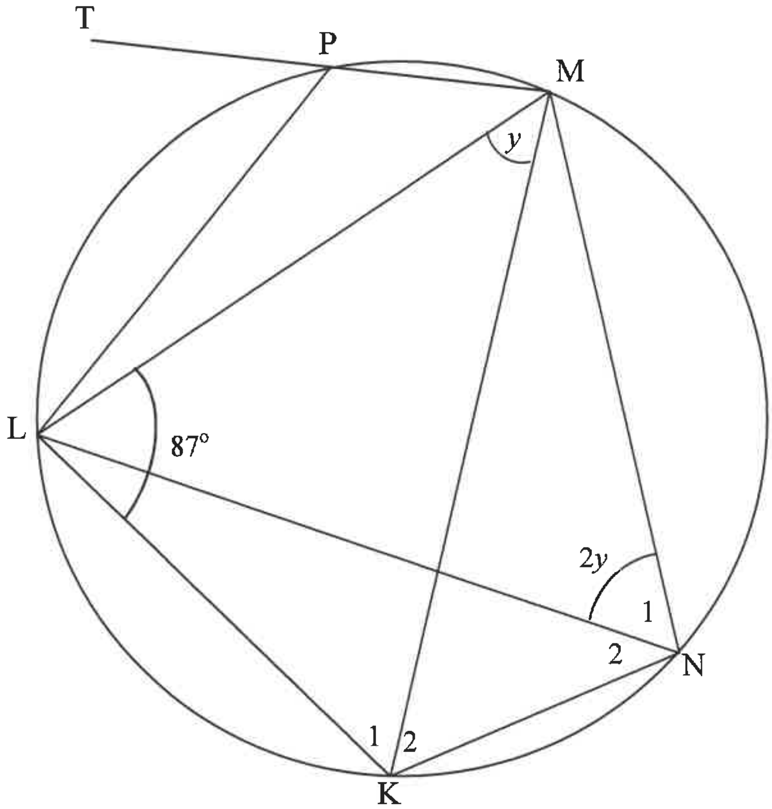
8.1 In the diagram, O is the centre of the circle. Radii OH and OJ are drawn. A tangent is drawn from K to touch the circle at H. ΔHGJ is drawn such that GJ $\parallel$ KH. $\hat{O}_1 = 2x$.



8.1.1 Name, giving reasons, THREE angles, each equal to x. (5)

8.1.2 Prove that $\hat{H}_2 = \hat{H}_3$. (3)

8.2 In the diagram, KLMN is a cyclic quadrilateral with $K\hat{L}M = 87°$. Diagonals LN and MK are drawn. P is a point on the circle and MP is produced to T, a point outside the circle. Chord LP is drawn. $L\hat{M}K = y$ and $\hat{N}_1 = 2y$.



8.2.1 Name, giving a reason, another angle equal to y. (2)

8.2.2 Calculate, giving reasons, the size of:

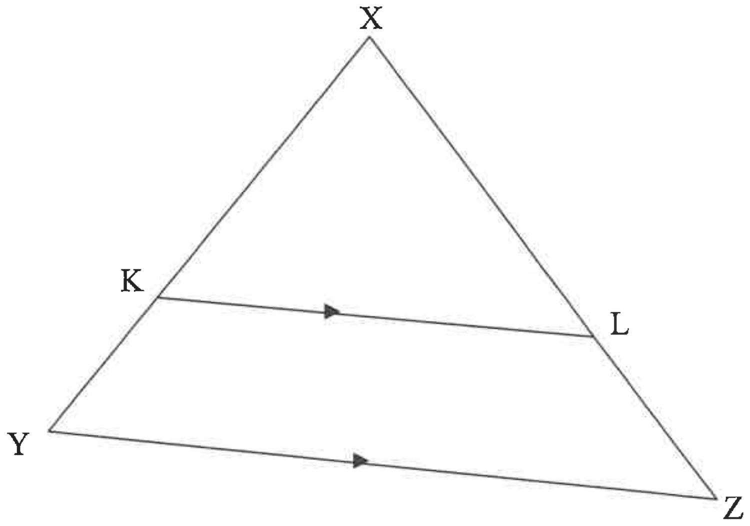
(a) y (3)

(b) $T\hat{P}L$ (2)

[15]

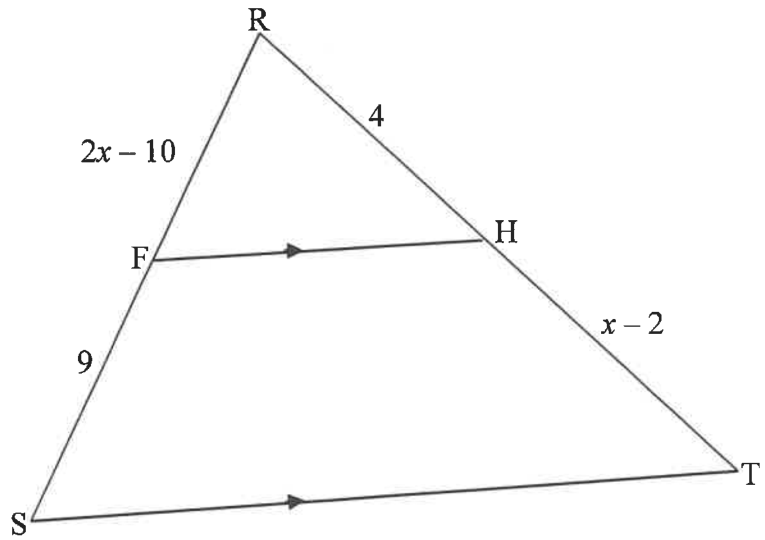
QUESTION 9

9.1 Use the diagram to prove the theorem which states that a line drawn parallel to one side of a triangle divides the other two sides proportionally, that is prove that $\frac{XK}{KY} = \frac{XL}{LZ}$.



(5)

9.2 In ΔRST, F is a point on RS and H is a point on RT such that $FH \parallel ST$.
$RF = 2x - 10$, $FS = 9$, $RH = 4$ and $HT = x - 2$.



9.2.1 Determine, giving a reason, the value of x. (5)

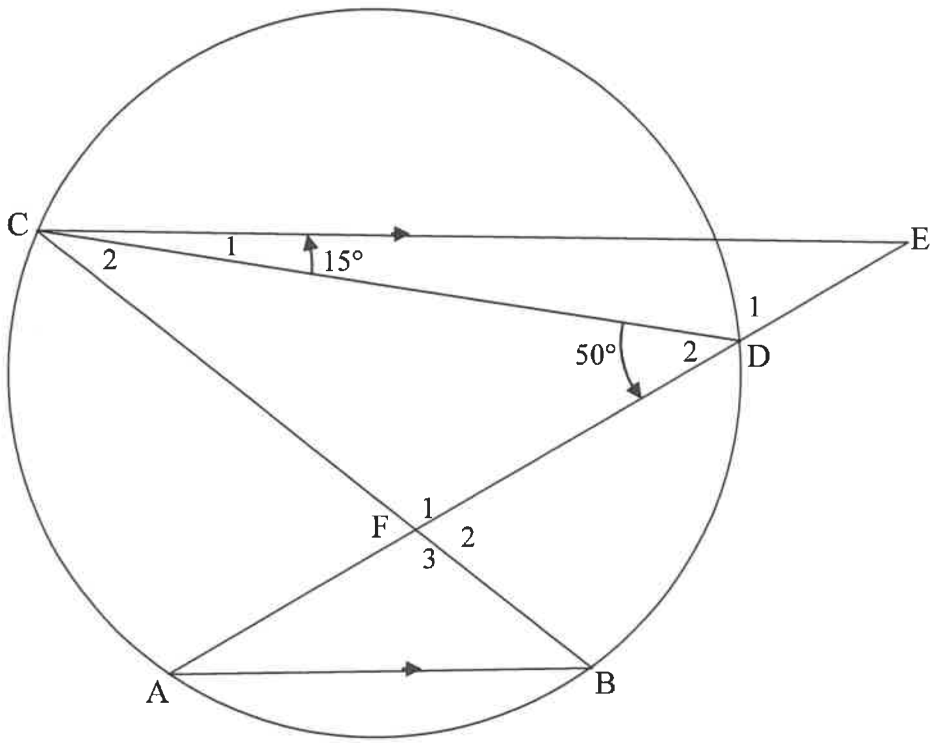
9.2.2 Determine the ratio: $\frac{\text{area } \Delta RFH}{\text{area } \Delta RST}$. (4)

[14]

Give reasons for your statements in QUESTIONS 8, 9, 10 and 11.

QUESTION 8

In the diagram, points A, B, D and C lie on a circle. CE || AB with E on AD produced. Chords CB and AD intersect at F. $\hat{D}_2 = 50°$ and $\hat{C}_1 = 15°$.



8.1 Calculate, with reasons, the size of:

8.1.1 $\hat{A}$ (3)

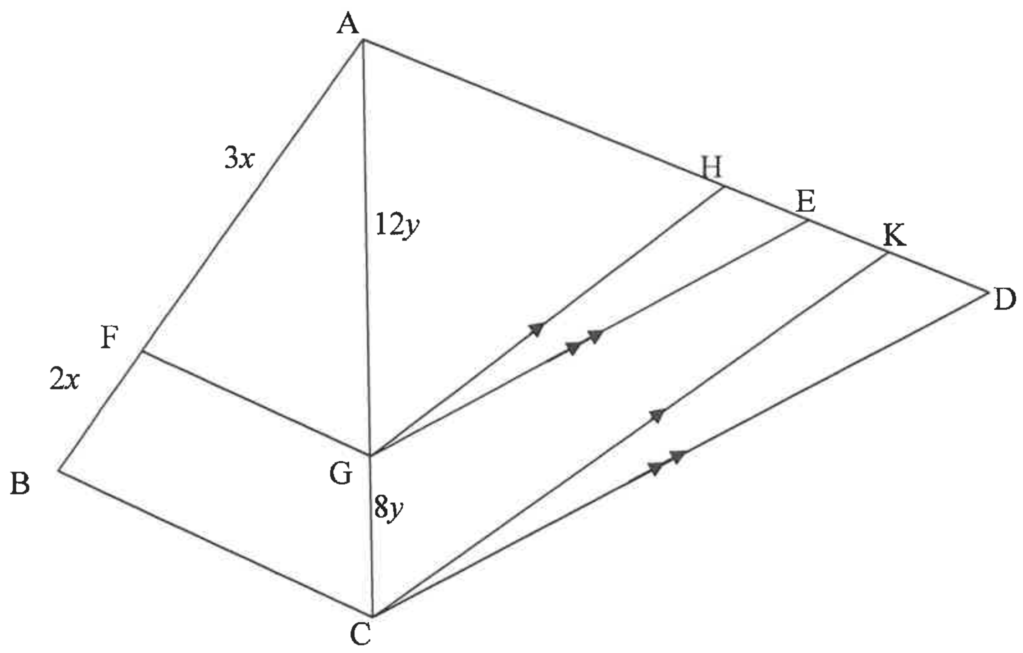
8.1.2 $\hat{C}_2$ (2)

8.2 Prove, with a reason, that CF is a tangent to the circle passing through points C, D and E. (2)

[7]

QUESTION 9

In the diagram, ΔABC and ΔACD are drawn. F and G are points on sides AB and AC respectively such that AF = $3x$, FB = $2x$, AG = $12y$ and GC = $8y$. H, E and K are points on side AD such that GH || CK and GE || CD.



9.1 Prove that:

9.1.1 FG || BC (2)

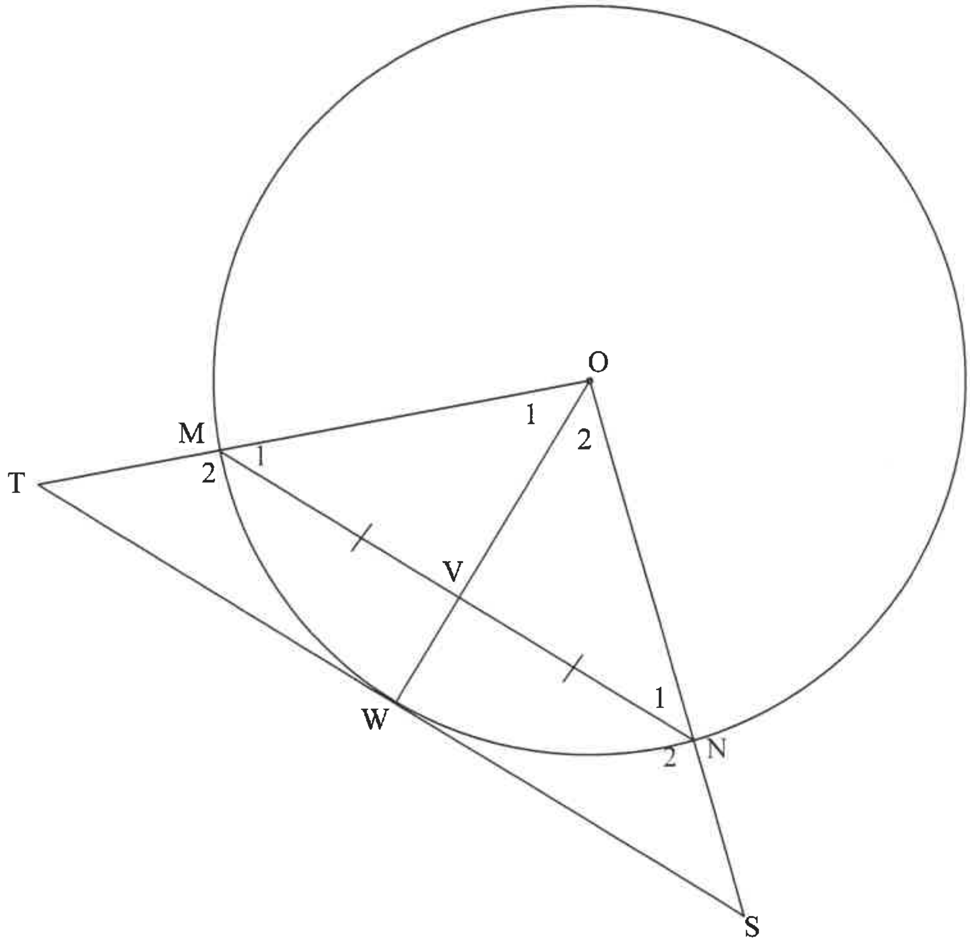
9.1.2 $\frac{AH}{HK} = \frac{AE}{ED}$ (3)

9.2 If it is further given that AH = 15 and ED = 12, calculate the length of EK. (5)

[10]

QUESTION 10

In the diagram, W is a point on the circle with centre O. V is a point on OW. Chord MN is drawn such that MV = VN. The tangent at W meets OM produced at T and ON produced at S.



10.1 Give a reason why $OV \perp MN$. (1)

10.2 Prove that:

10.2.1 $MN \parallel TS$ (2)

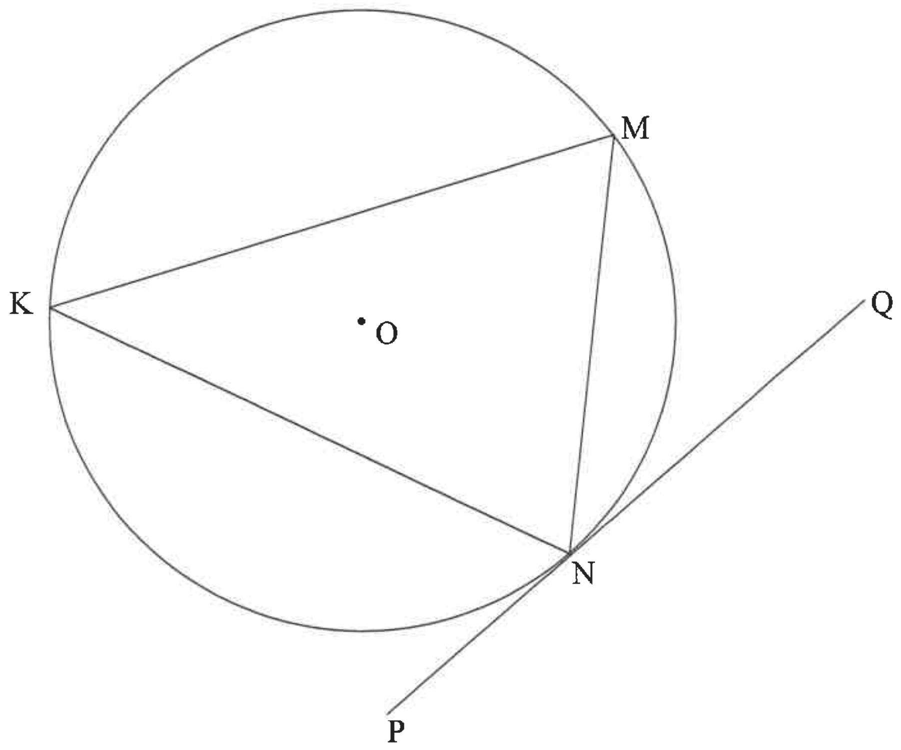
10.2.2 TMNS is a cyclic quadrilateral (4)

10.2.3 $OS \cdot MN = 2ON \cdot WS$ (5)

[12]

QUESTION 11

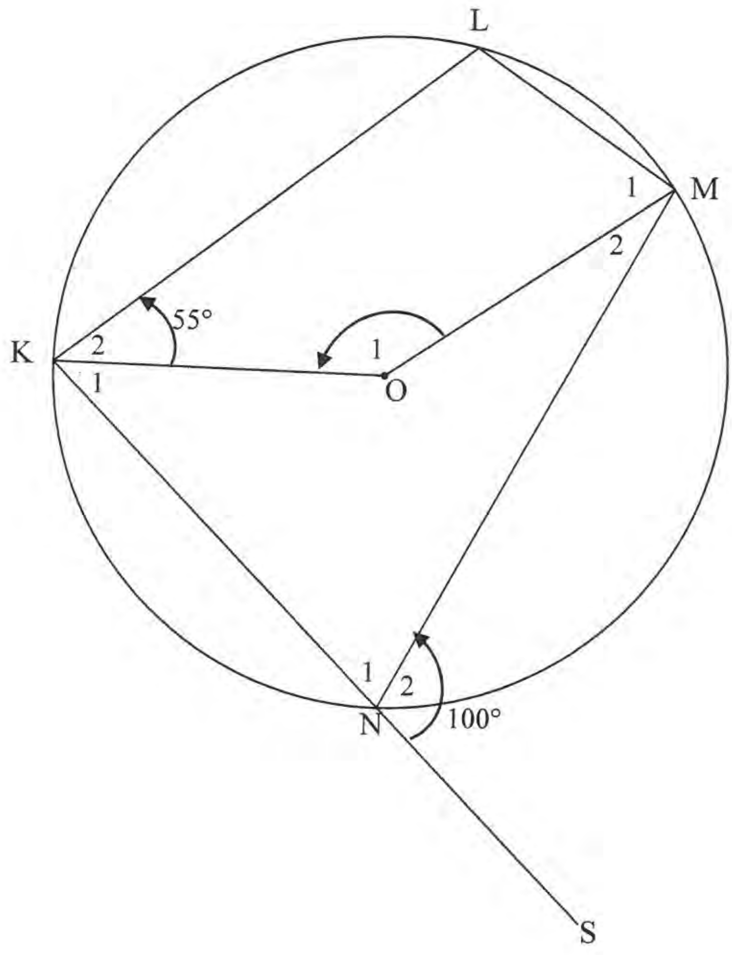
11.1 In the diagram, chords KM, MN and KN are drawn in the circle with centre O. PNQ is the tangent to the circle at N.



Prove the theorem which states that $M\hat{N}Q = \hat{K}$. (5)

QUESTION 8

In the diagram, O is the centre of circle KLMN and KO and OM are joined. Chord KN is produced to S. $\hat{K}_2 = 55°$ and $\hat{N}_2 = 100°$.



Determine, with reasons, the size of the following:

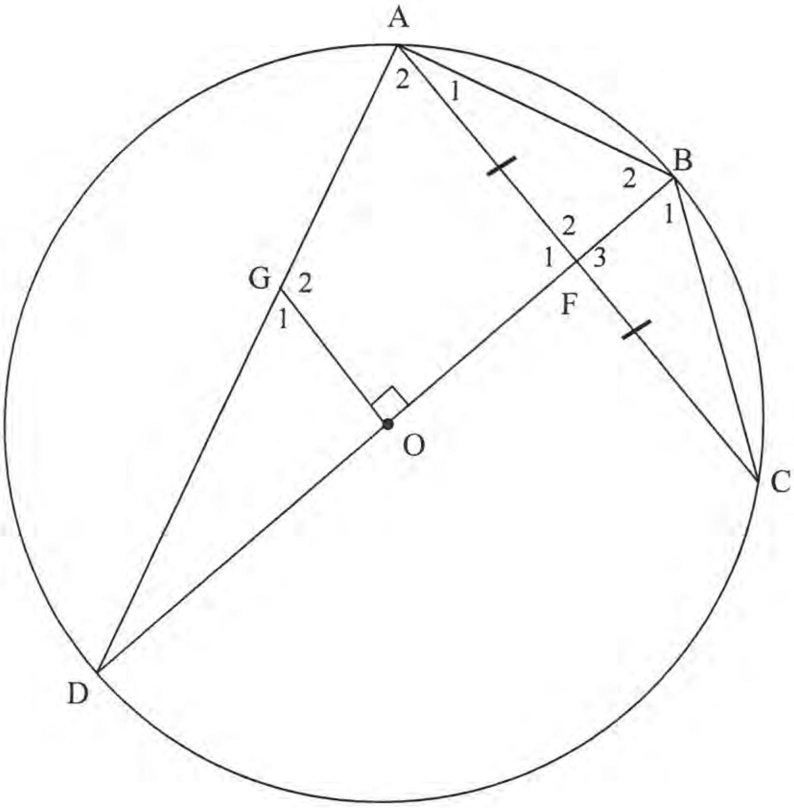
8.1 $\hat{L}$ (2)

8.2 $\hat{O}_1$ (3)

8.3 $\hat{M}_1$ (2)
[7]

QUESTION 9

In the diagram, O is the centre of circle ABCD and BOD is a diameter. F, the midpoint of chord AC, lies on BOD. G is a point on AD such that GO $\perp$ DB.



9.1 Give a reason why:

9.1.1 $D\hat{A}B = 90°$ (1)

9.1.2 AGOB is a cyclic quadrilateral (1)

9.2 Prove that:

9.2.1 AC $\parallel$ GO (3)

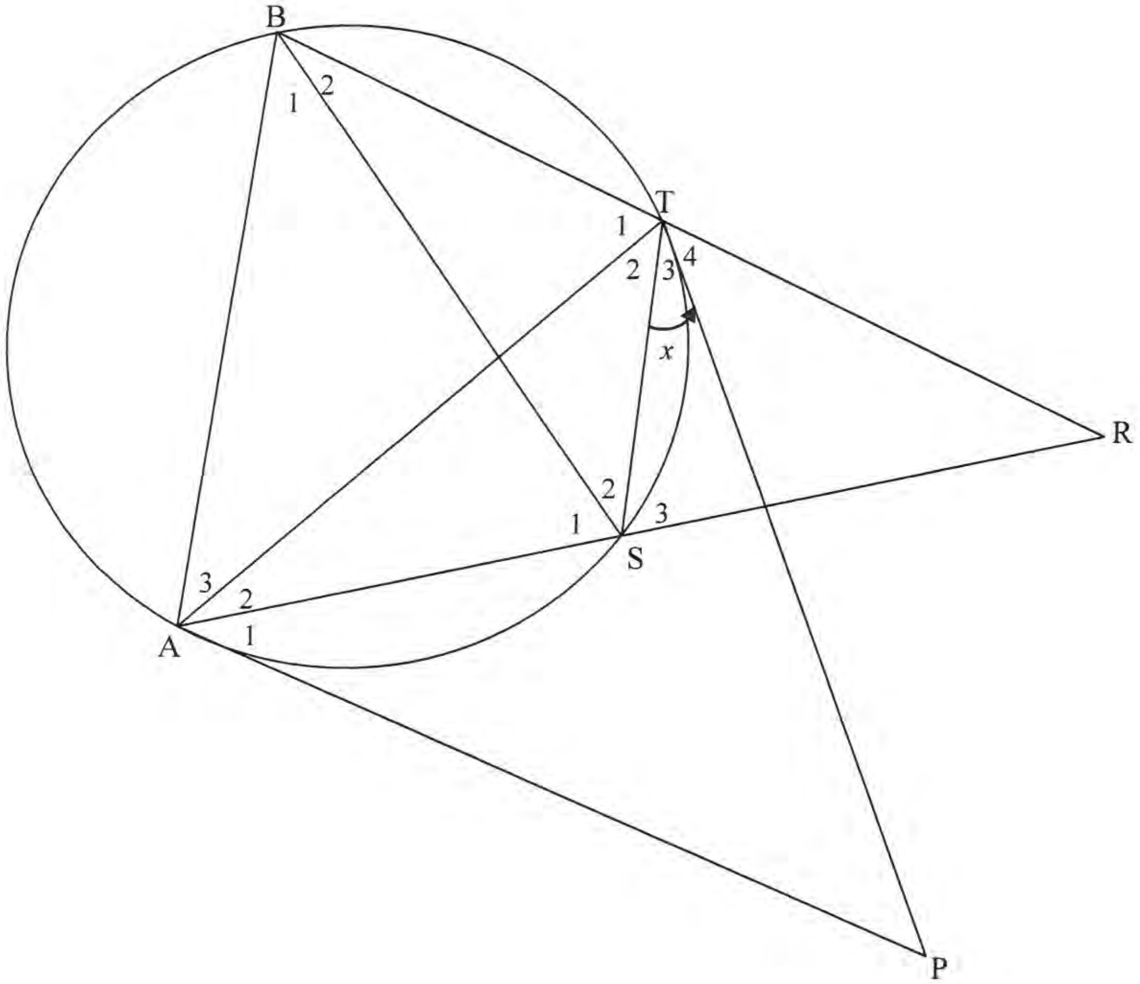
9.2.2 $\hat{G}_1 = \hat{B}_1$ (4)

9.3 If it is given that $FB = \frac{2}{5}r$, where r is the radius of the circle, determine, with reasons, the ratio of $\frac{DG}{DA}$. (3)

[12]

QUESTION 10

In the diagram, PA and PT are tangents to a circle at A and T respectively. B and S are points on the circle such that BT produced and AS produced meet at R and BR = AR.
BS, AT and TS are drawn. $\hat{T}_3 = x$.



10.1 Give a reason why $\hat{T}_3 = \hat{A}_2 = x$. (1)

10.2 Prove that:

10.2.1 AB || ST (5)

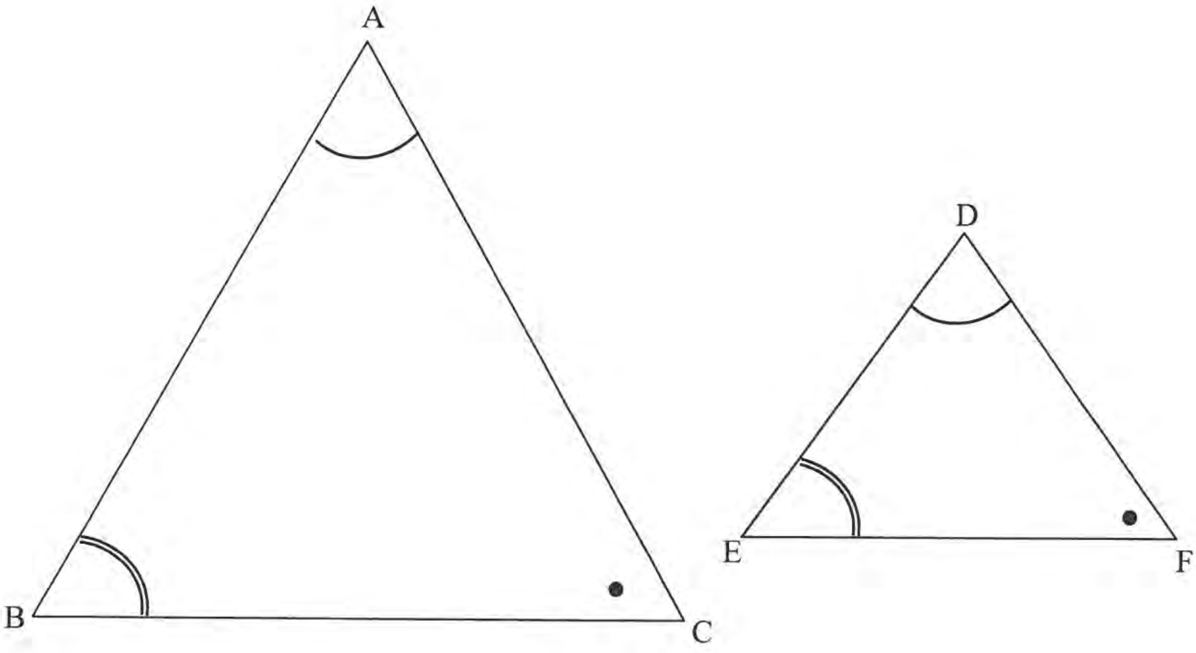
10.2.2 $\hat{T}_4 = \hat{A}_1$ (5)

10.2.3 RTAP is a cyclic quadrilateral (2)

[13]

QUESTION 11

11.1 In the diagram, ΔABC and ΔDEF are drawn with $\hat{A} = \hat{D}$, $\hat{B} = \hat{E}$ and $\hat{C} = \hat{F}$.

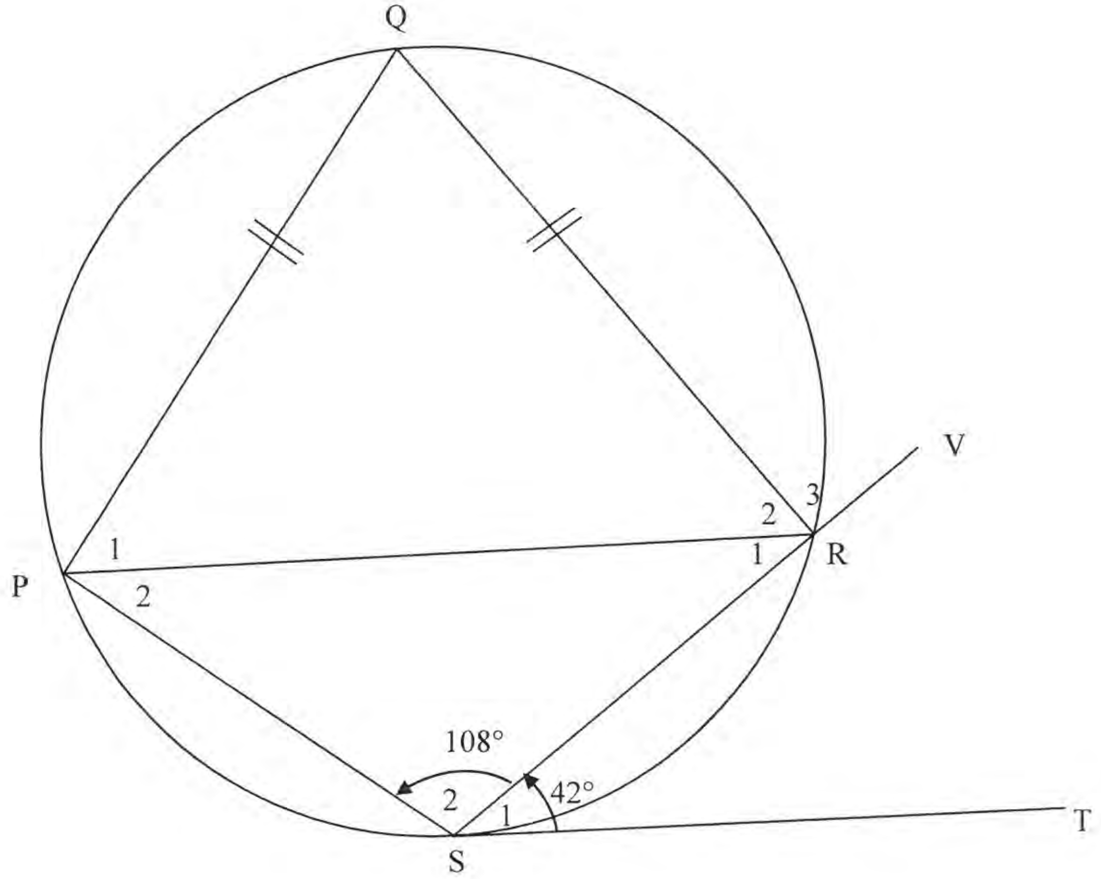


Prove the theorem which states that if two triangles, ΔABC and ΔDEF, are equiangular, then $\frac{DE}{AB} = \frac{DF}{AC}$. (6)

Give reasons for ALL statements and calculations in QUESTIONS 8, 9, 10 and 11.

QUESTION 8

In the diagram, PQRS is a cyclic quadrilateral. ST is a tangent to the circle at S and chord SR is produced to V. PQ = QR, $\hat{S}_1 = 42°$ and $\hat{S}_2 = 108°$.



Determine, with reasons, the size of the following angles:

8.1 $\hat{Q}$ (2)

8.2 $\hat{R}_2$ (2)

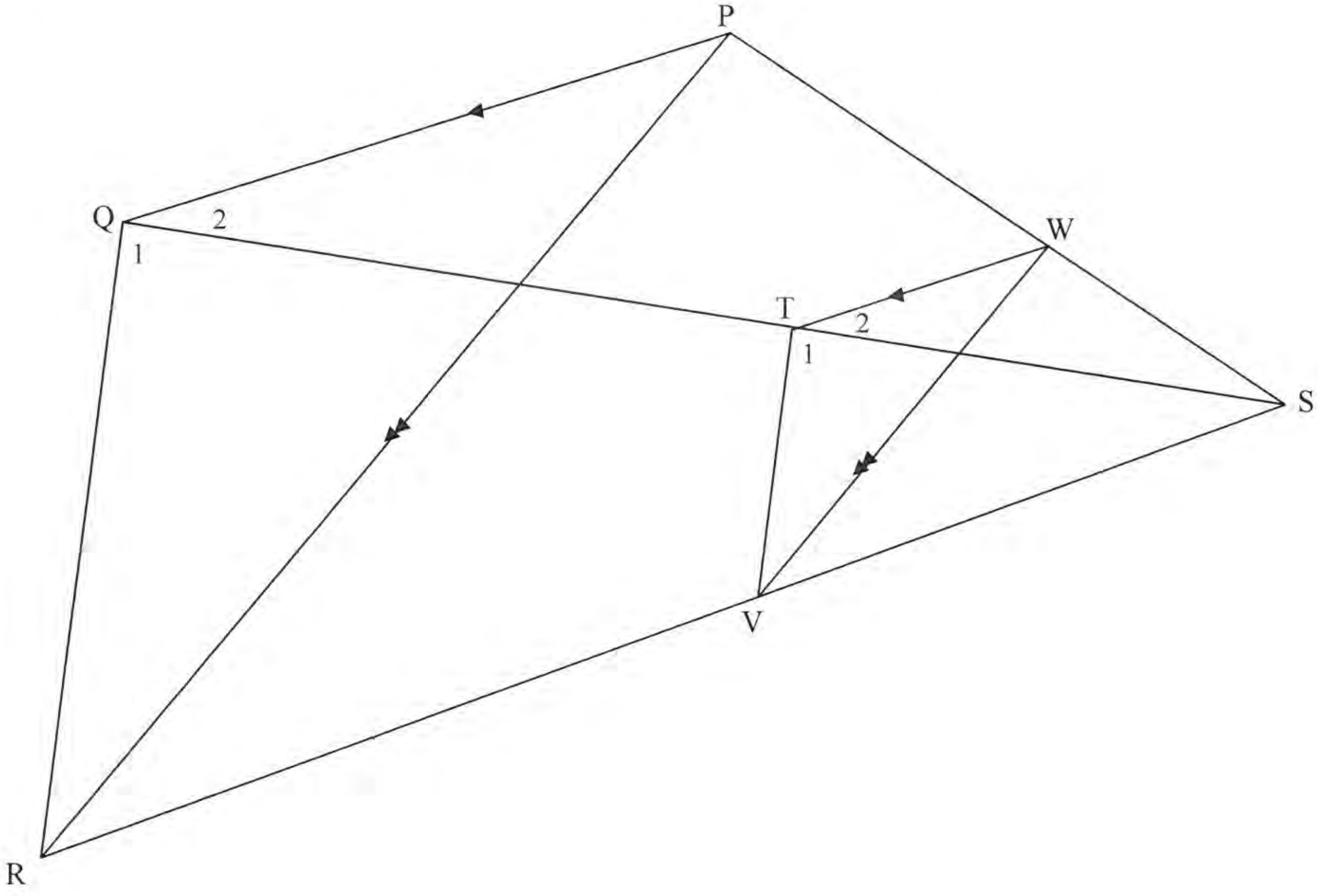
8.3 $\hat{P}_2$ (2)

8.4 $\hat{R}_3$ (2)

[8]

QUESTION 9

In the diagram, PQRS is a quadrilateral with diagonals PR and QS drawn. W is a point on PS. WT is parallel to PQ with T on QS. WV is parallel to PR with V on RS. TV is drawn. PW : WS = 3 : 2.



9.1 Write down the value of the following ratios:

9.1.1 $\frac{ST}{TQ}$ (2)

9.1.2 $\frac{SV}{VR}$ (1)

9.2 Prove that $\hat{T}_1 = \hat{Q}_1$. (4)

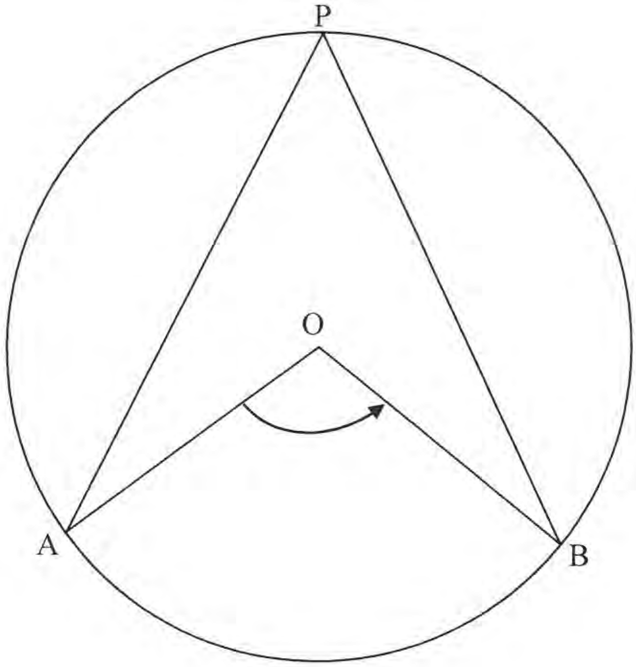
9.3 Complete the following statement: $\Delta VWS \ ||| \ \Delta \ldots$ (1)

9.4 Determine WV : PR. (2)

[10]

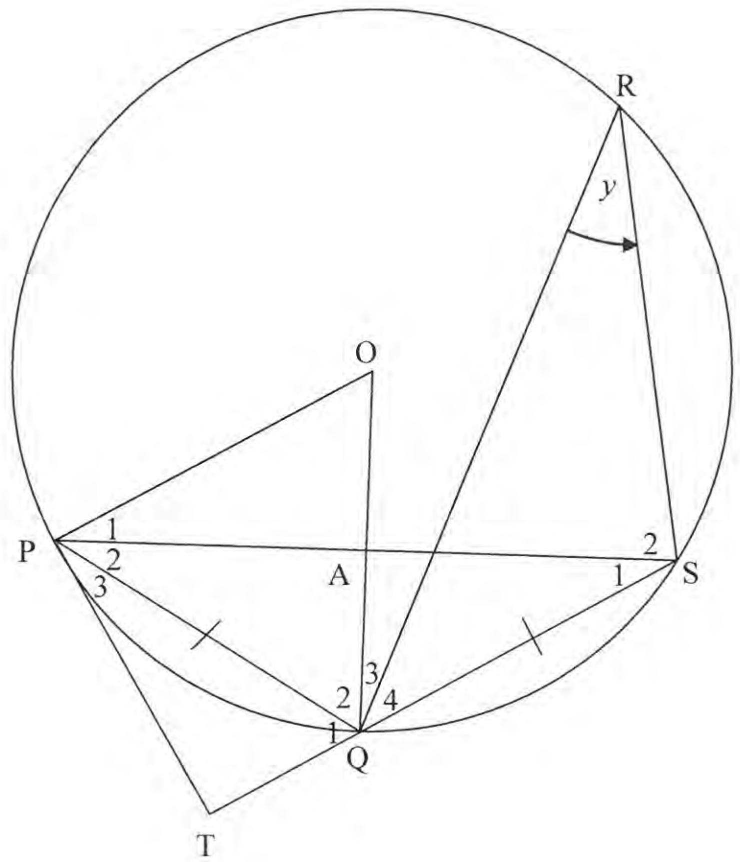
QUESTION 10

10.1 In the diagram, O is the centre of the circle and P is a point on the circumference of the circle. Arc AB subtends $A\hat{O}B$ at the centre of the circle and $A\hat{P}B$ at the circumference of the circle.



Use the diagram to prove the theorem that states that $A\hat{O}B = 2A\hat{P}B$. (5)

10.2 In the diagram, O is the centre of the circle and P, Q, S and R are points on the circle. PQ = QS and $Q\hat{R}S = y$. The tangent at P meets SQ produced at T. OQ intersects PS at A.



10.2.1 Give a reason why $\hat{P}_2 = y$. (1)

10.2.2 Prove that PQ bisects $T\hat{P}S$. (4)

10.2.3 Determine $P\hat{O}Q$ in terms of y. (2)

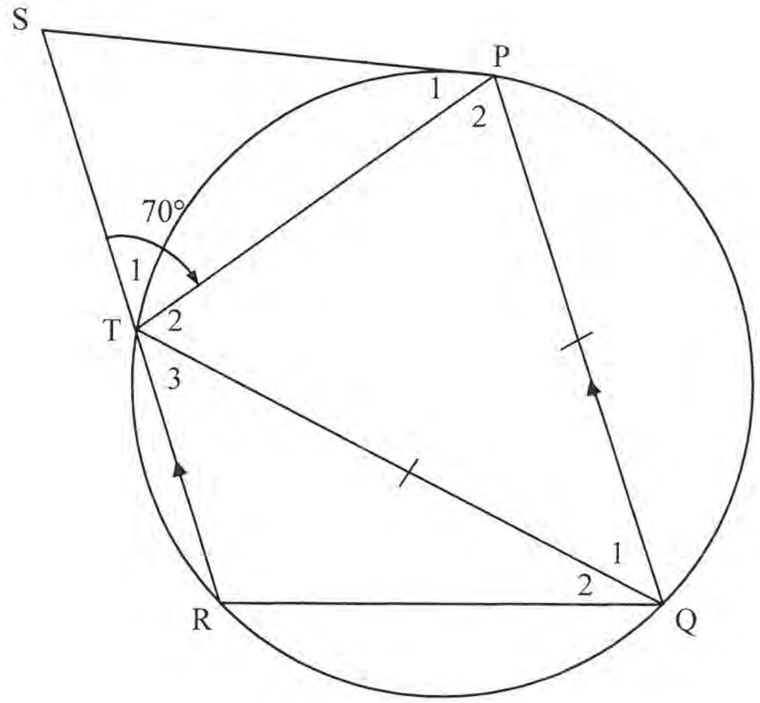
10.2.4 Prove that PT is a tangent to the circle that passes through points P, O and A. (2)

10.2.5 Prove that $O\hat{A}P = 90°$. (5)
[19]

Give reasons for ALL statements and calculations in QUESTIONS 8, 9 and 10.

QUESTION 8

8.1 In the diagram below PQRT is a cyclic quadrilateral having RT $\parallel$ QP. The tangent at P meets RT produced at S. QP = QT and $P\hat{T}S = 70°$.



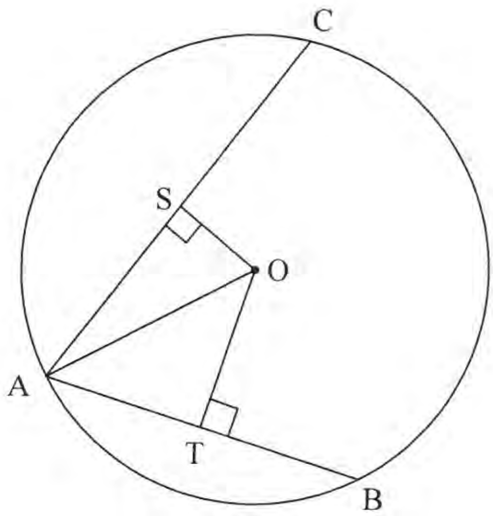
8.1.1 Give a reason why $\hat{P}_2 = 70°$. (1)

8.1.2 Calculate, with reasons, the size of:

(a) $\hat{Q}_1$ (3)

(b) $\hat{P}_1$ (2)

8.2 A, B and C are points on the circle having centre O. S and T are points on AC and AB respectively such that $OS \perp AC$ and $OT \perp AB$. AB = 40 and AC = 48.



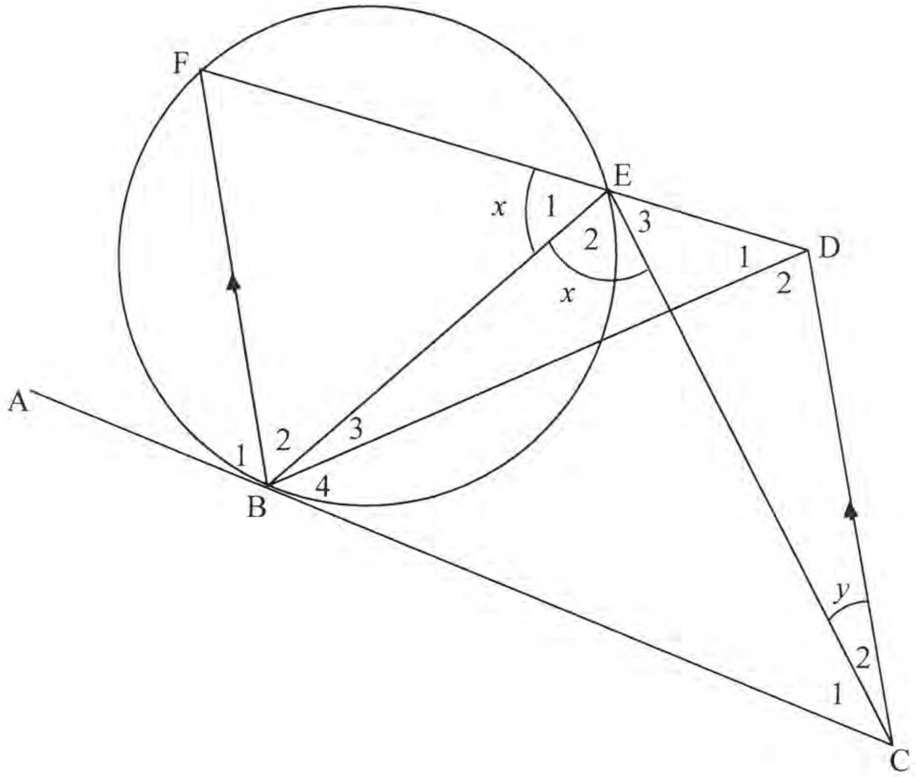
8.2.1 Calculate AT. (1)

8.2.2 If $OS = \frac{7}{15}OT$, calculate the radius OA of the circle. (5)

[12]

QUESTION 9

ABC is a tangent to the circle BFE at B. From C a straight line is drawn parallel to BF to meet FE produced at D. EC and BD are drawn. $\hat{E}_1 = \hat{E}_2 = x$ and $\hat{C}_2 = y$.



9.1 Give a reason why EACH of the following is TRUE:

9.1.1 $\hat{B}_1 = x$ (1)

9.1.2 $B\hat{C}D = \hat{B}_1$ (1)

9.2 Prove that BCDE is a cyclic quadrilateral. (2)

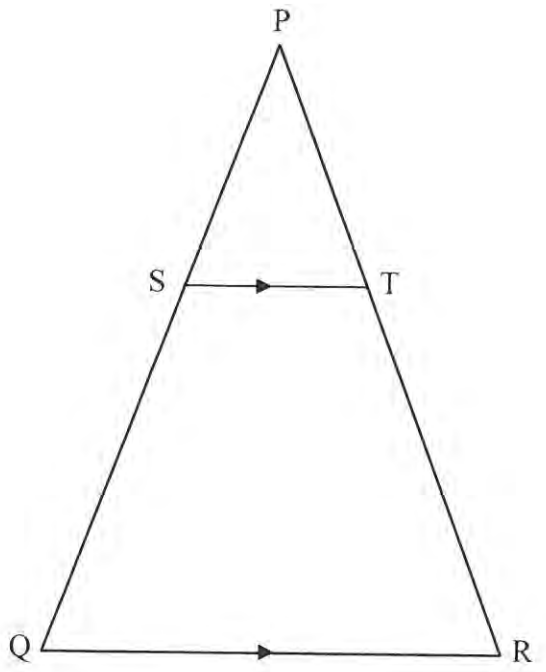
9.3 Which TWO other angles are each equal to x? (2)

9.4 Prove that $\hat{B}_2 = \hat{C}_1$. (3)

[9]

QUESTION 10

10.1 In the diagram ΔPQR is drawn. S and T are points on sides PQ and PR respectively such that $ST \parallel QR$.

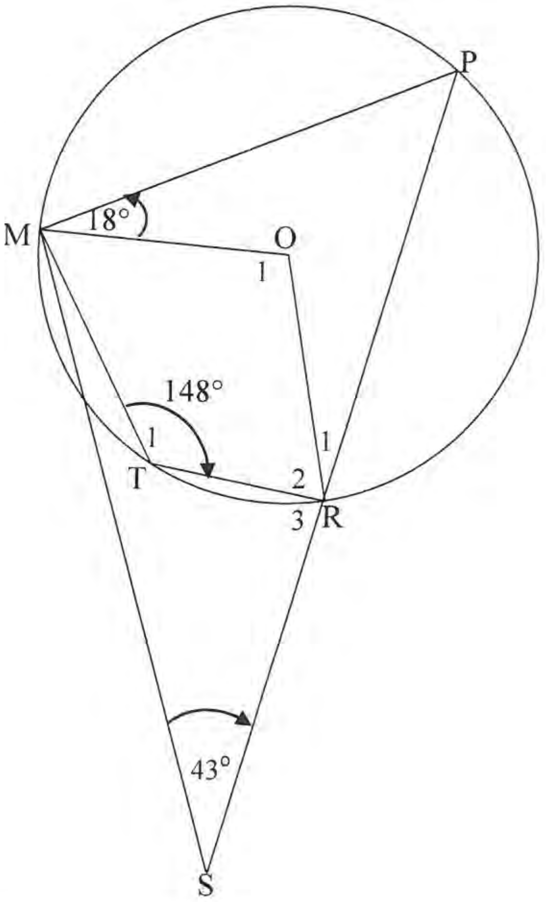


Prove the theorem which states that $\frac{PS}{SQ} = \frac{PT}{TR}$. (6)

Give reasons for ALL statements in QUESTIONS 8, 9, 10 and 11.

QUESTION 8

8.1 In the diagram below, P, M, T and R are points on a circle having centre O. PR produced meets MS at S. Radii OM and OR and the chords MT and TR are drawn. $\hat{T}_1 = 148°$, $P\hat{M}O = 18°$ and $\hat{S} = 43°$



Calculate, with reasons, the size of:

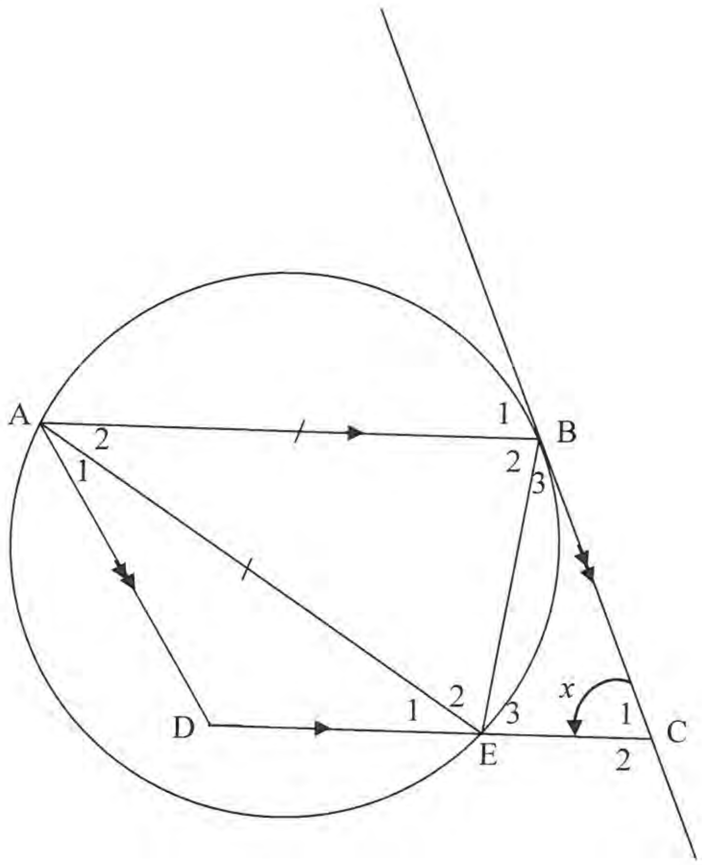
8.1.1 $\hat{P}$ (2)

8.1.2 $\hat{O}_1$ (2)

8.1.3 $O\hat{M}S$ (2)

8.1.4 $\hat{R}_3$ if it is given that $T\hat{M}S = 6°$ (2)

8.2 In the diagram below, the circle passes through A, B and E. ABCD is a parallelogram. BC is a tangent to the circle at B. AE = AB. Let $\hat{C}_1 = x$



8.2.1 Give a reason why $\hat{B}_1 = x$ (1)

8.2.2 Name, with reasons, THREE other angles equal in size to x. (6)

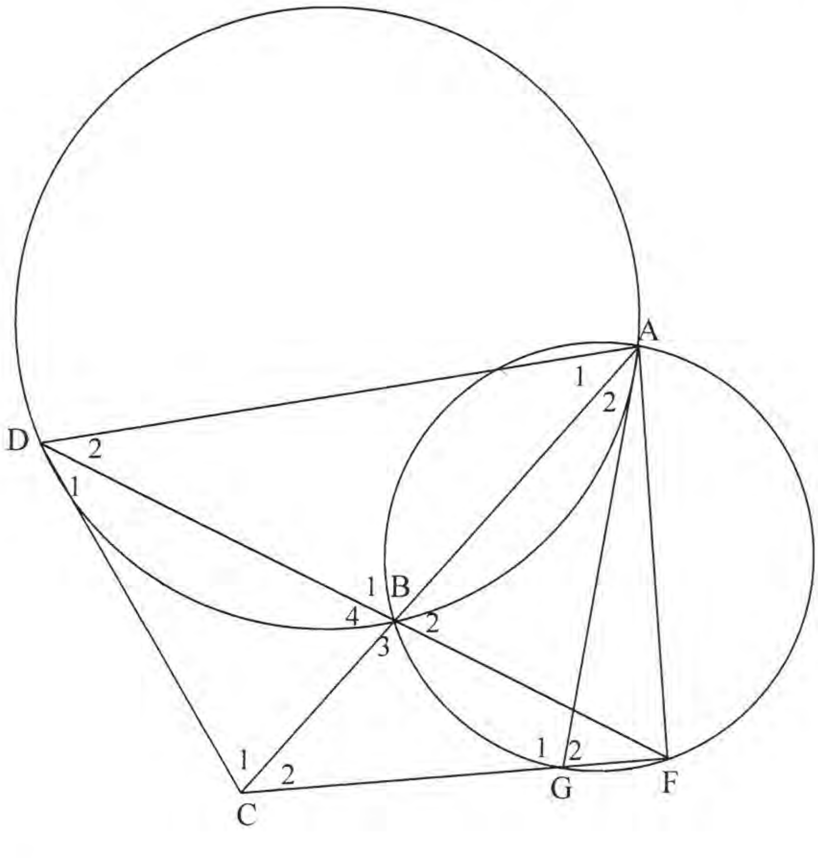
8.2.3 Prove that ABED is a cyclic quadrilateral. (3)
[18]

QUESTION 9

9.1 Complete the statement so that it is TRUE:

The angle between the tangent to a circle and the chord drawn from the point of contact is equal to the angle ... (1)

9.2 In the diagram below, two unequal circles intersect at A and B. AB is produced to C such that CD is a tangent to the circle ABD at D. F and G are points on the smaller circle such that CGF and DBF are straight lines. AD and AG are drawn.



Prove that:

9.2.1 $\hat{B}_4 = \hat{D}_1 + \hat{D}_2$ (4)

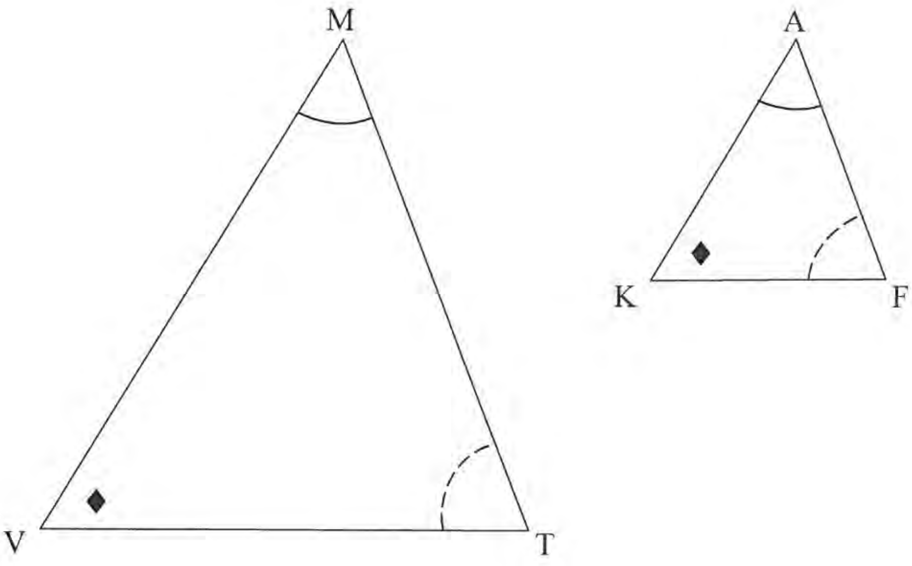
9.2.2 AGCD is a cyclic quadrilateral (4)

9.2.3 DC = CF (4)

[13]

QUESTION 10

10.1 In the diagram below, ΔMVT and ΔAKF are drawn such that $\hat{M} = \hat{A}$, $\hat{V} = \hat{K}$ and $\hat{T} = \hat{F}$

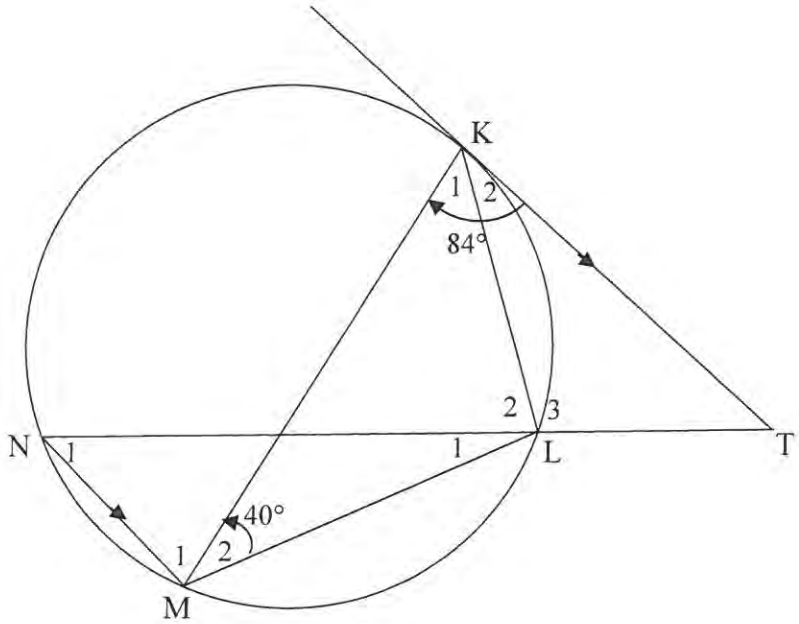


Use the diagram in the ANSWER BOOK to prove the theorem which states that if two triangles are equiangular, then the corresponding sides are in proportion, that is $\frac{MV}{AK} = \frac{MT}{AF}$ (7)

Give reasons for ALL statements in QUESTIONS 8, 9 and 10.

QUESTION 8

8.1 In the diagram below, tangent KT to the circle at K is parallel to the chord NM. NT cuts the circle at L. ΔKML is drawn. $\hat{M}_2 = 40°$ and $M\hat{K}T = 84°$.



Determine, giving reasons, the size of:

8.1.1 $\hat{K}_2$ (2)

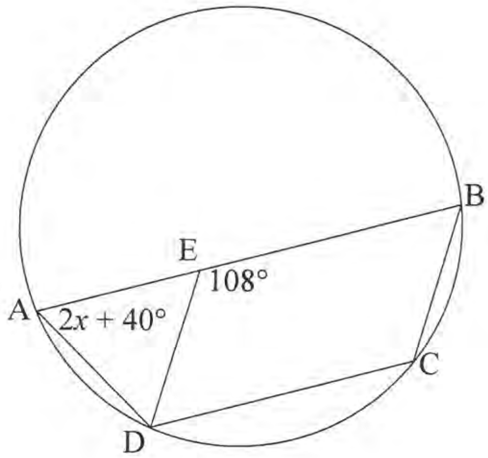
8.1.2 $\hat{N}_1$ (3)

8.1.3 $\hat{T}$ (2)

8.1.4 $\hat{L}_2$ (2)

8.1.5 $\hat{L}_1$ (1)

8.2 In the diagram below, AB and DC are chords of a circle. E is a point on AB such that BCDE is a parallelogram. $D\hat{E}B = 108°$ and $D\hat{A}E = 2x + 40°$.

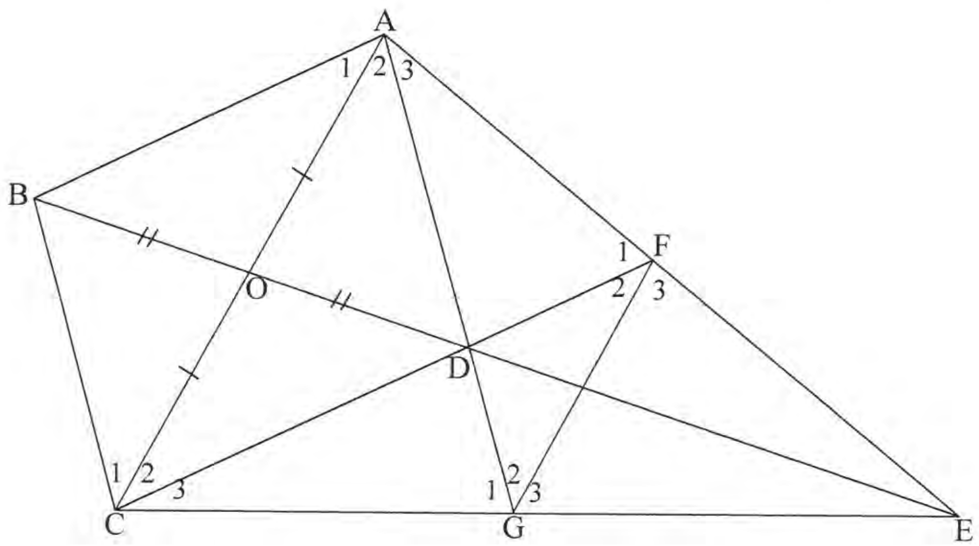


Calculate, giving reasons, the value of x. (5)

[15]

QUESTION 9

In the diagram below, EO bisects side AC of ΔACE. EDO is produced to B such that $BO = OD$. AD and CD produced meet EC and EA at G and F respectively.

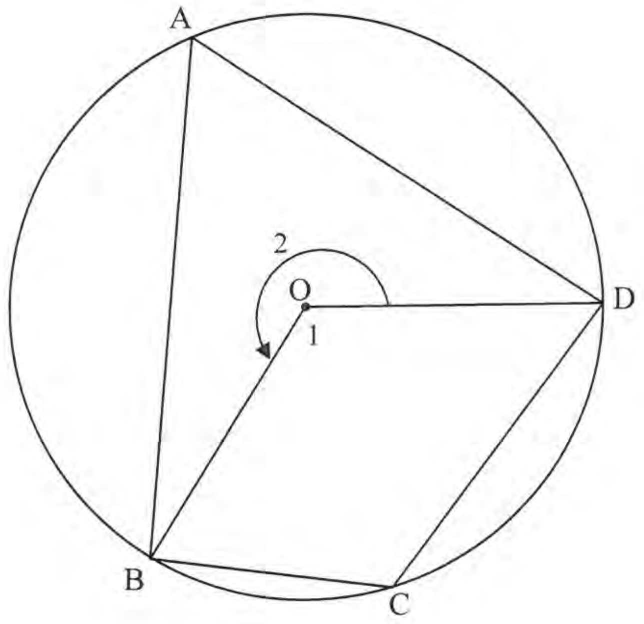


9.1	Give a reason why ABCD is a parallelogram.	(1)
9.2	Write down, with reasons, TWO ratios each equal to $\frac{ED}{DB}$.	(4)
9.3	Prove that $\hat{A}_1 = \hat{F}_2$.	(5)
9.4	It is further given that ABCD is a rhombus. Prove that ACGF is a cyclic quadrilateral.	(3)
		[13]

Give reasons for ALL statements in QUESTIONS 8, 9, 10 and 11.

QUESTION 8

8.1 In the diagram below, cyclic quadrilateral ABCD is drawn in the circle with centre O.

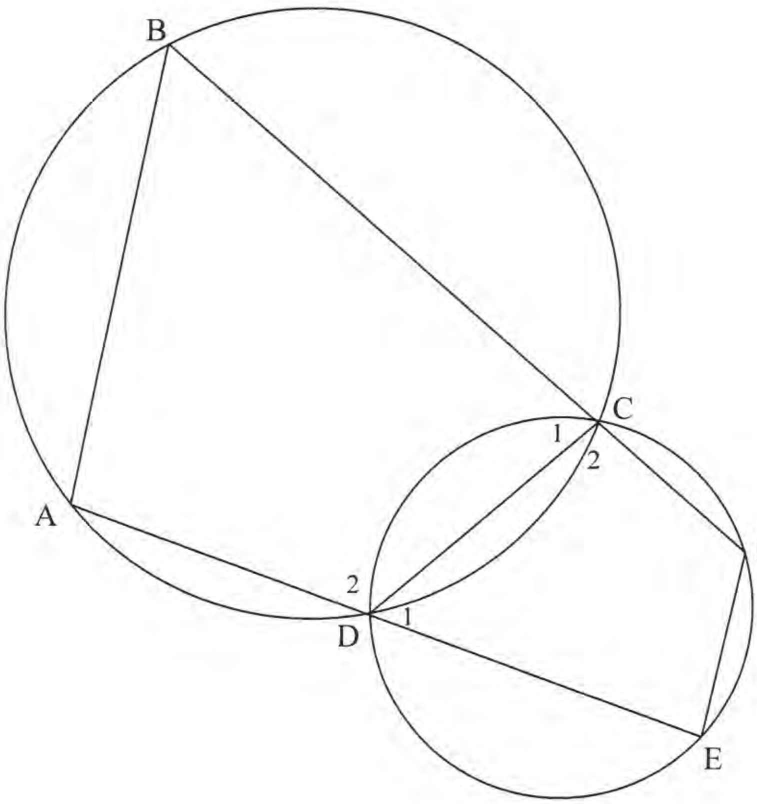


8.1.1 Complete the following statement:

The angle subtended by a chord at the centre of a circle is … the angle subtended by the same chord at the circumference of the circle. (1)

8.1.2 Use QUESTION 8.1.1 to prove that $\hat{A} + \hat{C} = 180°$. (3)

8.2 In the diagram below, CD is a common chord of the two circles. Straight lines ADE and BCF are drawn. Chords AB and EF are drawn.

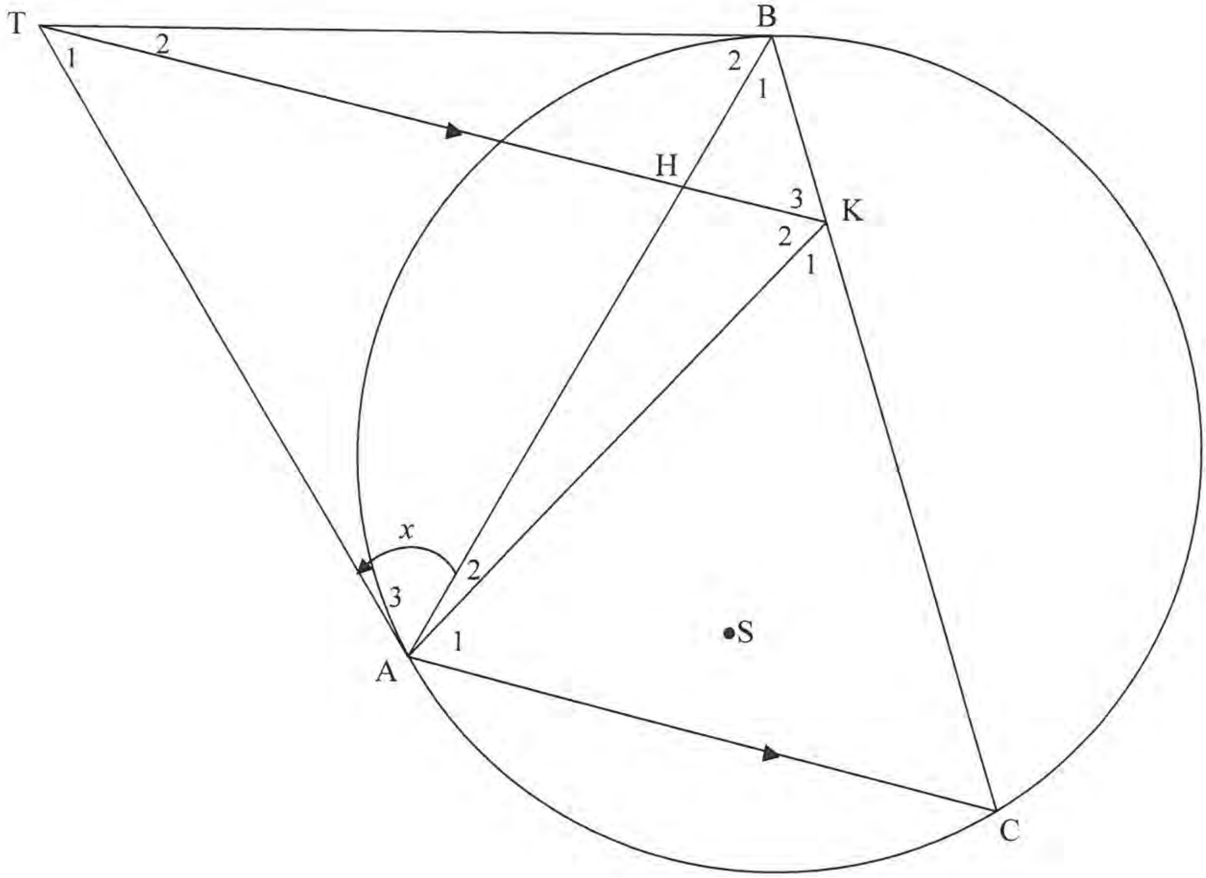


Prove that EF | | AB. (5)

[9]

QUESTION 9

In the diagram below, ΔABC is drawn in the circle. TA and TB are tangents to the circle. The straight line THK is parallel to AC with H on BA and K on BC. AK is drawn. Let $\hat{A}_3 = x$.



9.1 Prove that $\hat{K}_3 = x$. (4)

9.2 Prove that AKBT is a cyclic quadrilateral. (2)

9.3 Prove that TK bisects $A\hat{K}B$. (4)

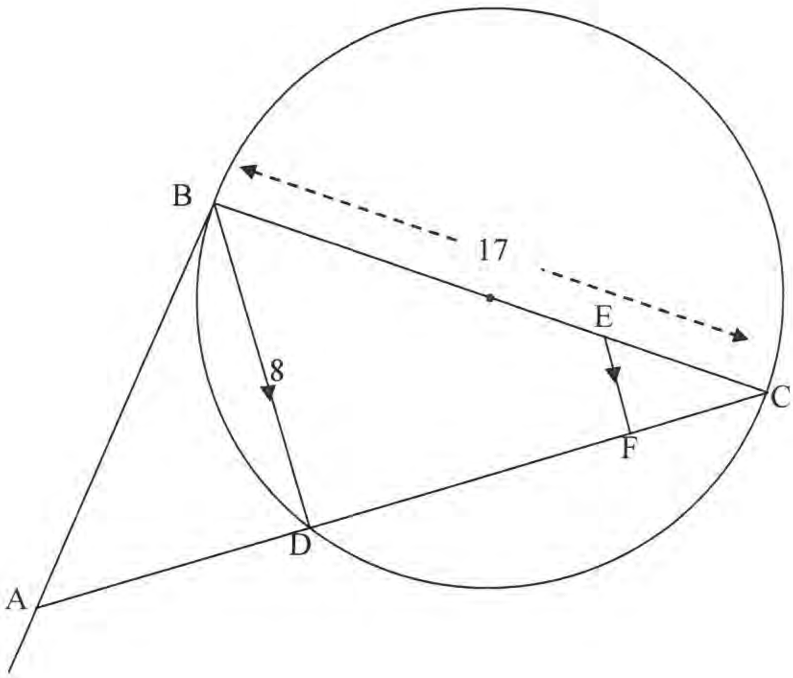
9.4 Prove that TA is a tangent to the circle passing through the points A, K and H. (2)

9.5 S is a point in the circle such that the points A, S, K and B are concyclic. Explain why A, S, B and T are also concyclic. (2)

[14]

QUESTION 10

In the diagram below, BC = 17 units, where BC is a diameter of the circle. The length of chord BD is 8 units. The tangent at B meets CD produced at A.



10.1 Calculate, with reasons, the length of DC. (3)

10.2 E is a point on BC such that BE : EC = 3 : 1. EF is parallel to BD with F on DC.

10.2.1 Calculate, with reasons, the length of CF. (3)

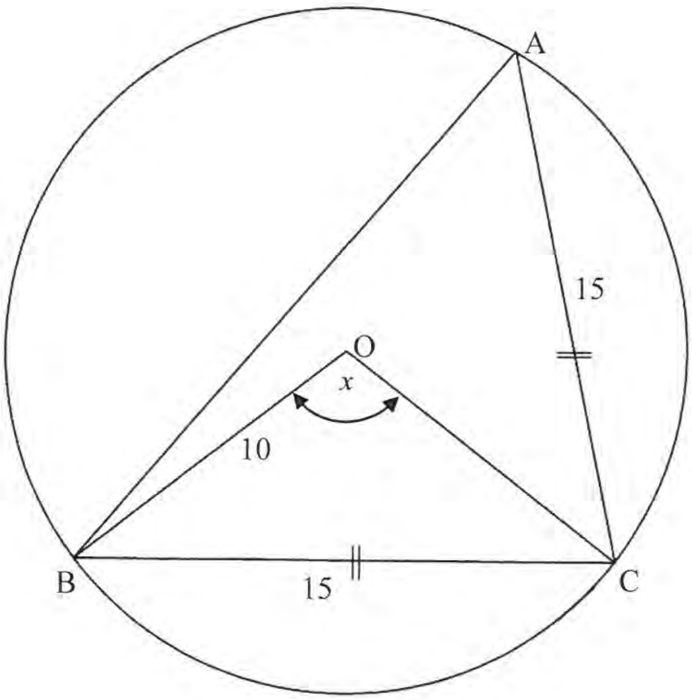
10.2.2 Prove that $\Delta BAC \mid\mid\mid \Delta FEC$. (5)

10.2.3 Calculate the length of AC. (4)

10.2.4 Write down, giving reasons, the radius of the circle passing through points A, B and C. (2)

[17]

6.2 In the diagram below, a circle with centre O passes through A, B and C. $BC = AC = 15$ units. BO and OC are joined. $OB = 10$ units and $B\hat{O}C = x$.



Calculate:

6.2.1 The size of x (4)

6.2.2 The size of $A\hat{C}B$ (3)

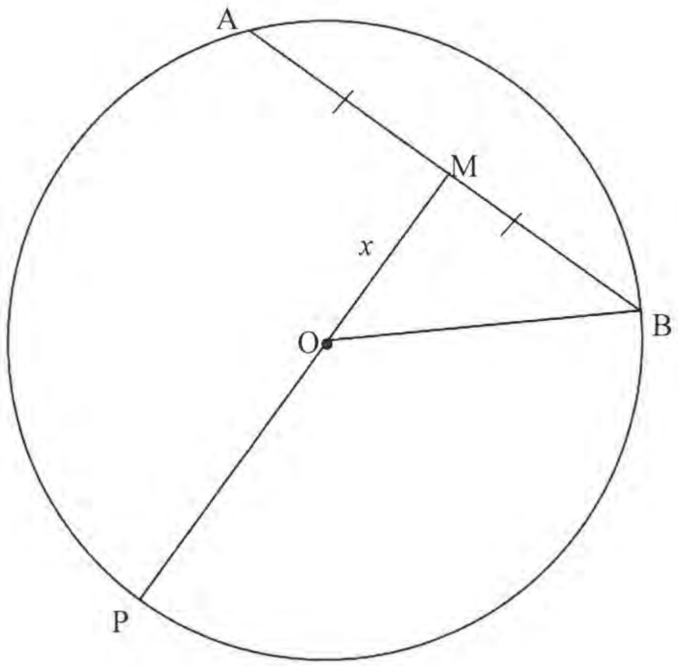
6.2.3 The area of ΔABC (2)

[16]

GIVE REASONS FOR YOUR ANSWERS IN QUESTIONS 7, 8, 9 AND 10.

QUESTION 7

In the diagram, AB is a chord of the circle with centre O. M is the midpoint of AB. MO is produced to P, where P is a point on the circle. OM = x units, AB = 20 units and $\frac{\text{PM}}{\text{OM}} = \frac{5}{2}$.



7.1 Write down the length of MB. (1)

7.2 Give a reason why OM $\perp$ AB. (1)

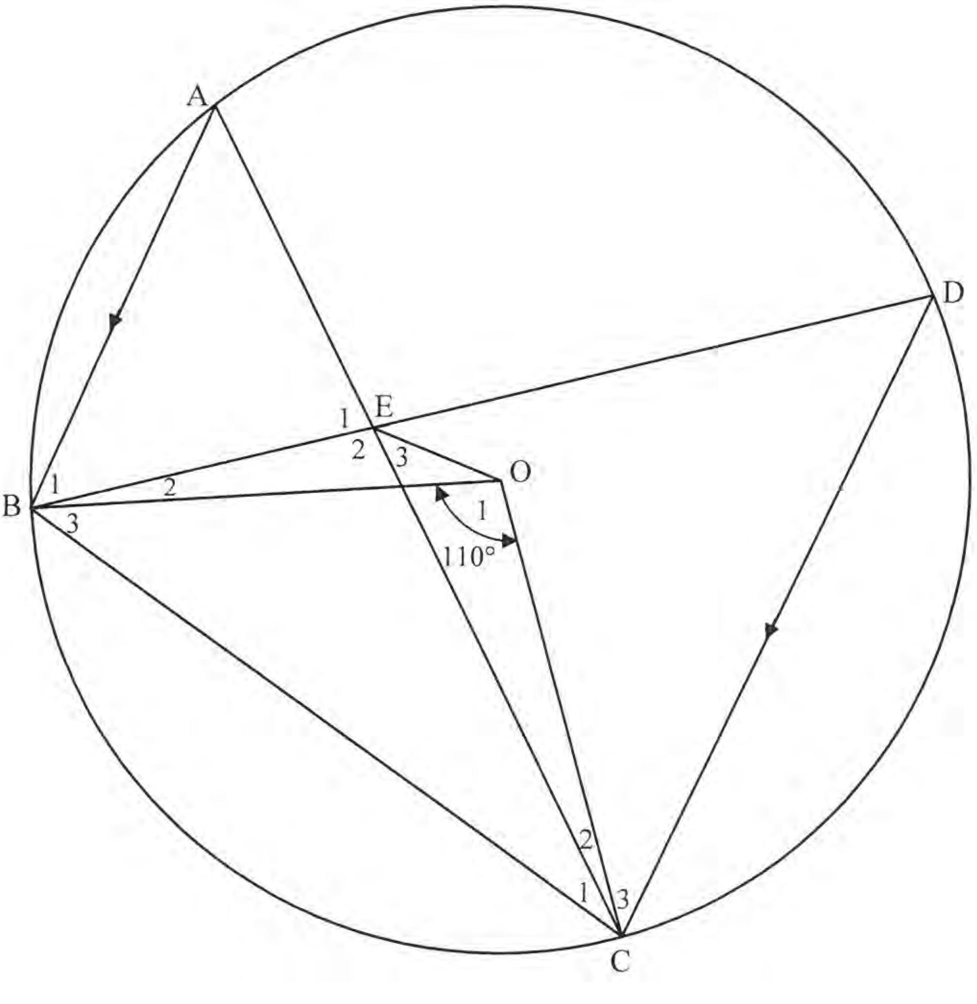
7.3 Show that $\text{OP} = \frac{3x}{2}$ units. (2)

7.4 Calculate the value of x. (3)

[7]

QUESTION 8

In the diagram below, the circle with centre O passes through A, B, C and D.
$AB \parallel DC$ and $B\hat{O}C = 110°$.
The chords AC and BD intersect at E.
EO, BO, CO and BC are joined.



8.1 Calculate the size of the following angles, giving reasons for your answers:

8.1.1 $\hat{D}$ (2)

8.1.2 $\hat{A}$ (2)

8.1.3 $\hat{E}_2$ (4)

8.2 Prove that BEOC is a cyclic quadrilateral. (2)

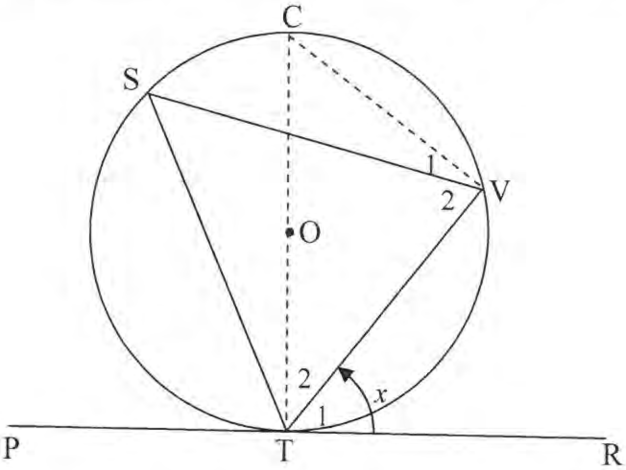
[10]

QUESTION 9

9.1 Complete the statement of the following theorem:

The exterior angle of a cyclic quadrilateral is equal to ... (1)

9.2 In the diagram below the circle with centre O passes through points S, T and V. PR is a tangent to the circle at T. VS, ST and VT are joined.



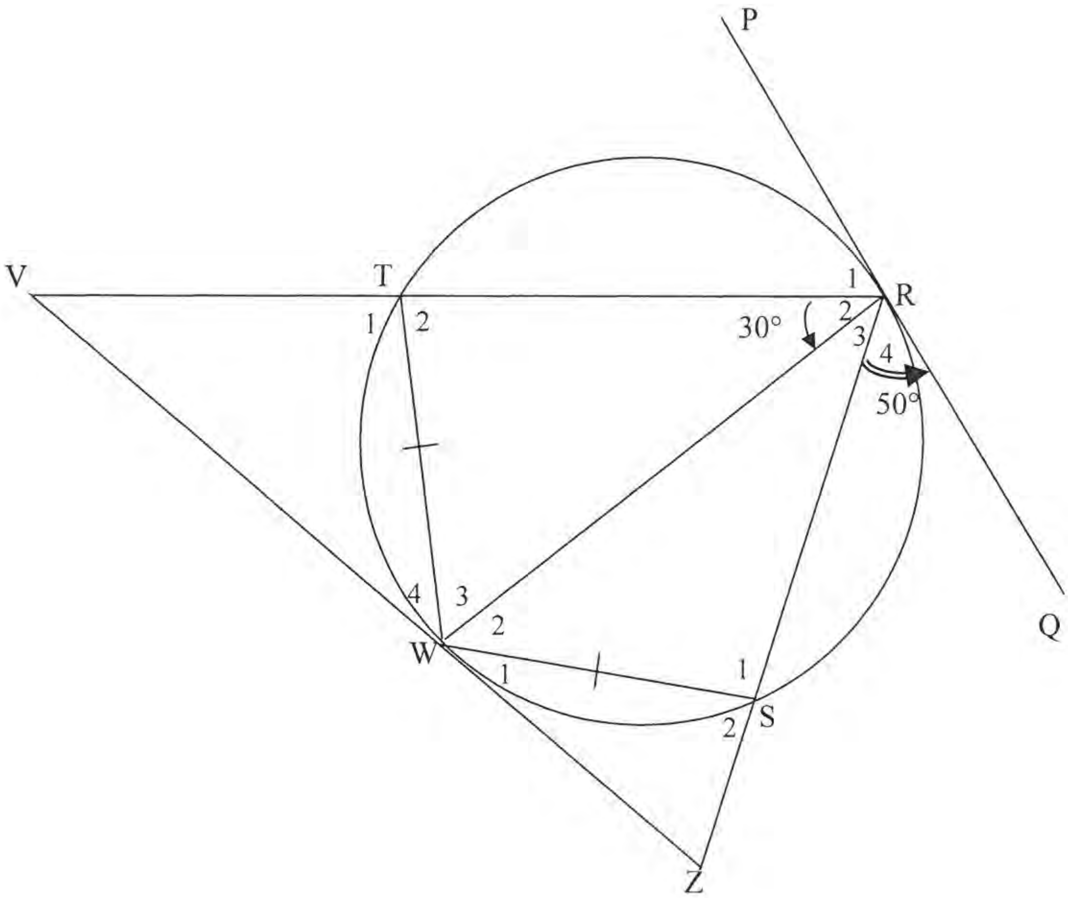
Given below is the partially completed proof of the theorem that states that $V\hat{T}R = \hat{S}$.
Using the above diagram, complete the proof of the theorem on DIAGRAM SHEET 3.

Construction: Draw diameter TC and join CV.

Statement	Reason
Let: $V\hat{T}R = \hat{T}_1 = x$	
$\hat{V}_1 + \hat{V}_2 =$..........	....
$\hat{T}_2 = 90° - x$	....
$\therefore \hat{C} =$	Sum of the angles of a triangle
$\therefore \hat{S} = x$	....
$\therefore V\hat{T}R = \hat{S}$	

(5)

9.3 In the figure, TRSW is a cyclic quadrilateral with TW = WS. RT and RS are produced to meet tangent VWZ at V and Z respectively. PRQ is a tangent to the circle at R. RW is joined. $\hat{R}_2 = 30°$ and $\hat{R}_4 = 50°$.



9.3.1 Give a reason why $\hat{R}_3 = 30°$. (1)

9.3.2 State, with reasons, TWO other angles equal to $30°$. (3)

9.3.3 Determine, with reasons, the size of:

(a) $\hat{S}_2$ (3)

(b) $\hat{V}$ (4)

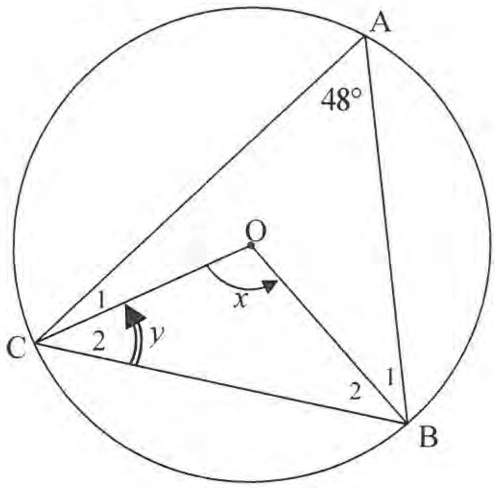
9.3.4 Prove that $WR^2 = RV \times RS$. (5)

[22]

GIVE REASONS FOR YOUR STATEMENTS IN QUESTIONS 8, 9 AND 10.

QUESTION 8

8.1 In the diagram, O is the centre of the circle passing through A, B and C. $C\hat{A}B = 48°$, $C\hat{O}B = x$ and $\hat{C}_2 = y$.

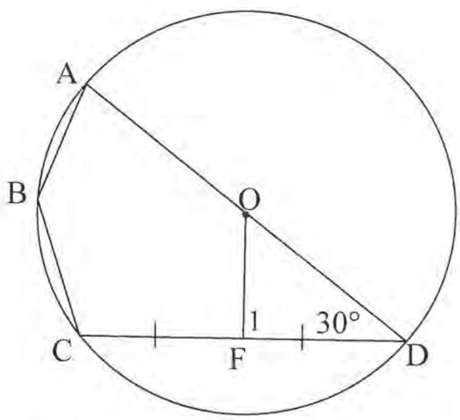


Determine, with reasons, the size of:

8.1.1 x (2)

8.1.2 y (2)

8.2 In the diagram, O is the centre of the circle passing through A, B, C and D. AOD is a straight line and F is the midpoint of chord CD. $O\hat{D}F = 30°$ and OF are joined.

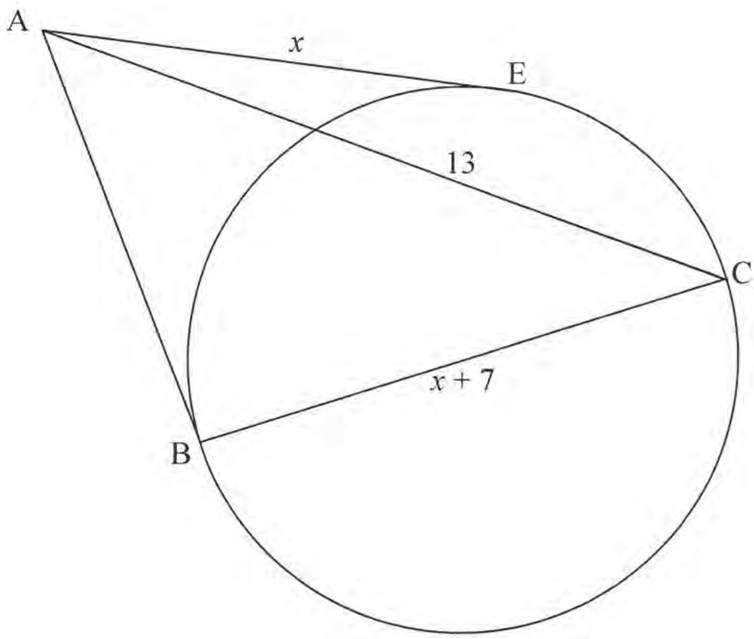


Determine, with reasons, the size of:

8.2.1 $\hat{F}_1$ (2)

8.2.2 $A\hat{B}C$ (2)

8.3 In the diagram, AB and AE are tangents to the circle at B and E respectively. BC is a diameter of the circle. AC = 13, AE = x and BC = $x + 7$.



8.3.1 Give reasons for the statements below.
Complete the table on DIAGRAM SHEET 3.

	Statement	**Reason**
(a)	$A\hat{B}C = 90°$	
(b)	$AB = x$	

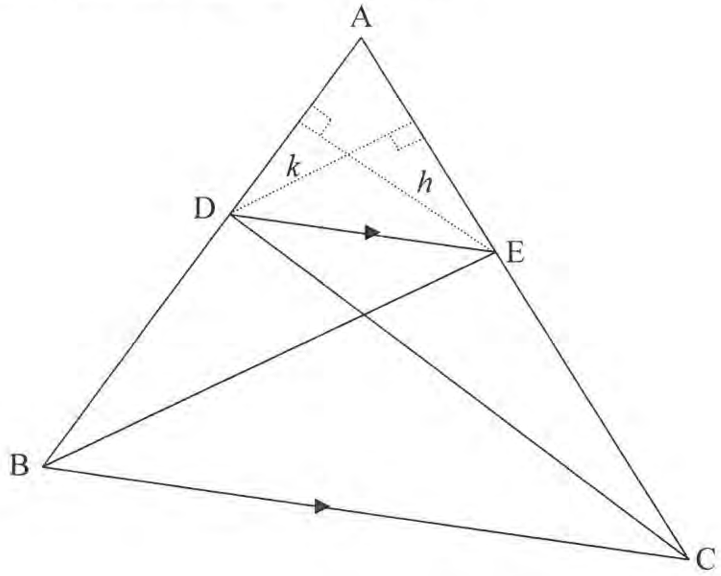
(2)

8.3.2 Calculate the length of AB. (4)

[14]

QUESTION 9

9.1 In the diagram, points D and E lie on sides AB and AC of ΔABC respectively such that DE | | BC. DC and BE are joined.



9.1.1 Explain why the areas of ΔDEB and ΔDEC are equal. (1)

9.1.2 Given below is the partially completed proof of the theorem that states that if in any ΔABC the line DE | | BC then $\frac{AD}{DB} = \frac{AE}{EC}$.

Using the above diagram, complete the proof of the theorem on DIAGRAM SHEET 4.

Construction: Construct the altitudes (heights) h and k in ΔADE.

$$\frac{\text{area } \Delta ADE}{\text{area } \Delta DEB} = \frac{\frac{1}{2}(AD)(h)}{\frac{1}{2}(BD)(h)} = \ldots\ldots\ldots$$

$$\frac{\text{area } \Delta ADE}{\text{area } \Delta DEC} = \ldots\ldots\ldots\ldots\ldots\ldots\ldots = \frac{AE}{EC}$$

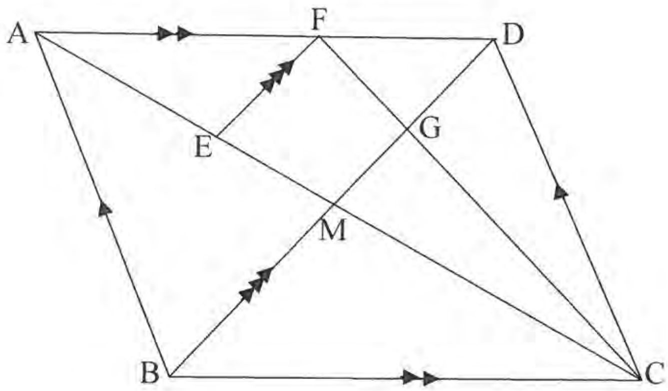
But area ΔDEB = ... (reason: ....)

$$\therefore \frac{\text{area } \Delta ADE}{\text{area } \Delta DEB} = \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$\therefore \frac{AD}{DB} = \frac{AE}{EC}$$

(5)

9.2 In the diagram, ABCD is a parallelogram. The diagonals of ABCD intersect in M. F is a point on AD such that AF : FD = 4 : 3. E is a point on AM such that EF | | BD. FC and MD intersect in G.



Calculate, giving reasons, the ratio of:

9.2.1 $\frac{EM}{AM}$ (3)

9.2.2 $\frac{CM}{ME}$ (3)

9.2.3 $\frac{\text{area } \Delta FDC}{\text{area } \Delta BDC}$ (4)

[16]

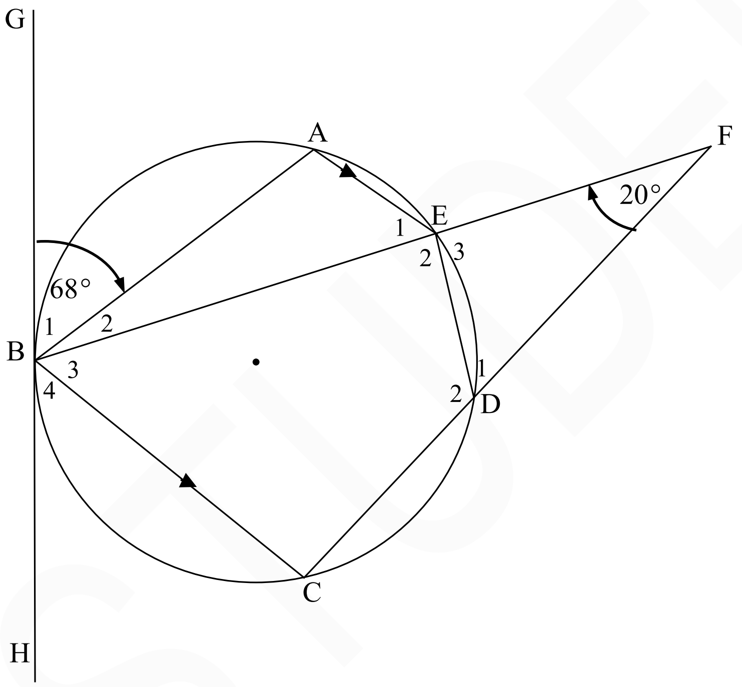
Give reasons for your statements in QUESTIONS 8, 9 and 10.

QUESTION 8

8.1 Complete the following statement:

The angle between the tangent and the chord at the point of contact is equal to ... (1)

8.2 In the diagram, A, B, C, D and E are points on the circumference of the circle such that AE | | BC. BE and CD produced meet in F. GBH is a tangent to the circle at B. $\hat{B}_1 = 68°$ and $\hat{F} = 20°$.



Determine the size of each of the following:

8.2.1 $\hat{E}_1$ (2)

8.2.2 $\hat{B}_3$ (1)

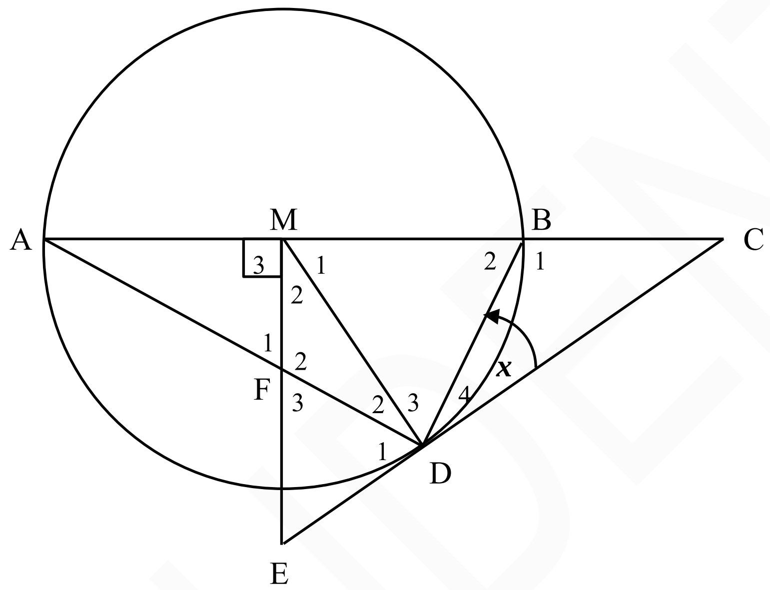
8.2.3 $\hat{D}_1$ (2)

8.2.4 $\hat{E}_2$ (1)

8.2.5 $\hat{C}$ (2)
[9]

QUESTION 9

In the diagram, M is the centre of the circle and diameter AB is produced to C. ME is drawn perpendicular to AC such that CDE is a tangent to the circle at D. ME and chord AD intersect at F. MB = 2BC.



9.1 If $\hat{D}_4 = x$, write down, with reasons, TWO other angles each equal to x. (3)

9.2 Prove that CM is a tangent at M to the circle passing through M, E and D. (4)

9.3 Prove that FMBD is a cyclic quadrilateral. (3)

9.4 Prove that $DC^2 = 5BC^2$. (3)

9.5 Prove that $\Delta DBC \mid\mid\mid \Delta DFM$. (4)

9.6 Hence, determine the value of $\frac{DM}{FM}$. (2)

[19]